The Ninth Grade Math Competition Class
Decimals
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1. Convert repeating decimal $0.\overline{3123}$ to fraction.

$x = 0.3123123123\ldots$

$1000x = 3123.123123\ldots$

$10000x = 3123.123123123\ldots$

$9999x = 3123$

$$x = \frac{3123}{9999} = \frac{347}{1111}$$

2. Compute $\frac{4!+3!}{3!+2!}$. Express your answer as a decimal to the nearest hundredth.

$\frac{4!+3!}{3!+2!} = \frac{30}{8} = \frac{15}{4} = 3\frac{3}{4} = 3.75$

24

6 2

3. What is the 4037^{th} digit following the decimal point in the expansion of $\frac{1}{111}$?

$\frac{1}{111} = \frac{9}{999} = 0.\overline{009}$

120120

$$
\begin{array}{r}
0.009009 \\
111\overline{)\,1.000} \\
\underline{-\,999} \\
1000 \\
\underline{999} \\
\end{array}
$$

$$
\begin{array}{r}
1345122 \\
3\overline{)\,4037} \\
\underline{-\,3} \\
10 \\
\underline{\,9} \\
13 \\
\underline{\,12} \\
17
\end{array}
$$

4. Evaluate the infinite geometric series

$$\frac{7^0}{100} + \frac{7^1}{100^2} + \frac{7^2}{100^3} + \cdots$$

as a fraction and find the first 6 digits in its ~~decimal~~ expansion.

0.01 0.0007 0.000049 $\frac{7^3}{100^4}$

0.01
0.0007
0.000049
0.000343
0.010749
0.010752

$$X = \frac{7^0}{100} + \frac{7^1}{100^2} + \frac{7^2}{100^3} + \cdots$$

$$\frac{7}{100}X = \frac{7^1}{100^2} + \frac{7^2}{100^3} + \frac{7^3}{100^4} + \cdots$$

$$\frac{93}{100}X = \frac{7^0}{100}$$

$$X = \frac{1}{93}$$

5. Let S be the set of real numbers that can be represented as repeating decimals of the form $0.\overline{abc}$, where a, b, c are distinct digits. Find the sum of the elements of S.

$0.\overline{012}, 0.\overline{013}, 0.\overline{014} \ldots$

$0.\overline{986}, 0.\overline{987}$

$\ldots$

$0.\overline{999}$

$10 \cdot 9 \cdot 8 = 720$

$= 360 \cdot 0.\overline{999}$

$= 360$

$x = 0.999999\ldots$

$10x = 9.999999\ldots$

$9x = 9$

$x = 1$

6. The rational number r is the largest number less than 1 whose base-7 expansion consists of two distinct digits, i.e., $r = 0.\overline{AB}$. Written as a reduced fraction, $r = \frac{p}{q}$, find $p + q$.

7. Express $0.72\overline{45}$ as a common fraction.

8. Let p be a prime number other than 2 or 5. What is the maximum possible number of digits in the repeating block of digits in $\frac{1}{p}$?