ALS+R1R3+KRR

Group 4

Model 2:Alternating Least Squares + Penalty of magnitudes + Temporal Dynamics + Kernel Ridge Regression

Step 1. Load Data and Train-Test Split

In this step, we loaded the data and added variable timediff for temporal dynamics, which is the difference between the user's rating time and the average rating time of this user. Then we splitted the original dataset to training set(80%) and test set(20%)

```
data <- read.csv("../data/ml-latest-small/ratings.csv")
# Convert timestamp and set up time difference
data$timestamp <- anydate(data$timestamp)
data <- data %>%
  group_by(userId) %>%
  mutate(timediff = (timestamp - mean(timestamp)) %>% as.numeric) %>%
  ungroup() %>%
  arrange(timestamp)

set.seed(1)

# train-test split (.8/.2)
train_ind <- createDataPartition(data$userId, p=.8, list=F)


data_train <- data[train_ind, ]
data_test <- data[-train_ind, ]
```

Step 2. Model Setup

1. Equations recap

Alternating Least Squares

$$min_{q^{\star}p^{\star}} \sum_{(u,i)\in K} (r_u i - q_i^T p_u)^2 + \lambda(\sum_i n_{q_i}\|q_i\|^2 + \sum_i n_{p_u}\|p_u\|^2)$$

ALS technique rotate between fixing the q_i's and fixing the p_u's. When all pu's are fixed, system recomputes the q_i's by solving a least-squares problem, and vice versa. This ensures that each step decreases object function until convergence.

f: dimension of latent factors

q_i: factors associated with item i , measures the extent to which items possesses those factors

p_u: factors associated with user u, measures the extent of interest that user has in an item are high on corresponding factors.

Regularizations

- Penalty of magnitudes:

$$\sum_{(u,i)\in K} \lambda(\|q_i\|^2 + \|p_u\|^2)$$

The constant lambda controls the extent of regularization and we determined it through cross validation.

- Temporal Dynamics

$$\hat{r}_{ui} = q_i^T p_u + \mu + b_u + b_i + \alpha_u dev_u(t)$$

The $dev_u(t)$ is the related to the difference between rating time of user and their average rating time, measured by

$$dev_u(t) = sign(t - t_u)\mathring{u}|t - t_u|^\beta$$

The final objective function of our model 2 is

$$min_{q,p} \sum_{(u,i)\in K} [r_{ui} - (q_i^T p_u + \mu + b_u + b_i + \alpha_u dev_u(t)]^2 + \lambda \sum (\|q_i\|^2 + \|p_u\|^2 + b_i^2 + b_u^2 + \alpha_u^2)$$

Post-processing

Kernel Ridge Regression:

$$\hat{\beta} = (X^T X + \lambda I)^{-1} X^T y$$
$$\hat{y_i} = {x_i}^T \hat{\beta}$$

Equilvalent to:

$$\hat{\beta} = X^T (X X^T + \lambda I)^{-1} y$$

$$\hat{y}_i = K(x_i^T, X)(K(X, X) + \lambda I)^{-1} y$$

y: vector of movies rated by user i X: a matrix of observations - each row of X is normalized vector of features of one movie j rated by user i:

$$x_{j_2} = \frac{q_j}{\|q_j\|}$$

K:

$$K(x_i^T, X) = x_i^T x_j$$
$$K(x_i^T, X) = \exp(2(x_i^T x_j - 1))$$

λ:

$$\lambda = 0.5$$

2. Algorithm Setup

For matrix factorization, with the method in paper4, to include b_u, b_i, and a_u in the iterations, we add one row to the movie matrix and two rows to the user matrix. To speed up the algorithm, we use parallel package to parallelize computations and tibble in the factorization function.

