

**The Ninth Grade Math Competition Class**  
**Quadratic Formula and Polynomial**  
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1. Find the value of  $x$  if  $x$  is positive and  $x - 1$  is the reciprocal of  $x + \frac{1}{2}$ .

2. It is given that one root of  $2x^2 + rx + s = 0$ , with  $r$  and  $s$  real numbers, is  $3 + 2i$ . Find  $s$ .

3. Find all values of  $k$  such that  $x^2 + kx + 27 = 0$  has two distinct real solutions for  $x$ .

$$k^2 - 4 \cdot 27 > 0$$

$$k^2 > 108$$

$$k > \sqrt{108} = 6\sqrt{3}$$

$$k < -\sqrt{108} = -6\sqrt{3}$$

**4.** Find all real solutions to  $(x^2 - 5x + 5)^{x^2 - 9x + 20} = 1$ .

**5.** Find all real solutions  $(x, y)$  of the system  $x^2 + y = 12 = y^2 + x$ .

6. Find all values of  $m$  for which the zeros of  $2x^2 - mx - 8$  differ by  $m - 1$ .

7. A polynomial of degree four with leading coefficient 1 and integer coefficients has two zeros, both of which are integers. Which of the following can also be a zero of the polynomial?

(A)  $\frac{1+i\sqrt{11}}{2}$       (B)  $\frac{1+i}{2}$       (C)  $\frac{1}{2} + i$       (D)  $1 + \frac{i}{2}$       (E)  $\frac{1+i\sqrt{13}}{2}$

**8.** Find the sum of all the roots of the equation  $x^{2001} + (\frac{1}{2} - x)^{2001} = 0$ .



9. Three of the roots of  $x^4 + ax^2 + bx + c = 0$  are  $-2$ ,  $-3$ ,  $5$ . Find the value of  $a + b + c$ .

- 10.** One root of the quadratic  $x^2 + bx + c = 0$  is  $1 - 3i$ . If  $b$  and  $c$  are real numbers, then what are  $b$  and  $c$ ?

- 11.** Suppose the roots of  $x^3 + 3x^2 + 4x - 11 = 0$  are  $a$ ,  $b$  and  $c$ , and the roots of  $x^3 + rx^2 + sx + t = 0$  are  $a + b$ ,  $b + c$ , and  $c + a$ , find the value of  $t$ .

**12.** Let  $a$ ,  $b$ , and  $c$  be the roots of  $x^3 - 3x^2 + 1$ .

- Find a polynomial whose roots are  $a + 3$ ,  $b + 3$  and  $c + 3$ .
- Find a polynomial whose roots are  $\frac{1}{a+3}$ ,  $\frac{1}{b+3}$ , and  $\frac{1}{c+3}$ .
- Compute  $\frac{1}{a+3} + \frac{1}{b+3} + \frac{1}{c+3}$ .
- Find a polynomial whose roots are  $a^2$ ,  $b^2$  and  $c^2$ .

**13.** The equation  $2^{333x-2} + 2^{111x+2} = 2^{222x+1} + 1$  has three real roots. Find their sum.

- 14.** If  $P(x)$  is a polynomial in  $x$  such that for all  $x$ ,  $x^{23} + 23x^{17} - 18x^{16} - 24x^{15} + 108x^{14} = (x^4 - 3x^2 - 2x + 9) \cdot P(x)$ , compute the sum of coefficients of  $P(x)$ .

**15.** The real number  $x$  satisfies the equation  $x + \frac{1}{x} = \sqrt{5}$ . What is the value of  $x^{11} - 7x^7 + x^3$ ?

- 16.** All the roots of the polynomial  $x^6 - 10z^5 + Az^4 + Bz^3 + cZ^2 + Dz + 16$  are positive integers, possibly repeated. What is the value of  $B$ ?