

Project4

In this project, you are going to explore matrix factorization methods for recommender system. The goal is to match consumers with most appropriate products. Matrix factorization methods characterize both items and users by vectors of factors inferred from item rating patterns. High correspondence between item and user factors leads to a recommendation. Matrix factorization generally has 3 parts:

- factorization algorithm
- regularization
- postprocessing

It is highly recommended to read this review paper.

Step 1 Load Data and Train-test Split

```
library(tidyverse)
library(ggplot2)
data <- read.csv("../data/ml-latest-small/ratings.csv")
set.seed(0)
test_idx <- sample(1:nrow(data), round(nrow(data)/5, 0))
train_idx <- setdiff(1:nrow(data), test_idx)
data_train <- data[train_idx,]
data_test <- data[test_idx,]
```

###Step 2 Matrix Factorization #### Step 2.1 Algorithm and Regularization

For matrix factorization, we used an Alternating Least Squares algorithm, with Penalty of Magnitudes regularization. The referenced paper are:

A3. Alternating Least Squares Section 3.1

R1. Penalty of Magnitudes Section: a Basic Matrix Factorization Model

```
U <- length(unique(data$userId))
I <- length(unique(data$movieId))
source("../lib/Matrix_Factorization.R")
```

Step 2.2 Parameter Tuning

Here we tune parameters, such as the dimension of factor and the penalty parameter λ by cross-validation.

```
source("../lib/cross_validation.R")
f_list <- seq(10, 20, 10)
l_list <- seq(-2, -1, 1)
f_l <- expand.grid(f_list, l_list)

result_summary <- array(NA, dim = c(nrow(f_l), 100, 4))
run_time <- system.time(for(i in 1:nrow(f_l)){
  par <- paste("f = ", f_l[i,1], ", lambda = ", 10^f_l[i,2])
  cat(par, "\n")
  current_result <- cv.function.als(data, K = 5, f = f_l[i,1], lambda = 10^f_l[i,2])
```

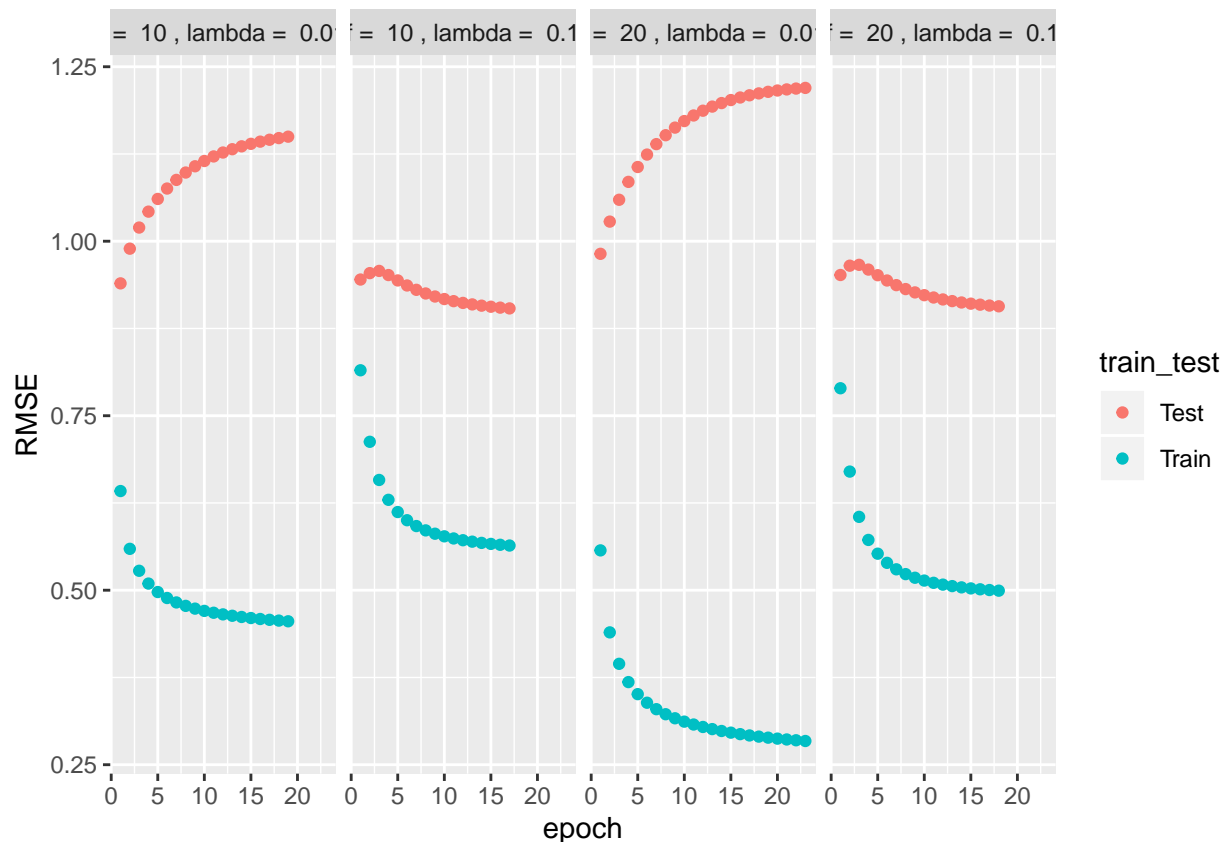
```

result_summary[,i] <- matrix(unlist(current_result), ncol = 100, byrow = T)
save(result_summary, file = "../output/rmse_als.Rdata")

})

load("../output/rmse_als.Rdata")
rmse <- data.frame(rbind(t(result_summary[1,,]), t(result_summary[2,,])),
  train_test = rep(c("Train", "Test"), each = 4),
  par = rep(paste("f = ", f_1[1], ", lambda = ", 10^f_1[2]), times = 2)) %>%
  gather("epoch", "RMSE", -train_test, -par)
rmse$epoch <- as.numeric(gsub("X", "", rmse$epoch))
rmse %>%
  drop_na(RMSE) %>%
  ggplot(aes(x = epoch, y = RMSE, col = train_test)) + geom_point() + facet_grid(~par)

