Computational Physics ps-6 Report

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https://github.com/TZW56203/phys-ga2000

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1 Problem 1

1.1 Part (a) (b) (c)

The flux and flux residue of galaxy number 2000 are shown in Figure 1.

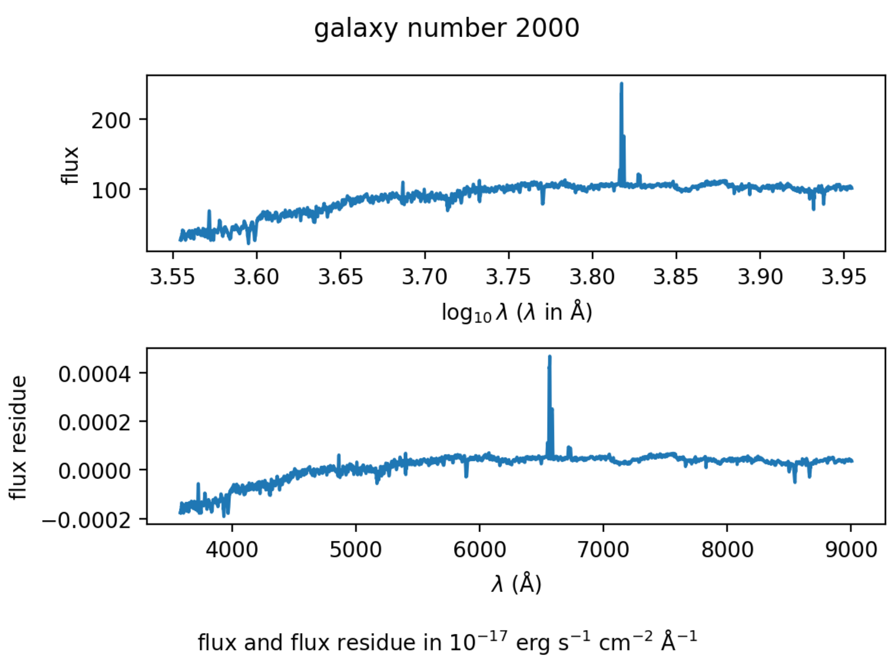

Figure 1: Galaxy number 2000.

We note that the range of wavelengths in Figure 1 overlaps with that of the Balmer series. Especially, the wavelength for the $n = 3$ to $n = 2$ transition, which is about 656 nm, is prominent in the spectrum.

1.2 Part (d)

The first five eigenvectors of the covariance matrix are show in Figure 2.

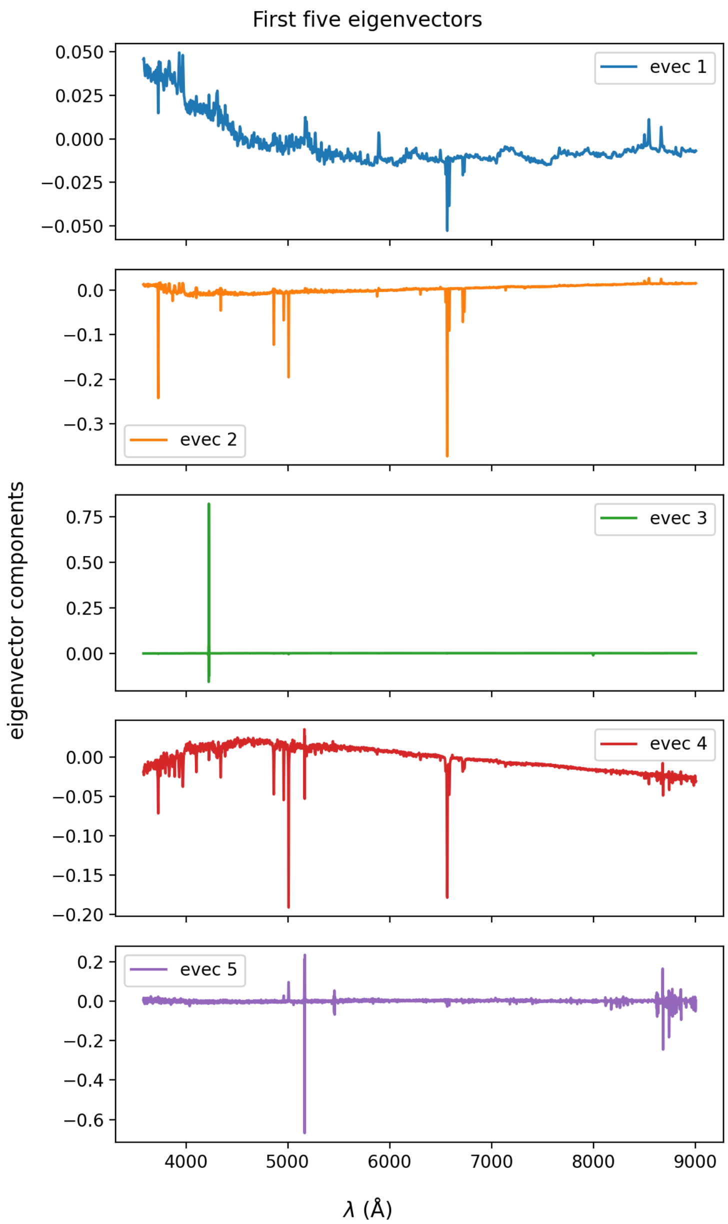

Figure 2: First five eigenvectors.

