

The Ninth Grade Math Competition Class
Factorials and Palindrome
Anthony Wang

1. What is the largest 4-digit palindrome that is the sum of 2 different 3-digit palindromes?

$$\begin{array}{r} 999 \\ + 666 \\ \hline 1665 \end{array}$$

$$\begin{array}{r} 1 \quad A \quad A \quad 1 \\ \hline \quad 5 \quad 5 \quad 5 \\ + \quad 6 \quad 6 \quad 6 \\ \hline \boxed{1 \quad 2 \quad 2 \quad 1} \end{array}$$

2. Find the largest n for which 12^n evenly divides $20!$. ≤ 18 ≤ 8

$$2n \leq 18$$

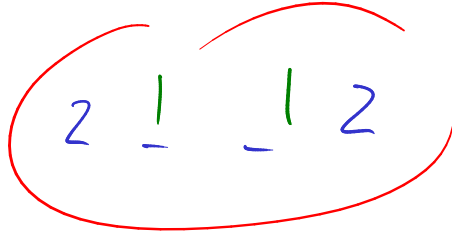
$$n \leq 8$$

$$(12)^n = (2^2 \cdot 3)^n = 2^{2n} \cdot 3^n$$

$$\left\lfloor \frac{20}{3} \right\rfloor = \left\lfloor \frac{6}{3} \right\rfloor = \left\lfloor \frac{2}{3} \right\rfloor = 0$$

$$\left\lfloor \frac{20}{2} \right\rfloor = \left\lfloor \frac{10}{2} \right\rfloor = \left\lfloor \frac{5}{2} \right\rfloor = \left\lfloor \frac{2}{2} \right\rfloor = \left\lfloor \frac{1}{2} \right\rfloor = 0$$

3. What is the first year after 2018 that is a palindrome?



4. What is the product of the largest 3 digit palindrome and the least 3 digit palindrome?

999

101

999

, 101

999

999

100899

5. How many 5-digit palindromes are there?

$$\begin{array}{c} \overline{A} \quad \overline{B} \\ 9 \quad 10 \end{array} \quad \begin{array}{c} \overline{C} \quad \overline{B} \\ 10 \quad 1 \end{array} \quad \begin{array}{c} \overline{A} \\ 1 \end{array} = \boxed{900}$$

6. Find the sum of all 3-digit palindromes.

$$1+2+3+4+5+6 = 7 \cdot 3 = 21$$

$$101, 111, 121, 131, \dots, 979, 989, 999$$

1100

$$1100$$

$$\begin{array}{ccc} A & B & A \\ 9 & 10 & 1 = 90 \end{array}$$

$$\underbrace{1100}_{\text{sum of pair}} \cdot \underbrace{\frac{90}{2}}_{\text{\# of pairs}} = 49500$$

7. Palindromic primes are numbers that are both palindromic and prime. Find the greatest 3-digit palindromic prime?

$$\times 999 = 9 \cdot 111 = 3 \cdot 333$$

$$\times 989 = 23 \cdot 43$$

$$\times 979 = 11 \cdot 89$$

$$\times 969 = 3 \cdot 323$$

$$\times 959 = 7 \cdot 137$$

$$\times 949 = 13 \cdot 73$$

$$\times 939 = 3 \cdot 313$$

$$\circledast 929$$

10001, 10101, ..., 99999, 99999

$$S = 10000 \cdot \frac{900}{2} = 495000.$$

8. A five-digit palindrome is a positive integer with respective digits $abcba$, where a is non-zero. Let S be the sum of all five-digit palindromes. What is the sum of the digits of S ?

9. h There are unique integers $a_2, a_3, a_4, \dots, a_7$ such that

$$\frac{5}{7} = \frac{a_2}{2!} + \frac{a_3}{3!} + \frac{a_4}{4!} + \frac{a_5}{5!} + \frac{a_6}{6!} + \frac{a_7}{7!},$$

with $0 \leq a_i \leq i$, for $i = 2, 3, \dots, 7$. Find $a_2 + a_3 + \dots + a_7$.

$1042 = 9$

$0 \leq a_7 < 7$

$$5 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 a_2 + 7 \cdot 6 \cdot 5 \cdot 4 a_3 + 7 \cdot 6 \cdot 5 a_4 + 7 \cdot 6 a_5 + 7 a_6 + a_7$$

$$51422 = 7(3600) + a_7$$

$$= 7(\underline{514}) + \underline{2}$$

$514 = 6 \cdot 5 \cdot 4 \cdot 3 a_2 + 6 \cdot 5 \cdot 4 a_3 + 6 \cdot 5 a_4 + 6 a_5 + a_6$

$514 = 6(\underline{85}) + \underline{4}$

$0 \leq a_6 < 6$