```



```

test_rmse <- rep(NA, 4)
for (i in 1:nrow(f_1)) {
  epoch_rmse <- result_summary[2,,i]
  epoch_rmse <- epoch_rmse[!is.na(epoch_rmse)]
  test_rmse[i] <- epoch_rmse[length(epoch_rmse)]
}
best_f <- f_1[which(test_rmse==min(test_rmse)), 1]
best_lambda <- 10**f_1[which(test_rmse==min(test_rmse)), 2]

```

Based on the cross-validation, the optimal values for the parameters (i.e. the lowest test error in cross-validation) are 10 factors, with $\lambda=0.1$.

Step 2.3 Evaluation on the Model without Postprocessing

```
result <- als(f = best_f, lambda = best_lambda, max.iter = 100, stopping.thres = 0.001,
             data = data, train = data_train, test = data_test)

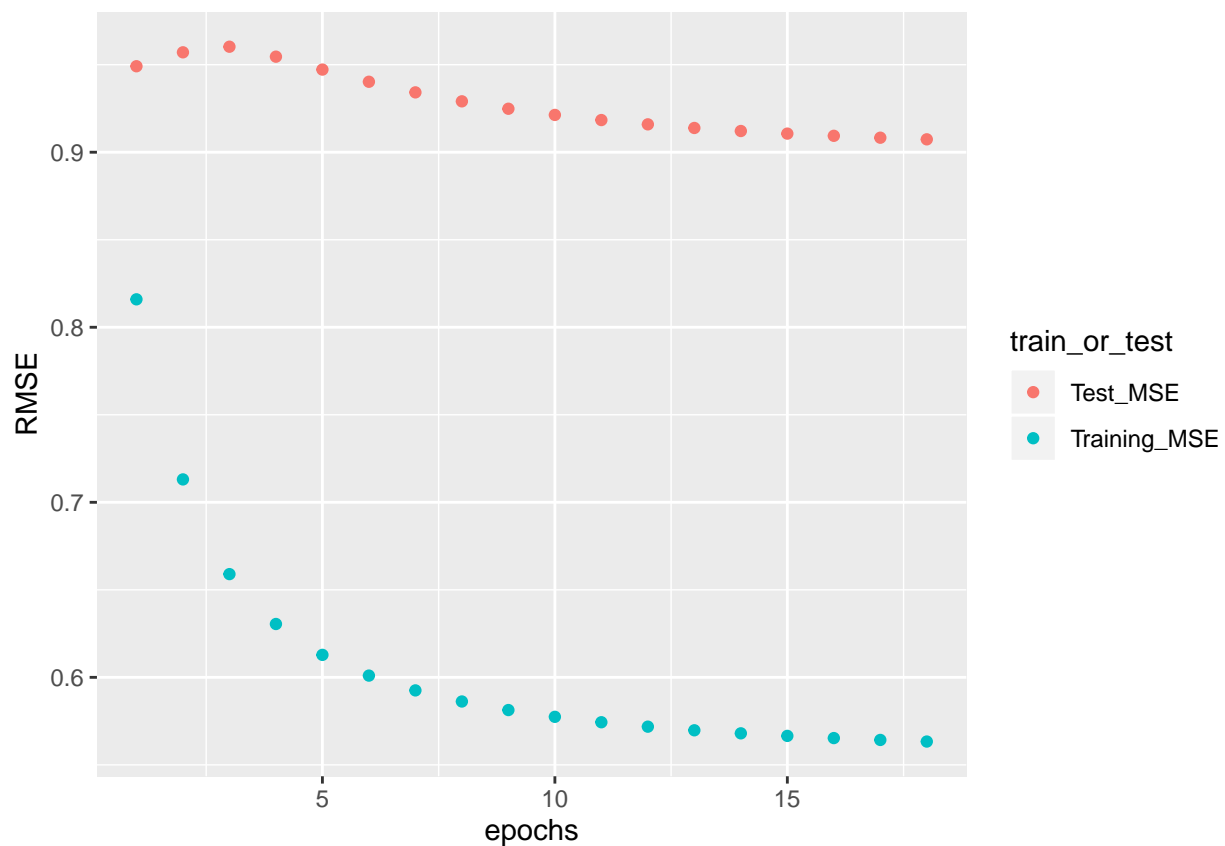
save(result, file = "../output/mat_fac_als.RData")
```

Here, we visualize training and testing RMSE by different epochs.

```
load(file = "../output/mat_fac_als.RData")

RMSE <- data.frame(epochs = seq(1, 100, 1), Training_MSE = result$train_RMSE,
                  Test_MSE = result$test_RMSE) %>%
  gather(key = train_or_test, value = RMSE, -epochs)

RMSE %>% drop_na(RMSE) %>% ggplot(aes(x = epochs, y = RMSE, col = train_or_test)) + geom_point()
```



```
initial_RMSE <- RMSE %>% drop_na(RMSE) %>% pull(RMSE)
cat("The RMSE of the model before postprocessing is", initial_RMSE[length(initial_RMSE)], "\n")
```

```
## The RMSE of the model before postprocessing is 0.9073578
```

Step 3 Postprocessing

After matrix factorization, postprocessing will be performed to improve accuracy.

KNN

The postprocessing method used here is SVD with KNN. The referenced paper is:

P2:Postprocessing SVD with KNN Section 3.5

Firstly, obtain the similarity data matrix and store it in RData file for further usage.

```
similarity.start <- proc.time()

m.s <- matrix(0,nrow = ncol(result$q), ncol = ncol(result$q))
for (i in 1:ncol(result$q)) {
  for (j in 1:ncol(result$q)) {
    m.s[i,j] = t(result$q[,i])%*%result$q[,j]/sqrt(sum(result$q[,i]^2))/sqrt(sum(result$q[,j]^2))
  }
}
colnames(m.s) <- colnames(result$q)
rownames(m.s) <- colnames(result$q)

similarity.stop <- proc.time()
similarity.time <- similarity.stop - similarity.start

save(m.s, file = "../output/movie_similarity.RData")
save(similarity.time, file = "../output/movie_similarity_time.RData")
```

