



# Causal Inference on Treatment Datasets

## Project4-Group3

Xiangning Han, Charles Shin, Sneha Swati, Shuyuan Wang, Minzhi Zhang

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**1**

**Overview**



## Goal

- Implementing, evaluating and comparing three algorithms for Causal Inference (i.e. estimating true ATE).
- The algorithms implemented are Stratification, Regression Adjustment and Weighted Regression, together with L2 penalized logistic regression as the propensity score estimation method.
- Performed the ATE estimation on two generated datasets, low dimensional and high dimensional dataset.



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## Propensity Score Estimation

## Propensity Score Estimation

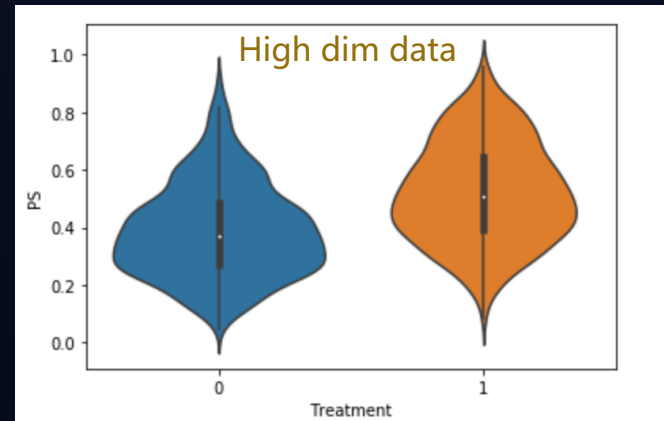
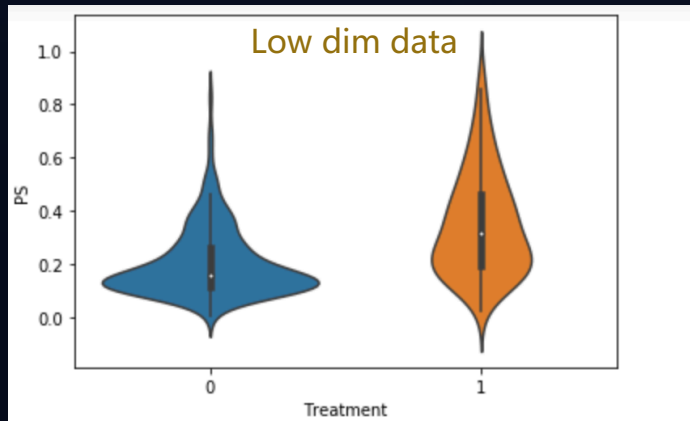
L2 penalized logistic regression as the propensity score estimation method.

The propensity score is defined by:  $e(x) = \Pr(T=1|X=x)$

L2 penalized logistic regression : modify the loss function with a penalty term, which is 'L2 norm' .

$$Q = -\frac{1}{n} \sum_{i=1}^n [y_i(\beta_0 + \beta_1 x_{i1} + \dots + \beta_p x_{ip}) + \log(1 + \exp(\beta_0 + \beta_1 x_{i1} + \dots + \beta_p x_{ip}))] + \lambda \sum_{j=1}^p \beta_j^2$$

For both low dimensional data and high dimensional data, with  $\lambda$  (parameter for the penalty term) chosen as 2, the violin plots show that the distribution of propensity scores for both groups are similar, indicating an acceptable overlapping.







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## Algorithm Overview

## Stratification

Data	Run Time	ATE Est
Low dim data	0.030781	2.879272
High dim data	0.013375	-2.783537

Divided subjects into five equal-size groups by the quintiles of the estimated propensity score.

The average of all the difference of average outcome between treated and untreated group for each stratum is the estimated ATE.



## Regression Adjustment

Data	Run Time	ATE Est
Low dim data	0.033268	2.502966
High dim data	0.130357	-2.999067

Remove the effects of confounding when estimating the effects of treatment on outcomes via covariate adjustment using the estimated propensity score.

The outcome variable  $Y$  is regressed on the indicator variable  $T$  and the estimated propensity score.

## Weighted Regression

Data	Run Time	ATE Est
Low dim data	0.136385	2.523955
High dim data	0.685741	-2.964053

Weighted least square estimation of the regression function:

$$Y_i = \alpha_0 + \tau * T_i + \alpha'_1 * T_i + \alpha'_2 * (Z_i - \bar{Z}) * T_i + \varepsilon_i$$

The outcome variable Y was regressed on the selected covariates Z.

To select Z, estimated  $K$  linear regressions of the type :

$$Y_i = \beta_0 + \beta_{k1} * T_i + \beta_{k2} * X_{ik} + \varepsilon_i , \text{ where } K \text{ is the total number of covariates.}$$

Calculate the t-statistic for the test of the null hypothesis that the slope coefficient  $\beta_{k2} = 0$  in each of these regressions, selected for Z all the covariates with a t-statistics larger in absolute value than  $t_{reg} = 2$ .

The estimated coefficient of the indicator variable T  $\hat{\tau}$  is the estimated ATE.



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## Evaluation and Comparison



## Evaluation and Comparison

Evaluation method:

Absolute error and relative error to evaluate the ATE estimates

Absolute error(ABSE) is given by:

$$ABSE = |ATE_{EST} - ATE_{True}|$$

Relative error is given by:

$$RE = |ATE_{EST} - ATE_{True}| / |ATE_{EST}|$$

## Evaluation and Comparison

Low dim data				
Algorithm	Run Time	ATE Est	Abs Error	Relative Error
Stratification	0.030781	2.879272	0.38	15.17%
Regression Adjustment	0.033268	2.502966	0.00	0.12%
Weighted Regression	0.136385	2.523955	0.02	0.96%

High dim data				
Algorithm	Run Time	ATE Est	Abs Error	Relative Error
Stratification	0.013375	-2.783537	0.22	7.22%
Regression Adjustment	0.130357	-2.999067	0.00	0.03%
Weighted Regression	0.685741	-2.964053	0.04	1.20%

### Conclusion:

- Regression Adjustment is the best method for both low dimensional data and high dimensional data.
- Stratification has a fastest run time among the three Algorithms.



## Reference

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Q & A



**Thank you!**