Quadratic Formula

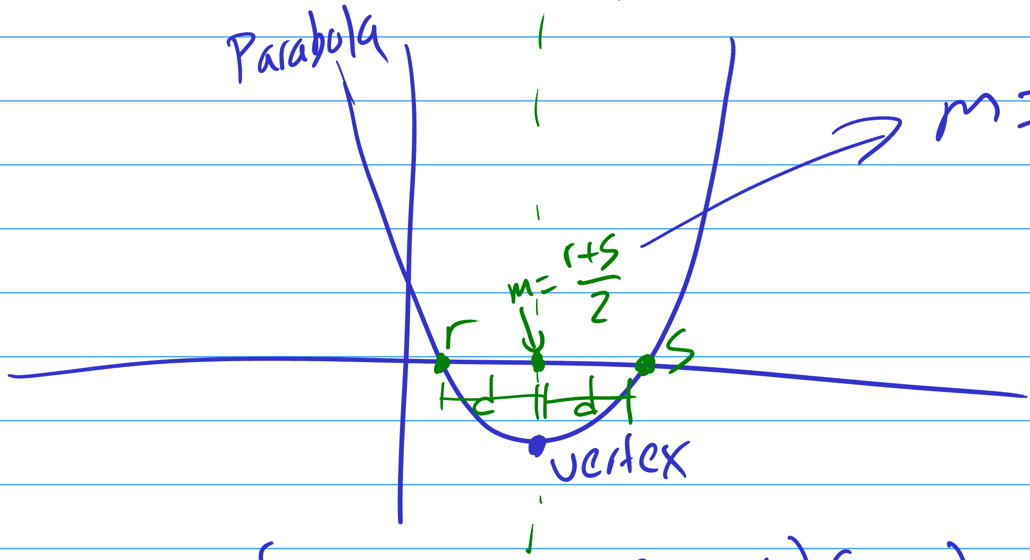
$$ax^2+bx+c=0$$

r, s roots

Vieta's form.

$$rs=\frac{c}{a}$$

$$r+s=\frac{-b}{a}$$



$$m=\frac{r+s}{2}=\frac{-b}{2a}$$

$$r=m-d$$
$$s=m+d$$

$$rs=(m-d)(m+d)=\frac{c}{a}$$

$$m^2-d^2=\frac{c}{a}$$

$$m^2-\frac{c}{a}=d^2$$

$$\sqrt{m^2-\frac{c}{a}}=d$$

$$r=m-\sqrt{m^2-\frac{c}{a}}$$

$$s=m+\sqrt{m^2-\frac{c}{a}}$$

$$\boxed{x=m\pm\sqrt{m^2-\frac{c}{a}}\,,\quad m=-\frac{b}{2a}}$$

$$x=-\frac{b}{2a}\pm\sqrt{\left(\frac{-b}{2a}\right)^2-\frac{c}{a}}=-\frac{b}{2a}\pm\sqrt{\frac{b^2}{4a^2}-\frac{c}{a}}$$

$$=-\frac{b}{2a}\pm\sqrt{\frac{b^2-4ac}{4a^2}}=\boxed{\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

$$ax^2+bx+c=0$$

$$\boxed{x^2+\frac{b}{a}x+\frac{c}{a}=0}$$

$$\frac{b}{a}=-2m \Rightarrow m=\frac{-b}{2a}$$

$$\frac{c}{a}=m^2+n \Rightarrow n=\frac{c}{a}-\frac{b^2}{4a^2}$$

$$(x-m)^2+n=0$$

$$x^2-2mx+m^2+n=0$$

$$\boxed{\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}=0}$$

vertex: $\left(-\frac{b}{2a}, \frac{c}{a}-\frac{b^2}{4a^2}\right)$

$$x^2+3x+1=0$$

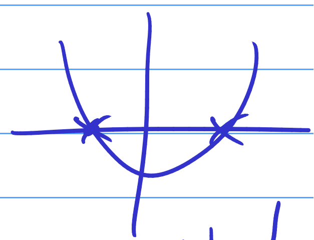
$$\left(x+\frac{3}{2}\right)^2+1-\frac{9}{4}=0$$

$$\left(x+\frac{3}{2}\right)^2-\frac{5}{4}=0$$

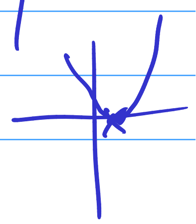
vertex $\left(-\frac{3}{2}, -\frac{5}{4}\right)$

Discriminant

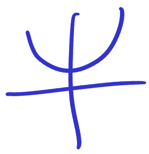
$b^2-4ac>0 \Rightarrow$ 2 real root



$b^2-4ac=0 \Rightarrow$ 1 double real root



$b^2-4ac<0 \Rightarrow$ 0 real roots, 2 complex roots



$x^2-4x+7=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{4\pm\sqrt{\overbrace{16-4\cdot7}^{-12}}}{2}=\underbrace{2}_{m}\pm\underbrace{\sqrt{3}}_{n}\,i$$

