

Bipartite entanglement

Quantum state in d^L dimensional Hilbert space

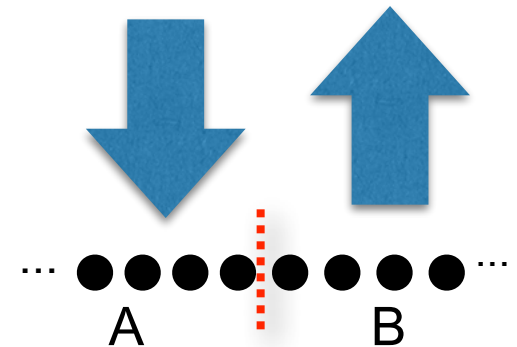
$$|\psi\rangle = \sum_{j_1, j_2, \dots, j_L} \psi_{j_1, j_2, \dots, j_L} |j_1\rangle |j_2\rangle \cdots |j_L\rangle, \quad j_n = 1 \dots d$$

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Decompose a state into a superposition of product states (**Schmidt decomposition**)



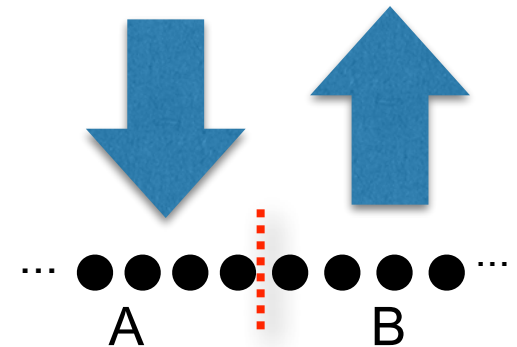
$$|\psi\rangle = \sum_{i,j} C_{i,j} |i\rangle_A \otimes |j\rangle_B = \sum_{\alpha} \Lambda_{\alpha} |\alpha\rangle_A \otimes |\alpha\rangle_B, \quad \langle \alpha | \alpha' \rangle = \delta_{\alpha \alpha'}$$

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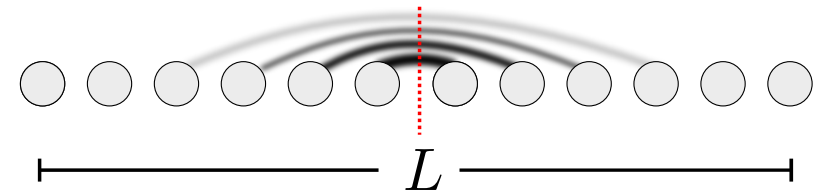
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Entanglement entropy as a measure for the amount of entanglement $S = - \sum_{\alpha} \Lambda_{\alpha}^2 \log \Lambda_{\alpha}^2$

Entanglement

Area law in one dimensional systems: Low depth circuit!

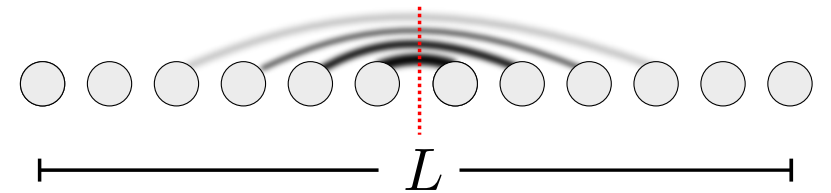
$$S(L) = \text{const.}$$



Entanglement

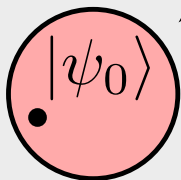
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Many body Hilbert space