Define the KNN_pred_rating function that will be used to derive the predicted rating for a given movie ID and user ID by using KNN method and the movie factor matrix obtained above.

```
load("../output/movie_similarity.RData")
load("../output/movie_similarity_time.RData")

pred_rating <- t(result$q) %*% result$p
k <- c(1,2,3,5,10,15)

#Define a function to extract the corresponding predicted rating by using KNN for the test set.
KNN_pred_rating <- function(test_set, pred, K){
  u <- test_set[1]
  m <- test_set[2]

  m_idx_urated <- unique(data$movieId[which(data$userId == u)])

  mid_ch <- c()
  for (i in 1:K) {
    m_idx <- ifelse(m %in% m_idx_urated,
      colnames(m.s)[which(m.s[as.character(m),] == sort(m.s[as.character(m),as.character(m_idx_urated)]
        decreasing = TRUE)[i+1])],
      colnames(m.s)[which(m.s[as.character(m),] == sort(m.s[as.character(m),as.character(m_idx_urated)]
        decreasing = TRUE)[i])])
    mid_ch <- c(mid_ch,m_idx)
  }

  pred_rating.um <- pred[mid_ch, as.character(u)]

  pred_rating <- mean(pred_rating.um)
  return(pred_rating)
}
```

Use partial cross validation method to obtain an optimal k that would minimize the RMSE of the predicted rating by using the training set.

```

rmse_tr_te_k <- c()

k_cv <- 5
for (i in 1:length(k)) {
  rmse_tr_te_j <- c()
  for (j in 1:k_cv) {
    tr.te.idx <- sample(1:nrow(data_train),0.2*nrow(data_train),replace = TRUE)
