Project 4: Causal Inference Algorithms Evaluation

Group 4: Zhenglei Chen, Jaival Desai, Qinzhe Hu, Levi Lee, Luyao Sun, Xinyi Wei

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Setup

First, we set working directories, install required libraries and import the data.

```
setwd("C:/Users/Charlie/Documents/GitHub/Fall2020-Project4-group-4")
```

```
packages.used <- c("dplyr", "ggplot2", "WeightedROC", "rpart", "rpart.plot")

# check packages that need to be installed.
packages.needed <- setdiff(packages.used, intersect(installed.packages()[,1], packages.used))

# install additional packages
if(length(packages.needed) > 0){
  install.packages(packages.needed, dependencies = TRUE)
}

library(dplyr)
```

```
## Warning: package 'dplyr' was built under R version 3.6.3

##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 3.6.3
```

```
library(WeightedROC)
```

```
## Warning: package 'WeightedROC' was built under R version 3.6.3
```

```r
library(rpart)
library(rpart.plot)
```

```
## Warning: package 'rpart.plot' was built under R version 3.6.3
```

```r
library(base)
```

```r
df_high <- read.csv("../data/highDim_dataset.csv")
df_low <- read.csv("../data/lowDim_dataset.csv")
```

Introduction

Description

About the Data

Description

Background: Trees

Description

Cross-Validation

Description

Step 1: Set Controls and Establish Hyperparameters

Description

```r
K <- 5  # number of CV folds
sample.reweight <- TRUE # run sample reweighting in model training

# setting the following to false loads data generated from a previous run
# this data is the same in each run due to a set seed

run.cv.trees_high <- FALSE # run cross-validation on the training set for trees on high dim data

run.cv.trees_low <- FALSE # run cross-validation on the training set for trees on low dim data
```

Description

```r
# hyperparameters for trees
hyper_grid_trees <- expand.grid(
  cp = c(2^(0), 2^(-1), 2^(-2), 2^(-3), 2^(-4),
         2^(-5), 2^(-6), 2^(-7), 2^(-8), 2^(-9),
         2^(-10), 2^(-11), 2^(-12), 2^(-13), 2^(-14),
         2^(-15), 2^(-16), 2^(-17), 0, -2^(0))
)
```

Step 2: Cross-Validate the Hyperparameters

Description

Description

```
# features are the predictors: V1 - Vp
# column 1 is the response Y
# column 2 is the treatment A

feature_train_high = df_high[, -1:-2]
label_train_high = df_high[, 2]

feature_train_low = df_low[, -1:-2]
label_train_low = df_low[, 2]
```

High Dimensional Data *Description*

```
set.seed(5243)

if(run.cv.trees_high){
  res_cv_trees_high <- matrix(0, nrow = nrow(hyper_grid_trees), ncol = 4)
  for(i in 1:nrow(hyper_grid_trees)){
    cat("complexity = ", hyper_grid_trees$cp[i], "\n", sep = "")
    res_cv_trees_high[i,] <- cv.function(features = feature_train_high, labels = label_train_high,
                                         cp = hyper_grid_trees$cp[i],
                                         K, reweight = sample.reweight)
  save(res_cv_trees_high, file = "../output/res_cv_trees_high.RData")
  }
}else{
  load("../output/res_cv_trees_high.RData")
}
```

Low Dimensional Data *Description*

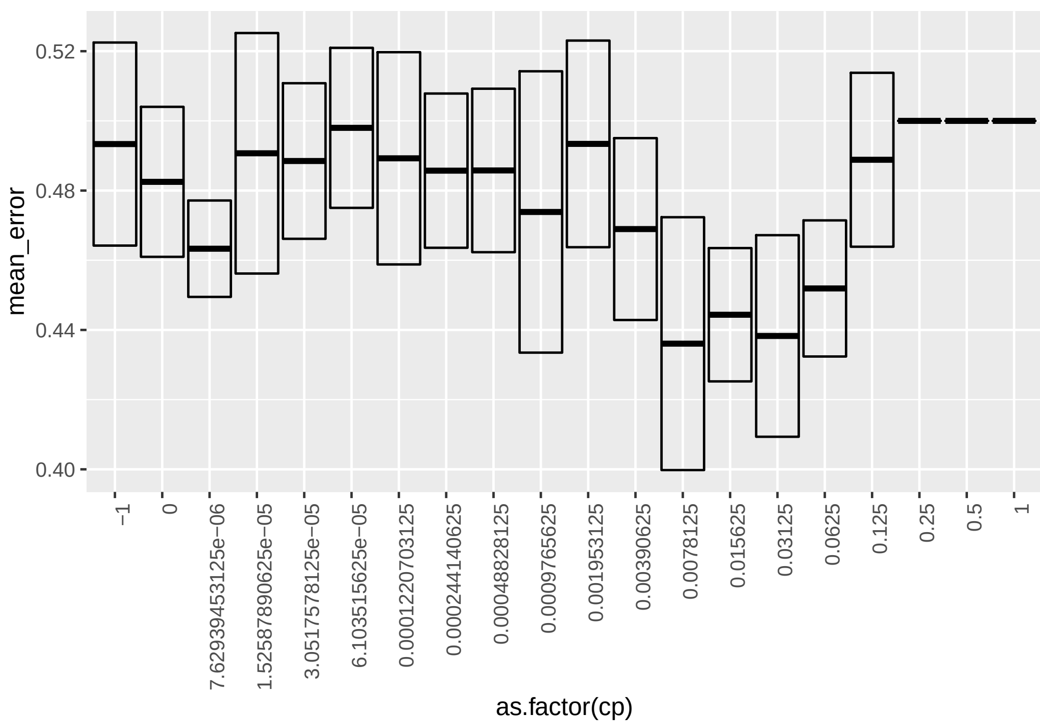
```
set.seed(5243)

if(run.cv.trees_low){
  res_cv_trees_low <- matrix(0, nrow = nrow(hyper_grid_trees), ncol = 4)
  for(i in 1:nrow(hyper_grid_trees)){
    cat("complexity = ", hyper_grid_trees$cp[i], "\n", sep = "")
    res_cv_trees_low[i,] <- cv.function(features = feature_train_low, labels = label_train_low,
                                        cp = hyper_grid_trees$cp[i],
                                        K, reweight = sample.reweight)
  save(res_cv_trees_low, file="../output/res_cv_trees_low.RData")
  }
}else{
  load("../output/res_cv_trees_low.RData")
}
```

