PROJECT 4: GROUP 3

Paper A4: Fairness Beyond Disparate Treatment & Disparate Impact: Learning Classification without Disparate Mistreatment vs Paper A1: Learning Fair Representations

The aim of the 2 papers is to formulate fairness as an optimization problem of finding a good representation of the data with two competing goals: to encode the data as well as possible, while simultaneously obfuscating any information about membership in the sensitive group.

Across the nation, judges, probation and parole officers are increasingly using algorithms to assess a criminal defendant's likelihood of becoming a recidivist – a term used to describe criminals who re-offend. There are dozens of these risk assessment algorithms in use. Many states have built their own assessments, and several academics have written tools.

The Northpointe's tool, called COMPAS (which stands for Correctional Offender Management Profiling for Alternative Sanctions), found that black defendants were far more likely than white defendants to be incorrectly judged to be at a higher risk of recidivism, while white defendants were more likely than black defendants to be incorrectly flagged as low risk.

Hence we use the Compas scores of 2 years and try to find a good representation of accuracy along with fairness and check if there is a discrimination or not

In [ ]:

```

```

In [ ]:
```
import pandas as pd
import numpy as np
import pickle
import scipy.optimize as optim
from sklearn import preprocessing
from sklearn.model_selection import train_test_split
from sklearn import feature_extraction
#from __future__ import division
import os,sys
import numpy as np
from collections import defaultdict
from random import seed, shuffle
from collections import defaultdict
from copy import deepcopy
import numpy.core.multiarray
import cvxpy as cvx
import dccp
from dccp.problem import is_dccp
import traceback
import matplotlib.pylab as plt
import math
import copy
```

In [ ]:
```
!pip install dccp
```

```
Looking in indexes: https://pypi.org/simple, (https://pypi.org/simple,) h
ttps://us-python.pkg.dev/colab-wheels/public/simple/ (https://us-python.p
kg.dev/colab-wheels/public/simple/)
Collecting dccp
  Downloading dccp-1.0.4.tar.gz (8.0 kB)
Requirement already satisfied: cvxpy>=0.3.5 in /usr/local/lib/python3.7/d
ist-packages (from dccp) (1.2.2)
Requirement already satisfied: numpy>=1.15 in /usr/local/lib/python3.7/di
st-packages (from cvxpy>=0.3.5->dccp) (1.21.6)
Requirement already satisfied: ecos>=2 in /usr/local/lib/python3.7/dist-p
ackages (from cvxpy>=0.3.5->dccp) (2.0.10)
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t-packages (from cvxpy>=0.3.5->dccp) (3.2.2)
Requirement already satisfied: scipy>=1.1.0 in /usr/local/lib/python3.7/d
ist-packages (from cvxpy>=0.3.5->dccp) (1.7.3)
Requirement already satisfied: qdldl in /usr/local/lib/python3.7/dist-pac
kages (from osqp>=0.4.1->cvxpy>=0.3.5->dccp) (0.1.5.post2)
Building wheels for collected packages: dccp
  Building wheel for dccp (setup.py) ... done
  Created wheel for dccp: filename=dccp-1.0.4-py3-none-any.whl size=7386
 sha256=518fa5a6964f826d958b50c8fef07c8ea8b36030bd539ba6297aea3950ad2eb9
  Stored in directory: /root/.cache/pip/wheels/44/a0/2b/8944fc49959e6ae8c
c9584719c236016c214a04baf6516e24d
Successfully built dccp
Installing collected packages: dccp
Successfully installed dccp-1.0.4
```

In [ ]:
```
import time
start_dm = time.time()
```

In [ ]:
```
#df = pd.read_csv('../data/compas-scores-two-years.csv')
df = pd.read_csv('compas-scores-two-years.csv')
df['length_of_stay']=df['c_jail_out'].apply(pd.to_datetime) - df['c_jail_
df['length_of_stay']=df['length_of_stay'].dt.days
df['length_of_stay'] = df.length_of_stay.apply(lambda x:'greater than 100
```

In [ ]:
```
#Selecting features
features = ["age_cat", "race", "sex", "priors_count", "c_charge_degree","
cont_feature = ["priors_count"]
sensitive_attributes = ["race"]

x_control = defaultdict(list)
data=df.copy()

# Data Filtering
idx = np.where((data['days_b_screening_arrest']<=30) & (data['days_b_scree
 & (data['is_recid']!=-1) & (data['c_charge_degree']!="O") & (data['score_
 ((data['race']=="African-American") |(data['race']=="Caucasian")))
data=data.iloc[idx]


# convert class label 0 to -1
y = data['two_year_recid']
y[y==0] = -1

#adding intercept
intercept = np.ones(data.shape[0]).reshape(data.shape[0], 1)
X=pd.DataFrame(intercept, columns=['intercept'])


feature_names = []
for attr in features:
    vals = data[attr]
    if attr in cont_feature:
        vals = [float(v) for v in vals]
        vals = preprocessing.scale(vals) #makes it into a 0 mean and vari
        vals = np.reshape(vals, (len(y), -1))

    else:
        lb = preprocessing.LabelBinarizer()
        lb.fit(vals)
        vals = lb.transform(vals)


# Creating list called feature_names which contains column names
    # Checking continuous features
    if attr in cont_feature:
        feature_names.append(attr)
    #Checking categorical features
    else:
        # Binary features
        if vals.shape[1] == 1:
            feature_names.append(attr)
        #Non Binary features - adding the names for every category
        else:
            for k in lb.classes_:
                feature_names.append(attr + "_" + str(k))

    X[feature_names]=vals
    feature_names=[]
```

```
# Splitting the data into train and test
from sklearn.model_selection import train_test_split
x_train, x_test, y_train, y_test = train_test_split(X, y, test_size=0.5, 
x_control_train=x_train[sensitive_attributes]
x_control_test =x_test[sensitive_attributes]
feature_names= x_train.columns
x_train,x_test, y_train, y_test,x_control_train,x_control_test=x_train.va

#Making x_control_train and x_control_test into a dictionary
x_control_train={sensitive_attributes[0]:x_control_train.flatten()}
x_control_test={sensitive_attributes[0]:x_control_test.flatten()}
```

```
In [ ]:
```

PAPER 4 : Fairness Beyond Disparate Treatment & Disparate Impact: Learning Classification without Disparate Mistreatment

The overall goal of this algorithm is to build a fair classifier. In pratical scenarios when we are building a predictive model, a lot of festures are sensitive like gender, race, religion which can lead to a biased model. One example can be creating a predictive model to predict salaries of individuals and gender is included in the model. To avoid this, we add constraints to the model when we are optimizing. Under an ideal scenario, while building any type of model, our aim is to minimize the loss function. To make the model fair, we add contraints to the model, i.e we minimize the loss function while also making sure the contraints are met. In this algorithm those constraints were FPR and FNR. We cannot directly add the FPR abd FNR contraints to optimization funtion because they make the optimization problem non convex which are difficult to solve. Therefore we use a substitute which is the covariance between the users' sensitive attributes and the signed distance between the fea- ture vectors of misclassified users and the classifier decision boundary. The result in the algorithm is that we can find a balance which balances accuracy and fairness

In [ ]:
```
def train_model(x, y, x_control, EPS, cons_params,tau, mu):

    max_iters = 100
    max_iter_dccp = 50

    num_points, num_features = x.shape
    w = cvx.Variable(num_features)
    np.random.seed(0)
    w.value = np.random.rand(x.shape[1])
    loss = cvx.sum(  cvx.logistic( cvx.multiply(-y, x*w) )  ) / num_point

    #loss =  cvx.sum(cvx.logistic(x @ w) - cvx.multiply(y, x @ w) )
    constraints = []
    if cons_params is not None:

        for attribute in cons_params["sensitive_attrs_to_cov_thresh"].key

            attribute_list = x_control_train[attribute]

            s_val_to_total = {param:{} for param in [0,1,2]}
            s_val_to_avg = {param:{} for param in [0,1,2]}
            cons_sum_dict = {param:{} for param in [0,1,2]}

            for val in set(attribute_list):
                s_val_to_total[0][val] = sum(x_control_train[attribute] =
                s_val_to_total[1][val] = sum(np.logical_and(x_control_tra
                s_val_to_total[2][val] = sum(np.logical_and(x_control_tra


            for param in [0,1,2]:
                s_val_to_avg[param][0] = s_val_to_total[param][1] / float
                s_val_to_avg[param][1] = 1.0 - s_val_to_avg[param][0]


            for v in set(attribute_list):

                idx = x_control_train[attribute] == v


                ##########################################################
                # #DCCP constraints
                dist_bound_prod = cvx.multiply(y_train[idx], x_train[idx]

                cons_sum_dict[0][v] = cvx.sum( cvx.minimum(0, dist_bound_
                cons_sum_dict[1][v] = cvx.sum( cvx.minimum(0, cvx.multipl
                cons_sum_dict[2][v] = cvx.sum( cvx.minimum(0, cvx.multipl
                ##########################################################


            if cons_params["cons_type"] == 4:
                params = [1,2]
            elif cons_params["cons_type"] in [0,1,2]:
                params = [cons_params["cons_type"]]


            ##############################################################
```

```
            for param in params:
                threshold = abs(cons_params["sensitive_attrs_to_cov_thresh
                constraints.append( cons_sum_dict[param][1] <= cons_sum_d
                constraints.append( cons_sum_dict[param][1] >= cons_sum_d

            ##############################################################

    if cons_params is not None:
        if cons_params.get("take_initial_sol") is None: # true by default
            take_initial_sol = True
        elif cons_params["take_initial_sol"] == False:
            take_initial_sol = False

        if take_initial_sol == True: # get the initial solution
            p = cvx.Problem(cvx.Minimize(loss), [])
            p.solve()
    prob = cvx.Problem(cvx.Minimize(loss), constraints)

    try:

        prob.solve(method='dccp', tau=tau, mu=mu, tau_max=1e10,
            solver='ECOS', verbose=False,
            feastol=EPS, abstol=EPS, reltol=EPS,feastol_inacc=EPS, abstol_
            max_iters=max_iters, max_iter=max_iter_dccp)


    except:
        traceback.print_exc()
        sys.stdout.flush()
        sys.exit(1)


    # check that the fairness constraint is satisfied
    for f_c in constraints:
       # assert(f_c.value == True) # can comment this out if the solver f
        pass


    w = np.array(w.value).flatten() # flatten converts it to a 1d array

    return w
```