1.3 Part (e)

The first five eigenvectors of the covariance matrix computed using SVD are show in Figure 3.

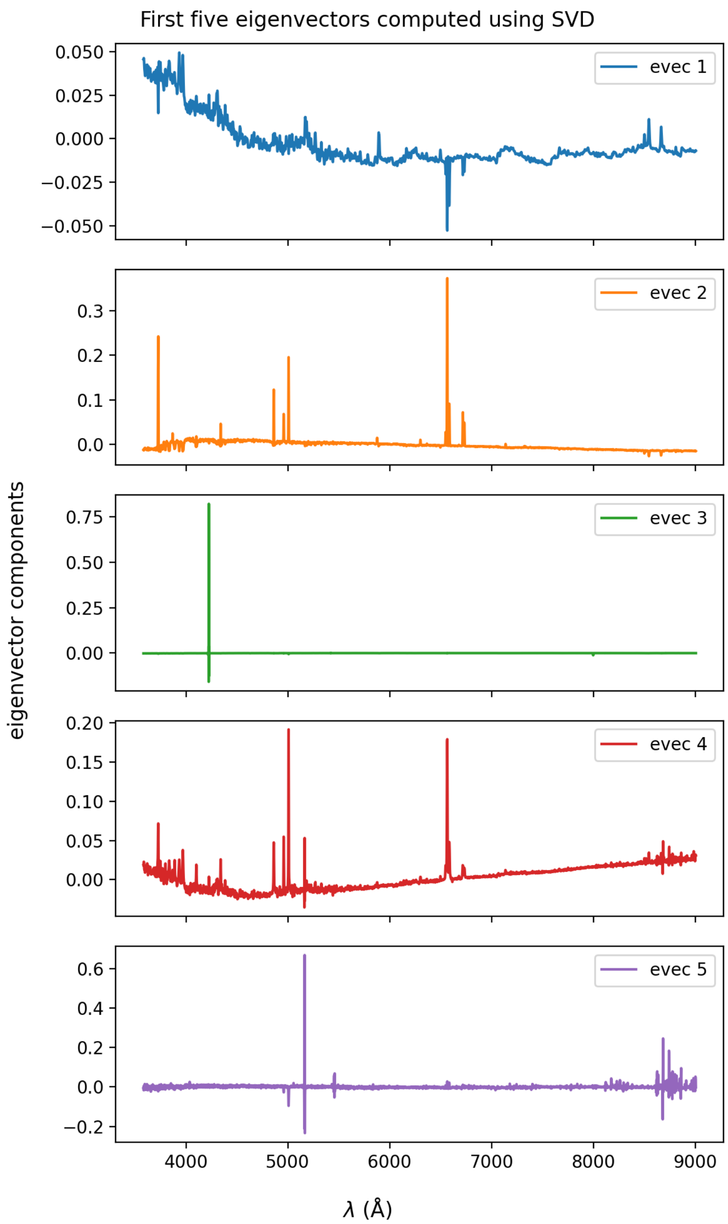

Figure 3: First five eigenvectors computed using SVD.

Listing 1: Eigenvalues.

```
first five ev1s: [0.12243894 0.04632396 0.00910877 0.00672037 0.00344144
   0.00323884]
first five ev2s: [0.12243894 0.04632396 0.00910877 0.00672037 0.00344144
   0.00323884]
```

As shown in Listing 1, the eigenvalues computed using the two methods are the same, which indicates that the eigenvectors are equivalent. We note that eigenvectors 2, 4, and 5 in Figure 2 and Figure 3 are negative to each other. But this is reasonable as one eigenvalue can associate with multiple eigenvectors that form an vector space known as eigenspace.

Listing 2 shows that using SVD is actually slower than directly finding the eigenvectors of the covariance matrix using `np.linalg.eig()`. Here, the units are seconds, and both methods are implemented twice in the shown time.

Listing 2: Computational cost.

```
t1 = 55.5503966670949
t2 = 112.83291685697623
```

1.4 Part (f)

Listing 3 shows that the condition number of the covariance matrix **C** is quite large, whereas the condition number of the flux residue matrix **R** is much smaller. This might be one reason we want to use the SVD method.

Listing 3: Condition number.

```
condition number of cov: 33166539000.0
condition number of flux_res: 6561841.5
```

1.5 Part (g)

Figure 4 shows the approximated flux with $N_c = 5$ for galaxy number 2000.