3. Parameter Tuning

To save the time, with the refult of parameters tuning of ALS and KRR from the former model and some attempts, we only tested the following parameters here. lambda_als = 1,5,10 beta = 0.2,0.4,0.5 And set f = 15 lambda_p = 1 sigmas = 1

```
fs <- c(15)
lambdas_P <- c(1)
sigmas = c(1)


lambdas_als <- c(1, 5,10)
betas <- c(0.2,0.4,0.5)

cv.df <- expand.grid("fs"=fs,
                     "lambdas_als"=lambdas_als,
                     "lambdas_p"=lambdas_P,
                     "sigmas" =sigmas,
                     "betas"=betas)
```

The code of parameter tuning

```
source("../lib/cross_validation.R")
source("../lib/ALS_R_KRR.R")
source("../lib/ALSwithR1R3.R")
source("../lib/ALSwithP3.R")

set.seed(1)
# cl <- makeCluster(4)
#
# arp_result=cv.df %>%
#   as_tibble() %>%
#   mutate(train_mean=NA,test_mean=NA)
#
# for (i in 1:dim(arp_result)[1]){
#
#   tmp <- cv.functionA3R3P3(dat_train=data,
#                         K=4,
#                         f=x[i,]$fs,
#                         maxIter=15,
#                         lambdas_als=x[i,]$lambdas_als,
#                         lambdas_p=x[i,]$lambdas_p,
#                         sigmas=x[i,]$sigmas,
#                         betas=x[i,]$betas)
#
#   arp_result[i,"train_mean"]=tmp$mean_train_rmse
#   arp_result[i,"test_mean"]=tmp$mean_test_rmse
#
#   rm(tmp)
# }
#
# save(arp_result, file="../output/cvARP.RData")
#
# stopCluster(cl)

load(file="../output/cvARP.RData")
arp_result
```

```
## # A tibble: 9 x 7
##      fs lambdas_als lambdas_p sigmas betas train_mean test_mean
```

```
##   <dbl>       <dbl>     <dbl>  <dbl> <dbl>      <dbl>     <dbl>
## 1    15           1         1      1   0.2      0.497      1.25
## 2    15           5         1      1   0.2      0.469      1.32
## 3    15          10         1      1   0.2      0.485      1.30
## 4    15           1         1      1   0.4      0.889      1.33
## 5    15           5         1      1   0.4      0.929      1.44
## 6    15          10         1      1   0.4      0.935      1.43
## 7    15           1         1      1   0.5      1.17       1.48
## 8    15           5         1      1   0.5      1.25       1.62
## 9    15          10         1      1   0.5      1.25       1.61
```

The optimal parameters for ALS + R1R3 + KRR is

```
arp_result[which.min(arp_result$test_mean),]
```

```
## # A tibble: 1 x 7
##      fs lambdas_als lambdas_p sigmas betas train_mean test_mean
##   <dbl>       <dbl>     <dbl>  <dbl> <dbl>      <dbl>     <dbl>
## 1    15           1         1      1   0.2      0.497      1.25
```