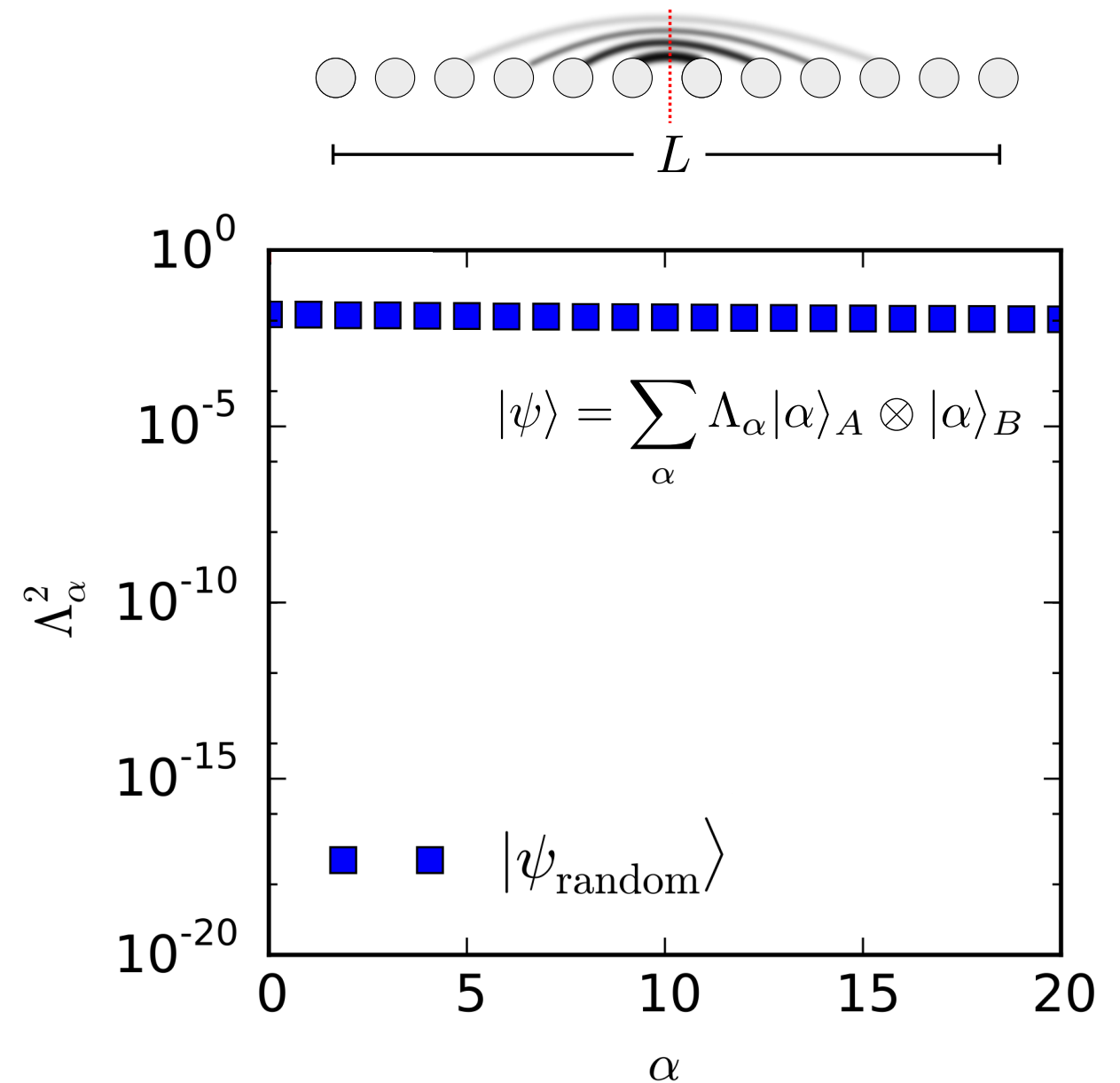
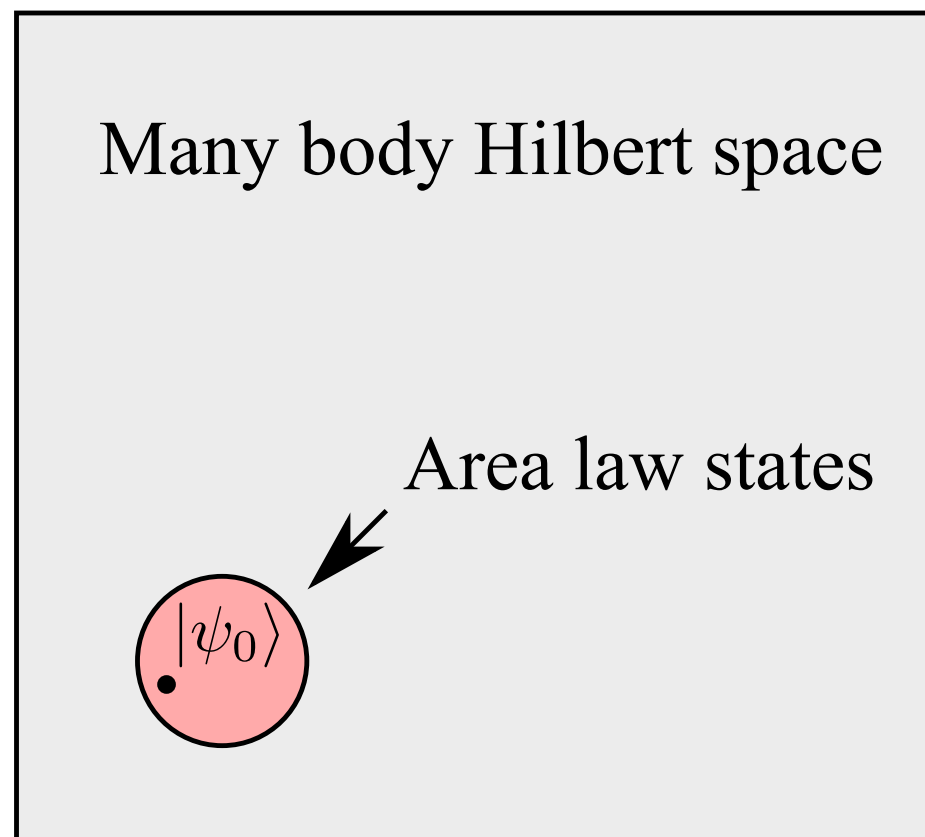
Area law states



Entanglement

Area law in one dimensional systems: Low depth circuit!