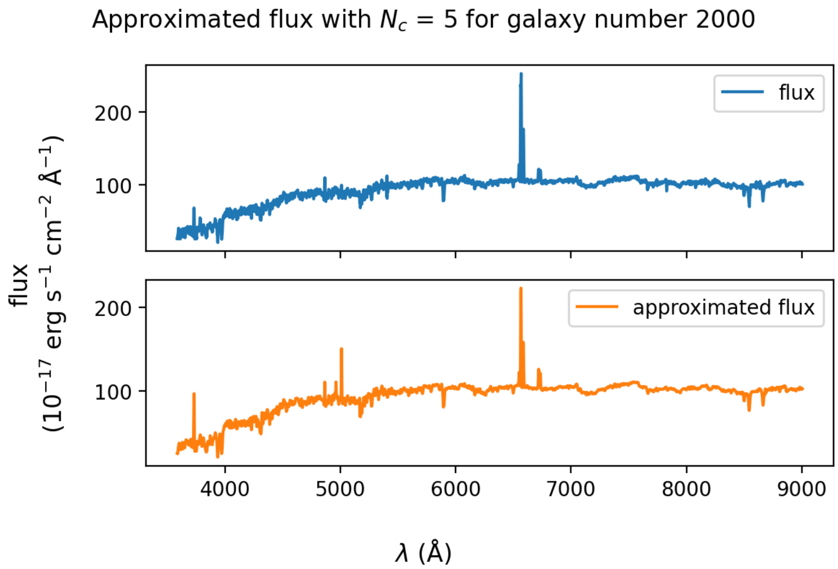

Figure 4: Approximated flux with $N_c = 5$ for galaxy number 2000.

1.6 Part (h)

Figure 5 shows the first three coefficients c_0, c_1, and c_2.

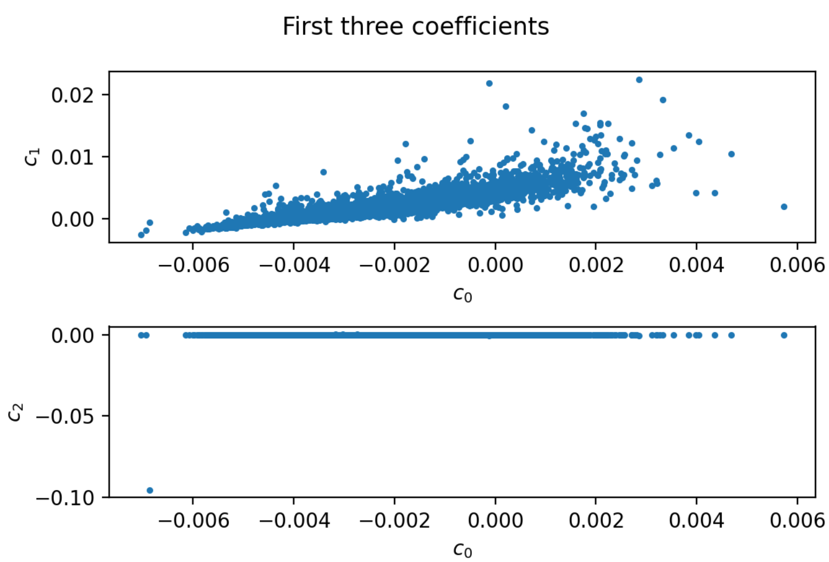

Figure 5: First three coefficients.

1.7 Part (i)

Figure 6 shows the root mean square error of the approximated flux as a function of N_c.

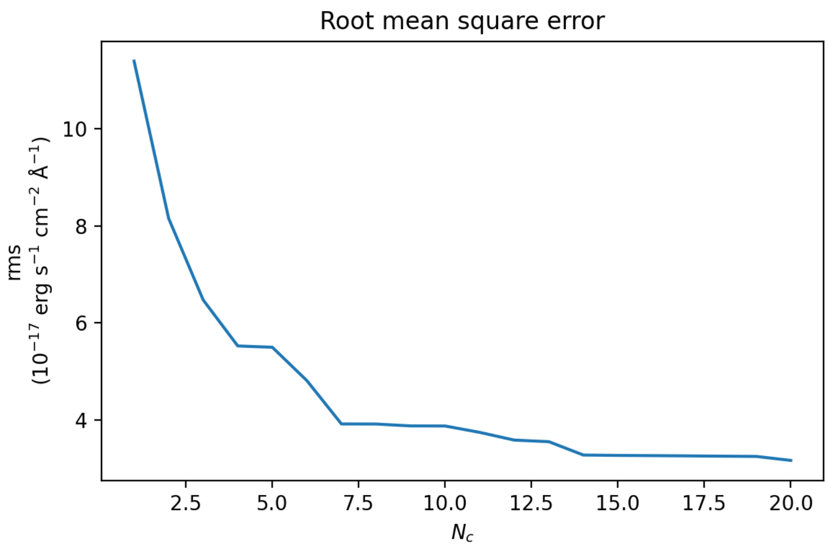

Figure 6: Root mean square error.

Listing 4 shows the root mean square error for $N_c = 20$.

Listing 4: Root mean square error for $N_c = 20$.

```
rms (Nc=20): 3.16158127784729
```