Computational Physics ps-9 Report

Tongzhou Wang,
GitHub account: TZW56203, repository: phys-ga2000.
https://github.com/TZW56203/phys-ga2000

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1 Problem 1

1.1 Part (a) and (b)

The second-order differential equation

$$\frac{d^2x}{dt^2} = -\omega^2 x. \tag{1}$$

can be put into two first-order differential equations

$$\begin{cases} \frac{dx}{dt} = y, \\ \frac{dy}{dt} = -\omega^2 x. \end{cases} \tag{2}$$

Figure 1 shows the motion of a harmonic oscillator. We can observe that as we increase the amplitude by making the initial value of x bigger, the period stays roughly the same.

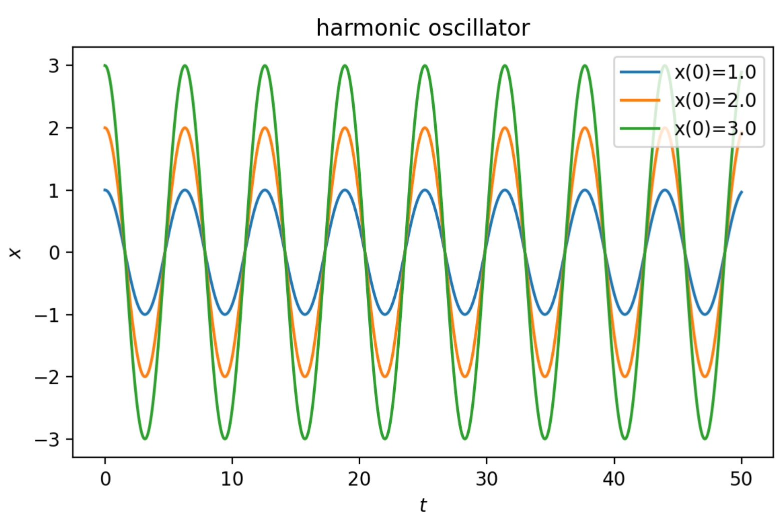

Figure 1: Harmonic oscillator.

1.2 Part (c) and (d)

Figure 2 shows the motion of an anharmonic oscillator. We can observe that the larger the amplitude, the shorter the period, that is, the faster the oscillation.

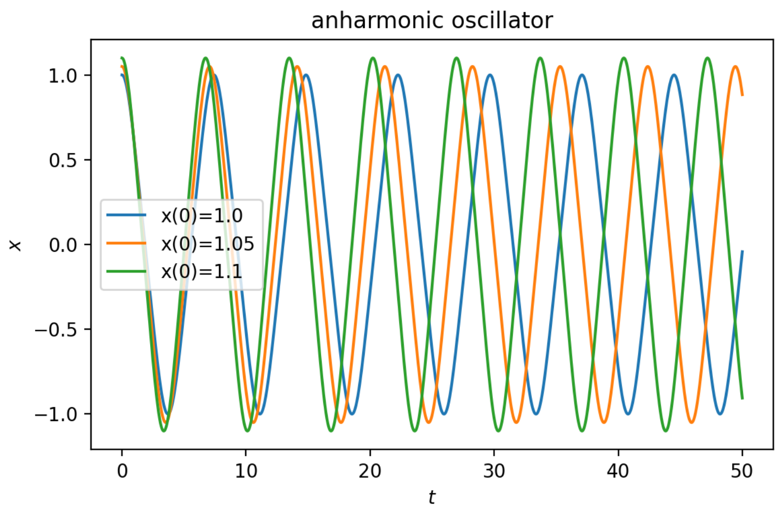

Figure 2: Anharmonic oscillator.

Figure 3 shows the phase space diagram of the anharmonic oscillator.

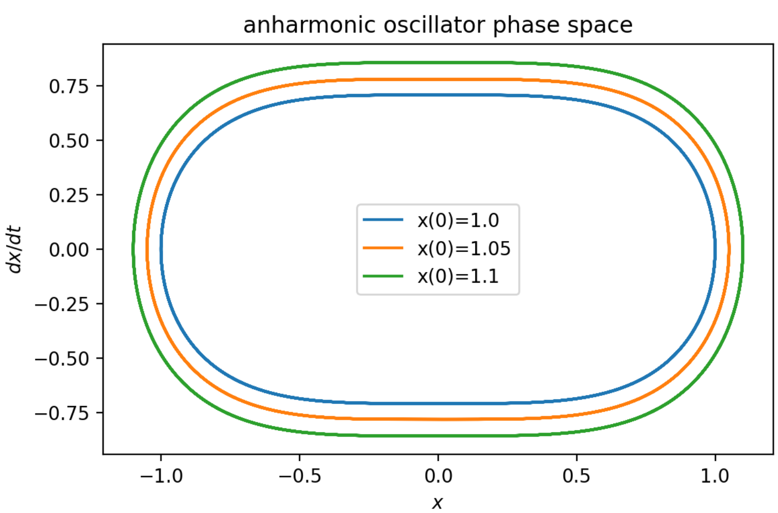

Figure 3: Anharmonic oscillator phase space.

1.3 Part (e)

Figure 4 shows the phase space diagram of the van der Pol oscillator.

