Computational Physics ps-3 Report

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1 Problem 1

The relation between the matrix multiplication computation time and the matrix size ($N \times N$) is shown below.

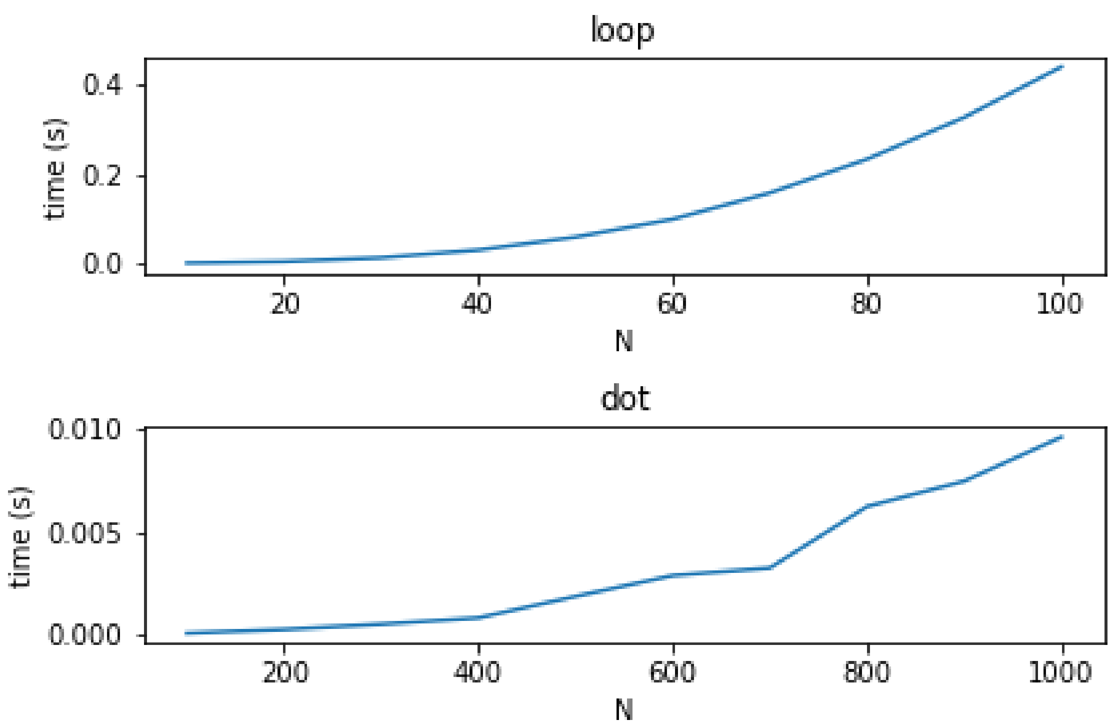

Figure 1: Matrix multiplication computation time vs matrix size.