    dt.tr.te <- data_train[tr.te.idx,]
    pred_tr_te_rating_k <- apply(dt.tr.te, 1, KNN_pred_rating, pred_rating,k[i])
    rmse_tr_te_j[j] <- sqrt(mean((dt.tr.te$rating - pred_tr_te_rating_k)^2))
  }
  rmse_tr_te_k[i] <- mean(rmse_tr_te_j)
  cat("The RMSE of the model for training cv using KNN postprocessing method while k = ",
      k[i]," is", rmse_tr_te_k[i],"\n")
}

save(rmse_tr_te_k, file = "../output/knn_cv_rmse.RData")

load("../output/knn_cv_rmse.RData")

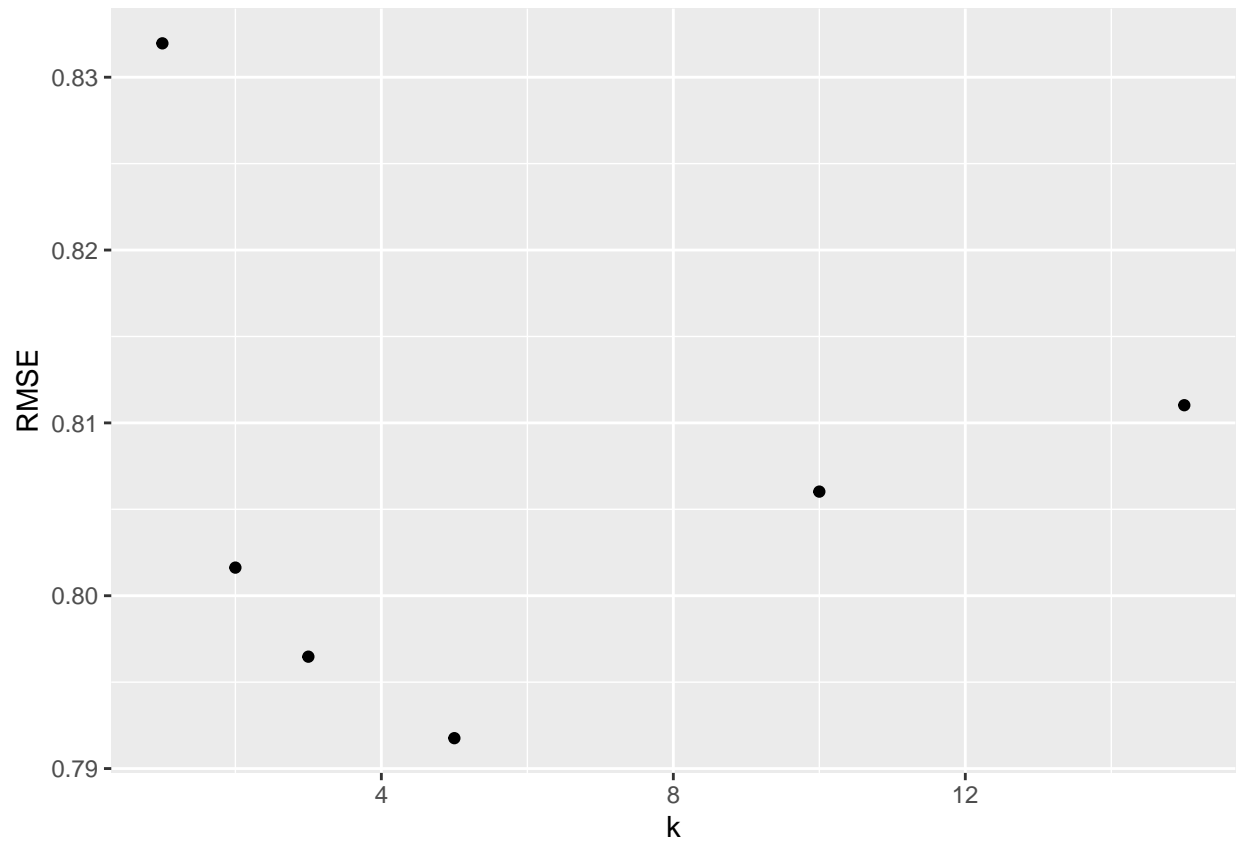
for (i in 1:length(k)) {
  cat("The cross-validation RMSE of the model for KNN postprocessing method while k = ",
      k[i]," is", rmse_tr_te_k[i],"\n")
}

## The cross-validation RMSE of the model for KNN postprocessing method while k = 1 is 0.8319596
## The cross-validation RMSE of the model for KNN postprocessing method while k = 2 is 0.801627
## The cross-validation RMSE of the model for KNN postprocessing method while k = 3 is 0.7964731
## The cross-validation RMSE of the model for KNN postprocessing method while k = 5 is 0.791759
## The cross-validation RMSE of the model for KNN postprocessing method while k = 10 is 0.8060235
## The cross-validation RMSE of the model for KNN postprocessing method while k = 15 is 0.8110235

RMSE.df <- data.frame(k = c(1,2,3,5,10,15),
                      RMSE = rmse_tr_te_k)

ggplot(RMSE.df) +
  geom_point(aes(k, RMSE))

