Bipartite entanglement

Quantum state in d^L dimensional Hilbert space

$$|\psi\rangle = \sum_{j_1, j_2, \ldots, j_L} \psi_{j_1, j_2, \ldots, j_L} |j_1\rangle |j_2\rangle \ldots |j_L\rangle \ , \ j_n = 1 \ldots d$$

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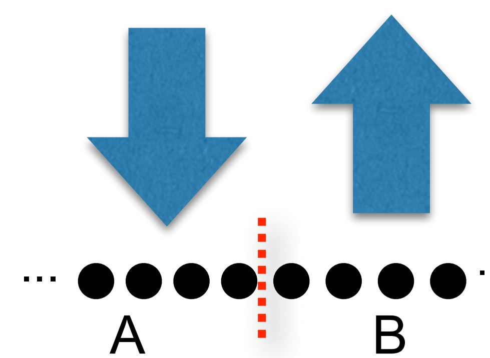
Decompose a state into a superposition of product states (**Schmidt decomposition**)



$$|\psi\rangle = \sum_{i,j} C_{i,j}|i\rangle_A \otimes |j\rangle_B = \sum_{\alpha} \Lambda_\alpha|\alpha\rangle_A \otimes |\alpha\rangle_B, \ \langle\alpha|\alpha'\rangle = \delta_{\alpha\alpha'}$$

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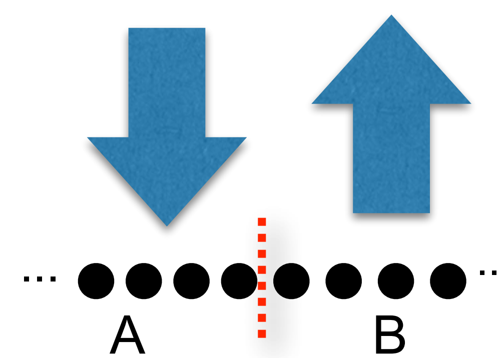

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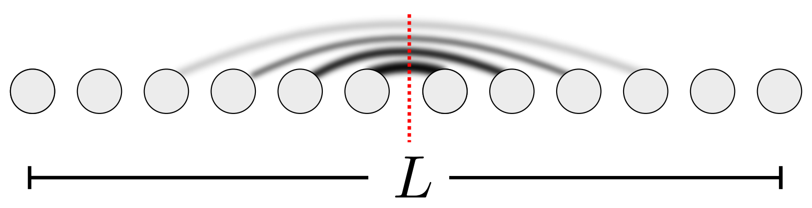
Entanglement entropy as a measure for the amount of entanglement $S = -\sum_\alpha \Lambda_\alpha^2 \log \Lambda_\alpha^2$

Entanglement

Area law in one dimensional systems: Low depth circuit!

$$S(L) = \text{const.}$$



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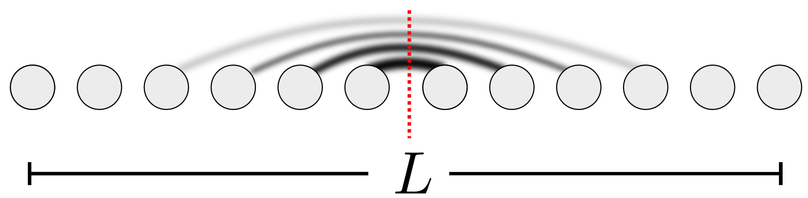

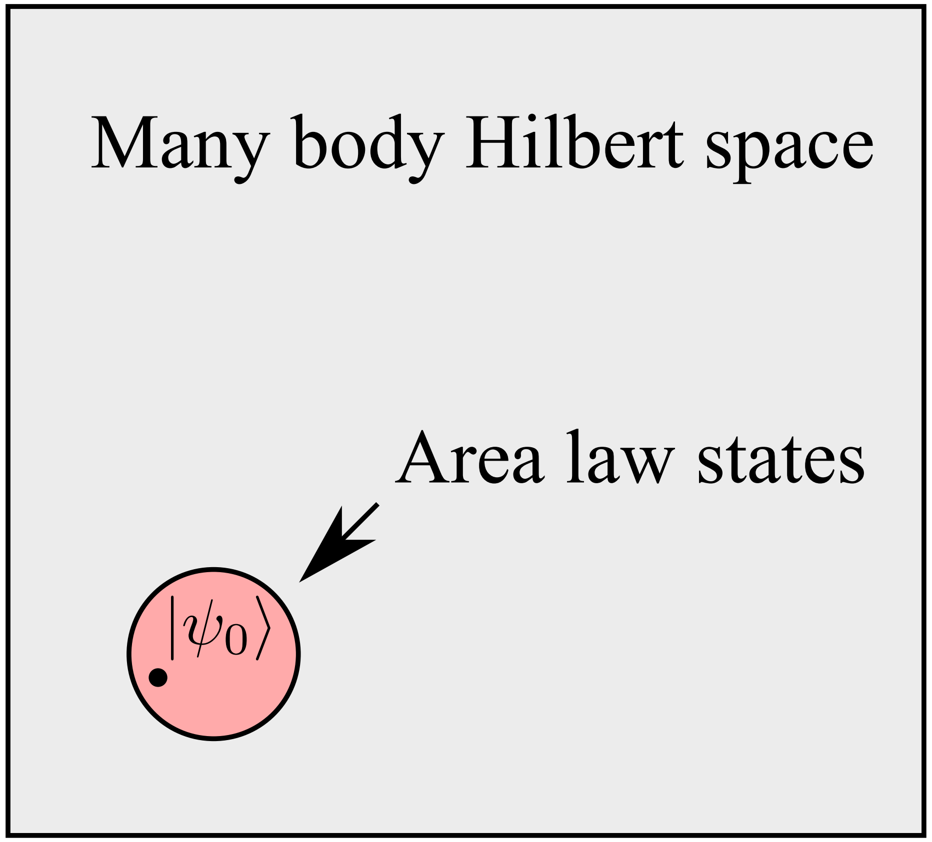

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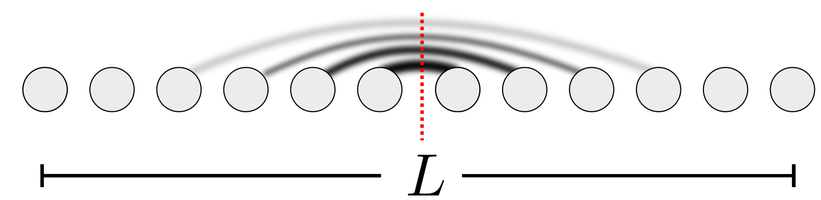