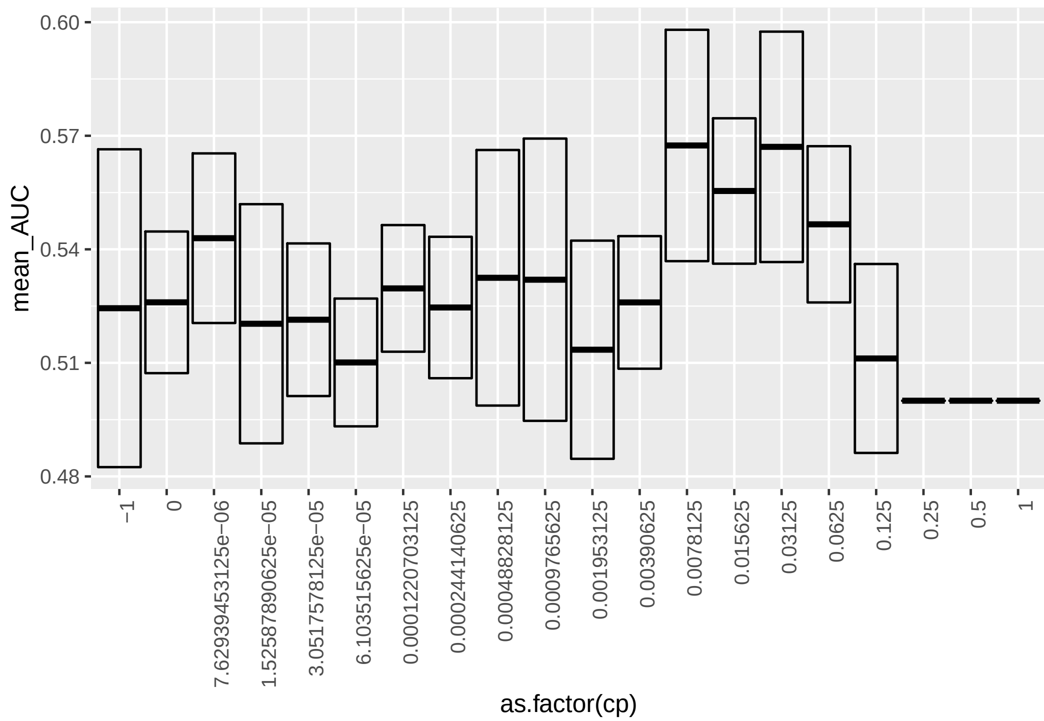
Step 3: Visualize CV Error and AUC

Description

```
# create data frame to organize results
res_cv_trees_high <- as.data.frame(res_cv_trees_high)
colnames(res_cv_trees_high) <- c("mean_error", "sd_error", "mean_AUC", "sd_AUC")
cv_results_trees_high = data.frame(hyper_grid_trees, res_cv_trees_high)

# look at top 5 models with highest AUC
cv_results_trees_high[order(cv_results_trees_high$mean_AUC, decreasing = TRUE), ][1:5, ]
```

```
##              cp mean_error   sd_error  mean_AUC     sd_AUC
## 8  7.812500e-03  0.4360668 0.03628091 0.5674343 0.03056252
## 6  3.125000e-02  0.4382665 0.02892494 0.5670732 0.03043522
## 7  1.562500e-02  0.4443627 0.01912652 0.5554088 0.01920727
## 5  6.250000e-02  0.4519064 0.01953579 0.5466031 0.02064263
## 18 7.629395e-06  0.4633120 0.01383644 0.5429351 0.02240466
```