```



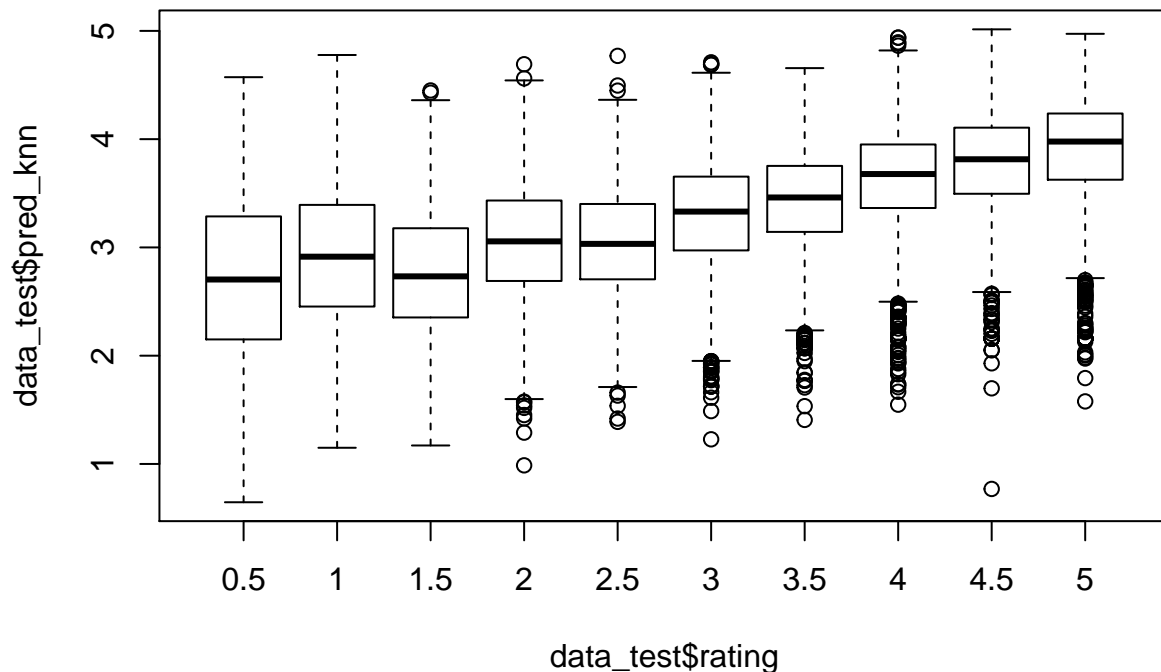
```
best_k <- k[which(rmse_tr_te_k==min(rmse_tr_te_k))]
```