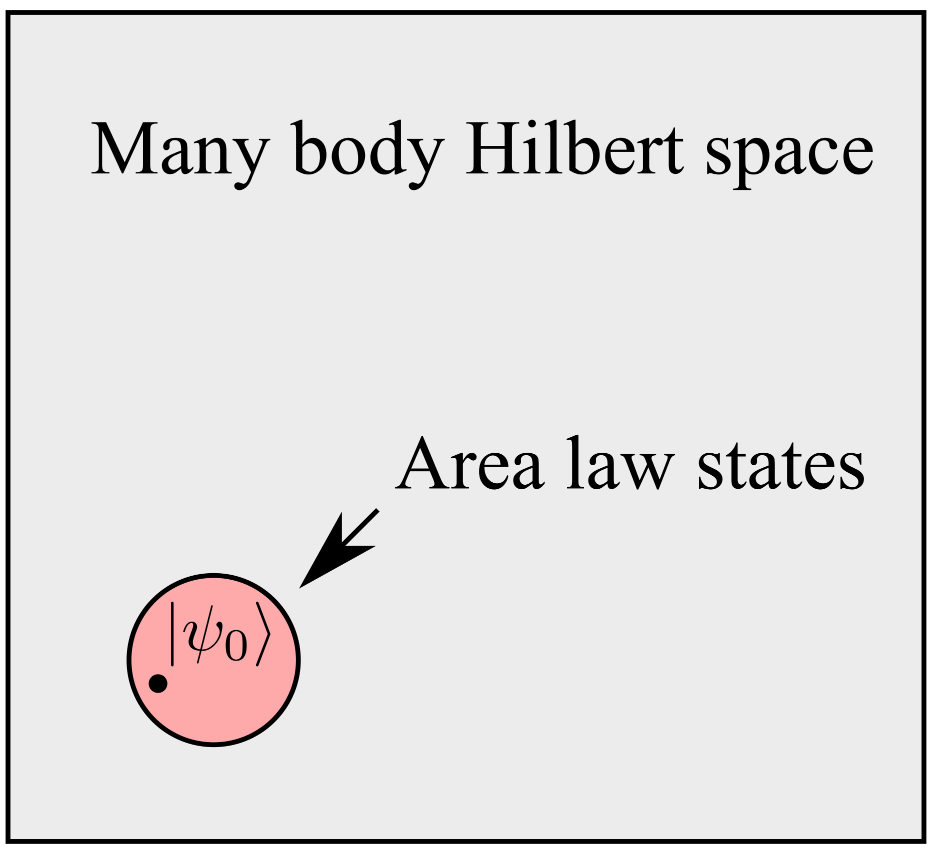

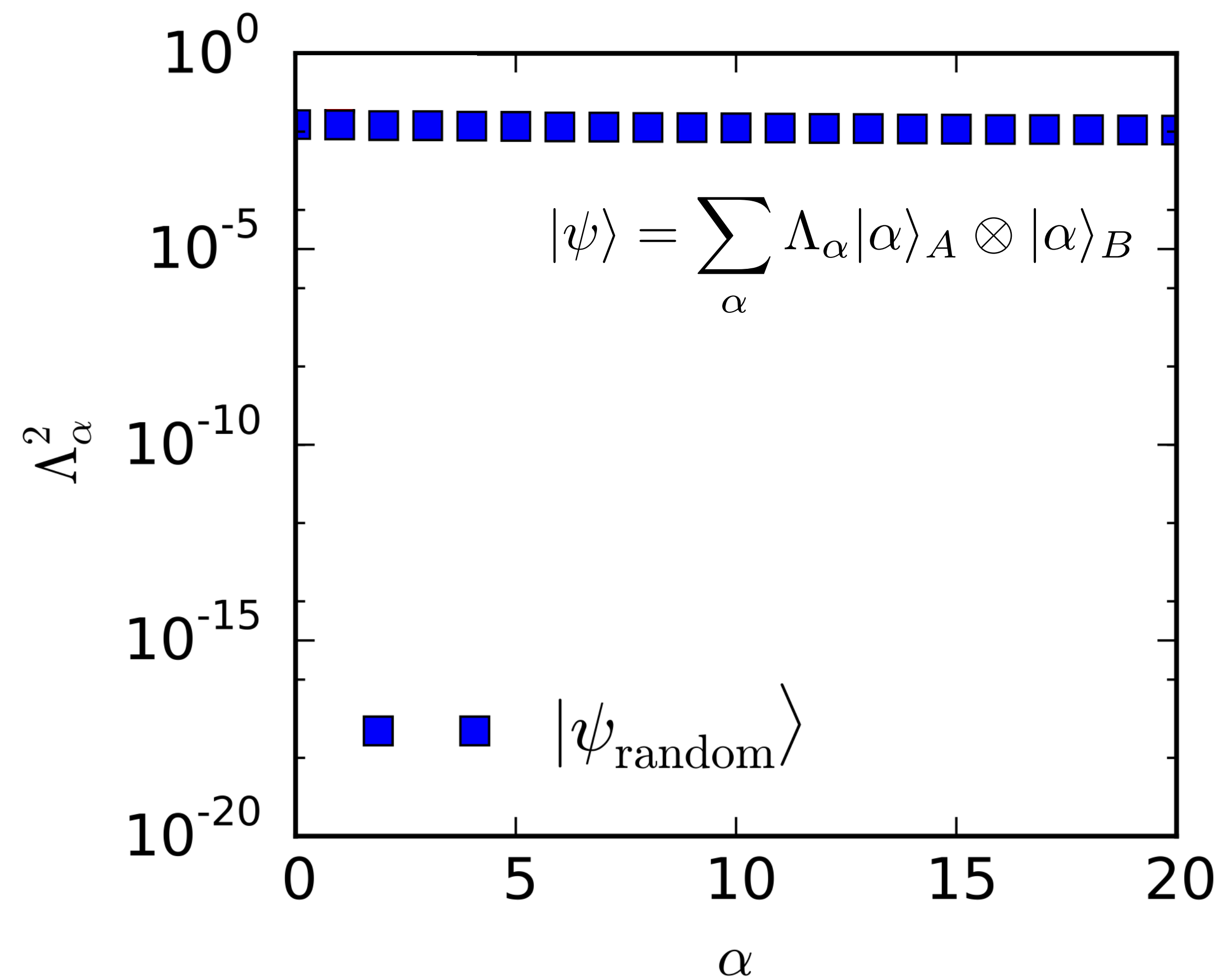

- Efficient compression by discarding small Schmidt values

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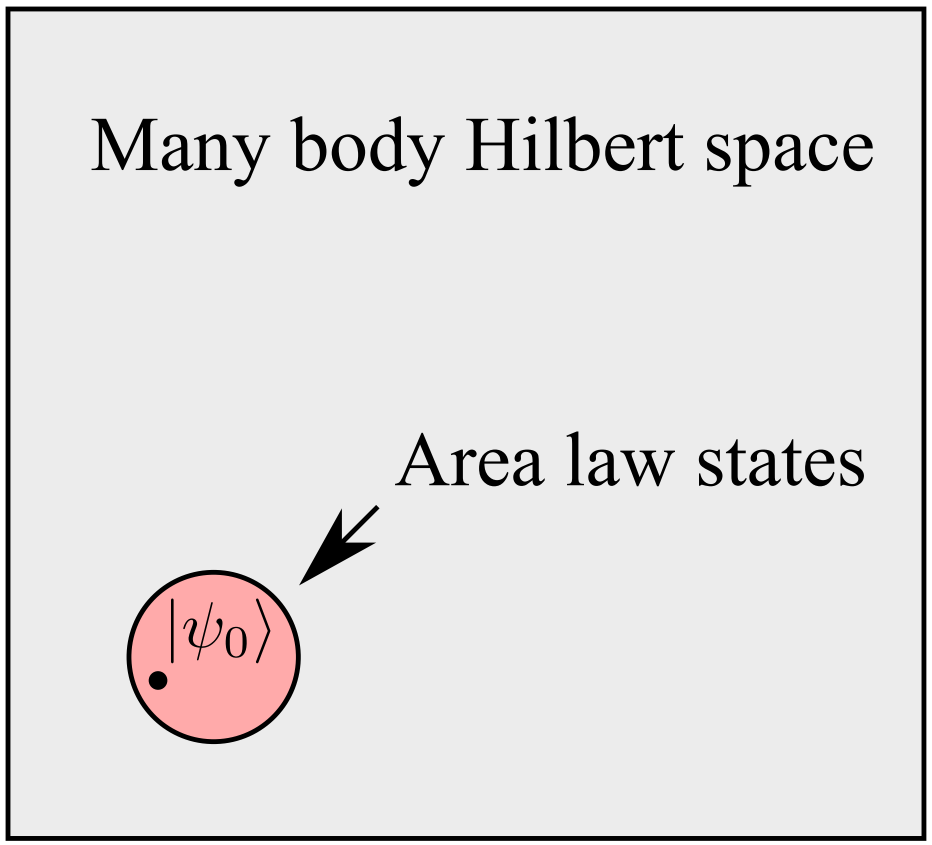

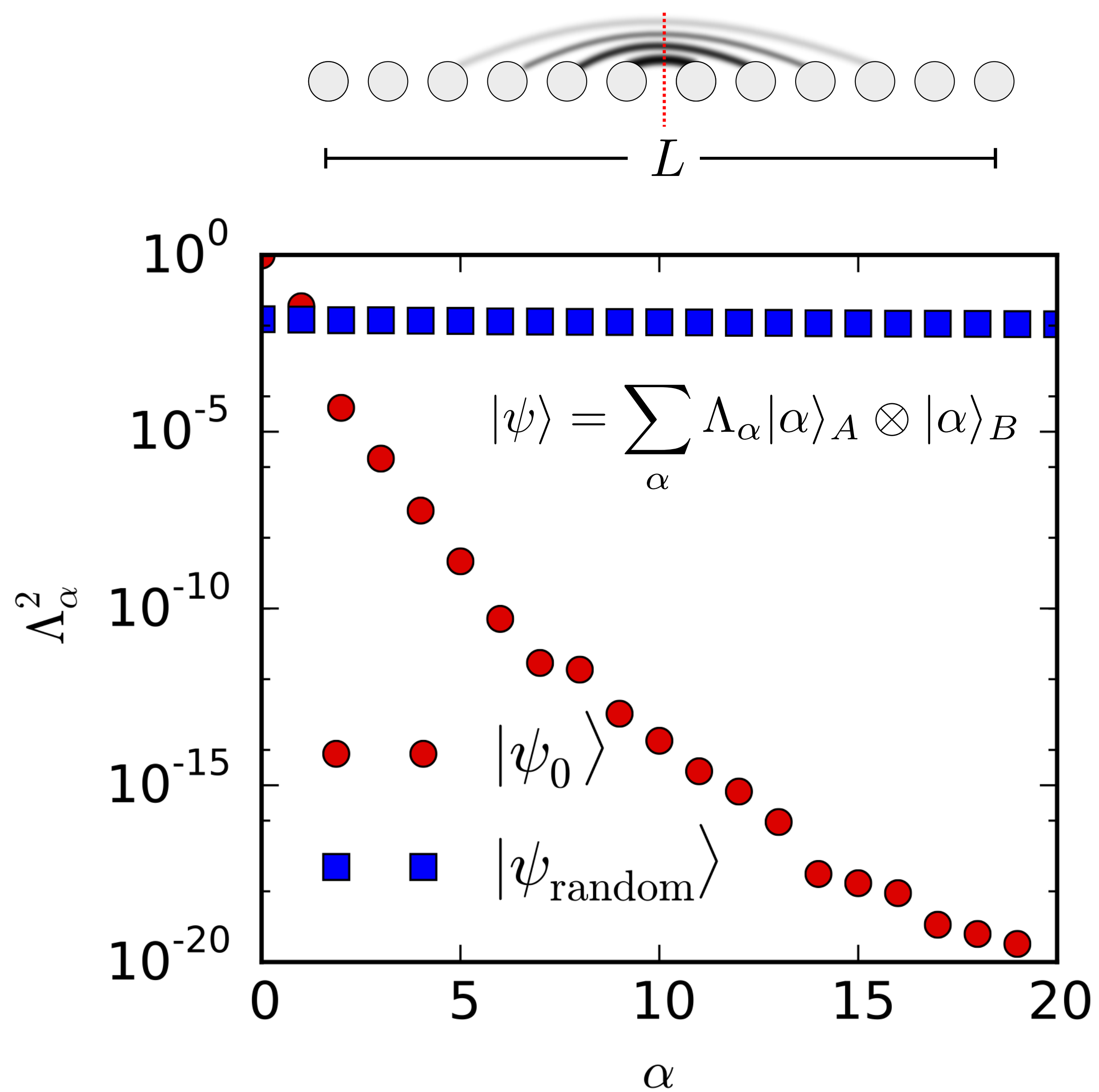

- Efficient compression by discarding small Schmidt values

Compression of quantum states

Example: $$|\psi\rangle = \sum_{i=1}^{N_A}\sum_{j=1}^{N_B} C_{ij}|i\rangle_A|j\rangle_B = \sum_{\gamma} \lambda_\gamma|\phi_\gamma\rangle_A|\phi_\gamma\rangle_B$$

