

**The Ninth Grade Math Competition Class**  
**Unit Digit**  
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1. What is the units digit of  $(3^{13})^3$ ?

$$(3^{13})^3 = 3^{39}$$

$$\begin{array}{l} 3^1 = 3 \\ 3^2 = 9 \\ 3^3 = \dots 7 \\ 3^4 = \dots 1 \\ 3^5 = \dots 3 \end{array} \quad \left. \vphantom{\begin{array}{l} 3^1 = 3 \\ 3^2 = 9 \\ 3^3 = \dots 7 \\ 3^4 = \dots 1 \\ 3^5 = \dots 3 \end{array}} \right\}$$

2. Find the units digit of  $n$  given that  $m \cdot n = 71^6$  and  $m$  has a units digit of 7.

7  
7, 3

**3. (a:)** Find the units digit of the sum

$$1! + 2! + 3! + \cdots + 2006!$$

**(b:)** Find the units digit of the above sum when it is expressed in base 7.

4. A positive two-digit integer is divisible by  $n$  and its units digit is  $n$ . What is the greatest possible value of  $n$ ?

5. Find the units digit of  $3^{2016} - 2^{2016}$ .

$3^{2016} - 2^{2016} = (-5) = 5$

$3^1 = 3$   
 $3^2 = 9$   
 $3^3 = 7$   
 $3^4 = 1$   
 $3^5 = 3$

$2^1 = 2$   
 $2^2 = 4$   
 $2^3 = 8$   
 $2^4 = 6$   
 $2^5 = 2$

6. The cube of the 3-digit natural number  $A7B$  is 108531333. What is  $A + B$ ?

7. How many of the positive divisors of  $6^{2006}$  have a units digit of 6?

8. (a) Convert 1599 to base 16.
- (b) Find all possible units digits of perfect fourth powers when written in base 16.
- (c) Determine all non-negative integral solutions  $(n_1, n_2, \dots, n_{14})$  if any, of the Diophantine equation.

$$n_1^4 + n_2^4 + \dots + n_{14}^4 = 1599.$$

(A Diophantine equation is an equation in which only integer solutions are allowed. )