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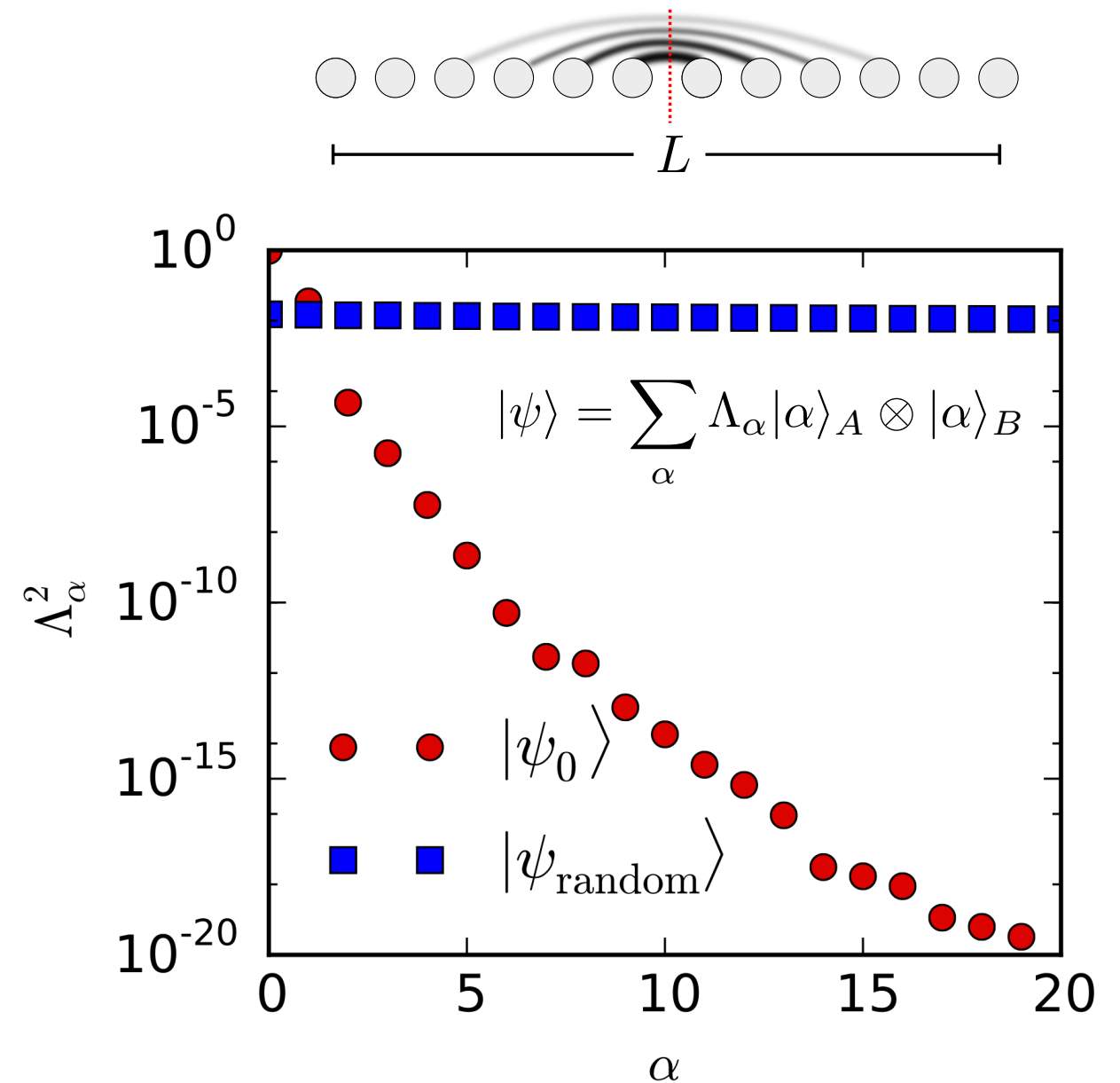
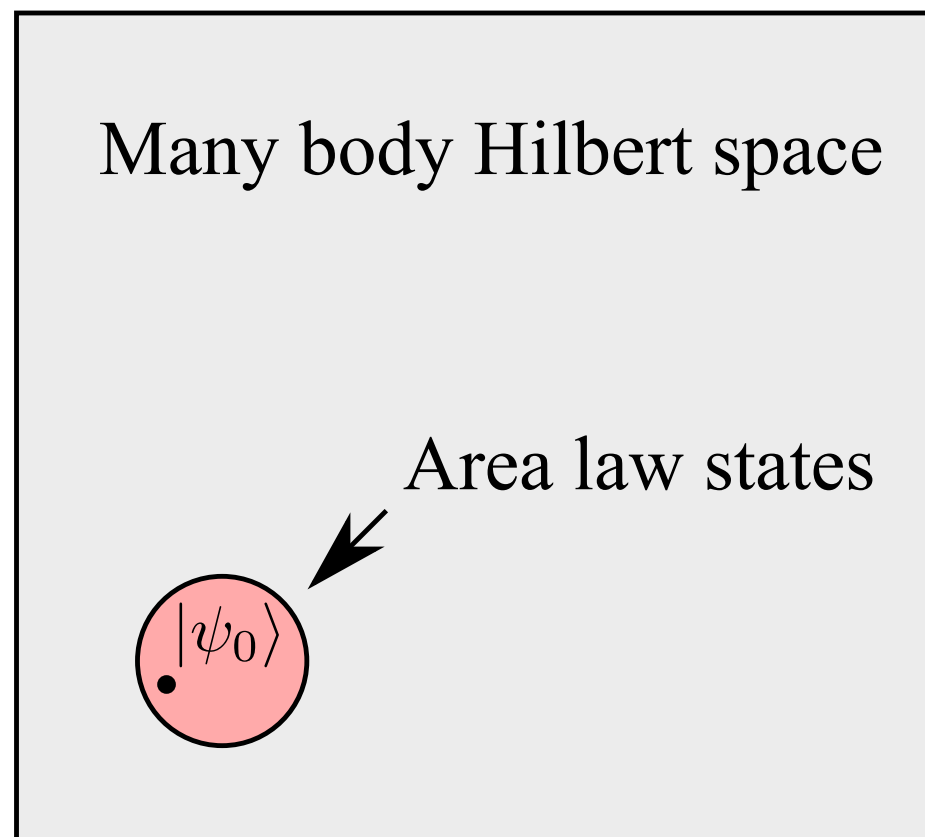