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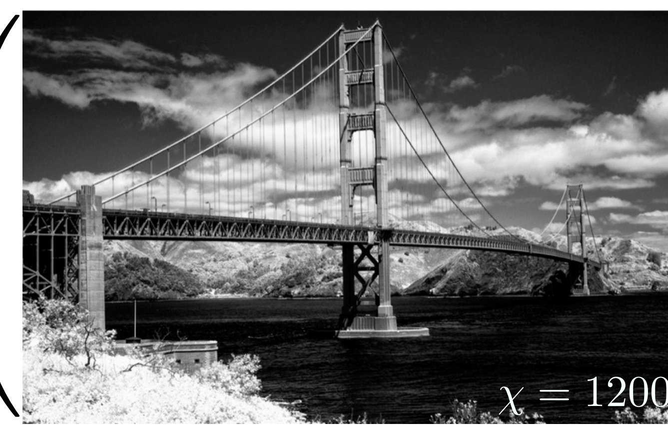
Matrix can represent an image (array of pixel)

$$C = \begin{pmatrix} 0.23 & \cdots & 0.56 \\ \vdots & \ddots & \vdots \\ 0.22 & \cdots & 0.34 \end{pmatrix} =$$



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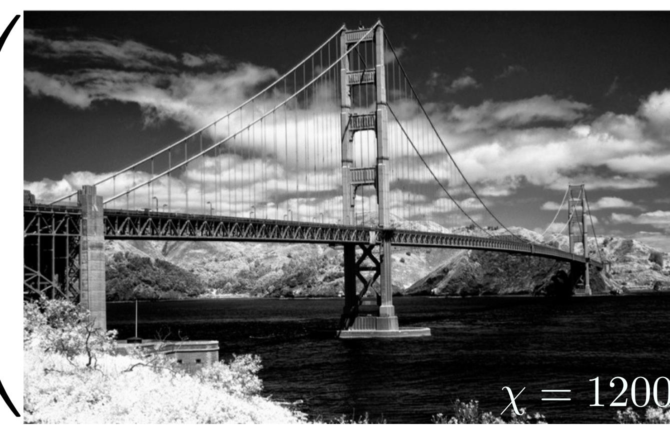
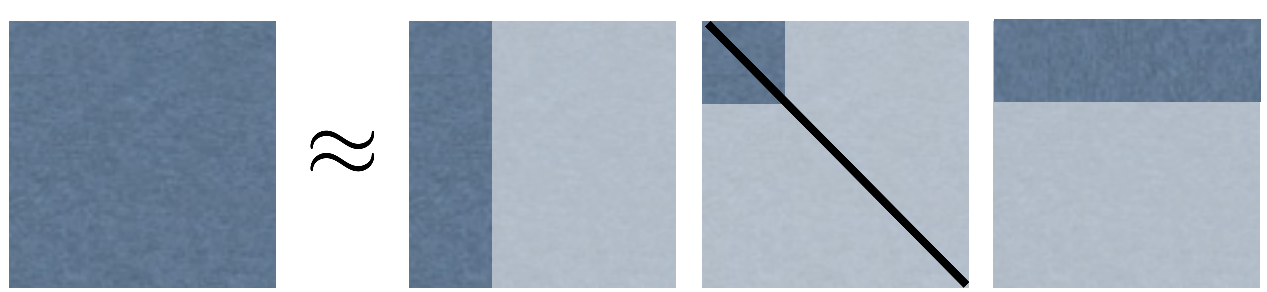


Reconstruction of the matrix (image) from a small number of Schmidt states (SVD):



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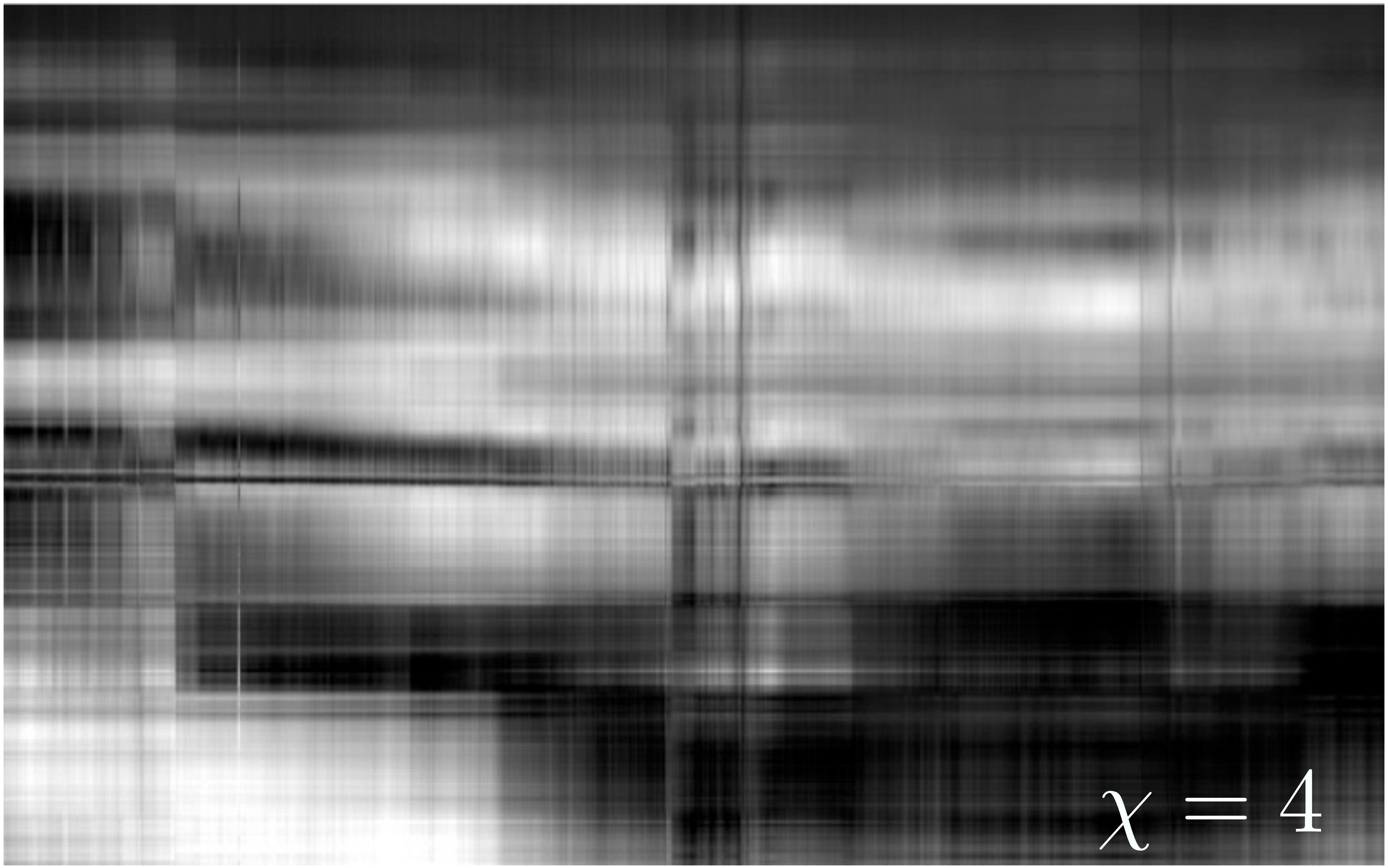