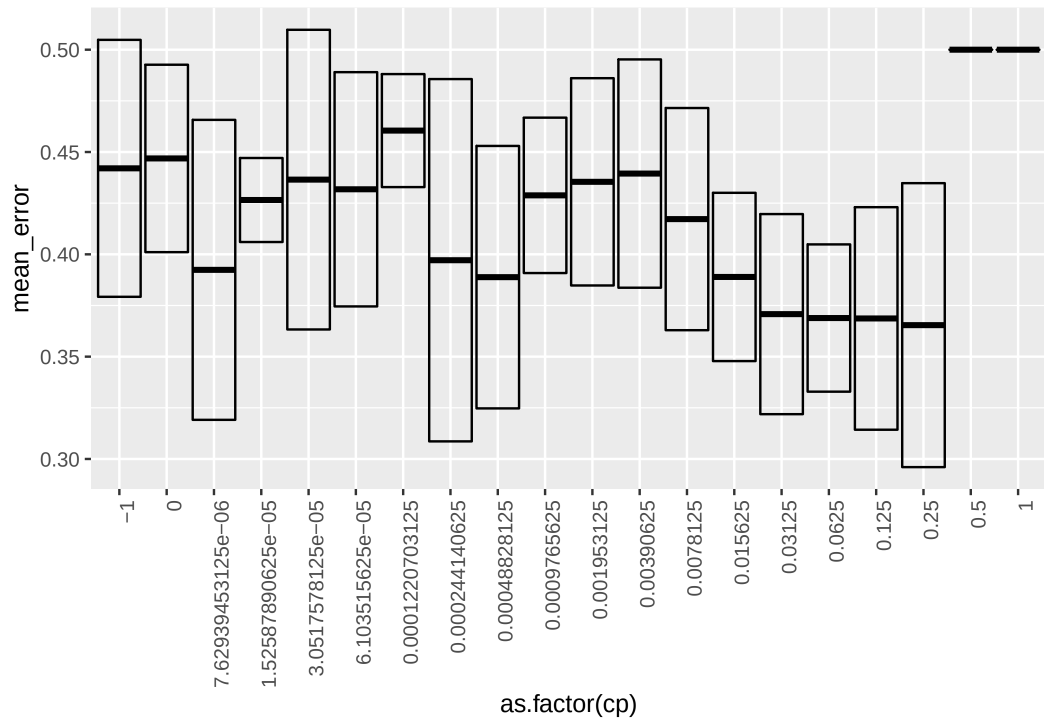

```r
best_cp_high <- cv_results_trees_high$cp[cv_results_trees_high$mean_AUC ==
                                           max(cv_results_trees_high$mean_AUC)]

best_cp_high
```