In [ ]:
```
def sensitive_features(y_true, y_pred, x_control, sensitive_attributes, f

    x_control_internal = deepcopy(x_control)

    data_dictionary = {}

    s = sensitive_attributes[0]

    s_vals = x_control_internal[s]
    for s_type in set(s_vals):
        data_dictionary[s_type] = {}
        y_true_local = y_true[s_vals==s_type]
        y_pred_local = y_pred[s_vals==s_type]

        acc = float(sum(y_true_local==y_pred_local)) / len(y_true_local)

        fp = sum(np.logical_and(y_true_local == -1.0, y_pred_local == +1.
        fn = sum(np.logical_and(y_true_local == +1.0, y_pred_local == -1.
        tp = sum(np.logical_and(y_true_local == +1.0, y_pred_local == +1.
        tn = sum(np.logical_and(y_true_local == -1.0, y_pred_local == -1.

        all_neg = sum(y_true_local == -1.0)
        all_pos = sum(y_true_local == +1.0)

        fpr = float(fp) / float(fp + tn)
        fnr = float(fn) / float(fn + tp)
        tpr = float(tp) / float(tp + fn)
        tnr = float(tn) / float(tn + fp)


        data_dictionary[s_type]["fp"] = fp
        data_dictionary[s_type]["fn"] = fn
        data_dictionary[s_type]["fpr"] = fpr
        data_dictionary[s_type]["fnr"] = fnr
        data_dictionary[s_type]["tpr"] = tpr
        data_dictionary[s_type]["tnr"] = tnr
        data_dictionary[s_type]["acc"] = (tp + tn) / (tp + tn + fp + fn)

    return data_dictionary
```

In [ ]:
```
def check_accuracy(x_train, y_train, x_test, y_test, y_train_predicted, y_


    correct_answers = (y_train_predicted == y_train).astype(int)
    train_score = float(sum(correct_answers)) / float(len(correct_answers
    correct_answers_train = sum(correct_answers)


    correct_answers = (y_test_predicted == y_test).astype(int)
    test_score = float(sum(correct_answers)) / float(len(correct_answers)
    correct_answers_test = sum(correct_answers)

    return train_score, test_score, correct_answers_train, correct_answer
```

In [ ]:
```
def distance_boundary(w, x, s_attr_arr):

    distances_boundary = np.zeros(x.shape[0])
    if isinstance(w, dict):
        for k in w.keys():
            d = np.dot(x, w[k])
            distances_boundary[s_attr_arr == k] = d[s_attr_arr == k]
    else:
        distances_boundary = np.dot(x, w)
    return distances_boundary
```

In [ ]:
```
def classification_stats(w, x_train, y_train, x_control_train, x_test, y_

    s_val = sensitive_attributes[0]

    distances_boundary_train = distance_boundary(w, x_train, x_control_tr
    distances_boundary_test = distance_boundary(w, x_test, x_control_test

    all_class_labels_assigned_train = np.sign(distances_boundary_train)
    all_class_labels_assigned_test = np.sign(distances_boundary_test)

    train_score, test_score, correct_answers_train, correct_answers_test 

    data_dictionary_train = sensitive_features(y_train, all_class_labels_

    data_dictionary_test = sensitive_features(y_test, all_class_labels_as

    return train_score, test_score, data_dictionary_train, data_dictionar
```

In [ ]:
```
def classifier(EPS,tau,mu,cons_params):

    w = train_model(x_train, y_train, x_control_train, EPS, cons_params,ta

    train_score, test_score, data_dictionary_train, data_dictionary_test =

    return w, test_score, data_dictionary_test
```

In [ ]:
```
def accuracy_1(threshold, flag):
    cons_type = 0 # No constraint
    sensitive_attrs_to_cov_thresh = {"race": {0:{0:0, 1:threshold}, 1:{0:
    cons_params = {"cons_type": cons_type, "tau": tau, "mu": mu, "sensiti
    w_uncons, acc_cons, data_dictionary = classifier(EPS,tau,mu,cons_para
    if flag == True:
        print("Unconstrained classifier")
        print("Accuracy:  " + str(round(acc_cons,4)))
        print("s     FPR.    FNR.    TNR.    TPR.   Accuracy")
        for key in data_dictionary:
            print("-----------------------------------------")
            print(str(key) + "      " + str(round(data_dictionary[key]['fp
              "    " +str(round(data_dictionary[key]['tpr'],2))+ "    "+ s

        print()

    return (acc_cons, abs(data_dictionary[0]['fpr']-data_dictionary[1]['f
    abs(data_dictionary[0]['fnr']-data_dictionary[1]['fnr']),data_diction
```

In [ ]:
```
def accuracy_2(threshold, flag):
    cons_type = 1 # FPR constraint
    sensitive_attrs_to_cov_thresh = {"race": {0:{0:0, 1:0}, 1:{0:0, 1:thr
    cons_params = {"cons_type": cons_type, "tau": tau, "mu": mu, "sensiti
    w_cons, acc_cons, data_dictionary  = classifier(EPS,tau,mu,cons_param
    if flag == True:
        print("FPR constraint classifier")
        print("Accuracy:  " + str(round(acc_cons,4)))
        print("s     FPR.    FNR.    TNR.    TPR.   Accuracy")
        for key in data_dictionary:
            print("-----------------------------------------")
            print(str(key) + "      " + str(round(data_dictionary[key]['fp
            + "    "+ str(data_dictionary[key]['acc']))
        print()
    #print("accuracy cons",acc_cons)
    return (acc_cons, abs(data_dictionary[0]['fpr']-data_dictionary[1]['f
    abs(data_dictionary[0]['fnr']-data_dictionary[1]['fnr']),data_diction
```

In [ ]:
```
def accuracy_3(threshold, flag):
    cons_type = 2 # FNR constraint
    sensitive_attrs_to_cov_thresh = {"race": {0:{0:0, 1:0}, 1:{0:0, 1:0},
    cons_params = {"cons_type": cons_type, "tau": tau, "mu": mu, "sensiti
    w_cons, acc_cons, data_dictionary  = classifier(EPS,tau,mu,cons_param
    if flag == True:
        print("FNR constraint classifier")
        print("Accuracy:  " + str(round(acc_cons,4)))
        print("s      FPR.     FNR.     TNR.     TPR.    Accuracy")
        for key in data_dictionary:
            print("-----------------------------------------")
            print(str(key) + "      " + str(round(data_dictionary[key]['fp
              "     " +str(round(data_dictionary[key]['tpr'],2))+ "    "+ s

            print()

    return (acc_cons, abs(data_dictionary[0]['fpr']-data_dictionary[1]['f
    abs(data_dictionary[0]['fnr']-data_dictionary[1]['fnr']),data_diction
```

In [ ]:
```
def accuracy_4(threshold, flag):
    cons_type = 4 # FPR & FNR constraint
    sensitive_attrs_to_cov_thresh = {"race": {0:{0:0, 1:0}, 1:{0:0, 1:thre
    cons_params = {"cons_type": cons_type, "tau": tau, "mu": mu, "sensiti
    w_cons, acc_cons, data_dictionary  = classifier(EPS,tau,mu,cons_param
    if flag == True:
        print("FPR & FNR constraint classifier")
        print("Accuracy:  " + str(round(acc_cons,4)))
        print("s      FPR.     FNR.     TNR.     TPR.    Accuracy")
        for key in data_dictionary:
            print("-----------------------------------------")
            print(str(key) + "      " + str(round(data_dictionary[key]['fp
              "     " +str(round(data_dictionary[key]['tpr'],2))+ "    "+ s

            print()

    return (acc_cons, abs(data_dictionary[0]['fpr']-data_dictionary[1]['f
    abs(data_dictionary[0]['fnr']-data_dictionary[1]['fnr']),data_diction
```