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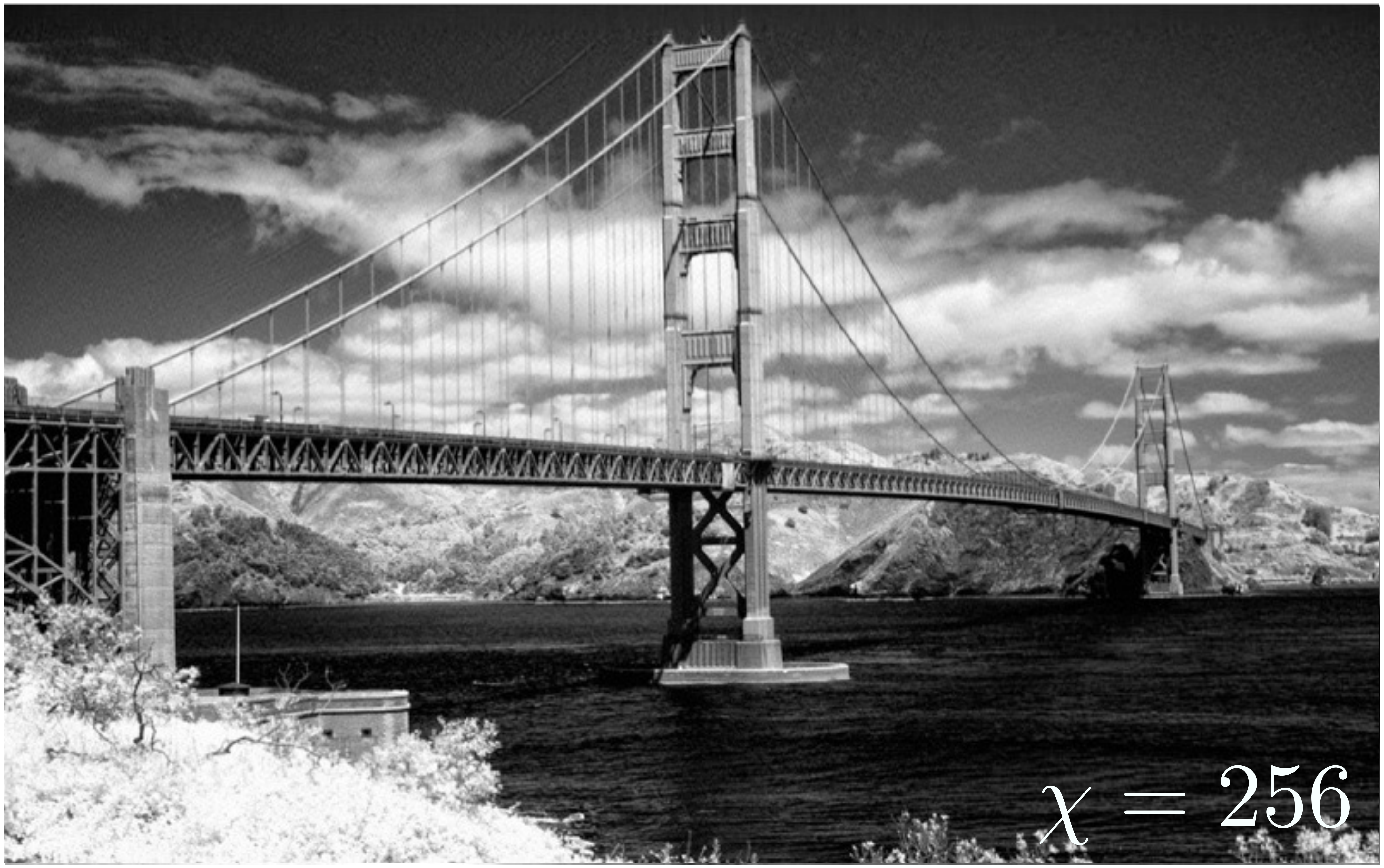
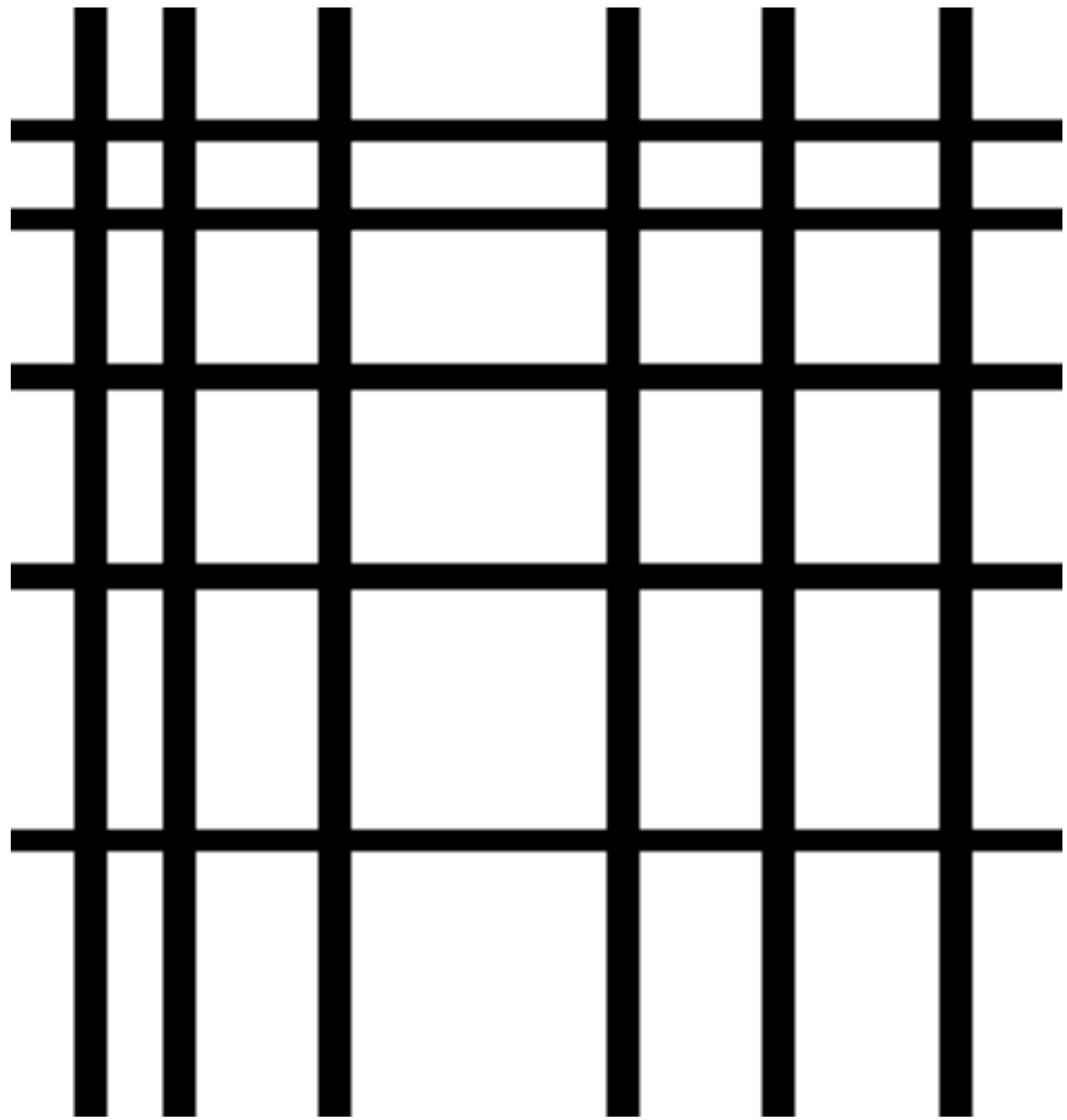