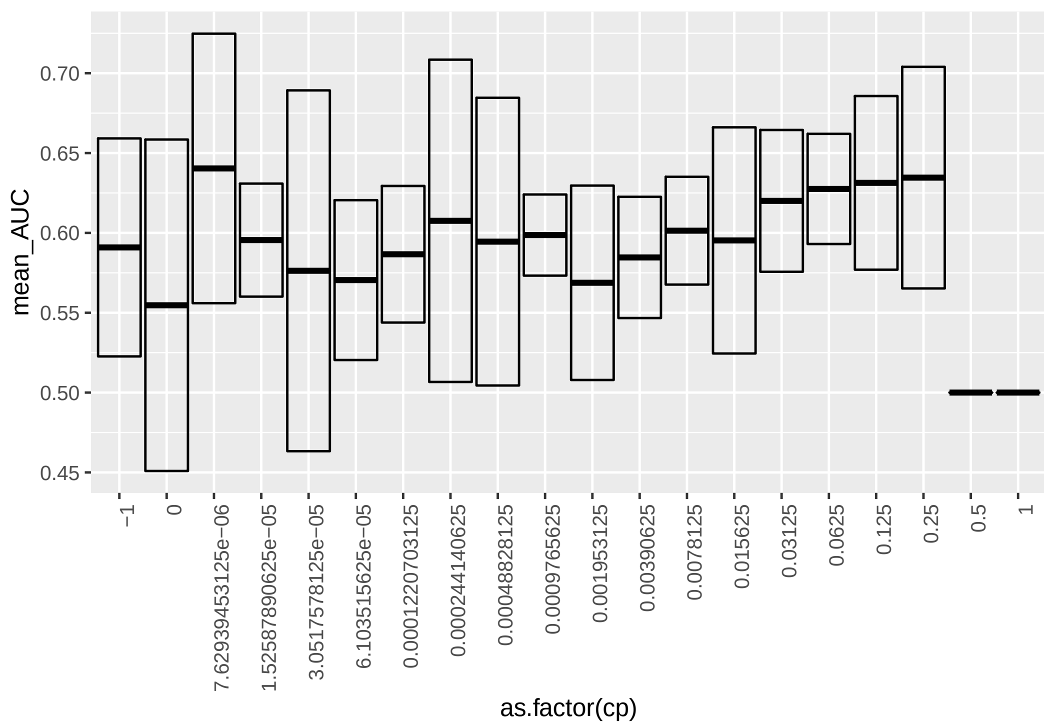
```
## [1] 0.0078125
```

Low Dimensional Data *Description*

```r
# create data frame to organize results
res_cv_trees_low <- as.data.frame(res_cv_trees_low)
colnames(res_cv_trees_low) <- c("mean_error", "sd_error", "mean_AUC", "sd_AUC")
cv_results_trees_low = data.frame(hyper_grid_trees, res_cv_trees_low)

# look at top 5 models with lowest AUC
cv_results_trees_low[order(cv_results_trees_low$mean_AUC, decreasing = TRUE), ][1:5, ]
```

```
##               cp mean_error   sd_error  mean_AUC     sd_AUC
## 18 7.629395e-06  0.3924126 0.07329778 0.6403787 0.08438962
## 3  2.500000e-01  0.3653965 0.06938198 0.6346035 0.06938198
## 4  1.250000e-01  0.3686625 0.05437925 0.6313375 0.05437925
## 5  6.250000e-02  0.3688654 0.03598451 0.6275346 0.03449488
## 6  3.125000e-02  0.3707763 0.04889355 0.6200507 0.04439815
```






```
best_cp_low <- cv_results_trees_low$cp[cv_results_trees_low$mean_AUC ==
                                         max(cv_results_trees_low$mean_AUC)]

best_cp_low
```

```
## [1] 7.629395e-06
```

Propensity Score Estimation

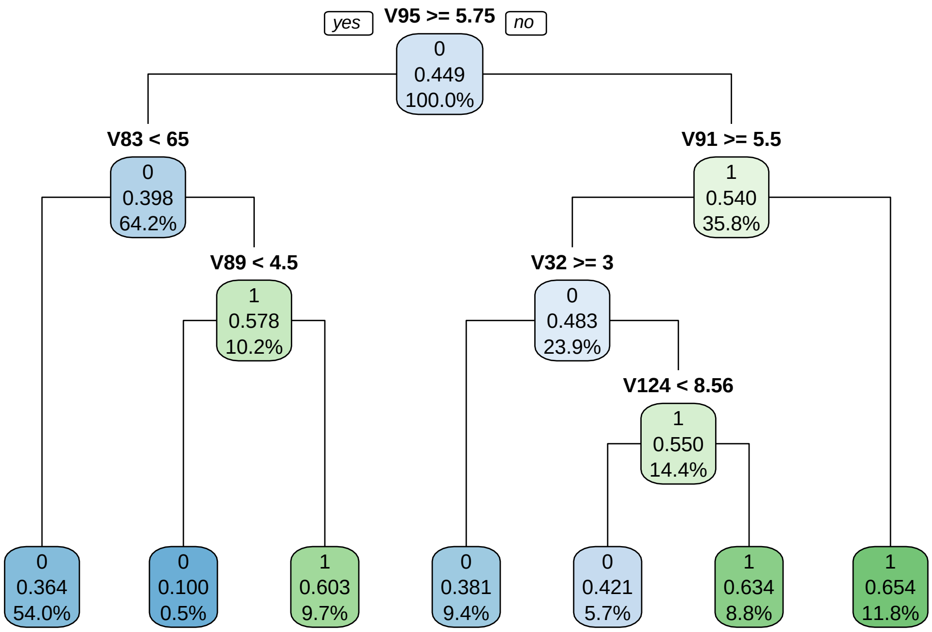
Description

```r
# imbalanced dataset requires weights
# to be used in the trained model

weights_high <- rep(NA, length(df_high$A))
for (v in unique(df_high$A)){
  weights_high[df_high$A == v] = 0.5 * length(df_high$A) / length(df_high$A[df_high$A == v])
}


weights_low <- rep(NA, length(df_low$A))
for (v in unique(df_low$A)){
  weights_low[df_low$A == v] = 0.5 * length(df_low$A) / length(df_low$A[df_low$A == v])
}
```

```r
start.time_propensity_score_high <- Sys.time()

# create tree model for high dimensional data with best cp parameter
tree_high <- rpart(A ~ . - Y, method = "class", data = df_high, cp = best_cp_high)

# calculate propensity scores
prop_score_high <- predict(tree_high, newdata = df_high[, -2], type = "prob")[, 2]

end.time_propensity_score_high <- Sys.time()
time_propensity_score_high <- end.time_propensity_score_high - start.time_propensity_score_high
time_propensity_score_high
```

