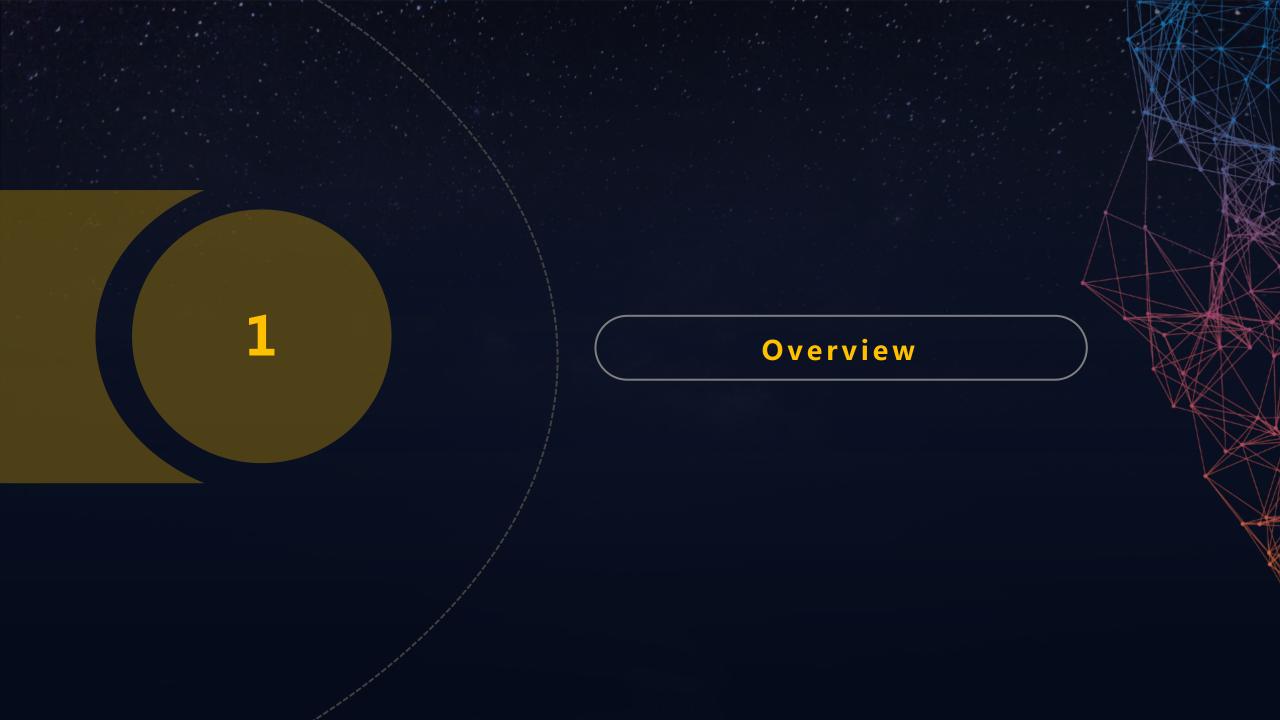
Causal Inference on Treatment Datasets

Project4-Group3

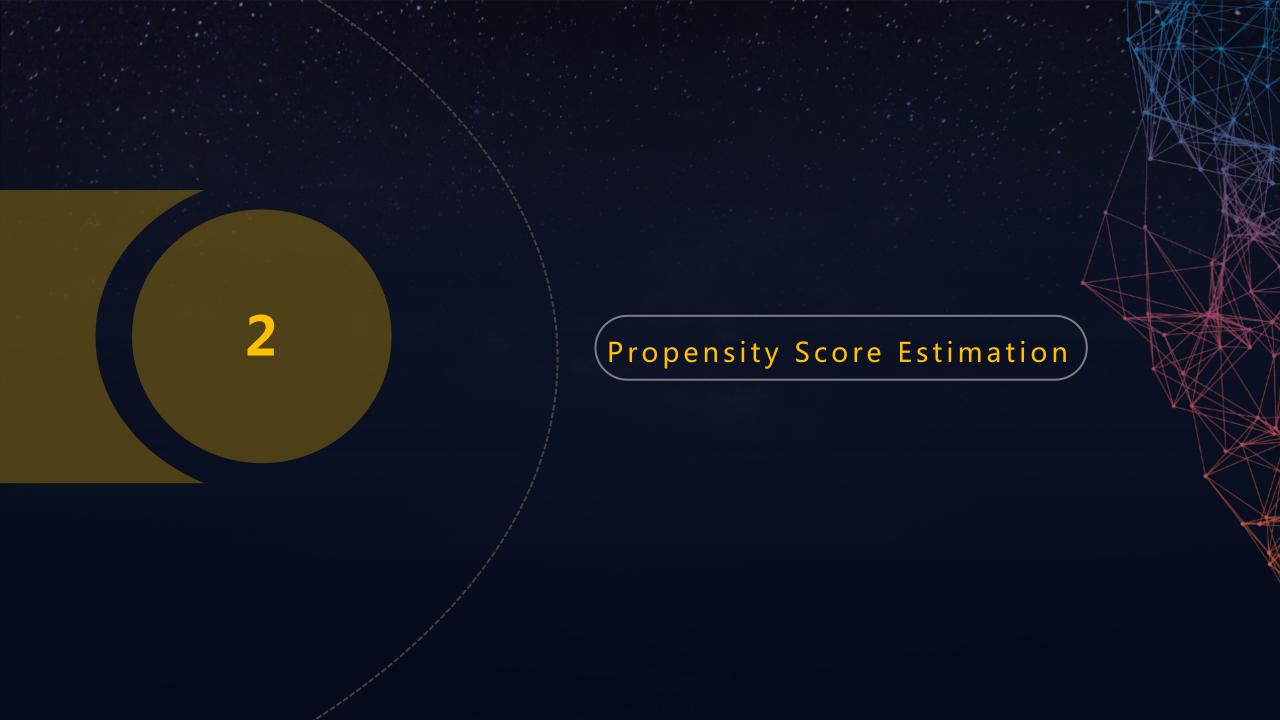
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Overview Propensity Score Estimation Agenda Algorithm Overview Evaluation and Comparison Q & A