Compression of quantum states

Compression of quantum states

Important features visible already for < 16 states!

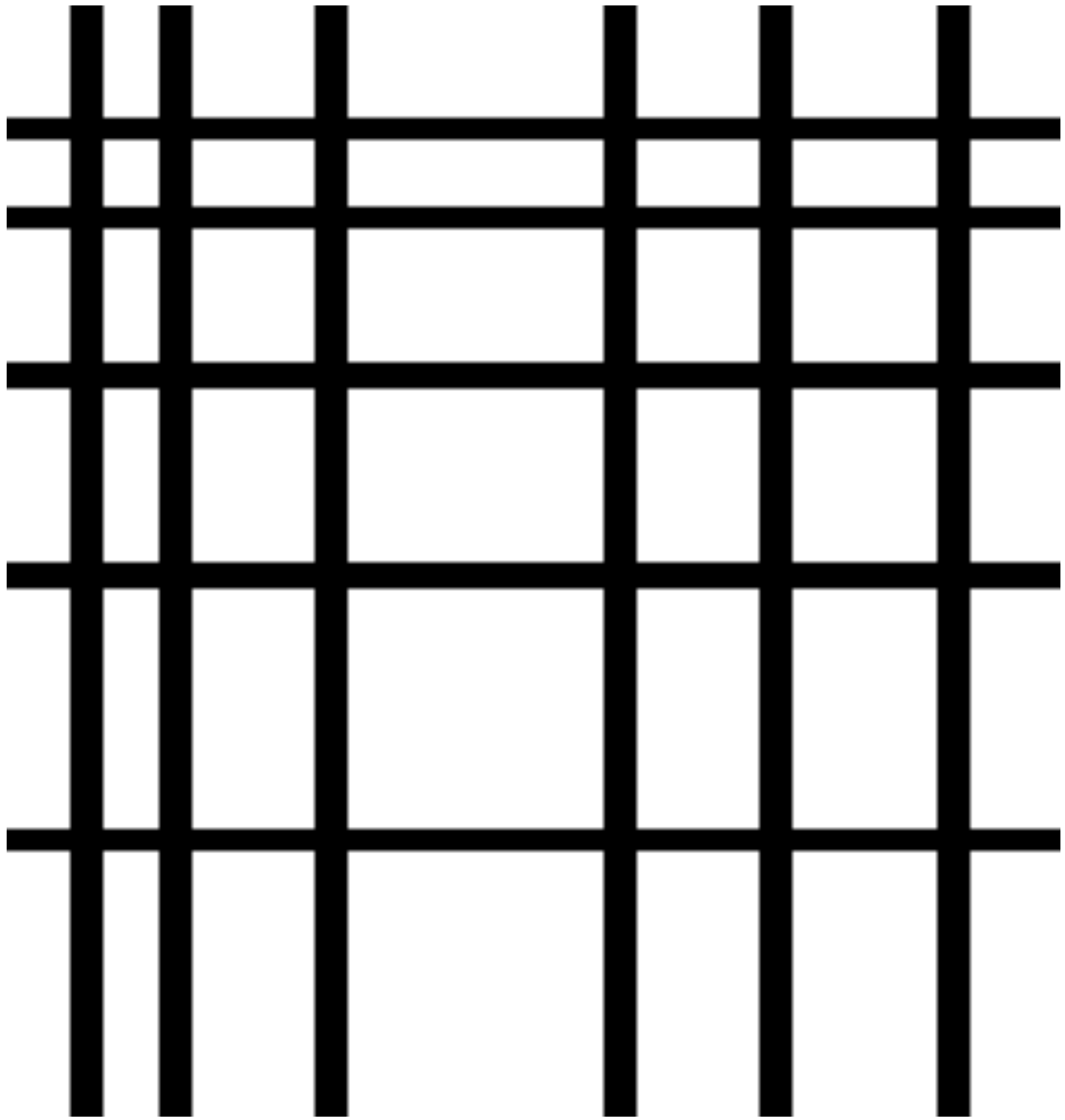
Compression of quantum states



[Mondrian]

https://colab.research.google.com/drive/1_4V66KqRXHMR-CQl18eqkQBu9EZqw2KM#scrollTo=hKqe82w-U-NM