High Dimensional Data

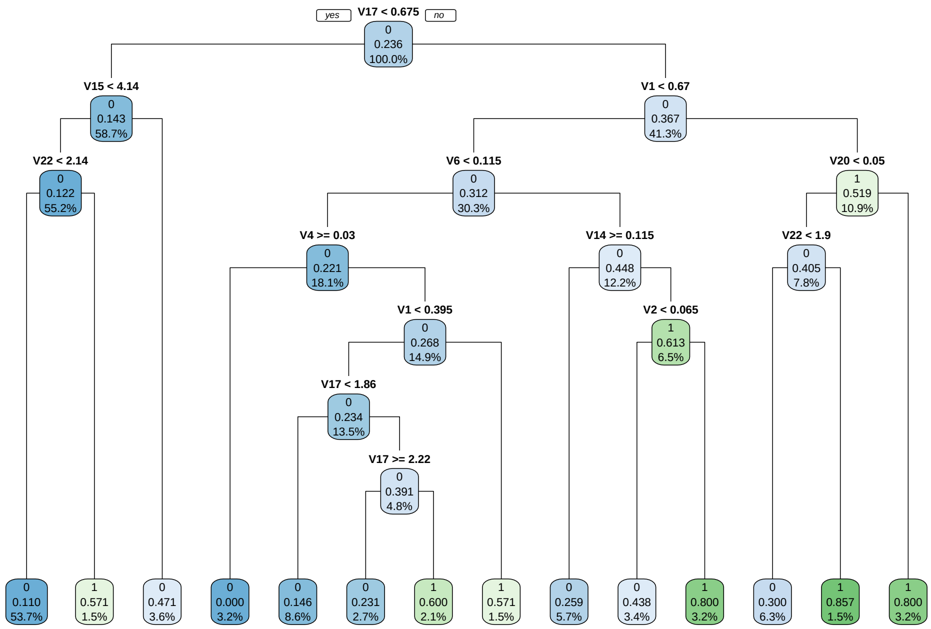
```
## Time difference of 0.6233349 secs
```




```
start.time_propensity_score_low <- Sys.time()

# create tree model for low dimensional data with best cp parameter
tree_low <- rpart(A ~ . - Y, method = "class", data = df_low, cp = best_cp_low)

# calculate propensity scores
prop_score_low <- predict(tree_low, newdata = df_low[, -2], type = "prob")[, 2]

end.time_propensity_score_low <- Sys.time()
time_propensity_score_low <- end.time_propensity_score_low - start.time_propensity_score_low
time_propensity_score_low
```

Low Dimensional Data

```
## Time difference of 0.02792501 secs
```



ATE Estimation

Description

Stratification

Description

```
K = 5
quintiles <- seq(0, 1, by = 1/K)
```

High Dimensional Data *Description*

```
start.time_stratification_high <- Sys.time()

df_high <- cbind(df_high, prop_score_high)
quintile_values_high <- rep(NA, length(quintiles))

for (i in 1:length(quintiles)){
  quintile_values_high[i] <- quantile(prop_score_high, quintiles[i])
}

# values of quintiles for high data
quintile_values_high
```

```
## [1] 0.1000000 0.3635523 0.3635523 0.3809524 0.6342857 0.6540084
```

```r
df_high$quintile_class_high <- rep(NA, nrow(df_high))

# assign quintile class to each observation
for (i in 1:nrow(df_high)){
  if ((quintile_values_high[1] <= df_high$prop_score_high[i]) &
      (df_high$prop_score_high[i] < quintile_values_high[2])) {
    df_high$quintile_class_high[i] <- 1
  } else if ((quintile_values_high[2] <= df_high$prop_score_high[i]) &
             (df_high$prop_score_high[i] < quintile_values_high[3])) {
    df_high$quintile_class_high[i] <- 2
  } else if ((quintile_values_high[3] <= df_high$prop_score_high[i]) &
             (df_high$prop_score_high[i] < quintile_values_high[4])) {
    df_high$quintile_class_high[i] <- 3
  } else if ((quintile_values_high[4] <= df_high$prop_score_high[i]) &
             (df_high$prop_score_high[i] < quintile_values_high[5])) {
    df_high$quintile_class_high[i] <- 4
  } else if ((quintile_values_high[5] <= df_high$prop_score_high[i]) &
             (df_high$prop_score_high[i] <= quintile_values_high[6])) {
    df_high$quintile_class_high[i] <- 5
  }
}

summary_high = expand.grid(
  A = c(0, 1),
  quintile = c(1, 2, 3, 4, 5),
  n = NA,
  prop = NA,
  avg_y = NA
)

for (i in 1:nrow(summary_high)) {
  subset <- df_high[(df_high$A == summary_high$A[i]) & (df_high$quintile_class_high == summary_high$qui
  summary_high$n[i] = nrow(subset)
  summary_high$prop[i] = summary_high$n[i]/nrow(df_high)
  summary_high$avg_y[i] = mean(subset$Y)
}


for (i in 1:nrow(summary_high)) {
  if (is.nan(summary_high$avg_y[i]) == TRUE) {
    summary_high$avg_y[i] <- 0
  }
}

# this table records the mean response in each quintile; needed for stratification
summary_high
```

```
##    A quintile   n   prop      avg_y
## 1  0        1   9 0.0045  -1.556754
## 2  1        1   1 0.0005   3.448809
## 3  0        2   0 0.0000   0.000000
## 4  1        2   0 0.0000   0.000000
## 5  0        3 688 0.3440 -13.637227
```

```
## 6   1         3 393 0.1965 -16.140803
## 7   0         4 260 0.1300 -10.267325
## 8   1         4 237 0.1185 -10.349496
## 9   0         5 146 0.0730  -6.152075
## 10  1         5 266 0.1330  -8.714239
```

```r
quntile_prop_high <- summary_high %>% group_by(quintile) %>% summarise(sum = sum(n)/nrow(df_high))
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```r
# this table records the proportions for each quintile; also needed for stratification
quntile_prop_high
```

```
## # A tibble: 5 x 2
##   quintile   sum
##      <dbl> <dbl>
## 1        1 0.005
## 2        2 0
## 3        3 0.540
## 4        4 0.248
## 5        5 0.206
```

```r
ATE_stratification_high = quntile_prop_high$sum[1]*(summary_high$avg_y[2] - summary_high$avg_y[1]) +
  quntile_prop_high$sum[2]*(summary_high$avg_y[4] - summary_high$avg_y[3]) +
  quntile_prop_high$sum[3]*(summary_high$avg_y[6] - summary_high$avg_y[5]) +
  quntile_prop_high$sum[4]*(summary_high$avg_y[8] - summary_high$avg_y[7]) +
  quntile_prop_high$sum[5]*(summary_high$avg_y[10] - summary_high$avg_y[9])


ATE_stratification_high
```

```
## [1] -1.87638
```

```r
end.time_stratification_high <- Sys.time()
time_stratification_high <- end.time_stratification_high - start.time_stratification_high
time_stratification_high
```

```
## Time difference of 0.27125 secs
```