In [ ]:

In [ ]:
```
import warnings
warnings.filterwarnings("ignore")
```

In [ ]:
```
tau = 3.0
mu = 1.5
EPS = 1e-6
```

In [ ]:
```
threshold = 0
print("Threshold is " + str(threshold))
print("------------------------------------------------------------------------
print()
acc_cons, diff_fpr, diff_fnr,acc_w,acc_b = accuracy_1(threshold, True)
acc_cons, diff_fpr, diff_fnr,acc_w,acc_b = accuracy_2(threshold, True)
acc_cons, diff_fpr, diff_fnr,acc_w,acc_b = accuracy_3(threshold, True)
acc_cons, diff_fpr, diff_fnr,acc_w,acc_b = accuracy_4(threshold, True)
print()
```

```
Threshold is 0
--------------------------------------------------------------------------
----------------------

Unconstrained classifier
Accuracy:  0.676
s      FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0      0.38    0.27    0.62    0.73   0.6719643992371265
-----------------------------------------
1      0.16    0.57    0.84    0.43   0.6819887429643527

FPR constraint classifier
Accuracy:  0.6669
s      FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0      0.27    0.39    0.73    0.61   0.6757787666878576
-----------------------------------------
1      0.32    0.4    0.68    0.6   0.6538461538461539

FNR constraint classifier
Accuracy:  0.6756
s      FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0      0.28    0.37    0.72    0.63   0.6745073108709473

-----------------------------------------
1      0.25    0.44    0.75    0.56   0.6772983114446529

FPR & FNR constraint classifier
Accuracy:  0.6707
s      FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0      0.24    0.4    0.76    0.6   0.6776859504132231

-----------------------------------------
1      0.29    0.42    0.71    0.58   0.6604127579737336
```

In [ ]:
```
end_dm = time.time()
runtime_dm = (end_dm-start_dm)
print("Runtime of the code is  " + str(runtime_dm) + ' seconds')
```

```
Runtime of the code is  113.56461477279663 seconds
```

In [ ]:
```
tau = 3.0
mu = 1.5
EPS = 1e-6
cons_type7 = 0 # No constraint
sensitive_attrs_to_cov_thresh7 = {"race": {0:{0:0, 1:threshold}, 1:{0:0,
cons_params7 = {"cons_type": cons_type7, "tau": tau, "mu": mu, "sensitive_
w_uncons7, acc_cons7, data_dictionary7 = classifier(EPS,tau,mu,cons_param
data_dictionary7[1]['acc']
```

Out[203]:
```
0.6819887429643527
```

Analysis on varying the 'c' constant

In [ ]:
```
for threshold in range(0, 50, 5):
    threshold = threshold/1000
    print("Threshold is " + str(threshold))
    print("----------------------------------------------------------------------
    print()
    acc_cons, diff_fpr, diff_fnr,acc_w,acc_b = accuracy_1(threshold, True
    acc_cons, diff_fpr, diff_fnr,acc_w,acc_b = accuracy_2(threshold, True
    acc_cons, diff_fpr, diff_fnr,acc_w,acc_b = accuracy_3(threshold, True
    acc_cons, diff_fpr, diff_fnr,acc_w,acc_b = accuracy_4(threshold, True
    print()
```

```
Threshold is 0.0
--------------------------------------------------------------------------
----------------------

Unconstrained classifier
Accuracy:  0.676
s      FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0      0.38    0.27    0.62    0.73   0.6719643992371265
-----------------------------------------
1      0.16    0.57    0.84    0.43   0.6819887429643527

FPR constraint classifier
Accuracy:  0.6669
s      FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0      0.27    0.39    0.73    0.61   0.6757787666878576
-----------------------------------------
1      0.32    0.4    0.68    0.6   0.6538461538461539

FNR constraint classifier
Accuracy:  0.6756
s      FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0      0.28    0.37    0.72    0.63   0.6745073108709473

-----------------------------------------
1      0.25    0.44    0.75    0.56   0.6772983114446529

FPR & FNR constraint classifier
Accuracy:  0.6707
s      FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0      0.24    0.4    0.76    0.6   0.6776859504132231

-----------------------------------------
1      0.29    0.42    0.71    0.58   0.6604127579737336


Threshold is 0.005
--------------------------------------------------------------------------
----------------------

Unconstrained classifier
Accuracy:  0.6745
s      FPR.    FNR.    TNR.    TPR.   Accuracy
```

```
-----------------------------------------
0     0.38    0.27    0.62    0.73   0.6706929434202161
-----------------------------------------
1     0.17    0.57    0.83    0.43   0.6801125703564728

FPR constraint classifier
Accuracy:  0.6707
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.31    0.33    0.69    0.67   0.684043229497775
-----------------------------------------
1     0.33    0.38    0.67    0.62   0.651031894934334

FNR constraint classifier
Accuracy:  0.6722
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.3    0.36    0.7    0.64   0.6719643992371265

-----------------------------------------
1     0.25    0.45    0.75    0.55   0.6726078799249531

FPR & FNR constraint classifier
Accuracy:  0.6726
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.31    0.33    0.69    0.67   0.684043229497775

-----------------------------------------
1     0.33    0.37    0.67    0.63   0.6557223264540337


Threshold is 0.01
-------------------------------------------------------------------------
----------------------

Unconstrained classifier
Accuracy:  0.6745
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.38    0.27    0.62    0.73   0.6706929434202161
-----------------------------------------
1     0.17    0.57    0.83    0.43   0.6801125703564728

FPR constraint classifier
Accuracy:  0.6715
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.3    0.34    0.7    0.66   0.677050222504768
-----------------------------------------
1     0.3    0.4    0.7    0.6   0.6632270168855535

FNR constraint classifier
Accuracy:  0.6745
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.31    0.35    0.69    0.65   0.673871582962492
```

```
-----------------------------------------
1     0.24    0.46    0.76    0.54   0.6754221388367729

FPR & FNR constraint classifier
Accuracy:  0.6715
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.3    0.34    0.7    0.66   0.677050222504768

-----------------------------------------
1     0.3    0.4    0.7    0.6   0.6632270168855535


Threshold is 0.015
--------------------------------------------------------------------------
----------------------

Unconstrained classifier
Accuracy:  0.6745
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.38    0.27    0.62    0.73   0.6706929434202161
-----------------------------------------
1     0.17    0.57    0.83    0.43   0.6801125703564728

FPR constraint classifier
Accuracy:  0.673
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.31    0.34    0.69    0.66   0.6776859504132231
-----------------------------------------
1     0.28    0.42    0.72    0.58   0.6660412757973734

FNR constraint classifier
Accuracy:  0.6749
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.31    0.34    0.69    0.66   0.673871582962492

-----------------------------------------
1     0.23    0.47    0.77    0.53   0.6763602251407129

FPR & FNR constraint classifier
Accuracy:  0.6726
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.31    0.34    0.69    0.66   0.677050222504768

-----------------------------------------
1     0.28    0.42    0.72    0.58   0.6660412757973734


Threshold is 0.02
--------------------------------------------------------------------------
----------------------
```

```
Unconstrained classifier
Accuracy:  0.6745
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.38    0.27    0.62    0.73   0.6706929434202161
-----------------------------------------
1     0.17    0.57    0.83    0.43   0.6801125703564728

FPR constraint classifier
Accuracy:  0.6756
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.3    0.34    0.69    0.66   0.6789574062301335
-----------------------------------------
1     0.25    0.45    0.75    0.55   0.6707317073170732

FNR constraint classifier
Accuracy:  0.6741
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.33    0.33    0.68    0.67   0.6713286713286714

-----------------------------------------
1     0.22    0.48    0.78    0.52   0.6782363977485929

FPR & FNR constraint classifier
Accuracy:  0.6756
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.3    0.34    0.69    0.66   0.6789574062301335

-----------------------------------------
1     0.25    0.45    0.75    0.55   0.6707317073170732


Threshold is 0.025
-------------------------------------------------------------------------
----------------------

Unconstrained classifier
Accuracy:  0.6745
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.38    0.27    0.62    0.73   0.6706929434202161
-----------------------------------------
1     0.17    0.57    0.83    0.43   0.6801125703564728

FPR constraint classifier
Accuracy:  0.6787
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.3    0.33    0.69    0.67   0.6815003178639543
-----------------------------------------
1     0.23    0.47    0.77    0.53   0.6744840525328331

FNR constraint classifier
Accuracy:  0.6772
```

```
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.33    0.32    0.67    0.68   0.6726001271455817

-----------------------------------------
1     0.2    0.5    0.8    0.5   0.6838649155722326

FPR & FNR constraint classifier
Accuracy:  0.6787
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.3    0.33    0.69    0.67   0.6815003178639543

-----------------------------------------
1     0.23    0.47    0.77    0.53   0.6744840525328331


Threshold is 0.03
-------------------------------------------------------------------------
----------------------

Unconstrained classifier
Accuracy:  0.6745
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.38    0.27    0.62    0.73   0.6706929434202161
-----------------------------------------
1     0.17    0.57    0.83    0.43   0.6801125703564728

FPR constraint classifier
Accuracy:  0.6775
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.32    0.33    0.68    0.67   0.6789574062301335
-----------------------------------------
1     0.21    0.51    0.79    0.49   0.6754221388367729

FNR constraint classifier
Accuracy:  0.679
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.34    0.31    0.66    0.69   0.6745073108709473

-----------------------------------------
1     0.2    0.5    0.8    0.5   0.6857410881801126

FPR & FNR constraint classifier
Accuracy:  0.6772
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.32    0.32    0.68    0.68   0.6795931341385887

-----------------------------------------
1     0.21    0.51    0.79    0.49   0.6735459662288931


Threshold is 0.035
```

```
--------------------------------------------------------------------------
----------------------

Unconstrained classifier
Accuracy:  0.6745
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.38    0.27    0.62    0.73   0.6706929434202161
-----------------------------------------
1     0.17    0.57    0.83    0.43   0.6801125703564728

FPR constraint classifier
Accuracy:  0.676
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.33    0.32    0.67    0.68   0.673871582962492
-----------------------------------------
1     0.19    0.53    0.81    0.47   0.6791744840525328

FNR constraint classifier
Accuracy:  0.6779
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.35    0.3    0.65    0.7   0.6745073108709473

-----------------------------------------
1     0.2    0.51    0.8    0.49   0.6829268292682927

FPR & FNR constraint classifier
Accuracy:  0.6768
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.33    0.32    0.67    0.68   0.673871582962492

-----------------------------------------
1     0.19    0.53    0.81    0.47   0.6810506566604128


Threshold is 0.04
--------------------------------------------------------------------------
----------------------

Unconstrained classifier
Accuracy:  0.6745
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.38    0.27    0.62    0.73   0.6706929434202161
-----------------------------------------
1     0.17    0.57    0.83    0.43   0.6801125703564728

FPR constraint classifier
Accuracy:  0.6753
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.34    0.32    0.66    0.68   0.6726001271455817
-----------------------------------------
1     0.18    0.55    0.82    0.45   0.6791744840525328
```

```
FNR constraint classifier
Accuracy:  0.6775
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.36    0.29    0.64    0.71   0.677050222504768

-----------------------------------------
1     0.19    0.53    0.81    0.47   0.6782363977485929

FPR & FNR constraint classifier
Accuracy:  0.6753
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.34    0.31    0.66    0.69   0.6732358550540368

-----------------------------------------
1     0.19    0.54    0.81    0.46   0.6782363977485929


Threshold is 0.045
--------------------------------------------------------------------------
----------------------

Unconstrained classifier
Accuracy:  0.6745
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.38    0.27    0.62    0.73   0.6706929434202161
-----------------------------------------
1     0.17    0.57    0.83    0.43   0.6801125703564728

FPR constraint classifier
Accuracy:  0.6783
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.36    0.29    0.64    0.71   0.677050222504768
-----------------------------------------
1     0.17    0.56    0.83    0.44   0.6801125703564728

FNR constraint classifier
Accuracy:  0.6756
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.38    0.27    0.62    0.73   0.673871582962492

-----------------------------------------
1     0.18    0.55    0.82    0.45   0.6782363977485929

FPR & FNR constraint classifier
Accuracy:  0.6775
s     FPR.    FNR.    TNR.    TPR.   Accuracy
-----------------------------------------
0     0.36    0.29    0.64    0.71   0.677050222504768

-----------------------------------------
1     0.18    0.55    0.82    0.45   0.6782363977485929
```