$m \pm n\, i$

$x^2-3x+10=0$

$$x=\frac{3\pm\sqrt{9-40}}{2}=\frac{3}{2}\pm\frac{\sqrt{31}}{2}\,i$$

$m \pm n\, i$

$2+\sqrt{3}\,i$ complex conjugates
$2-\sqrt{3}\,i$

ax^2+bx+c, a,b,c are real numbers

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\underbrace{\frac{-b}{2a}}_{m}\pm\underbrace{\frac{\sqrt{b^2-4ac}}{2a}}_{n}\,i$$

$m \pm n\, i$

Complex Conjugates Theorem:
roots of ax^2+bx+c, a,b,c real, are complex conjugates

$m+ni \longleftrightarrow m-ni$

$$x^2 - 4x - 7 = 0$$

$$x = \frac{4 \pm \sqrt{4^2 + 4 \cdot 7}}{2} = 2 \pm \sqrt{11}$$

Radical conjugate Theorem

$ax^2 + bx + c$, a, b, c rational:

roots are $m - \sqrt{n}$, $m + \sqrt{n}$

Ex: Find a polynomial of minimum deg. with rationals roots, that has $2 - \sqrt{3}$ and $3 + 2\sqrt{5}$ as roots

$2 - \sqrt{3}$ $\quad$ $3 + 2\sqrt{5}$

$2 + \sqrt{3}$ $\quad$ $3 - 2\sqrt{5}$

$$((x-2)+\sqrt{3})((x-2)-\sqrt{3})((x-3)-2\sqrt{5})((x-3)+2\sqrt{5})$$

$$(x^2 + 4x + 1) \quad (x^2 - 6x - 11)$$

$$= x^4 - 10x^3 + 14x^2 + 38x - 11$$

Ex: $x^2 + bx + c = 0$, b, c are rational, $2 - \sqrt{3}$ is a root

Find b and c

$$2 - \sqrt{3} + 2 + \sqrt{3} = \frac{-b}{1} \Rightarrow \boxed{4 = -b,\ b = -4}$$

$$(2 - \sqrt{3})(2 + \sqrt{3}) = \frac{c}{1} \Rightarrow \boxed{1 = c}$$

Vieta's for polynomials

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x + a_0 = 0$$

$$a_n (x - r_1)(x - r_2) \cdots (x - r_n) = 0$$

$$a_n \Big(x^n - (r_1 + r_2 + r_3 \ldots + r_n) x^{n-1} + (r_1 r_2 + r_1 r_3 + \ldots) x^{n-2}$$
$$+ (r_1 r_2 r_3 + r_1 r_2 r_4 + \ldots) x^{n-3} + \ldots$$
$$\underbrace{(-r_1)(-r_2) \cdots (-r_n)}_{(-1)^n r_1 r_2 r_3 \cdots r_n} \Big)$$

$$a_{n-1} x^{n-1} = a_n \left(-(r_1 + r_2 + r_3 + \ldots + r_n) x^{n-1} \right)$$

$$\boxed{\begin{aligned} -\frac{a_{n-1}}{a_n} &= r_1 + r_2 + \ldots + r_n \\ \frac{a_{n-2}}{a_n} &= r_1 r_2 + r_1 r_3 + r_1 r_4 + \ldots r_{n-1} r_n \\ &\vdots \\ (-1)^n \frac{a_0}{a_n} &= r_1 r_2 r_3 \cdots r_n \end{aligned}}$$

Ex: Let p, q, r are the roots of $x^3 - 4x^2 + 15x - 7 = 0$

Find $p + q + r = -\frac{-4}{1} = 4$

$$pqr = -\frac{-7}{1} = 7$$

$$pq+qr+rp = \frac{15}{1} = 15$$

Ex: Let r,s,t be the solutions to $3x^3-4x^2+5x+7=0$

$$r+s+t = -\frac{-4}{3} = \frac{4}{3}$$

$$r^2+s^2+t^2 = \boxed{-\frac{14}{9}}$$

$$r+s+t = \frac{4}{3}$$

$$r^2+s^2+t^2+\underbrace{2rs+2st+2rt}_{2\cdot\frac{5}{3}} = \frac{16}{9}$$

$$\Rightarrow r^2+s^2+t^2 = \frac{16}{9} - \frac{10}{3} = -\frac{14}{9}$$

$$\frac{1}{r}+\frac{1}{s}+\frac{1}{t} = \frac{st+rt+rs}{rst} = \frac{\frac{5}{3}}{-\frac{7}{3}} = \boxed{-\frac{5}{7}}$$