The Ninth Grade Math Competition Class
Prime Factorization 1
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1. What is the smallest positive integer N such that the value $7 + 30N$ is not a prime number?

$N=1$ 37
2 67
3 97
4 127
5 157
6 187 $= 11 \cdot 17$

2. The product of a set of positive integers is 140. What is their least possible sum?

$$140 = 2^2 \cdot 5 \cdot 7 = 2 \cdot 2 \cdot 5 \cdot 7$$

$$2+2+5+7 = 16$$

$$ab \geq a+b \quad (a, b \geq 2)$$

3. Find the greatest natural number that must be a divisor of any common multiple of 14, 26 and 66.

$k \cdot \operatorname{lcm}(14, 26, 66)$

4. The product of any two of the possible integers 30, 72 and N is divisible by the third. What is the smallest possible value of N?

$30 = 2 \cdot 3 \cdot 5$

$72 = 2^3 \cdot 3^2 \cdot 5^0$

$N = 2^{\overset{2}{a}} \cdot 3^{\overset{1}{b}} \cdot 5^{\overset{1}{c}} = 60$

$N \mid 30 \cdot 72$ $\quad a \leq 4,\ b \leq 3,\ c \leq 1$

$30 \mid 72N$ $\quad c \geq 1$

$72 \mid 30N$ $\quad a \geq 2,\ b \geq 1$

5. How many divisors of 5400 are not multiples of any perfect square greater than 1?

$5400 = 2^3 \cdot 3^3 \cdot 5^2$

$2^0 \quad 3^0 \quad 5^0$

$2^1 \quad 3^1 \quad 5^1$

$d = 2^a \, 3^b \, 5^c$

$2 \cdot 2 \cdot 2 = \boxed{8}$

6. How many of positive divisors of 45000 themselves have exactly 12 positive divisor?

$45000 = 2^3 \cdot 3^2 \cdot 5^4$

$d = 2^a 3^b 5^c$

$(a+1)(b+1)(c+1) = 12$

12	1	1	✗
6	2	1	✗
4	3	1	✓
4	1	3	✓
3	1	4	✓
1	3	4	✓
3	2	2	✓
2	3	2	✓
2	2	3	✓

7

2^9 3^5

$t(m)=10$ $t(n)=6$

7. If m has 10 positive divisors, n has 6 positive divisors, and $gcd(m,n) = 1$, how many positive divisors does mn have?

$t(mn) = t(m)\,t(n) = 60$

$2^9 3^5$

8. If n has exactly 7 positive divisors, how many positive divisors does n^2 have?

$256 = 2^8$ 9 div.

$64 = 2^6$ 7 div.

$64^2 = (2^6)^2 = 2^{12}$ 13 div.

$n = a^e b^f \dots = a^6$

$t(n) = (\overset{6}{e}+1)(\overset{0}{f}+1) \dots = 7$

$\ 7 \quad 1 \quad 1$

$n^2 = (a^6)^2 = a^{12}$ 13 div.

9. How many of the positive divisors of 168 are even?

$168 =$

$84 = 2^2 \cdot 3^1 \cdot 7^1$

$(2+1)(1+1)(1+1) = \boxed{12}$

10. Show that any positive perfect square has an odd number of positive divisors?

n^2

$n = p_1^{e_1} p_2^{e_2} \dots p_k^{e_k}$

$n^2 = (p_1^{e_1} p_2^{e_2} \dots p_k^{e_k})^2 = p_1^{2e_1} p_2^{2e_2} \dots p_k^{2e_k}$

$$t(n^2) = \underbrace{\underbrace{(2e_1+1)}_{\text{odd}}\underbrace{(2e_2+1)}_{\text{odd}}\dots\underbrace{(2e_k+1)}_{\text{odd}}}_{\text{odd}}$$