PLOTS

Constraints on FPR

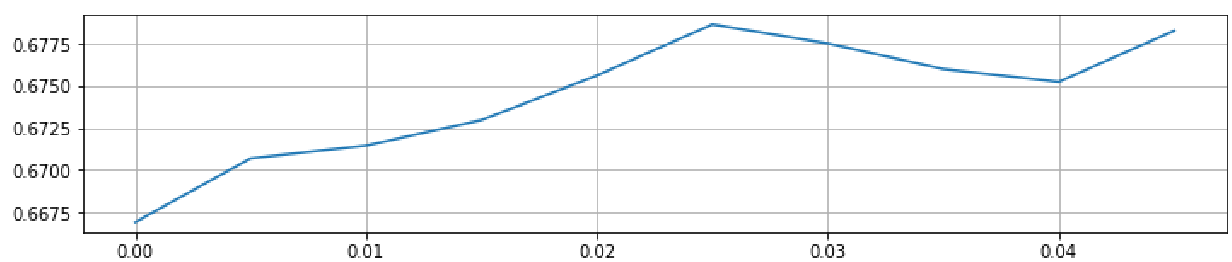
In [ ]:

```
accuracy_dictionary = {}
diff_fpr_dictionary = {}
diff_fnr_dictionary = {}

for threshold in range(0, 50, 5):

    threshold = threshold/1000

    acc_cons, diff_fpr, diff_fnr = accuracy_2(threshold, False)
    diff_fpr_dictionary[threshold] = diff_fpr
    diff_fnr_dictionary[threshold] = diff_fnr
    accuracy_dictionary[threshold] = acc_cons
```

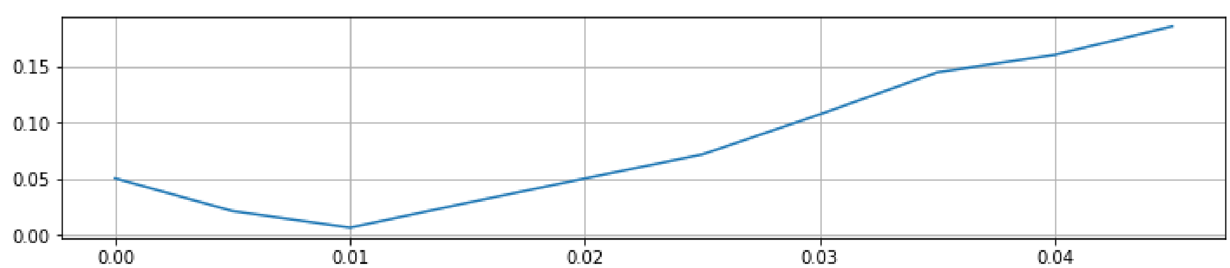
In [ ]:

```
import seaborn as sns
print("Constraints on FPR")
print("Accuracy Plot vs Threshold")
lists = sorted(accuracy_dictionary.items()) # sorted by key, return a lis
fig=plt.figure(figsize=(12, 8))


x, y = zip(*lists) # unpack a list of pairs into two tuples
plt.subplot( 3, 1,1)
plt.plot(x, y)
plt.grid()
plt.show()

print("Differnce between FPR in Sensitive Variables Plot vs Threshold")
lists = sorted(diff_fpr_dictionary.items()) # sorted by key, return a lis
plt.figure(figsize=(12, 8))
x, y = zip(*lists) # unpack a list of pairs into two tuples
plt.subplot(3, 1,2)
plt.plot(x, y)
plt.grid()
plt.show()

print("Differnce between FNR in Sensitive Variables Plot vs Threshold")
lists = sorted(diff_fnr_dictionary.items()) # sorted by key, return a lis
plt.figure(figsize=(12, 8))
x, y = zip(*lists) # unpack a list of pairs into two tuples
plt.subplot( 3,1, 3)
plt.plot(x, y)
plt.grid()
plt.show()
```

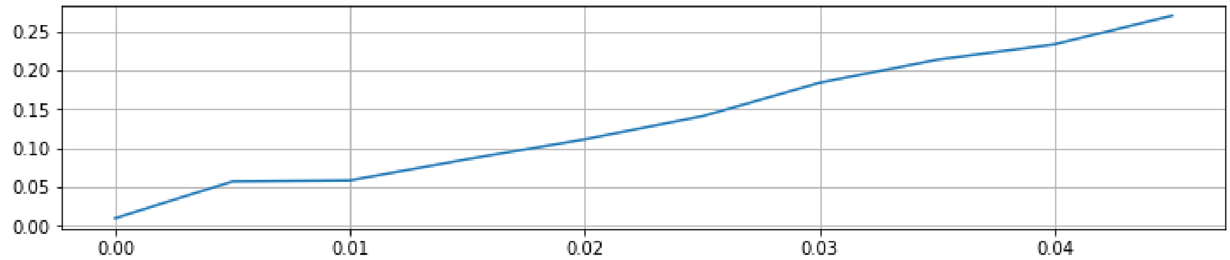
```
Constraints on FPR
Accuracy Plot vs Threshold
```




```
Differnce between FPR in Sensitive Variables Plot vs Threshold
```




```
Differnce between FNR in Sensitive Variables Plot vs Threshold
```