Goal

- Implementing, evaluating and comparing three algorithms for Causal Inference (i.e. estimating true ATE).
- The algorithms implemented are Stratification, Regression Adjustment and Weighted
 Regression, together with L2 penalized logistic regression as the propensity score
 estimation method.
- Performed the ATE estimation on two generated datasets, low dimensional and high dimensional dataset.



Propensity Score Estimation

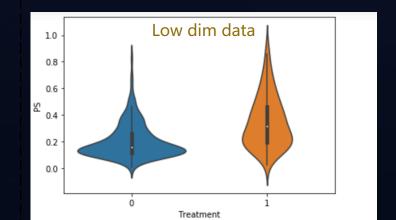
L2 penalized logistic regression as the propensity score estimation method.

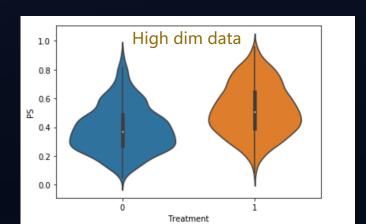
The propensity score is defined by: e(x)=Pr(T=1|X=x)

L2 penalized logistic regression modify the loss function with a penalty term, which is 'L2 norm'.

$$Q = -\frac{1}{n} \sum_{i=1}^{n} \left[y_i (\beta_0 + \beta_0 x_{i1} + \dots + \beta_{ip}) + \log(1 + \exp(\beta_0 + \beta_0 x_{i1} + \dots + \beta_{ip})) \right] + \lambda \sum_{j=1}^{p} \beta_j^2$$

For both low dimensional data and high dimensional data, with λ (parameter for the penalty term) chosen as 2, the violin plots show that the distribution of propensity scores for both groups are similar, indicating an acceptable overlapping.







Stratification

| Data | Run Time | ATE Est |
|---------------|----------|-----------|
| Low dim data | 0.030781 | 2.879272 |
| High dim data | 0.013375 | -2.783537 |

Divided subjects into five equal-size groups by the quintiles of the estimated propensity score.

The average of all the difference of average outcome between treated and untreated group for each stratum is the estimated ATE.



Regression Adjustment

| Data | Run Time ATE Est | |
|---------------|------------------|-----------|
| Low dim data | 0.033268 | 2.502966 |
| High dim data | 0.130357 | -2.999067 |

Remove the effects of confounding when estimating the effects of treatment on outcomes via covariate adjustment using the estimated propensity score.

The outcome variable Y is regressed on the indicator variable T and the estimated propensity score.

Weighted Regression

| Data | Run Time | ATE Est |
|---------------|----------|-----------|
| Low dim data | 0.136385 | 2.523955 |
| High dim data | 0.685741 | -2.964053 |

Weighted least square estimation of the regression function:

 $Y_i = \alpha_0 + \tau * T_i + \alpha'_1 * T_i + \alpha'_2 * (Z_i - \bar{Z}) * T_i + \varepsilon_i$

The outcome variable Y was regressed on the selected covariates Z.

To select Z, estimated K linear regressions of the type: $Y_i = \beta_0 + \beta_{k1} * T_i + \beta_{k2} * X_{ik} + \varepsilon_i$, where K is the total number of covariates.

Calculate the t-statistic for the test of the null hypothesis that the slope coefficient $\beta_{k2} = 0$ in each of these regressions, selected for Z all the covariates with a t-statistics larger in absolute value than $t_{reg} = 2$.

The estimated coefficient of the indicator variable T $\hat{\tau}$ is the estimated ATE.



Evaluation method:

Absolute error and relative error to evaluate the ATE estimates

Absolute error(ABSE) is given by:

$$ABSE = |ATE_{EST} - ATE_{True}|$$

Relative error is given by:

$$RE = |ATE_{EST} - ATE_{True}| / |ATE_{EST}|$$



Evaluation and Comparison

| Low dim data | | | | |
|--------------------------|-------------|----------|--------------|-------------------|
| Algorithm | Run Time | ATE Est | Abs Error | Relative Error |
| Stratification | 0.030781 | 2.879272 | 0.38 | 15.17% |
| Regression Adjustment | 0.033268 | 2.502966 | 0.00 | 0.12% |
| Weighted Regression | 0.136385 | 2.523955 | 0.02 | 0.96% |

| | High o | dim data | | |
|--------------------------|----------|-----------|--------------|-------------------|
| Algorithm | Run Time | ATE Est | Abs Error | Relative Error |
| Stratification | 0.013375 | -2.783537 | 0.22 | 7.22% |
| Regression Adjustment | 0.130357 | -2.999067 | 0.00 | 0.03% |
| Weighted Regression | 0.685741 | -2.964053 | 0.04 | 1.20% |

Conclusion:

Regression Adjustment is the best method for both low dimensional data and high

dimensional data.

• Stratification has a fastest run time among the three Algorithms.

Reference

- Austin, Peter C. 2011. "An Introduction to Propensity Score Methods for Reducing the Effects of Confounding in Observational Studies." Multivariate Behavioral Research 46 (3): 399–424.
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Thank you!