► Efficient compression by discarding small Schmidt values

Entanglement

Area law in one dimensional systems: Low depth circuit!

$$S(L) = \text{const.}$$



► Efficient compression by discarding small Schmidt values

Compression of quantum states

Example: $|\psi\rangle = \sum_{i=1}^{N_A} \sum_{j=1}^{N_B} C_{ij} |i\rangle_A |j\rangle_B = \sum_{\gamma} \lambda_{\gamma} |\phi_{\gamma}\rangle_A |\phi_{\gamma}\rangle_B$

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Matrix can represent an image (array of pixel)

$$C = \begin{pmatrix} 0.23 & \cdots & 0.56 \\ \vdots & \ddots & \vdots \\ 0.22 & \cdots & 0.34 \end{pmatrix} = \left(\begin{array}{c} \text{Image of the Golden Gate Bridge} \\ \chi = 1200 \end{array} \right)$$

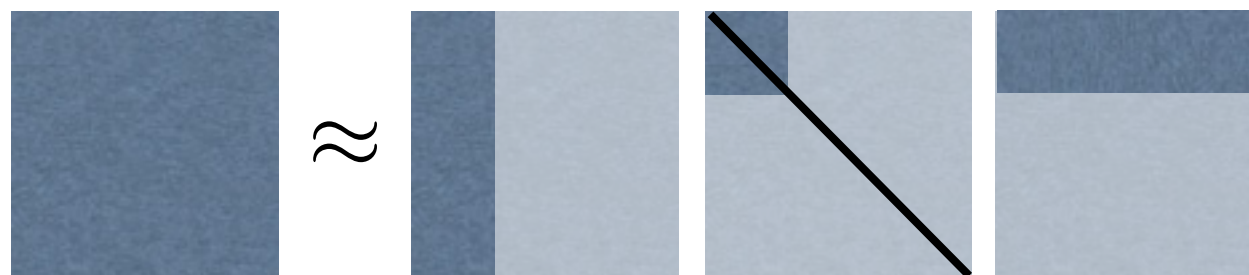
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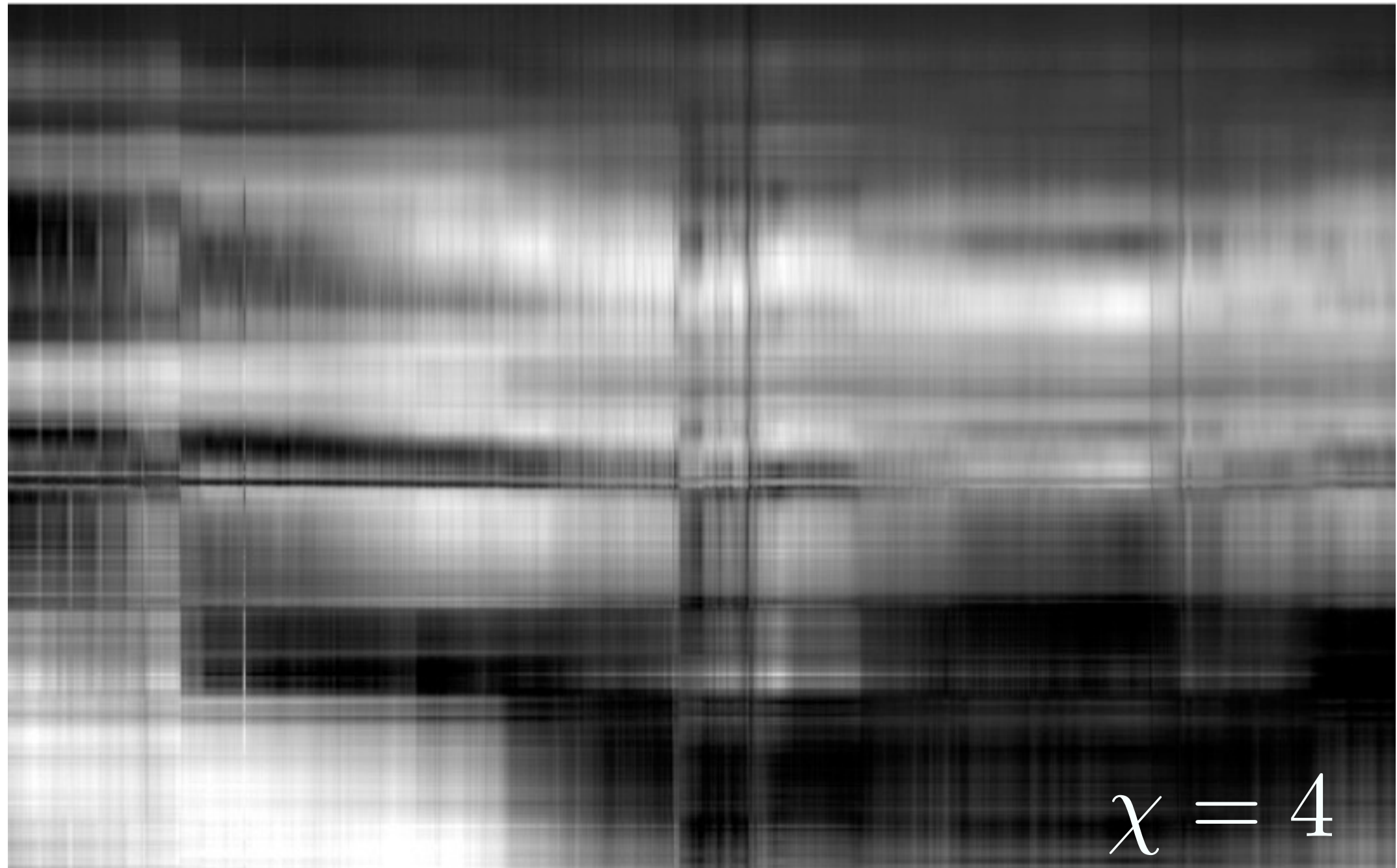
$$C = \begin{pmatrix} 0.23 & \cdots & 0.56 \\ \vdots & \ddots & \vdots \\ 0.22 & \cdots & 0.34 \end{pmatrix} = \left(\text{Image of Golden Gate Bridge} \right)_{\chi = 1200}$$

Reconstruction of the matrix (image) from a small number of Schmidt states (SVD):



Compression of quantum states

Compression of quantum states



Compression of quantum states

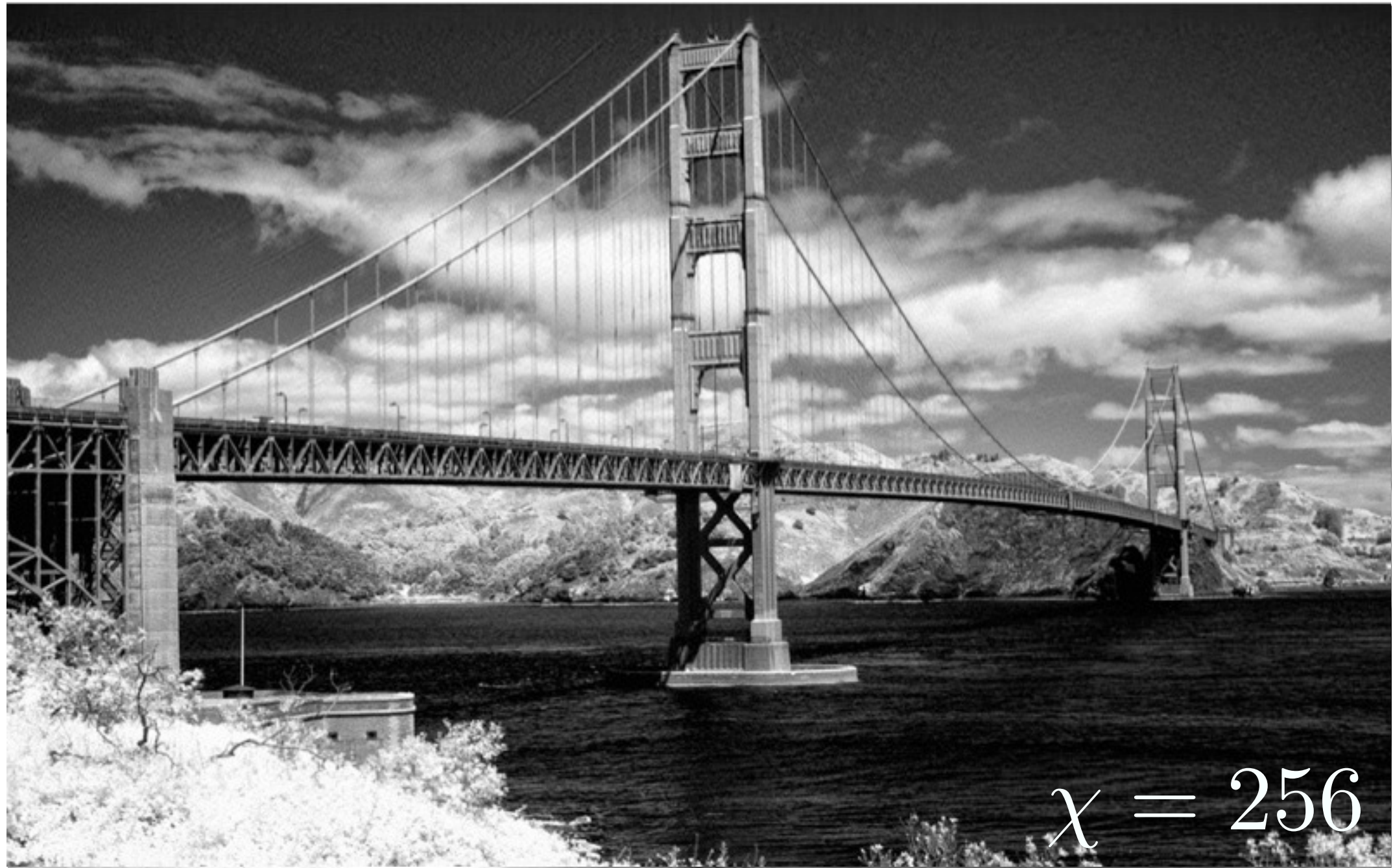


Compression of quantum states



$$\chi = 64$$

Compression of quantum states



Compression of quantum states



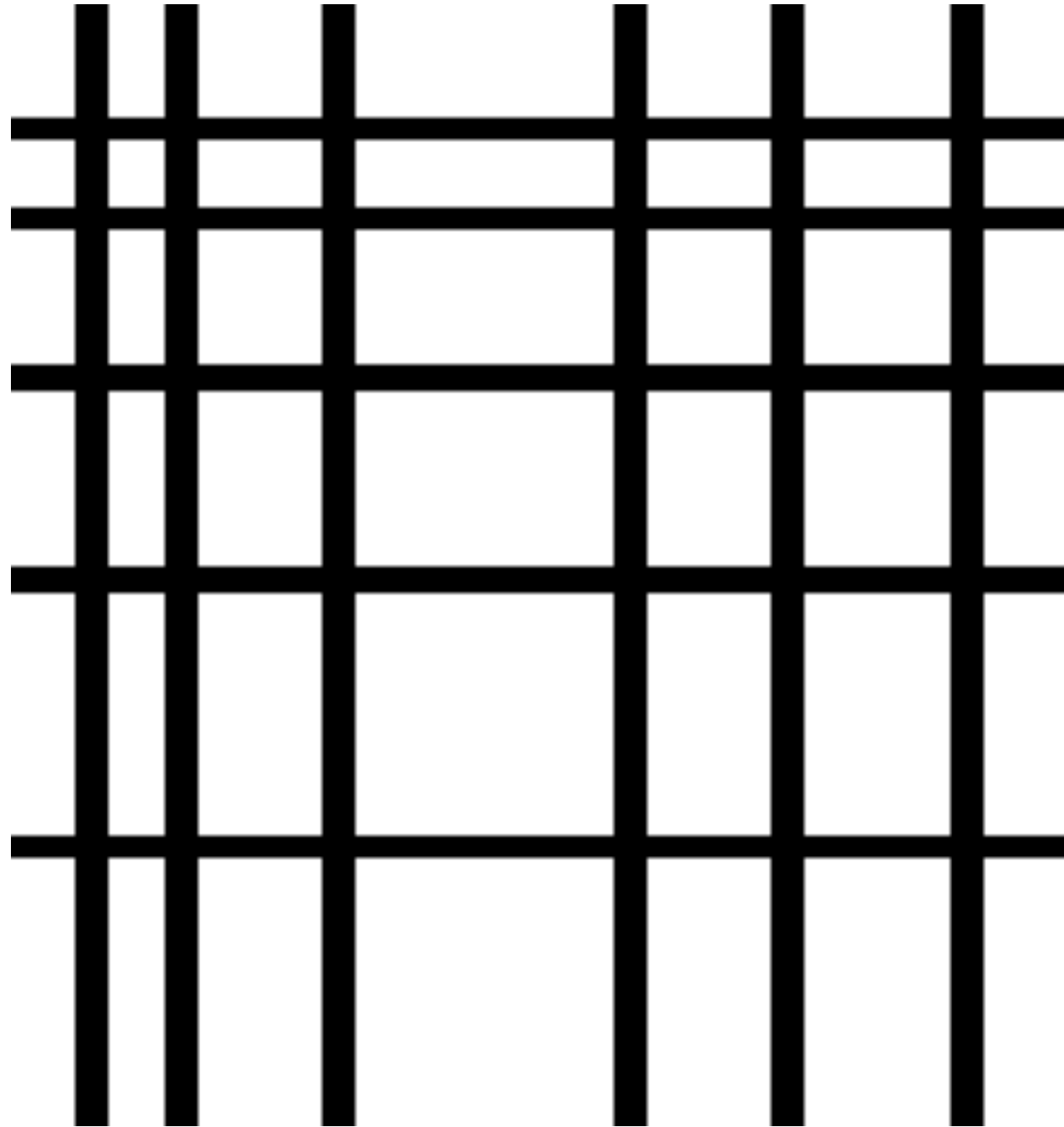
Compression of quantum states