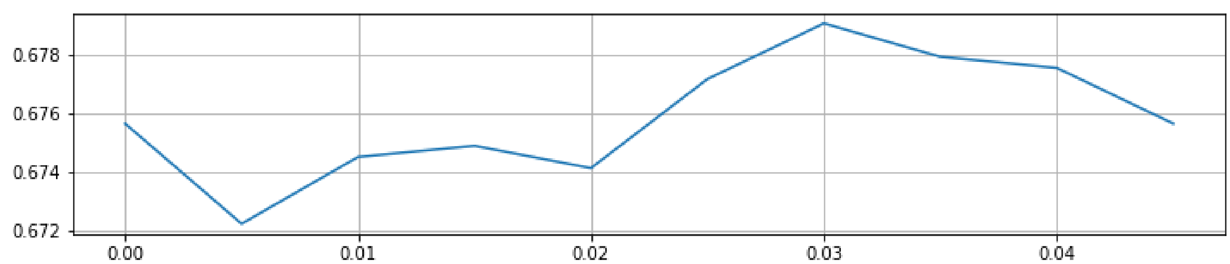
Here we see that as threshold(unfairness) rises the accuracy increases. With rise of unfairness the fpr and well as fnr increases

Constraints on FNR

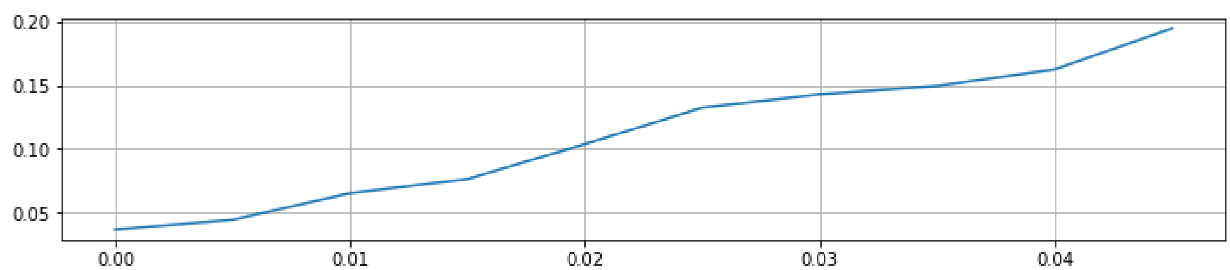
In [ ]:
```
accuracy_dictionary = {}
diff_fpr_dictionary = {}
diff_fnr_dictionary = {}

for threshold in range(0, 50, 5):

    threshold = threshold/1000

    acc_cons, diff_fpr, diff_fnr = accuracy_3(threshold, False)
    diff_fpr_dictionary[threshold] = diff_fpr
    diff_fnr_dictionary[threshold] = diff_fnr
    accuracy_dictionary[threshold] = acc_cons
```

In [ ]:
```
print("Constraint of FNR")
print("Accuracy Plot vs Threshold")
lists = sorted(accuracy_dictionary.items()) # sorted by key, return a lis
plt.figure(figsize=(12, 8))
x, y = zip(*lists) # unpack a list of pairs into two tuples
plt.subplot( 3, 1,1)
plt.plot(x, y)
plt.grid()
plt.show()

print("Differnce between FPR in Sensitive Variables Plot vs Threshold")
lists = sorted(diff_fpr_dictionary.items()) # sorted by key, return a lis
plt.figure(figsize=(12, 8))
x, y = zip(*lists) # unpack a list of pairs into two tuples
plt.subplot(3, 1,2)
plt.plot(x, y)
plt.grid()
plt.show()

print("Differnce between FNR in Sensitive Variables Plot vs Threshold")
lists = sorted(diff_fnr_dictionary.items()) # sorted by key, return a lis
plt.figure(figsize=(12, 8))
x, y = zip(*lists) # unpack a list of pairs into two tuples
plt.subplot( 3,1, 3)
plt.plot(x, y)
plt.grid()
plt.show()
```

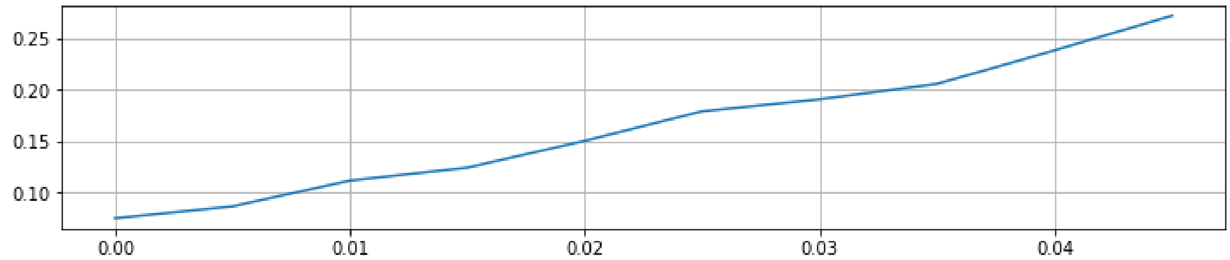
```
Constraint of FNR
Accuracy Plot vs Threshold
```




```
Differnce between FPR in Sensitive Variables Plot vs Threshold
```




```
Differnce between FNR in Sensitive Variables Plot vs Threshold
```



Here we see that as threshold(unfairness) rises the accuracy increases in first half but slightly decreases after threshold of 0.03. With rise of unfairness the fpr and well as fnr increases

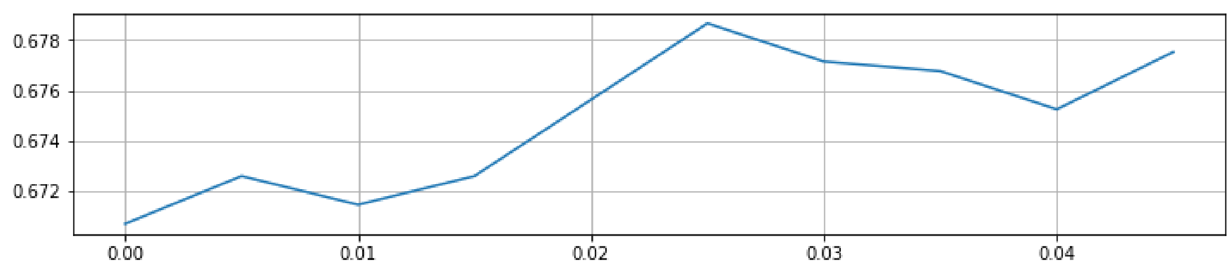
Constraints on FPR and FNR

In [ ]:

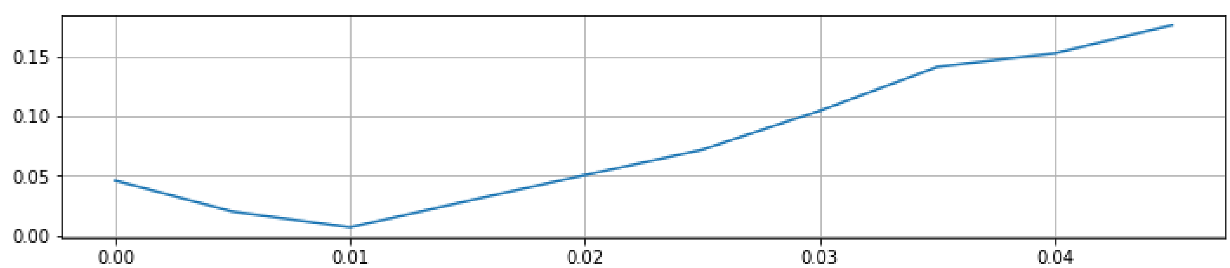
```
accuracy_dictionary = {}
diff_fpr_dictionary = {}
diff_fnr_dictionary = {}

for threshold in range(0, 50, 5):

    threshold = threshold/1000

    acc_cons, diff_fpr, diff_fnr = accuracy_4(threshold, False)
    diff_fpr_dictionary[threshold] = diff_fpr
    diff_fnr_dictionary[threshold] = diff_fnr
    accuracy_dictionary[threshold] = acc_cons
```