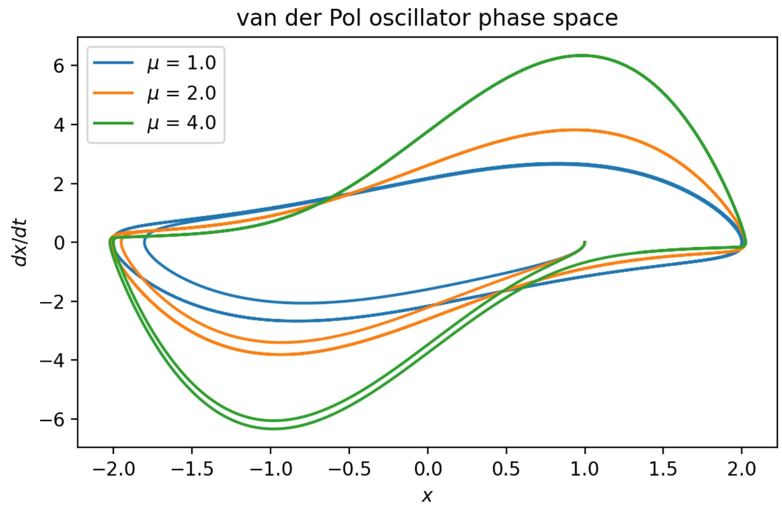

Figure 4: van der Pol oscillator phase space.

2 Problem 2

2.1 Part (a)

The Newton's second law gives

$$\begin{aligned} -\frac{1}{2}\pi R^2\rho C(\dot{x}^2+\dot{y}^2)\frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}} &= m\ddot{x}, \\ -mg-\frac{1}{2}\pi R^2\rho C(\dot{x}^2+\dot{y}^2)\frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}} &= m\ddot{y}. \end{aligned} \tag{3}$$

Rearranging, we get

$$\begin{aligned} \ddot{x} &= -\frac{\pi R^2\rho C}{2m}\dot{x}\sqrt{\dot{x}^2+\dot{y}^2}, \\ \ddot{y} &= -g-\frac{\pi R^2\rho C}{2m}\dot{y}\sqrt{\dot{x}^2+\dot{y}^2}. \end{aligned} \tag{4}$$

We can take

$$t' = \frac{t}{T},\ x' = \frac{x}{gT^2},\ y' = \frac{y}{gT^2}. \tag{5}$$

For convenience, we define

$$\begin{aligned} \frac{R^2 \rho C g T^2}{m} &\equiv k, \\ \frac{dx'}{dt'} &\equiv v_x, \\ \frac{dy'}{dt'} &\equiv v_y. \end{aligned} \tag{6}$$

Thus, the equations are rescaled to become

$$\begin{aligned} \frac{dv_x}{dt'} &= -\frac{\pi}{2} k v_x \sqrt{v_x^2 + v_y^2}, \\ \frac{dv_y}{dt'} &= -1 - \frac{\pi}{2} k v_y \sqrt{v_x^2 + v_y^2}. \end{aligned} \tag{7}$$

2.2 Part (b) and (c)

Figure 5 and 6 show the trajectories of the cannonball in the rescaled and original variables respectively. We can observe that the heavier the cannonball (the smaller the value of k), the further it travels.

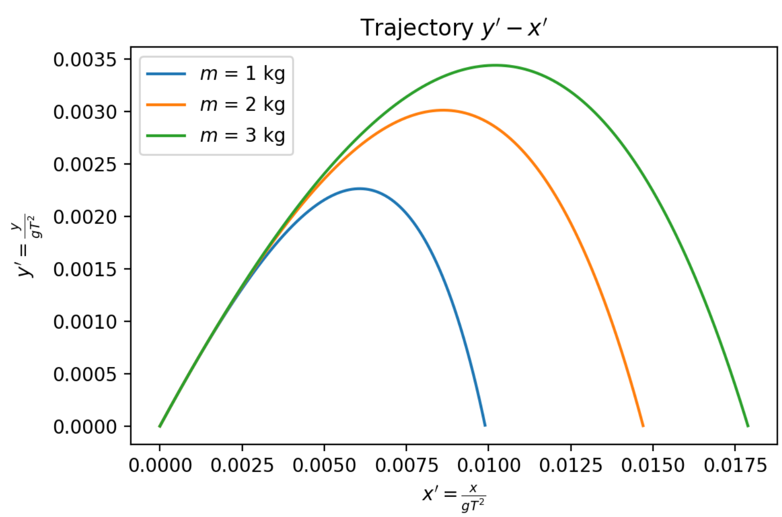

Figure 5: Trajectory in rescaled variables.

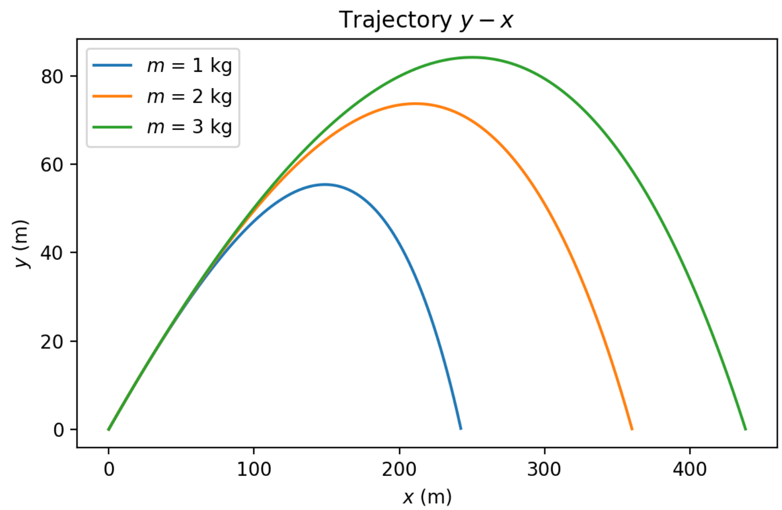

Figure 6: Trajectory in original variables.