Exponents

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2$$

$$\underbrace{}_{4 \text{ times}}$$

Def: $a^b = \underbrace{a \cdot a \cdot \ldots \cdot a}_{b \text{ times}}$

$$2^3 \cdot 2^4 = \underbrace{2 \cdot 2 \cdot 2}_{3 \text{ times}} \cdot \underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{4 \text{ times}} = 2^7$$

$$\boxed{a^m \cdot a^n = a^{m+n}} \quad \text{Rule 1}$$

$$\frac{3^6}{3^4} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3} = 3^2$$

$$\boxed{\frac{a^m}{a^n} = a^{m-n}} \quad \text{Rule 2}$$

$$3^0 = 1$$

$$\frac{3^1}{3^1} = 3^{1-1} = 3^0 = 1 \qquad \frac{3^1}{3^1} = \frac{3}{3} = 1$$

$$a^0 = 1$$

$$a^{-n}$$

$$\frac{a^0}{a^n} = a^{0-n} = \boxed{a^{-n} = \frac{1}{a^n}} \quad \text{Rule 3}$$

$$3^{2x} = 3^{x-5}$$

$$2x = x - 5 \implies x = -5$$

$$(2^3)^5 = 2^3 \cdot 2^3 \cdot 2^3 \cdots 2^3 = 2^{3\cdot5} = 2^{15}$$

$$\underbrace{}_{5 \text{ times}}$$

$$\boxed{(a^m)^n = a^{mn}} \quad \text{Rule 4}$$

$$(4^{-3})^{-2} = 4^6$$

$$(ab)^n = a^n b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$25^{\frac{1}{2}} = x$$

$$(25^{\frac{1}{2}})^2 = x^2 \qquad 25 = x^2$$

$$25 = x \cdot x \qquad x = \sqrt{25}$$

$$\boxed{a^{\frac{1}{2}} = \sqrt{a}} \qquad \boxed{a^{\frac{m}{n}} = \sqrt[n]{a^m}}$$

Ex:
$$(81^{-3})^{\frac{1}{4}} = \left((3^4)^{-3}\right)^{\frac{1}{4}} = 3^{4\cdot(-3)\cdot\frac{1}{4}} = \frac{1}{27}$$

$$\frac{4^{\frac{2}{3}} \, 2^{\frac{1}{6}} \, 3^{\frac{3}{2}}}{2^{-\frac{1}{2}} \, 3^{\frac{1}{2}}} = \frac{2^{\frac{4}{3}} \, 2^{\frac{1}{6}} \, 3^{\frac{3}{2}}}{2^{-\frac{1}{2}} \, 3^{\frac{1}{2}}} = 2^{\underbrace{\frac{4}{3}+\frac{1}{6}-\left(-\frac{1}{2}\right)}_{=2}} \, 3^{\underbrace{\frac{3}{2}-\frac{1}{2}}_{=1}}$$

$$= 2^2 \cdot 3^1 = 12$$