In [ ]:
```
print("Constraints of FNR and FPR")
print("Accuracy Plot vs Threshold")
lists = sorted(accuracy_dictionary.items()) # sorted by key, return a lis
plt.figure(figsize=(12, 8))
x, y = zip(*lists) # unpack a list of pairs into two tuples
plt.subplot( 3, 1,1)
plt.plot(x, y)
plt.grid()
plt.show()

print("Differnce between FPR in Sensitive Variables Plot vs Threshold")
lists = sorted(diff_fpr_dictionary.items()) # sorted by key, return a lis
plt.figure(figsize=(12, 8))
x, y = zip(*lists) # unpack a list of pairs into two tuples
plt.subplot(3, 1,2)
plt.plot(x, y)
plt.grid()
plt.show()

print("Differnce between FNR in Sensitive Variables Plot vs Threshold")
lists = sorted(diff_fnr_dictionary.items()) # sorted by key, return a lis
plt.figure(figsize=(12, 8))
x, y = zip(*lists) # unpack a list of pairs into two tuples
plt.subplot(3, 1,3)
plt.plot(x, y)
plt.grid()
plt.show()
```

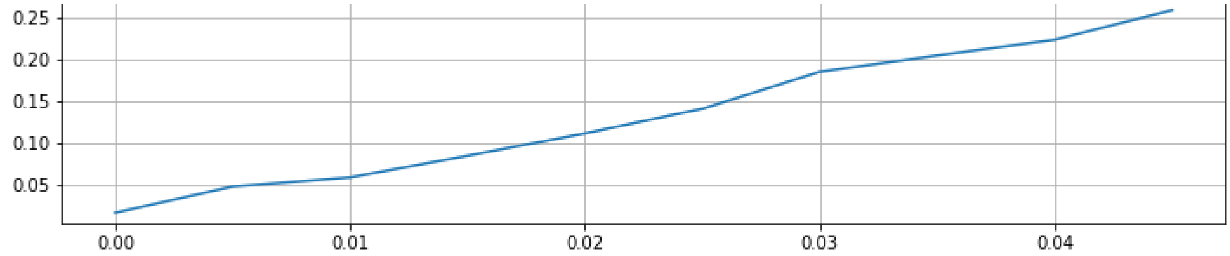
```
Constraints of FNR and FPR
Accuracy Plot vs Threshold
```




```
Differnce between FPR in Sensitive Variables Plot vs Threshold
```




```
Differnce between FNR in Sensitive Variables Plot vs Threshold
```



Here also we see that as threshold(unfairness) rises the accuracy increases. With rise of unfairness the fpr and well as fnr increases

In [ ]:

PAPER 1: Learning Fair Representations

This is a feature selection algorithm. Given data of x's, y and a sensitive variable (in the case of Compas, race), our goal was to calculate shapely values using observed conditional probabilities and the set structure of the data to estimate marginal accuracy and discrimination trade-off if a target variable is eliminated from the x matrix.

To build the code we basically tried to translate the proofs in the appendix to code, and were able to successfully implement this algorithm.

We ran this algorithm on a set of five candidate variables, and determined, on the basis of a high discrimination coefficient, we could likely remove priors_count from training sets.

Following this prescription modeled a logistic regression problem on the edited data. We looked at train and test accuracy and found that Black and White accuracies were comparable, but African American test accuracies were about 3.5% lower than those of White group.

In [ ]:
```
#helper function testing environment::
def helper_data_cleaning(x):
    #This code subsets the dataframe to relevant fields for the purpose o
    df = x[x['race'].isin(['African-American','Caucasian'])]
    idx = np.where((df['days_b_screening_arrest']<=30) & (df['days_b_scree
                  (df['is_recid']!=-1) & (df['c_charge_degree']!="O") &
                  ((df['race']=="African-American") |(df['race']=="Cauca
    df = df.iloc[idx]
    df=df.filter(items=['raw_data', 'age', 'c_charge_degree', 'race', 'age
                  'days_b_screening_arrest', 'decile_score', 'is_recid'

    return df

def encoding_data(x):

    x['race'] = x['race'].apply(lambda x: 1 if x == "Caucasian" else 0)
    #print(x['race'])
    x['sex'] = x['sex'].apply(lambda x: 1 if x == "Female" else 0)

    df1 = x
    df1['length_of_stay']=df1['c_jail_out'].apply(pd.to_datetime) - df1['
    df1['length_of_stay']=df1['length_of_stay'].dt.days
    df1['length_of_stay'] = df1.length_of_stay.apply(lambda x: 2  if x >1
    df1['priors_count'] = df1['priors_count'].apply(lambda x: 0 if x == 0
    df1['priors_count'] = df1['priors_count'].apply(lambda x: 1 if (1 <= 
    df1['priors_count'] = df1['priors_count'].apply(lambda x: 2 if x > 3 
    # Label Encoding
    #df1['race'] = df1['race'].apply(lambda x: 1 if x == 'Caucasian' else
    #df1['race'] = df1['race_2']
    #df1.loc[df1.race=='Caucasian']=1
    #df1.loc[df1.race=='African-American']=0

    categorical_variables = ['c_charge_degree','sex','age_cat','score_tex
    for var in categorical_variables:
        df1[var] = df1[var].astype('category').cat.codes
    final_vars = ['sex','age_cat','race','priors_count','c_charge_degree'
    df1 = df1[final_vars]
    y = x['two_year_recid']
    return y,df1
```

In [ ]:
```
#cleaning/prepping data
df_1 = helper_data_cleaning(df)
y, df_1 = encoding_data(df_1)
```

In [ ]:
```
def unique_information_coef(x, y):
    import itertools
    nrow = x.shape[0]
    x_shap = x.shape[1]
    z = np.concatenate((x,y), axis=1)

    #unique combinations of possible encodings
    unique_combo = []
    for r in z.T:
        unique_combo.append(np.unique(r).tolist())

    cartesian = list(itertools.product(*unique_combo))

    running_p = 0


    for possible in cartesian:
        #print(ff," ", possible)
        mutual_ct, r_1_ct, r_2_ct = 0,0,0
        mutual_ct = np.sum(np.all(possible == z, axis=1))
        r_1_ct = np.sum(np.all(possible[:x_shap] == z[:,:x_shap], axis=1)
        r_2_ct = np.sum(np.all(possible[x_shap:] == z[:,x_shap:], axis=1)

        #for row in z:
            #checking if joint
         #   if possible == row:
          #      mutual_ct = mutual_ct+1
            #checking x
           # if possible[:x_shap]== z[:x_shap]:
            #    r_1_ct = r_1_ct +1
            #checking y
            #if possible[x_shap:]== z[x_shap:]:
             #   r_2_ct = r_2_ct +1


        #saving computation in the event there's a zero result
        if (mutual_ct == 0 or r_1_ct == 0 or r_2_ct == 0):
            intermed = 0
        else:
            mutual_p = mutual_ct/nrow
            pr1 = r_1_ct/nrow
            pr2 = r_2_ct/nrow
            intermed = mutual_p * np.log(mutual_p / pr1) / pr1
        running_p += abs(intermed)


    return running_p

def conditional_info_coef(x,y,c):
    import itertools
    nrow = x.shape[0]
    x_shap = x.shape[1]
    y_shap = y.shape[1]
    #print(x.shape,y.shape,c.shape)
    z = np.concatenate((y,x,c), axis=1)
```

```
    #unique combinations of possible encodings
    unique_combo = []
    for r in z.T:
        unique_combo.append(np.unique(r).tolist())

    cartesian = list(itertools.product(*unique_combo))

    running_p = 0


    for possible in cartesian:

        #print(ff," ", possible)
        mutual_ct, r_1_ct, r_2_ct, r_cond = 0,0,0,0
        mutual_ct = np.sum(np.all(possible == z, axis=1))
        # r_1_ct = np.sum(np.all(possible[:1] == z[:,:1], axis=1))
        # r_2_ct = np.sum(np.all(possible[1:-x_shap] == z[:,1:-x_shap], a
        r_1_ct = np.sum(np.all(possible[:y_shap] == z[:,:y_shap], axis=1)
        r_2_ct = np.sum(np.all(possible[y_shap:-x_shap] == z[:,y_shap:-x_
        cond_ct_num = np.sum(np.where((possible[:y_shap] == z[:,:y_shap])
        cond_ct_den = np.sum(np.where((possible[-x_shap:] == z[:,-x_shap:
        if cond_ct_den == 0:
            r_cond = 0
        else:
            r_cond = cond_ct_num/cond_ct_den



        #saving computation in the event there's a zero result
        if (mutual_ct == 0 or r_1_ct == 0 or r_2_ct == 0 or r_cond ==0):
            intermed = 0
        else:
            mutual_p = mutual_ct/nrow
            pr1 = r_1_ct/nrow
            pr2 = r_2_ct/nrow
            intermed = mutual_p * np.log(mutual_p / pr2) / r_cond

        running_p += abs(intermed)


    return running_p

def powerset(seq):
    if len(seq) <= 1:
        yield seq
        yield []
    else:
        for item in powerset(seq[1:]):
            yield [seq[0]]+item
            yield item
```