$$\chi = 1200$$

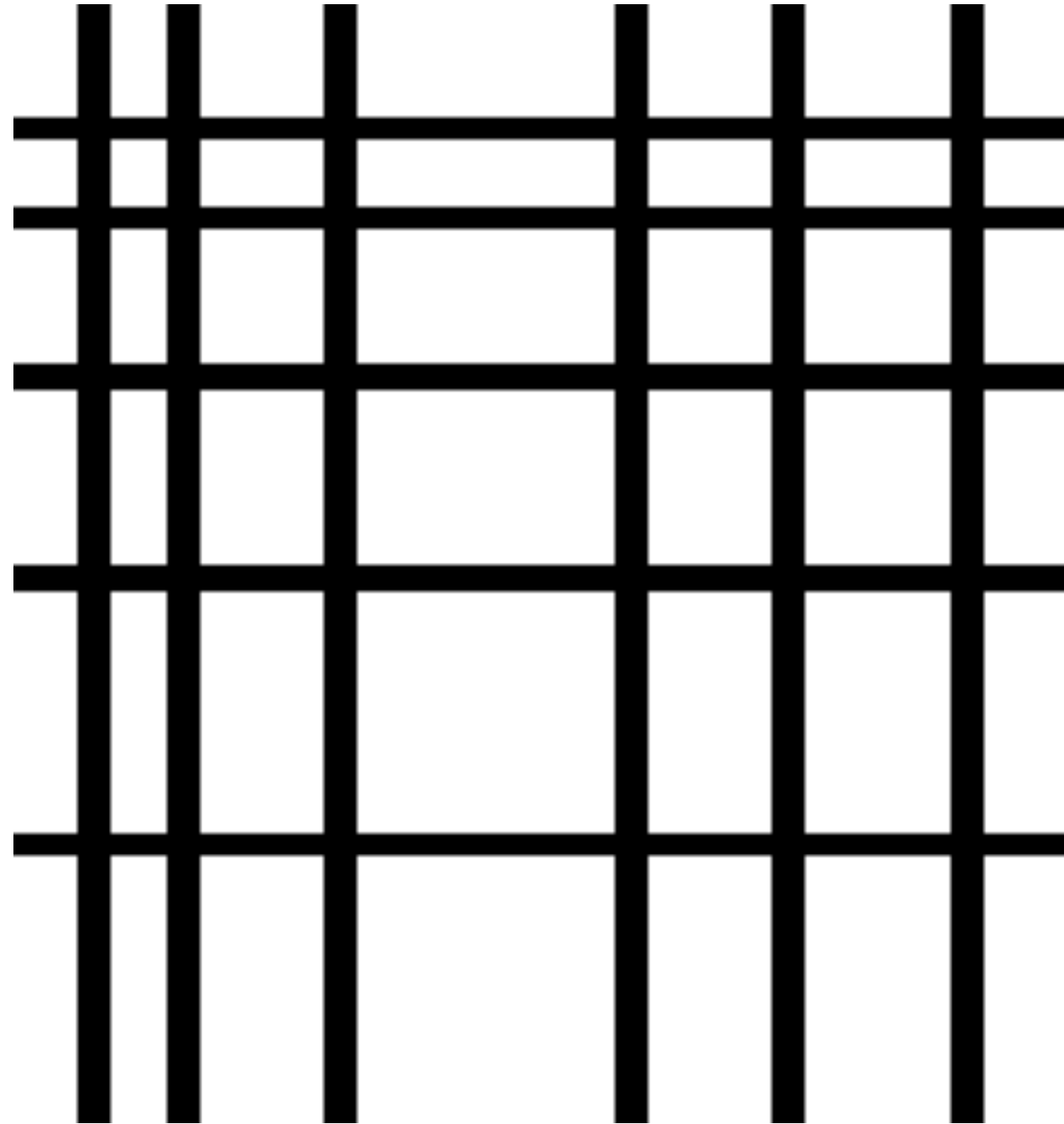
Important features visible already for < 16 states!

Compression of quantum states



[Mondrian]

Compression of quantum states



Exact for $\chi = 1$ state!

[Mondrian]

https://colab.research.google.com/drive/1_4V66KqRXHMR-CQI18eqkQBu9EZqw2KM#scrollTo=hKqe82w-U-NM

Matrix-Product States

Coefficients in the many-body wave function:


$$|\psi\rangle = \sum_{j_1, j_2, \dots, j_L} \psi_{j_1, j_2, \dots, j_L} |j_1\rangle |j_2\rangle \dots |j_L\rangle, \quad j_n = 1 \dots d$$

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Rank- L tensor: diagrammatic representation

$$\psi_{j_1, j_2, j_3, j_4, j_5} =$$


The diagram shows a horizontal gray bar with five vertical lines (legs) extending downwards from its center. The symbol ψ is positioned above the bar.