Compression of quantum states



[Mondrian]

Exact for $\chi = 1$ state!

https://colab.research.google.com/drive/1_4V66KqRXHMR-CQI18eqkQBu9EZqw2KM#scrollTo=hKqe82w-U-NM

Matrix-Product States

Coefficients in the many-body wave function:

$$|\psi\rangle = \sum_{j_1, j_2, \ldots, j_L} \psi_{j_1, j_2, \ldots, j_L} |j_1\rangle |j_2\rangle \ldots |j_L\rangle \ , \ j_n = 1 \ldots d$$

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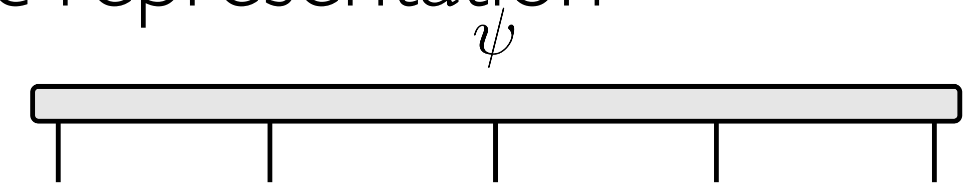
Rank-L tensor: diagrammatic representation

$$\psi_{j_1,j_2,j_3,j_4,j_5} =$$



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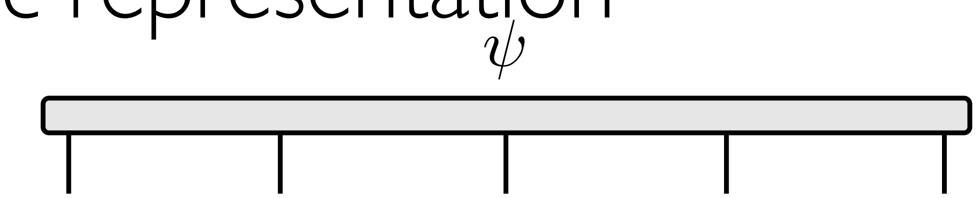
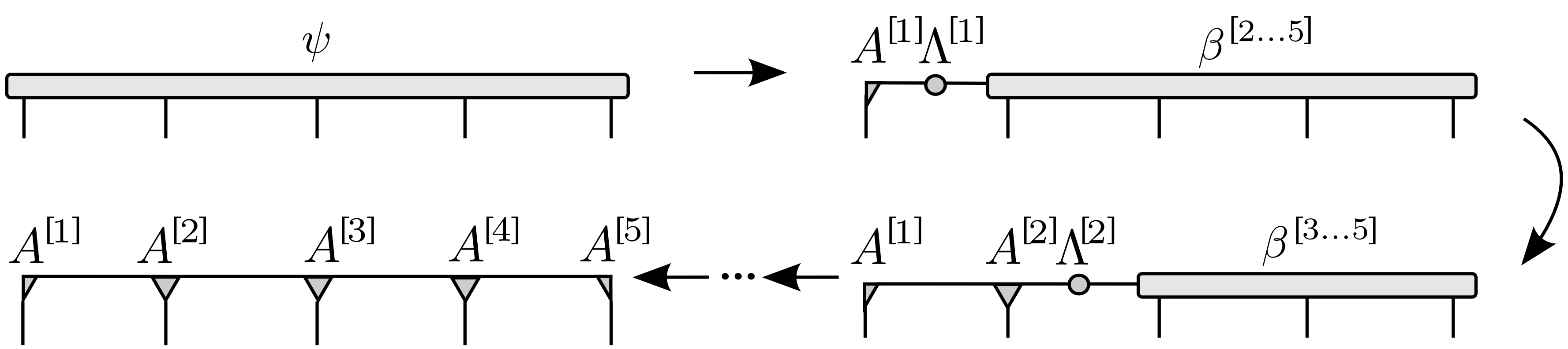
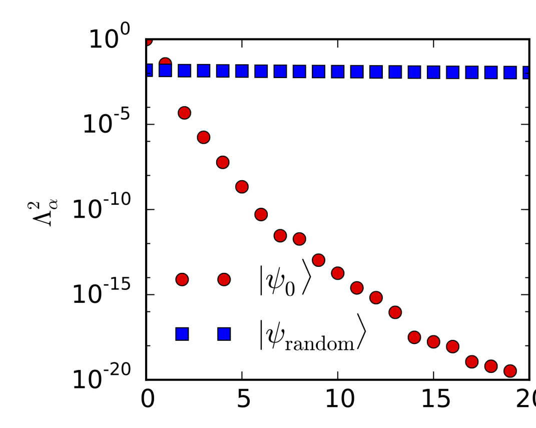


Successive Schmidt decompositions: **matrix-product states**

$$|\psi\rangle = \sum_{j_1=1}^{d} \sum_{\beta=1}^{d} A_{\beta}^{[1] j_1} \Lambda_{\beta}^{[1]} |j_1\rangle |\beta\rangle_{[2,\ldots N]}$$





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Matrix-Product States: Reduction of #variables $d^L \to Ld\chi^2$

(subsequent SVDs on each bond)

$$\psi_{j_1,j_2,\ldots,j_L} \approx \sum_{\alpha_1,\alpha_2,\ldots,\alpha_{L-1}} A^{j_1}_{\alpha_1} A^{j_2}_{\alpha_1,\alpha_2} \ldots A^{j_L}_{\alpha_{L-1}} \qquad \alpha_j = 1\ldots\chi$$

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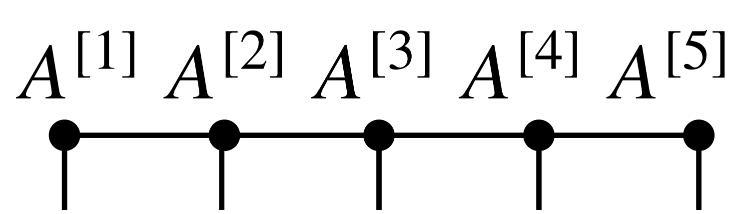
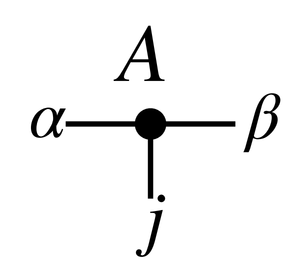
$\psi_{j_1,j_2,j_3,j_4,j_5} =$ 