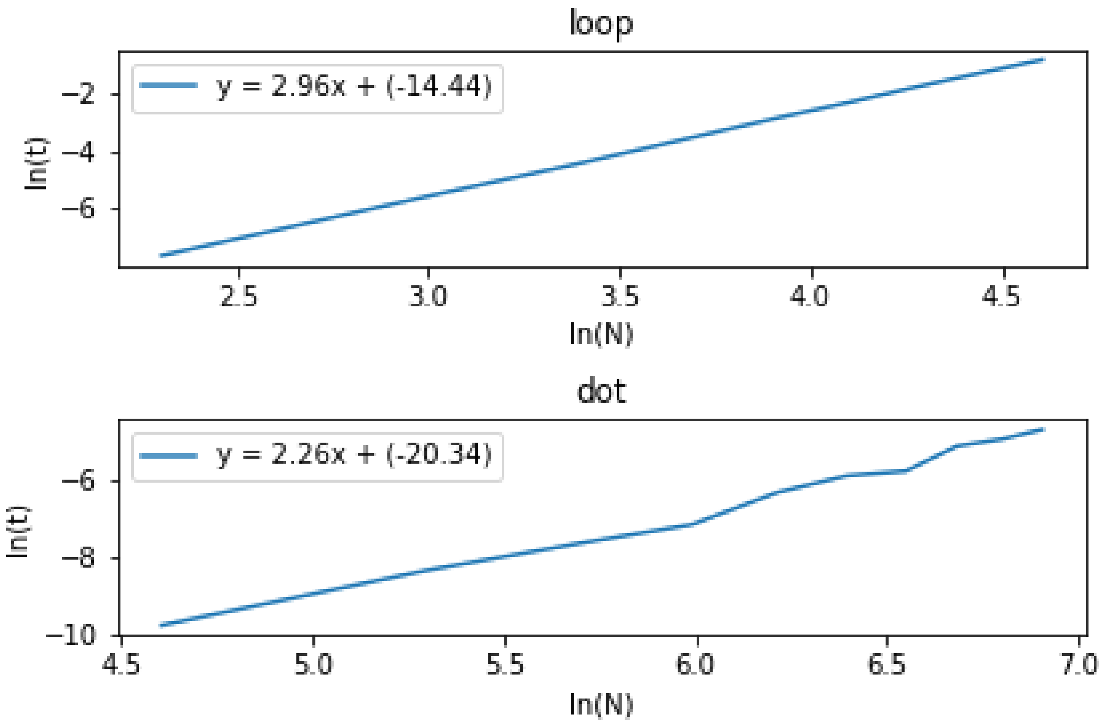

Figure 2: Logarithmic time vs logarithmic size.

The computation time of the loop function rises as N^3 as predicted. This can be concluded from Figure 2, where we have for the loop function $\ln(t) = 2.96 \ln(N) - 14.44$. The coefficient 2.96 indicates that the computation time rises as N^3.

Similarly, we can conclude that the computation time of the dot function does not rises as N^3.

The main difference between the two function is that the dot function is much faster, which can be seen from Figure 1.

2 Problem 2

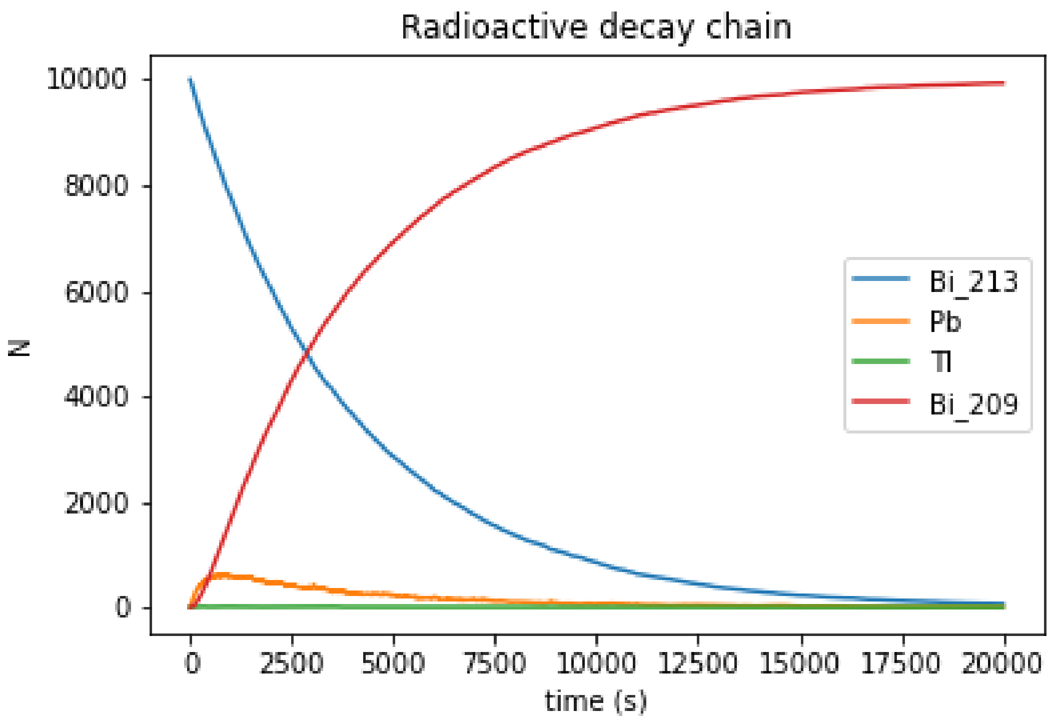

Figure 3: Radioactive decay chain.

3 Problem 3

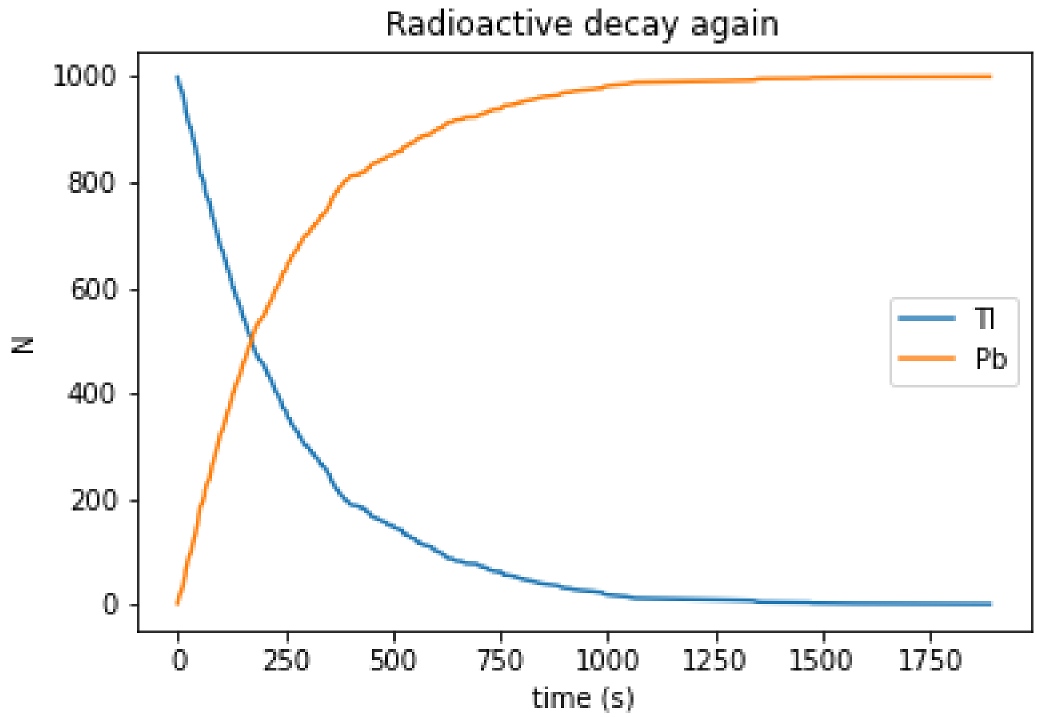

Figure 4: Radioactive decay again.

4 Problem 4

To find the mean and variance of the random variable $y = N^{-1} \sum_{i=1}^{N} x_i$, we first look at the independent and identically distributed random variables x_i, which have probability density $f = e^{-x}$. It is straight forward to find the mean and variance of x_i.

$$\begin{aligned} \mu \equiv \mathbb{E}(x) &= \int_0^\infty x e^{-x} dx = 1, \\ \sigma^2 \equiv \mathrm{Var}(x) &= \mathbb{E}(x^2) - \mathbb{E}^2(x) = 1. \end{aligned} \tag{1}$$

We then proceed to obtain

$$\begin{aligned}
\mathbb{E}(y) =& \mathbb{E}\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right) \\
=& \frac{1}{N}\mathbb{E}\left(\sum_{i=1}^{N} x_i\right) \\
=& \frac{1}{N}\sum_{i=1}^{N}\mathbb{E}(x_i) \\
=& \frac{N\mu}{N} \\
=& \mu = 1,
\end{aligned} \tag{2}$$

and

$$\begin{aligned}
\mathrm{Var}(y) =& \mathrm{Var}\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right) \\
=& \frac{1}{N^2}\mathrm{Var}\left(\sum_{i=1}^{N} x_i\right) \\
=& \frac{1}{N^2}\sum_{i=1}^{N}\mathrm{Var}(x_i) \\
=& \frac{N\sigma^2}{N^2} \\
=& \frac{\sigma^2}{N} = \frac{1}{N}.
\end{aligned} \tag{3}$$