Low Dimensional Data *Description*

```r
start.time_stratification_low <- Sys.time()

df_low <- cbind(df_low, prop_score_low)
quintile_values_low <- rep(NA, length(quintiles))

for (i in 1:length(quintiles)){
  quintile_values_low[i] <- quantile(prop_score_low, quintiles[i])
}

# values of quintiles for low data
quintile_values_low
```

```
## [1] 0.0000000 0.1098039 0.1098039 0.1463415 0.3000000 0.8571429
```

```r
df_low$quintile_class_low <- rep(NA, nrow(df_low))

# assign quintile class to each observation
for (i in 1:nrow(df_low)){
  if ((quintile_values_low[1] <= df_low$prop_score_low[i]) &
      (df_low$prop_score_low[i] < quintile_values_low[2])) {
    df_low$quintile_class_low[i] <- 1
  } else if ((quintile_values_low[2] <= df_low$prop_score_low[i]) &
             (df_low$prop_score_low[i] < quintile_values_low[3])) {
    df_low$quintile_class_low[i] <- 2
  } else if ((quintile_values_low[3] <= df_low$prop_score_low[i]) &
             (df_low$prop_score_low[i] < quintile_values_low[4])) {
    df_low$quintile_class_low[i] <- 3
  } else if ((quintile_values_low[4] <= df_low$prop_score_low[i]) &
             (df_low$prop_score_low[i] < quintile_values_low[5])) {
    df_low$quintile_class_low[i] <- 4
  } else if ((quintile_values_low[5] <= df_low$prop_score_low[i]) &
             (df_low$prop_score_low[i] <= quintile_values_low[6])) {
    df_low$quintile_class_low[i] <- 5
  }
}


summary_low = expand.grid(
  A = c(0, 1),
  quintile = c(1, 2, 3, 4, 5),
  n = NA,
  prop = NA,
  avg_y = NA
)

for (i in 1:nrow(summary_low)) {
  subset <- df_low[(df_low$A == summary_low$A[i]) &
                     (df_low$quintile_class_low == summary_low$quintile[i]), ]
  summary_low$n[i] = nrow(subset)
  summary_low$prop[i] = summary_low$n[i]/nrow(df_low)
  summary_low$avg_y[i] = mean(subset$Y)
}


for (i in 1:nrow(summary_low)) {
  if (is.nan(summary_low$avg_y[i]) == TRUE) {
    summary_low$avg_y[i] <- 0
  }
}

# this table records the mean response in each quintile; needed for stratification
summary_low
```

```
##    A quintile  n       prop    avg_y
## 1  0        1 15 0.03157895 18.25654
## 2  1        1  0 0.00000000  0.00000
```

```
## 3   0          2    0 0.00000000  0.00000
## 4   1          2    0 0.00000000  0.00000
## 5   0          3  227 0.47789474 15.06273
## 6   1          3   28 0.05894737 18.56302
## 7   0          4   65 0.13684211 17.99996
## 8   1          4   16 0.03368421 20.74083
## 9   0          5   56 0.11789474 19.23768
## 10 1          5   68 0.14315789 22.65577
```

```r
quntile_prop_low <- summary_low %>% group_by(quintile) %>% summarise(sum = sum(n)/nrow(df_low))
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```r
# this table records the proportions for each quintile; also needed for stratification
quntile_prop_low
```

```
## # A tibble: 5 x 2
##   quintile    sum
##      <dbl>  <dbl>
## 1        1 0.0316
## 2        2 0
## 3        3 0.537
## 4        4 0.171
## 5        5 0.261
```

```r
ATE_stratification_low = quntile_prop_low$sum[1]*(summary_low$avg_y[2] - summary_low$avg_y[1]) +
  quntile_prop_low$sum[2]*(summary_low$avg_y[4] - summary_low$avg_y[3]) +
  quntile_prop_low$sum[3]*(summary_low$avg_y[6] - summary_low$avg_y[5]) +
  quntile_prop_low$sum[4]*(summary_low$avg_y[8] - summary_low$avg_y[7]) +
  quntile_prop_low$sum[5]*(summary_low$avg_y[10] - summary_low$avg_y[9])

ATE_stratification_low
```

```
## [1] 2.662275
```

```r
end.time_stratification_low <- Sys.time()
time_stratification_low <- end.time_stratification_low - start.time_stratification_low
time_stratification_low
```

```
## Time difference of 0.06584597 secs
```

