Quadratic Formula

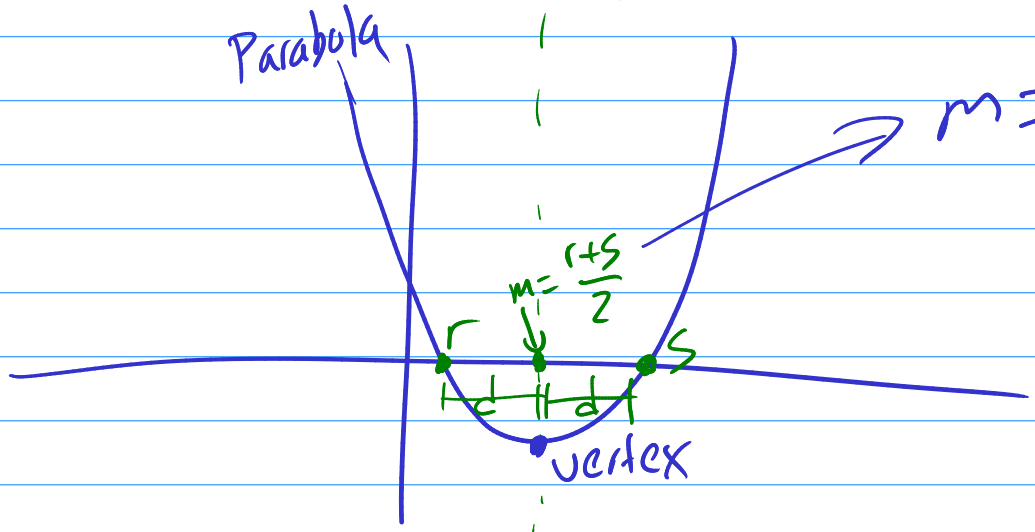
$ax^2+bx+c=0$

r, s roots

Vieta's form.

$$rs = \frac{c}{a}$$

$$r+s = -\frac{b}{a}$$



$$m = \frac{r+s}{2} = \frac{-b}{2a}$$

$r = m-d$

$s = m+d$

$$rs = (m-d)(m+d) = \frac{c}{a}$$

$$m^2 - d^2 = \frac{c}{a}$$

$$m^2 - \frac{c}{a} = d^2$$

$$\sqrt{m^2 - \frac{c}{a}} = d$$

$r = m - \sqrt{m^2 - \frac{c}{a}}$

$s = m + \sqrt{m^2 - \frac{c}{a}}$

$$\boxed{x = m \pm \sqrt{m^2 - \frac{c}{a}}, \quad m = -\frac{b}{2a}}$$

$$x = -\frac{b}{2a} \pm \sqrt{\left(\frac{-b}{2a}\right)^2 - \frac{c}{a}} = -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$$

$$= -\frac{b}{2a} \pm \sqrt{\frac{b^2-4ac}{4a^2}} = \boxed{\frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

$$ax^2+bx+c=0$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$\frac{b}{a}=-2m \Rightarrow m=\frac{-b}{2a}$$

$$\frac{c}{a}=m^2+n \Rightarrow n=\frac{c}{a}-\frac{b^2}{4a^2}$$

$$(x-m)^2+n=0$$

$$x^2-2mx+m^2+n=0$$

$$\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}=0$$

vertex: $\left(-\frac{b}{2a}, \frac{c}{a}-\frac{b^2}{4a^2}\right)$

$$x^2+3x+1=0$$

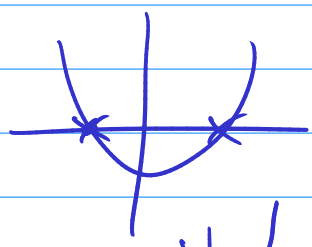
$$\left(x+\frac{3}{2}\right)^2+\left(-\frac{9}{4}\right)=0$$

$$\left(x+\frac{3}{2}\right)^2-\frac{5}{4}=0$$

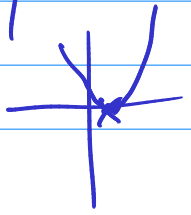
vertex $\left(-\frac{3}{2}, -\frac{5}{4}\right)$

Discriminant

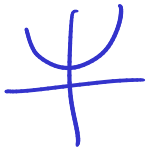
$b^2-4ac>0 \Rightarrow$ 2 real root



$b^2-4ac=0 \Rightarrow$ 1 doubl real root



$b^2-4ac<0 \Rightarrow$ 0 real roots, 2 complex rools



$x^2 - 4x + 7 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{4 \pm \sqrt{16-4\cdot 7}}{2} = \frac{4 \pm \sqrt{-12}}{2} = 2 \pm \sqrt{3}\,i$

$2 \pm \sqrt{3}\,i$: $m \pm n\,i$

$x^2 - 3x + 10 = 0$

$x = \frac{3 \pm \sqrt{9-40}}{2} = \frac{3}{2} \pm \frac{\sqrt{31}\,i}{2}$

$\frac{3}{2} \pm \frac{\sqrt{31}}{2}\,i$: $m \pm n\,i$

$2 + \sqrt{3}\,i$ complex conjugates
$2 - \sqrt{3}\,i$

$ax^2 + bx + c$, a, b, c are real numbers

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

$m \pm n\,i$

Complex Conjugates Theorem:
roots of $ax^2 + bx + c$, a, b, c real, are complex conjugates

$m + n\,i \longleftrightarrow m - n\,i$

$x^2 - 4x - 7 = 0$

$x = \frac{4 \pm \sqrt{4^2 + 4 \cdot 7}}{2} = 2 \pm \sqrt{11}$

Radical conjugate Theorem

$ax^2 + bx + c$, a, b, c rational:

roots are $m - \sqrt{n}$, $m + \sqrt{n}$

Ex: Find a polynomial of minimum deg. with rational roots, that has $2 - \sqrt{3}$ and $3 + 2\sqrt{5}$ as roots

$2 - \sqrt{3}$	$3 + 2\sqrt{5}$
$2 + \sqrt{3}$	$3 - 2\sqrt{5}$

$((x - 2) + \sqrt{3})((x - 2) - \sqrt{3})((x - 3) - 2\sqrt{5})((x - 3) + 2\sqrt{5})$

$(x^2 + 4x + 1)(x^2 - 6x - 11)$

$= x^4 - 10x^3 + 14x^2 + 38x - 11$

Ex: $x^2 + bx + c = 0$, b, c are rational, $2 - \sqrt{3}$ is a root. Find b and c

$2 - \sqrt{3} + 2 + \sqrt{3} = -\frac{b}{1} \Rightarrow 4 = -b,\ b = -4$

$(2 - \sqrt{3})(2 + \sqrt{3}) = \frac{c}{1} \Rightarrow 1 = c$