Figure 5 and 6 visually show that for large N the distribution of y tends toward Gaussian. Here we define

$$z = \frac{\sqrt{N}(y-\mu)}{\sigma}, \tag{4}$$

which according to the central limit theorem should have a standard normal distribution.

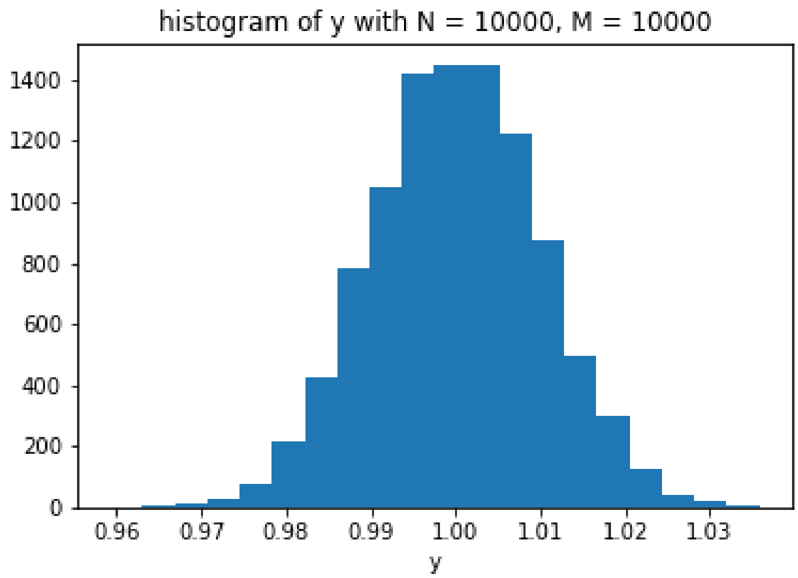

Figure 5: Histogram of y.

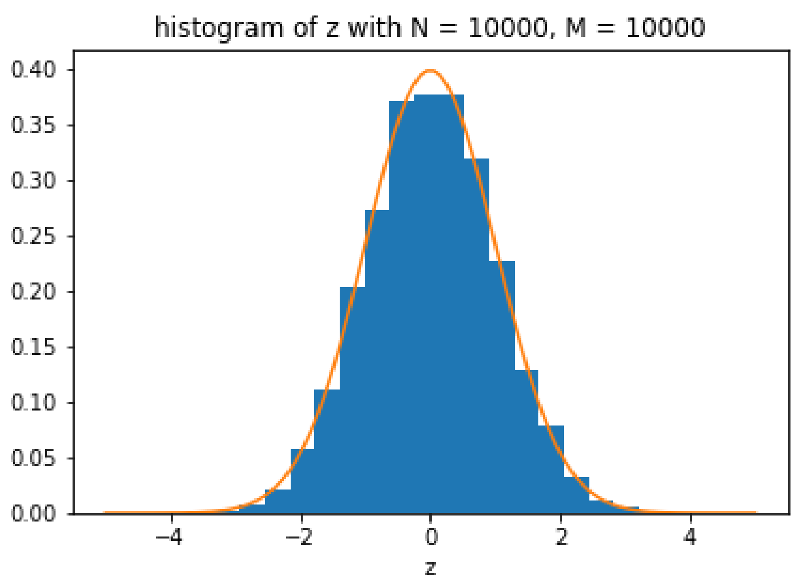

Figure 6: Histogram of z.

