

Decimals

$$\frac{1}{2} = 0.5$$

$$\frac{1}{4} = 0.25$$

$$\frac{1}{8} = 0.125$$

$$\frac{1}{16} = 0.0625$$

$$\frac{1}{32} = 0.03125$$

$$\frac{1}{2^n} = \frac{5^n}{10^n}$$

$\underbrace{\hspace{1cm}}_{2^n \cdot 5^n}$

$$\frac{1}{5^n} = \frac{2^n}{10^n}$$

$$\frac{2^a 5^b}{n}$$

How many
digits: $\frac{n}{2^a 5^b}$

$\max(a, b)$

$$\frac{2^2 5^3}{3}$$

$$\frac{3}{2^2 5^3} = 0.006 = \frac{6}{1000}$$

$\underbrace{\hspace{1cm}}_{500}$

Repeating Decimals

$$\frac{1}{3} = 0.\overline{3}$$

[illegible]

$$\frac{1}{6} = 0.166666\ldots$$
$$= 0.\overline{16}$$

$$\frac{1}{7} = 0.\underbrace{142857}\underbrace{142857}\underbrace{14}\dots = 0.\overline{142857}$$

$$\frac{1}{12} = 0.0\overline{83} \quad 12 \sqrt{1.00}$$

$$\begin{array}{r} 0.0\overline{83} \\ - 96 \\ \hline 40 \\ - 36 \\ \hline 40 \\ - 36 \\ \hline 4 \end{array}$$

Find the smallest n s.t. $\frac{10^n}{12}$ reduces to a frac. with denom that's divisible by neither 2 nor 5

$$\frac{10^n}{2^2 \cdot 3}$$

$$n=1$$

$$\frac{10}{2^2 \cdot 3} = \frac{5}{2 \cdot 3}$$

$$n=2$$

$$\frac{10^2}{2^2 \cdot 3} = \frac{25}{3}$$

$\frac{a}{b}$ this terminates if and only if b has only 2s and 5s in its prime factorization

Ex: Find the 13th digit past the dec. point of $\frac{5}{14}$

$$\frac{5}{14} = \frac{1 \cdot 5}{2 \cdot 7} = \frac{5}{2} \cdot \frac{1}{7} = 0.35741258$$

↑ 1 2 3 4 5 6 7 8 9 10 11 12 13 14

digit past here

$$7 \overline{) 2.5}$$

$$\begin{array}{r} 7 \overline{) 2.5} \\ -21 \\ \hline 40 \\ -35 \\ \hline 50 \end{array}$$

$$\begin{array}{r}
 218 \text{ (RS)} \\
 6 \overline{) 1313} \\
 \underline{-12} \\
 11 \\
 6 \\
 \underline{53} \\
 48
 \end{array}$$

Ex: $0.\overline{19} \rightarrow \text{fraction}$

$$x = 0.\overline{19} = 0.1919191919 \dots$$

$$100x = 19.1919191919 \dots$$

$$\Rightarrow 99x = 19$$

$$x = \frac{19}{99}$$

Ex: $0.2\overline{76} \rightarrow \text{Fraction}$

$$x = 0.276767676 \dots$$

$$1000x = 276.76767676$$

$$100x = 27.676767676$$

$$99x = 27.4 \Rightarrow$$

$$x = \frac{274}{990} = \frac{137}{495}$$

Ex: $0.12\overline{345} \rightarrow \text{fraction}$

$$x = 0.12345345345 \dots$$

$$1000x = 123.45345345345 \dots$$

$$999x = 123.33$$

$$x = \frac{12333}{99900}$$