Based on cross-validation, it turns out that $k=5$ would minimize the RMSE in predicting the movie ratings. Therefore, we choose $k=5$ as final parameter for KNN postprocessing method.

```
prediction.time <- system.time(pred_test_rating <- apply(data_test, 1, KNN_pred_rating,
                                                         pred_rating, best_k))
```

```
data_test$pred_knn <- pred_test_rating
```

```
boxplot(data_test$pred_knn ~ data_test$rating)
```



```
#calculate RMSE
rmse <- sqrt(mean((data_test$rating - data_test$pred_knn)^2))
cat("The RMSE of the model with KNN is", rmse, "\n")

## The RMSE of the model with KNN is 0.8979842
cat("The time for calculating similarity is", similarity.time[1], "s\n")

## The time for calculating similarity is 969.35 s
cat("The time for making predictions is", prediction.time[1], "s")

## The time for making predictions is 144.53 s
```

Kernel Ridge Regression

The postprocessing method used here is SVD with kernel ridge regression. The referenced paper is:

P3:Postprocessing SVD with kernel ridge regression Section 3.6

To improve efficiency, we will limit the ridge regression to 500 observations per user, taking the 500 most frequently-rated movies per user (as suggested in the reference paper). The `select_movie()` function below selects the list of rated movies to be used as ridge regression input for a given user.

```
topnum <- 500

movies_ranked_by_ratings <- data_train%>%
  count(movieId)%>%
  arrange(desc(n))%>%
  pull(movieId)
```

```

select_movie <- function(i){
  movies_i <- data_train%>%
    filter(userId == i)%>%
    pull(movieId)
  if (length(movies_i) <= topnum) {
    sel_list <- c(movies_i)
  }
  else {
    movies_i_ranked <- movies_ranked_by_ratings[movies_ranked_by_ratings %in% movies_i]
    sel_list <- movies_i_ranked[1:topnum]
  }
  return(sel_list)
}

```

We explore kernel ridge regression using both a Gaussian kernel and a linear kernel.