$A^j_{\alpha,\beta} =$ 

$\alpha,\beta = 1\ldots\chi$

$j = 1\ldots d$

Matrix-Product States

Matrix-product states (MPS): Reduction of the number of variables: $d^L \rightarrow L d \chi^2$ [M. Fannes et al. 92]

$$\psi_{j_1,j_2,j_3,j_4,j_5} = \sum_{\alpha_1,\alpha_2,\ldots,\alpha_4}^{\chi} M^{j_1}_{\alpha_1} M^{j_2}_{\alpha_1,\alpha_2} M^{j_3}_{\alpha_2,\alpha_3} M^{j_4}_{\alpha_3,\alpha_4} M^{j_5}_{\alpha_4}$$

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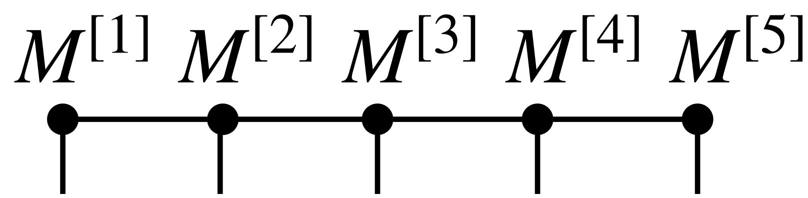
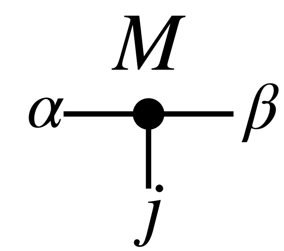
$\psi_{j_1,j_2,j_3,j_4,j_5} =$ 

$M^j_{\alpha,\beta} =$ 

$\alpha, \beta = 1 \ldots \chi$

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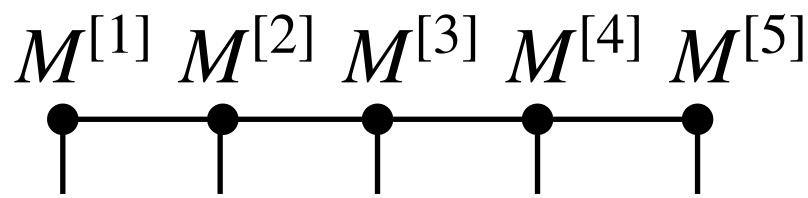
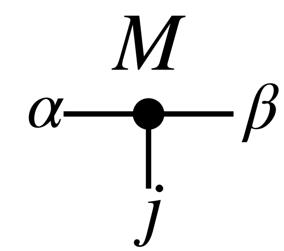
$\psi_{j_1,j_2,j_3,j_4,j_5} =$ 

$M^j_{\alpha,\beta} =$ 

$\alpha, \beta = 1 \ldots \chi$

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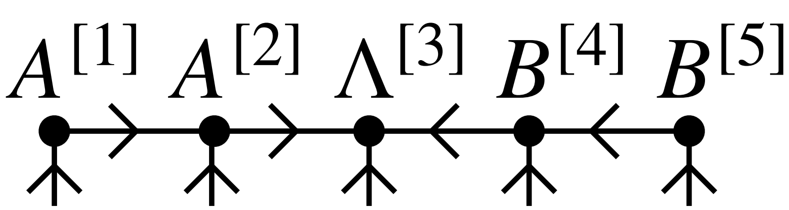
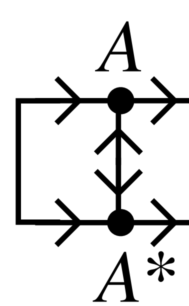
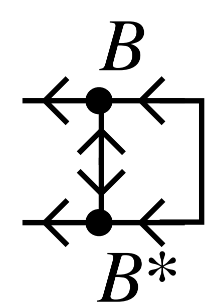
Canonical form: Use the gauge degree of freedom ($A^j = X M^j X^{-1}$) to find a convenient representation





$= \mathbb{1}$



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(Isometries)

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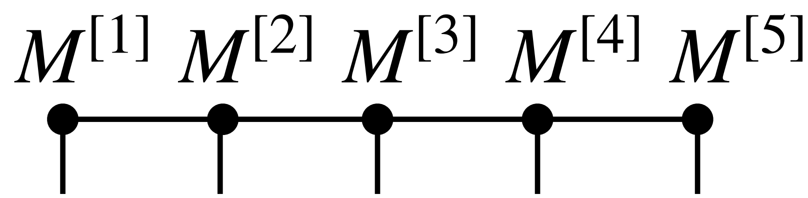
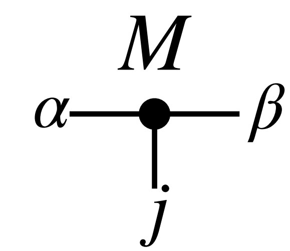
$\psi_{j_1,j_2,j_3,j_4,j_5} =$ 

$M^j_{\alpha,\beta} =$ 

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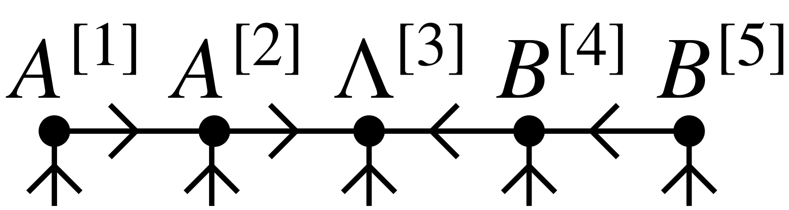

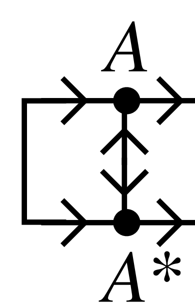

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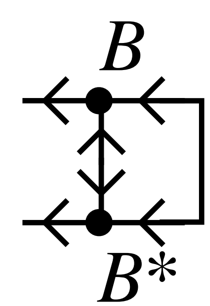

$= \mathbb{1}$ (Isometries)

Center matrix Λ represents wave function

$$|\psi\rangle = \sum_{\alpha,\beta,j} \Lambda^j_{\alpha,\beta} |\alpha\rangle |j\rangle |\beta\rangle$$

(orthogonal states $|j\rangle$, $|\alpha\rangle$, $|\beta\rangle$)