Matrix-Product States

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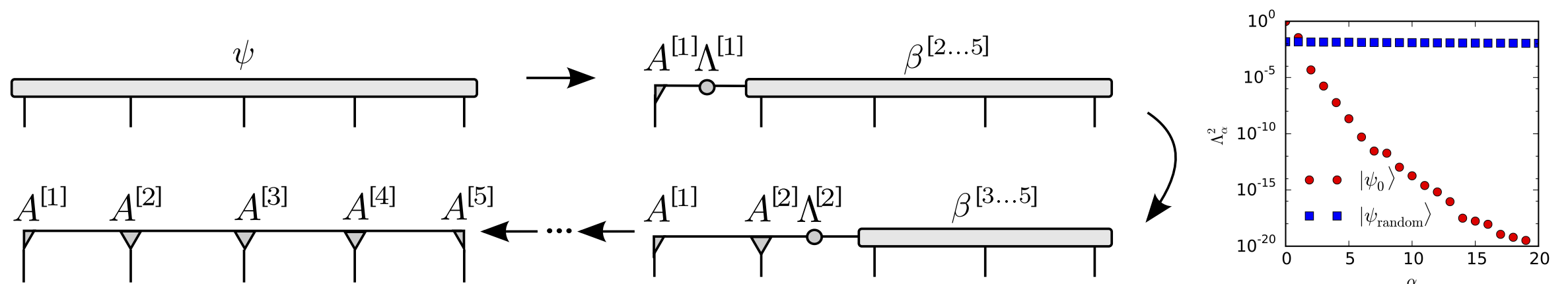
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Rank- L tensor: diagrammatic representation

$$\psi_{j_1, j_2, j_3, j_4, j_5} = \text{Diagram: A horizontal bar with 5 vertical legs, representing a rank-5 tensor. The label } \psi \text{ is above the bar.}$$

Successive Schmidt decompositions: **matrix-product states**

$$|\psi\rangle = \sum_{j_1=1}^d \sum_{\beta=1}^d A_{\beta}^{[1]j_1} \Lambda_{\beta}^{[1]} |j_1\rangle |\beta\rangle_{[2, \dots, N]}$$



Matrix-Product States

Many-body Hilbert space

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Matrix-Product States: Reduction of #variables $d^L \rightarrow Ld\chi^2$
(subsequent SVDs on each bond)

$$\psi_{j_1, j_2, \dots, j_L} \approx \sum_{\alpha_1, \alpha_2, \dots, \alpha_{L-1}} A_{\alpha_1}^{j_1} A_{\alpha_1, \alpha_2}^{j_2} \cdots A_{\alpha_{L-1}}^{j_L} \quad \alpha_j = 1 \dots \chi$$

Matrix-Product States

Many-body Hilbert space

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Diagrammatic representation

$$\psi_{j_1, j_2, j_3, j_4, j_5} = \begin{array}{c} A^{[1]} \quad A^{[2]} \quad A^{[3]} \quad A^{[4]} \quad A^{[5]} \\ \bullet \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \bullet \\ | \quad | \quad | \quad | \quad | \end{array}$$

$$A_{\alpha, \beta}^j = \begin{array}{c} A \\ \alpha \text{---} \bullet \text{---} \beta \\ | \\ j \end{array}$$

$\alpha, \beta = 1 \dots \chi$
 $j = 1 \dots d$

Matrix-Product States

Matrix-product states (MPS): Reduction of the number of variables: $d^L \rightarrow Ld\chi^2$ [M. Fannes et al. 92]

$$\psi_{j_1, j_2, j_3, j_4, j_5} = \sum_{\alpha_1, \alpha_2, \dots, \alpha_4}^{\chi} M_{\alpha_1}^{j_1} M_{\alpha_1, \alpha_2}^{j_2} M_{\alpha_2, \alpha_3}^{j_3} M_{\alpha_3, \alpha_4}^{j_4} M_{\alpha_4}^{j_5}$$

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Canonical form: Use the gauge degree of freedom ($A^j = XM^jX^{-1}$) to find a convenient representation

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$$\begin{array}{c} A \\ \text{---} \bullet \text{---} \\ \updownarrow \\ \text{---} \bullet \text{---} \\ A^* \end{array} = \mathbb{1}$$

$$\begin{array}{c} B \\ \text{---} \bullet \text{---} \\ \updownarrow \\ \text{---} \bullet \text{---} \\ B^* \end{array} = \mathbb{1} \quad (\text{Isometries})$$

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(Isometries)

Center matrix Λ represents wave function

$$|\psi\rangle = \sum_{\alpha, \beta, j} \Lambda_{\alpha, \beta}^j |\alpha\rangle |j\rangle |\beta\rangle \quad (\text{orthogonal states } |j\rangle, |\alpha\rangle, |\beta\rangle)$$