Regression Adjustment

Description

High Dimensional Data

Description

```r
start.time_regression_adjustment_high <- Sys.time()

ps_RA_high <- predict(tree_high, df_high, type = "prob")
high_data_ps <- cbind(ps_RA_high, df_high)
pred_high <- lm(Y ~ A + ps_RA_high, data = high_data_ps)
summary(pred_high)
```

```
##
## Call:
## lm(formula = Y ~ A + ps_RA_high, data = high_data_ps)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -17.4713  -3.4878  -0.6694   2.7522  30.0897
##
## Coefficients: (1 not defined because of singularities)
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   5.9564     0.6207   9.596   <2e-16 ***
## A            -2.5271     0.2569  -9.836   <2e-16 ***
## ps_RA_high0 -30.5718     1.0322 -29.617   <2e-16 ***
## ps_RA_high1       NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.535 on 1997 degrees of freedom
## Multiple R-squared:  0.3068, Adjusted R-squared:  0.3061
## F-statistic: 441.8 on 2 and 1997 DF,  p-value: < 2.2e-16
```

```r
ATE_regression_adjustment_high = pred_high$coefficients[2]
ATE_regression_adjustment_high
```

```
##         A
## -2.527116
```

```r
end.time_regression_adjustment_high <- Sys.time()
time_regression_adjustment_high <- end.time_regression_adjustment_high - start.time_regression_adjustme
time_regression_adjustment_high
```

```
## Time difference of 0.02493596 secs
```

Low Dimensional Data *Description*

```r
start.time_regression_adjustment_low <- Sys.time()

ps_RA_low <- predict(tree_low, df_low, type = "prob")
low_data_ps <- cbind(ps_RA_low, df_low)
pred_low <- lm(Y ~ A + ps_RA_low, data = low_data_ps)
summary(pred_low)
```

```
##
```

```
## Call:
## lm(formula = Y ~ A + ps_RA_low, data = low_data_ps)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.5153  -2.6807  -0.5655   1.8184  26.9282
##
## Coefficients: (1 not defined because of singularities)
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  22.5693     0.8535  26.445  < 2e-16 ***
## A             3.0532     0.5089   5.999 3.95e-09 ***
## ps_RA_low0   -7.5200     1.0017  -7.507 3.04e-13 ***
## ps_RA_low1        NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.055 on 472 degrees of freedom
## Multiple R-squared:  0.2829, Adjusted R-squared:  0.2798
## F-statistic: 93.08 on 2 and 472 DF,  p-value: < 2.2e-16
```

```
ATE_regression_adjustment_low = pred_low$coefficients[2]
ATE_regression_adjustment_low
```

```
##       A
## 3.05324
```

```
end.time_regression_adjustment_low <- Sys.time()
time_regression_adjustment_low <- end.time_regression_adjustment_low - start.time_regression_adjustment_
time_regression_adjustment_low
```

```
## Time difference of 0.009945154 secs
```

Stratification and Regression Adjustment

Description

High Dimensional Data

Description

```
start.time_stratification_regression_adjustment_high <- Sys.time()

lm_beta_high <- rep(NA, K)

for (i in 1:K){
  subset <- df_high[df_high$quintile_class_high == i, ]

  if (nrow(subset) == 0) {
    # if the quintile is empty, let the coefficient for A automatically be 0
    lm_beta_high[i] <- 0
  } else if (sum(subset$prop_score_high) == 0) {
    # if the propensity scores in the quintile are all 0, let the coefficient for A automatically be 0
    lm_beta_low[i] <- 0
```

```
  } else {
    # otherwise, run a linear model on the subset
    lm <- lm(Y ~ A + prop_score_high, data = subset)
    lm_beta_high[i] <- as.numeric(lm$coefficients[2])
  }
}

lm_beta_high
```

```
## [1]  5.005563  0.000000 -2.503576 -2.516297 -2.675199
```

```
ATE_stratification_regression_adjustment_high <- quntile_prop_high$sum[1]*lm_beta_high[1] +
  quntile_prop_high$sum[2]*lm_beta_high[2] +
  quntile_prop_high$sum[3]*lm_beta_high[3] +
  quntile_prop_high$sum[4]*lm_beta_high[4] +
  quntile_prop_high$sum[5]*lm_beta_high[5]


ATE_stratification_regression_adjustment_high
```

```
## [1] -2.504545
```

```
end.time_stratification_regression_adjustment_high <- Sys.time()
time_stratification_regression_adjustment_high <- end.time_stratification_regression_adjustment_high - s
time_stratification_regression_adjustment_high
```

```
## Time difference of 0.02393603 secs
```