Gaussian kernel

For each user, we use the training data to fit a kernel ridge regression using a Gaussian kernel, to predict the ratings for the test set.

```

krr.gaussian.start <- proc.time()

lambda <- 0.5 #Tuning parameter
X_full <- t(scale(result$q, center=FALSE)) #Scaled observation matrix for every movie in the data set
test_preds <- data.frame(userId=c(), movieId=c(), pred_krr_gaussian=c())

for (i in unique(data_test$userId)){
  user_train_data <- data_train %>%
    filter(userId == i, movieId %in% select_movie(i))
  y <- user_train_data$rating
  X <- X_full[as.character(user_train_data$movieId), ]
  K <- exp(2 * (X %*% t(X) - 1))
  b <- solve(K + lambda * diag(nrow(K))) %*% y

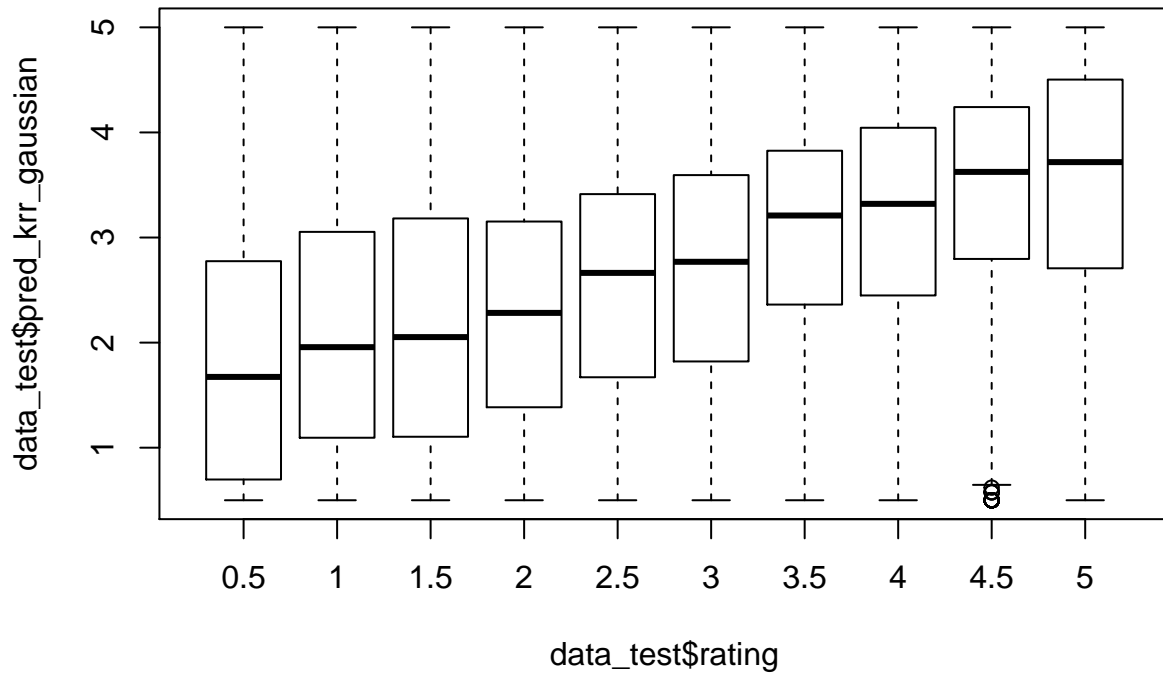
  user_test_data <- data_test %>%
    filter(userId == i) %>%
    select(userId, movieId)
  X_test <- X_full[as.character(user_test_data$movieId), ]
  user_test_data$pred_krr_gaussian <- exp(2 * (X_test %*% t(X) - 1)) %*% b
  test_preds <- rbind(test_preds, user_test_data)
}

#Since ratings cannot be below 0.5 or above 5.0,
#we cap the predictions at these minimum and maximum values.
test_preds$pred_krr_gaussian <- ifelse(test_preds$pred_krr_gaussian < 0.5, 0.5,
  ifelse(test_preds$pred_krr_gaussian > 5, 5, test_preds$pred_krr_gaussian))
data_test <- data_test %>% left_join(test_preds, by=c('userId', 'movieId'))
rmse_KRR_gaussian <- sqrt(mean((data_test$rating - data_test$pred_krr_gaussian)^2))

krr.gaussian.stop <- proc.time()
krr.gaussian.time <- krr.gaussian.stop - krr.gaussian.start

boxplot(data_test$pred_krr_gaussian ~ data_test$rating)

```

```
cat("The RMSE of the model with Gaussian kernel ridge regression is", rmse_KRR_gaussian, "\n")

## The RMSE of the model with Gaussian kernel ridge regression is 1.414612
cat("The time for training the model and making predictions is", krr.gaussian.time[1], "s")

## The time for training the model and making predictions is 13.25 s
```