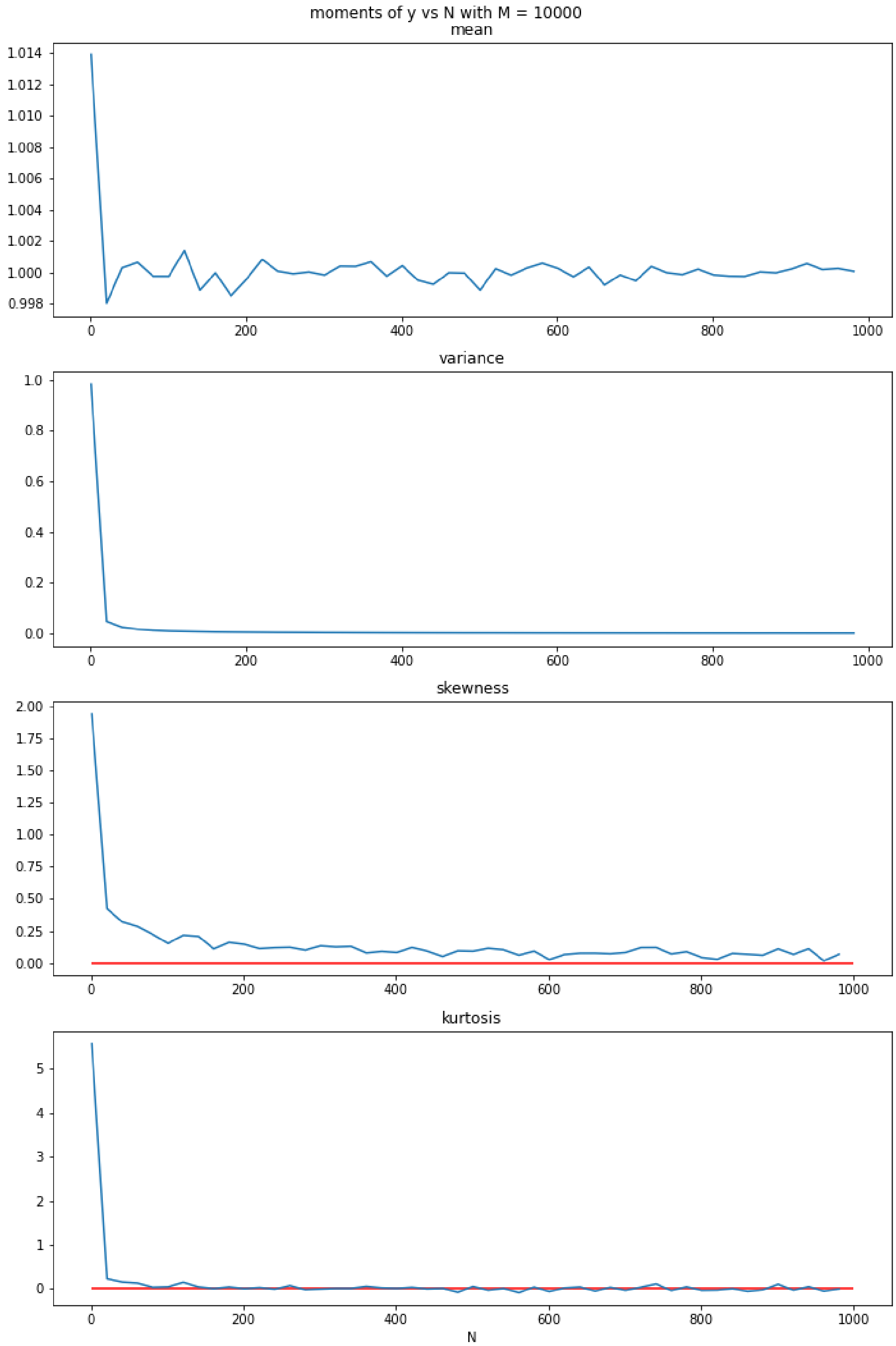

Figure 7: Moments of y vs N.

Figure 7 shows the mean, variance, skewness, and kurtosis of y as a function of N.
Around $N = 600$, the skewness reached about 1% of its value at $N = 1$.
Around $N = 100$, the kurtosis reached about 1% of its value at $N = 1$.