Low Dimensional Data *Description*

```
start.time_stratification_regression_adjustment_low <- Sys.time()

lm_beta_low <- rep(NA, 5)

for (i in 1:K){
  subset <- df_low[df_low$quintile_class_low == i, ]

  if (nrow(subset) == 0) {
    # if the quintile is empty, let the coefficient for A automatically be 0
    lm_beta_low[i] <- 0
  } else if (sum(subset$prop_score_low) == 0) {
    # if the propensity scores in the quintile are all 0, let the coefficient for A automatically be 0
    lm_beta_low[i] <- 0
  } else {
    # otherwise, run a linear model on the subset
    lm <- lm(Y ~ A + prop_score_low, data = subset)
    lm_beta_low[i] <- as.numeric(lm$coefficients[2])
  }
}

lm_beta_low
```

```
## [1] 0.000000 0.000000 3.500291 2.452154 2.916085
```

```
ATE_stratification_regression_adjustment_low <- quntile_prop_low$sum[1]*lm_beta_low[1] +
  quntile_prop_low$sum[2]*lm_beta_low[2] +
  quntile_prop_low$sum[3]*lm_beta_low[3] +
  quntile_prop_low$sum[4]*lm_beta_low[4] +
  quntile_prop_low$sum[5]*lm_beta_low[5]

ATE_stratification_regression_adjustment_low
```

```
## [1] 3.058512
```

```
end.time_stratification_regression_adjustment_low <- Sys.time()
time_stratification_regression_adjustment_low <- end.time_stratification_regression_adjustment_low - sta
time_stratification_regression_adjustment_low
```

```
## Time difference of 0.01795292 secs
```

Results

Insert Comparison of ATE and all Runtimes Here 8 Runtime values 6 ATE estimations + 2 true ATE create table and analyze results

#Runtime results

```
time <- matrix(c(time_propensity_score_high, time_stratification_high,
                 time_regression_adjustment_high, time_stratification_regression_adjustment_high,
                 time_propensity_score_low, time_stratification_low, time_regression_adjustment_low,
                 time_stratification_regression_adjustment_low), ncol = 2, byrow = F)
colnames(time) <- c("High Dimensional Data", "Low Dimensional Data")
rownames(time) <- c("Propensity Score Estimation", "Stratification", "Regression Adjustment",
                    "Stratification + Regression Adjustment")
time <- as.table(time)
time
```

```
##                                        High Dimensional Data
## Propensity Score Estimation                      0.623334885
## Stratification                                   0.271250010
## Regression Adjustment                            0.024935961
## Stratification + Regression Adjustment           0.023936033
##                                        Low Dimensional Data
## Propensity Score Estimation                     0.027925014
## Stratification                                  0.065845966
## Regression Adjustment                           0.009945154
## Stratification + Regression Adjustment          0.017952919
```

#ATE Results

```
ATE_true_high <- -3
ATE_true_low <- 2.5
ATE <- matrix(c(ATE_true_high, ATE_stratification_high, ATE_regression_adjustment_high,
```

```r
                ATE_stratification_regression_adjustment_high, ATE_true_low, ATE_stratification_low,
                ATE_regression_adjustment_low, ATE_stratification_regression_adjustment_low), ncol = 2,
colnames(ATE) <- c("High Dimensional Data", "Low Dimensional Data")
rownames(ATE) <- c("True", "Stratification", "Regression Adjustment",
                   "Stratification + Regression Adjustment")
ATE <- as.table(ATE)
ATE
```

```
##                                        High Dimensional Data
## True                                               -3.000000
## Stratification                                     -1.876380
## Regression Adjustment                              -2.527116
## Stratification + Regression Adjustment             -2.504545
##                                        Low Dimensional Data
## True                                               2.500000
## Stratification                                     2.662275
## Regression Adjustment                              3.053240
## Stratification + Regression Adjustment             3.058512
```

Conclusion

Runtimes of propensity Score Estimation for high dimensional data is 0.704 seconds, while for low dimensional data, it's 0.095 seconds. Among three methods, Stratification + Regression has the shortest runtimes: 0.284 seconds for high dimensinal data, and 0.335 seconds for low dimensional data.

By comparing ATE from three methods with the true ATE, we concluded that regression adjustment is the best method which estimates ATE for high dimensional data and low dimensional data that are the closest to the true ATEs. *Description*

References

Description