Linear kernel

For each user, we use the training data to fit a kernel ridge regression using a linear kernel, to predict the ratings for the test set.

```
krr.linear.start <- proc.time()

lambda <- 0.5 #Tuning parameter
X_full <- t(scale(result$q, center=FALSE))
test_preds <- data.frame(userId=c(), movieId=c(), pred_krr_linear=c())

for (i in unique(data_test$userId)){
  user_train_data <- data_train %>%
    filter(userId == i, movieId %in% select_movie(i))
  y <- user_train_data$rating
  X <- X_full[as.character(user_train_data$movieId), ]
  K <- X %*% t(X)
  b <- solve(K + lambda * diag(nrow(K))) %*% y

  user_test_data <- data_test %>%
```

```

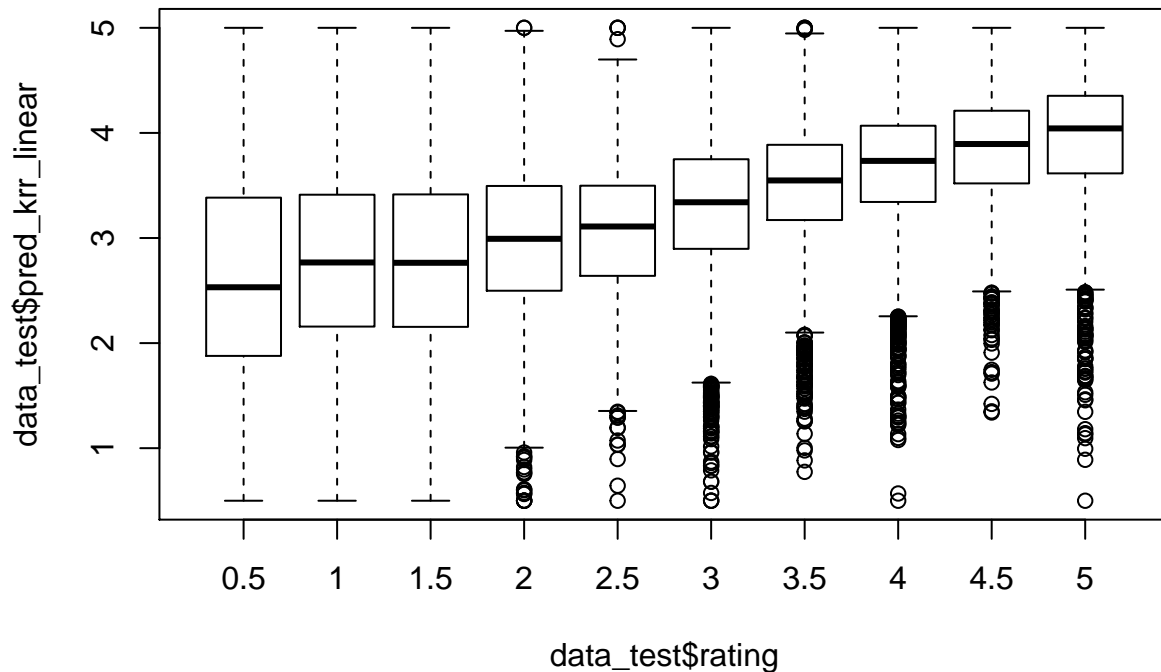
    filter(userId == i) %>%
    select(userId, movieId)
X_test <- X_full[as.character(user_test_data$movieId), ]
user_test_data$pred_krr_linear <- X_test %*% t(X) %*% b
test_preds <- rbind(test_preds, user_test_data)
}

#Since ratings cannot be below 0.5 or above 5.0,
#we cap the predictions at these minimum and maximum values.
test_preds$pred_krr_linear <- ifelse(test_preds$pred_krr_linear < 0.5, 0.5,
    ifelse(test_preds$pred_krr_linear > 5, 5, test_preds$pred_krr_linear))
data_test <- data_test %>% left_join(test_preds, by=c('userId', 'movieId'))
rmse_KRR_linear <- sqrt(mean((data_test$rating - data_test$pred_krr_linear)^2))

krr.linear.stop <- proc.time()
krr.linear.time <- krr.linear.stop - krr.linear.start

boxplot(data_test$pred_krr_linear ~ data_test$rating)

```



```

cat("The RMSE of the model with linear kernel ridge regression is", rmse_KRR_linear, "\n")

## The RMSE of the model with linear kernel ridge regression is 0.9477164
cat("The time for training the model and making predictions is", krr.linear.time[1], "s")

## The time for training the model and making predictions is 12.35 s

```

Conclusion

Given matrix factorization based on Alternating Least Squares with Penalty of Magnitudes, the KNN model outperforms kernel ridge regression for postprocessing (regardless of the choice of Gaussian vs. linear kernel). The KNN model has a test RMSE of 0.8980, while the Gaussian and linear kernel ridge regressions have test RMSE of 1.4146 and 0.9477, respectively.

The KNN model does take more computation time, however, including over 16 minutes for measuring similarities between movies and between 2-3 minutes for making predictions. The kernel ridge regression models, on the other hand, each take less than half a minute to train the model and make predictions.