4. Optimal Model

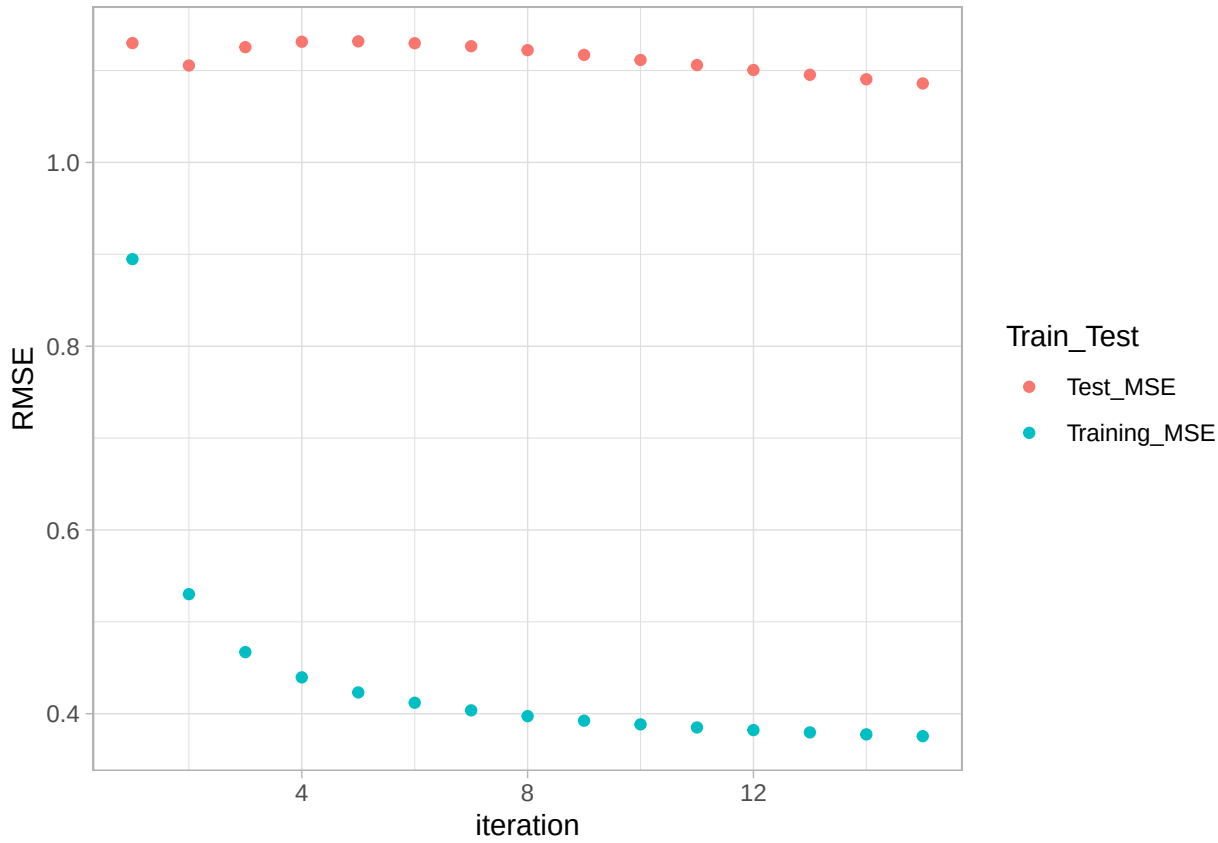
- Model Fitting To see the how ALS with R1R3 works, we first conducted the matrix factorization separately

```
# ALS with R1R3
# cl = makeCluster(8)
# ALS.R1R3.model <- ALS.R1R3(f = 15, lambda = 1, beta = 0.2, maxIters = 15 , data = data,train = data_t
# stopCluster(cl)
# save(ALS.R1R3.model, file="../output/ALS_R1R3_model.RData")
# stopCluster(cl)

load(file="../output/ALS_R1R3_model.RData")
```

Visualization of training and test RMSE by different iterations

```
g1 <- data.frame(iteration = c(1:15),Training_MSE = ALS.R1R3.model$`Train RMSE`,
                 Test_MSE = ALS.R1R3.model$`Test RMSE`) %>%
  gather(key = Train_Test,value = RMSE,-iteration) %>%
  ggplot(aes(x = iteration, y = RMSE, col = Train_Test)) + geom_point() +
  theme_light()
g1
```



Here, we fit the complete model wiht post-processing, KRR

```
# cl = makeCluster(8)
# ALS.R1R3.KRR.model <- ALS_R_KRR(data, data_train, data_test, f = 15, maxIters = 15, lambda_als = 1, l
# stopCluster(cl)
# save(ALS.R1R3.KRR.model, file="../output/ALS_R1R3_KRR_model.RData")
#
# stopCluster(cl)

load(file="../output/ALS_R1R3_KRR_model.RData")
```

- Model Evaluation We can see that when we add post-processing to the matrix factorization, the performance decrease. The final result of our model 2 is

```
cat("RMSE for train set:", round(ALS.R1R3.KRR.model$train_RMSE, 4), "  RMSE for test set:", round(ALS.R
```

```
## RMSE for train set: 0.5002   RMSE for test set: 1.2418
```