In [ ]:
```
def shapelydiscrimination(x,y):

    features = list(x.columns)
    features = [ele for ele in features if ele != 'race'] #removing race 
    nfeat = len(features)

    shapely_coeff = []

    #identifying a feature sequentially to exclude from infoset calc
    for i in range(nfeat):

        features_2 = features.copy()
        rem = features_2.pop(i) #removing a feature i programmatically
        pow_set = [elm for elm in powerset(features_2)] #generating power

        shapely_inter = 0 #for summing the shapelys
        #now, iterating over the subsets
        for subset in pow_set:
            s_len = len(subset)
            f_len = nfeat - 1 - len(subset)

            m_coefficient = math.factorial(s_len) * math.factorial(f_len)

            #inclusive discrimination metric
            inc_sub = subset.copy()
            inc_sub.append(rem)

            x_1 = np.array(x[inc_sub])
            #print("x:",)
            protected_attribute = np.array(x['race']).reshape(-1,1)
            #print(protected_attribute.shape)
            y_1 = np.array(y).reshape(-1,1)

            info_in_ = np.concatenate((x_1,protected_attribute), axis = 1

            incl_a = unique_information_coef(info_in_,y_1)
            incl_b = unique_information_coef(x_1,protected_attribute)
            incl_c = conditional_info_coef(x_1,protected_attribute,y_1)

            incl = incl_a * incl_b * incl_c
            #call info right here

            #exclusive discrimination metric
            x_2 = np.array(x[subset])
            info_ex_ = np.concatenate((x_2,protected_attribute), axis = 1
            excl_a = unique_information_coef(info_ex_,y_1)
            excl_b = unique_information_coef(x_2,protected_attribute)
            excl_c = conditional_info_coef(x_2,protected_attribute,y_1)
            excl = excl_a * excl_b * excl_c

            #call info right here
            marginal_discrimination = incl-excl
            #marginal = disc_incl - disc_excl
            #shapley_coeff[i] = shapley_coeff[i] + coef * marginal
            shapely_inter = shapely_inter + m_coefficient * marginal_disc
        shapely_coeff.append(shapely_inter)
```

```
    return shapely_coeff

def shapelyaccuracy(x,y):
    features = list(x.columns)
    features = [ele for ele in features if ele != 'race'] #removing race
    nfeat = len(features)

    shapely_coeff = []

    #identifying a feature sequentially to exclude from infoset calc
    for i in range(nfeat):

        features_2 = features.copy()
        rem = features_2.pop(i) #removing a feature i programmatically
        pow_set = [elm for elm in powerset(features_2)] #generating power

        shapely_inter = 0 #for summing the shapelys
        #now, iterating over the subsets
        for subset in pow_set:
            s_len = len(subset)
            f_len = nfeat - 1 - len(subset)

            m_coefficient = math.factorial(s_len) * math.factorial(f_len)

            #inclusive accuracy metric
            inc_sub = subset.copy()
            inc_sub.append(rem)
            inc_sub_diff = list(set(features)-set(inc_sub))
            x_1c = np.array(x[inc_sub_diff])
            x_1 = np.array(x[inc_sub])

            protected_attribute = np.array(x['race']).reshape(-1,1)

            y_1 = np.array(y).reshape(-1,1)

            c = np.concatenate((x_1c,protected_attribute), axis = 1)

            #print( "out2: ",get_conditional_info_coef(y_1,x_1,c))
            accuracy_inclusive = conditional_info_coef(x_1,y_1,c)
            #call info right here

            #exclusive accuracy metric
            x_2 = np.array(x[subset])
            excl_sub = features_2
            excl_sub_diff = list(set(features_2)-set(subset))
            x_2c = np.array(x[excl_sub_diff])
            d = np.concatenate((x_2c,protected_attribute), axis = 1)
            accuracy_exclusive = conditional_info_coef(x_2,y_1,d)

            #marginal accuracy by including
            marginal_acc = accuracy_inclusive - accuracy_exclusive
            shapely_inter = shapely_inter + m_coefficient * marginal_acc

        if shapely_inter> 0:
            shapely_coeff.append(shapely_inter)
    return shapely_coeff
```

In [ ]:
```
import time
start = time.time()
accuracy = shapelyaccuracy(df_1,y)
discrimination = shapelydiscrimination(df_1,y)
end = time.time()
print(end - start)
```

```
26.75325107574463
```

In [ ]:
```
features = list(df_1.columns)
features = [ele for ele in features if ele != 'race']
acc_disc_pretty = pd.DataFrame(list(zip(features,accuracy,discrimination)
acc_disc_pretty.columns = ['variable','accuracy','discrimination']
print(acc_disc_pretty)
varlist_for_model = list(acc_disc_pretty['variable'])
varlist_for_model.remove('priors_count')
print(varlist_for_model)
```

```
          variable  accuracy  discrimination
0              sex  0.607707     5933.746182
1          age_cat  0.892485     8871.607730
2     priors_count  0.984446     9024.280430
3  c_charge_degree  0.730598     7155.980280
4   length_of_stay  0.668507     8352.821957
['sex', 'age_cat', 'c_charge_degree', 'length_of_stay']
```

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Logistic regression

We now remove variable with highest discrimination and are left with tolerble or unaviodable

In [ ]:
```
from sklearn.linear_model import LogisticRegression
from sklearn.model_selection import train_test_split
from sklearn.metrics import accuracy_score
from sklearn.metrics import confusion_matrix
df_m = df_1[varlist_for_model]
x_train, x_test, y_train, y_test = train_test_split(df_m, y, test_size=0.
model = LogisticRegression()
model.fit(x_train,y_train)
y_hat = model.predict(x_test)
test_acc = accuracy_score(y_hat,y_test)
y_hat_train = model.predict(x_train)
train_acc = accuracy_score(y_hat_train,y_train)
```

In [ ]:
```
x_train = pd.merge(x_train, df_1, left_index=True, right_index=True)
x_train['pred'] = y_hat_train
x_train['act'] = y_train
x_test = pd.merge(x_test, df_1, left_index=True, right_index=True)
x_test['pred'] = y_hat
x_test['act'] = y_test
```

In [ ]:
```
x_train_c = x_train[x_train['race'] == 1]
x_test_c = x_test[x_test['race'] == 1]
x_train_nc = x_train[x_train['race'] == 0]
x_test_nc = x_test[x_test['race'] == 0]
White_train_acc = accuracy_score(x_train_c['pred'],x_train_c['act'])
Black_train_acc = accuracy_score(x_train_nc['pred'],x_train_nc['act'])
white_test_acc = accuracy_score(x_test_c['pred'],x_test_c['act'])
Black_test_acc = accuracy_score(x_test_nc['pred'],x_test_nc['act'])
```

In [ ]:
```


print("Race.          Train                          Test")
print("Overall.      ", train_acc,"     ",test_acc)
print("White(0)      ",White_train_acc,"     ",white_test_acc)
print("Black(1)      ",Black_train_acc,"     ",Black_test_acc)
```

```
Race.          Train                      Test
Overall.       0.5709258256632377       0.5555555555555556
White(0)       0.5740868366643694       0.5782208588957055
Black(1)       0.5688809629959876       0.5396995708154506
```

In [ ]:
```
# Overall confusion matrix
tn, fp, fn, tp = confusion_matrix(x_test['act'],x_test['pred']).ravel()
print("True Negative: ",tn," True Positive: ",tp," False Negative: ", fn,
print('-------------------------------')
print("FPR                                FNR")
print(fp/(fp+tp),'       ',fn/(fn+tn))
```

```
True Negative:  598  True Positive:  282  False Negative:  432 False Posi
tive:  272
------------------------------
FPR                            FNR
0.49097472924187724         0.41941747572815535
```

In [ ]:

```
# Races confusion matrix
# White
w_tn, w_fp, w_fn, w_tp= confusion_matrix(x_test_c['act'],x_test_c['pred']
#Black
b_tn, b_fp, b_fn, b_tp= confusion_matrix(x_test_nc['act'],x_test_nc['pred

#Print
print(print("Race.    FPR                              FNR"))
print("White(0)",'    ',w_fp/(w_fp+w_tp),'        ',w_fn/(w_fn+w_tn))
print("Black(1)",'    ',b_fp/(b_fp+b_tp),'        ',b_fn/(b_fn+b_tn))
```

```
Race.    FPR                              FNR
None
White(0)     0.5566037735849056         0.3568181818181818
Black(1)     0.4502923976608187         0.4661016949152542
```

In [ ]:

COMPARISON BETWEEN THE 2 PAPERS

From Paper A4

In [ ]:

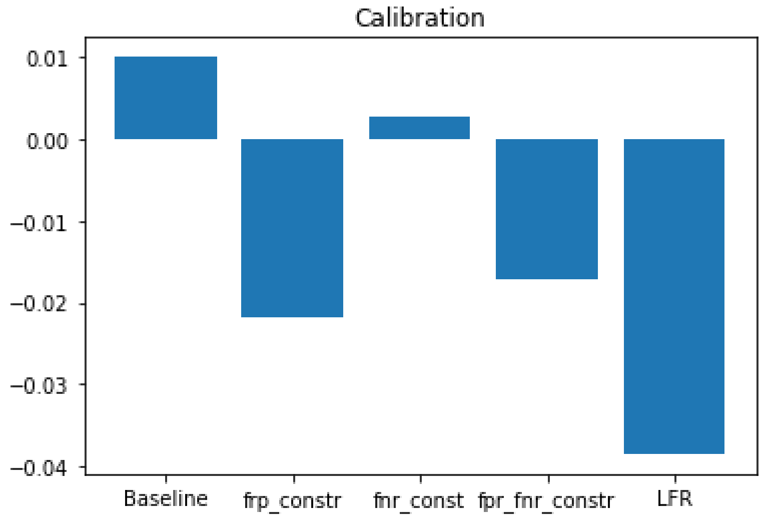
```
threshold = 0
print("Threshold is " + str(threshold))
print("-----------------------------------------------------------------
print()
# Baseline
acc_cons_bas, diff_fpr_bas, diff_fnr_bas, acc_w_bas,acc_b_bas = accuracy_
# With constraint
acc_cons_fpr, diff_fpr_fpr, diff_fnr_fpr, acc_w_fpr,acc_b_fpr= accuracy_2
acc_cons_fnr, diff_fpr_fnr, diff_fnr_fnr,acc_w_fnr,acc_b_fnr = accuracy_3
acc_cons_fpr_fnr, diff_fpr_fpr_fnr, diff_fnr_fpr_fnr,acc_w_fpr_fnr,acc_b_
```

```
Threshold is 0
--------------------------------------------------------------------------
----------------------

Unconstrained classifier
Accuracy:  0.676
s      FPR.     FNR.     TNR.     TPR.    Accuracy
-----------------------------------------
0      0.38     0.27     0.62     0.73    0.6719643992371265
-----------------------------------------
1      0.16     0.57     0.84     0.43    0.6819887429643527

FPR constraint classifier
Accuracy:  0.6669
s      FPR.     FNR.     TNR.     TPR.    Accuracy
-----------------------------------------
0      0.27     0.39     0.73     0.61    0.6757787666878576
-----------------------------------------
1      0.32     0.4     0.68     0.6    0.6538461538461539

FNR constraint classifier
Accuracy:  0.6756
s      FPR.     FNR.     TNR.     TPR.    Accuracy
-----------------------------------------
0      0.28     0.37     0.72     0.63    0.6745073108709473

-----------------------------------------
1      0.25     0.44     0.75     0.56    0.6772983114446529

FPR & FNR constraint classifier
Accuracy:  0.6707
s      FPR.     FNR.     TNR.     TPR.    Accuracy
-----------------------------------------
0      0.24     0.4     0.76     0.6    0.6776859504132231

-----------------------------------------
1      0.29     0.42     0.71     0.58    0.6604127579737336
```

Callibration

In [ ]:
```
#  Callibration

plt.bar(np.arange(5),[acc_b_bas-acc_w_bas,acc_b_fpr-acc_w_fpr,acc_b_fnr-ac
plt.xticks(np.arange(5),['Baseline','frp_constr','fnr_const','fpr_fnr_cons
plt.title("Calibration")
```

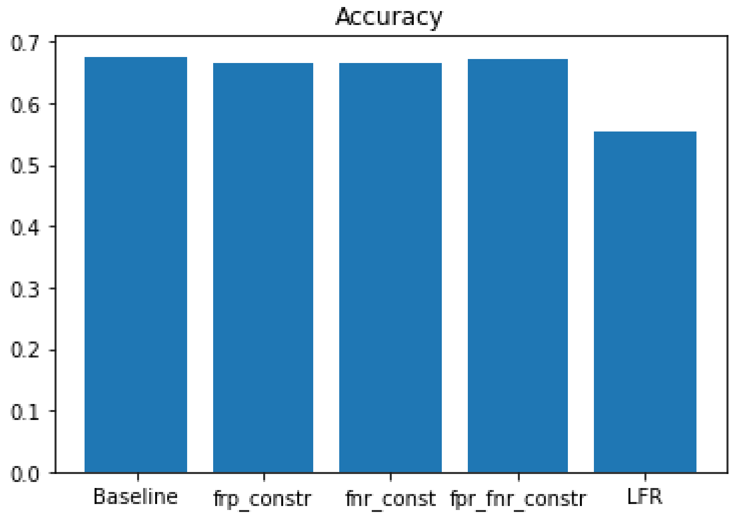
Out[224]:
```
Text(0.5, 1.0, 'Calibration')
```



Accuracy

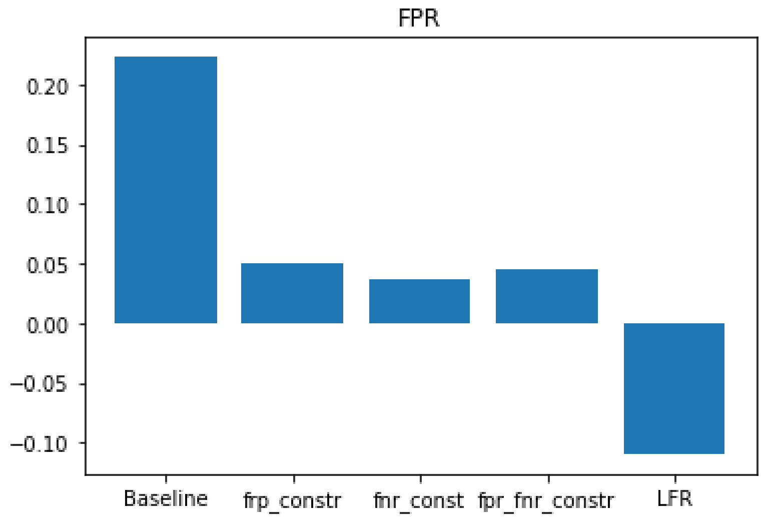
In [ ]:
```
# Accuracy
plt.bar(np.arange(5),[acc_cons_bas,acc_cons_fpr,acc_cons_fpr,acc_cons_fpr_
plt.xticks(np.arange(5),['Baseline','frp_constr','fnr_const','fpr_fnr_cons
plt.title("Accuracy")
```

Out[225]:
```
Text(0.5, 1.0, 'Accuracy')
```



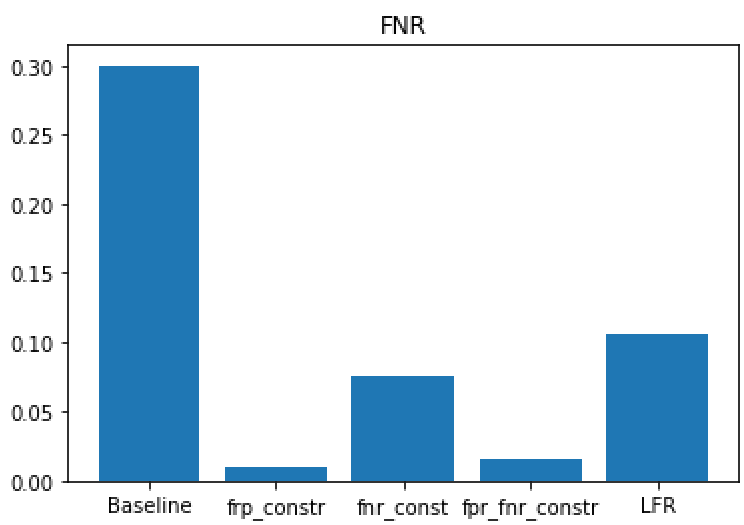
In [ ]:
```
# FPR
plt.bar(np.arange(5),[diff_fpr_bas,diff_fpr_fpr,diff_fpr_fnr,diff_fpr_fpr_
plt.xticks(np.arange(5),['Baseline','frp_constr','fnr_const','fpr_fnr_cons
plt.title("FPR").   # (black -white>)
```

Out[247]: Text(0.5, 1.0, 'FPR')



In [ ]:
```
# FNR
plt.bar(np.arange(5),[diff_fnr_bas,diff_fnr_fpr,diff_fnr_fnr,diff_fnr_fpr_
plt.xticks(np.arange(5),['Baseline','frp_constr','fnr_const','fpr_fnr_cons
plt.title("FNR")
```

Out[248]: Text(0.5, 1.0, 'FNR')



In [ ]:

In [ ]:

CONCLUSION:

1. We see that usually when unfairness rises accuracy increases.
2. Paper A4 algorithms(Having fpr constraint,fnr constraint, fpr and fnr constraint) provides a better accuracy as well as better calibration than LFR algorithm
3. We see fpr and fnr is lesser than the baseline models

In [ ]:

In [ ]: