## The Ninth Grade Math Competition Class Prime Factorization 1 Anthony Wang

1. What is the smallest positive integer N such that the value 7 + 30N is not a prime number?

N=1 37

- 2 67
- 3 47
- 4 127
- 5 157
- 6 187 = 11,17

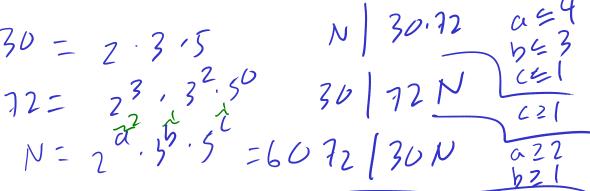
2. The product of a set of positive integers is 140. What is their least possible sum?

$$140 = 2^{2} \cdot 5.7 = 2 \cdot 2 \cdot 5.7$$
 $21215+7=16$ 
 $ab = a+b \quad (a,622)$ 

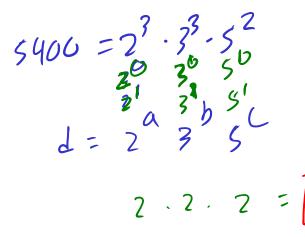
3. Find the greatest natural number that must be a divisor of any common multiple of 14, 26 and 66.

K. 1cm (14, 26, 66)

**4.** The product of any two of the possible integers 30, 72 and N is divisible by the third. What is the smallest possible value of N?



**5.** How many divisors of 5400 are not multiples of any perfect square greater than 1?



**6.** How many of positive divisors of 45000 themselves have exactly 12 positive divisor?

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$$45000 = 2^{\frac{3}{2}} \cdot 3^{\frac{2}{3}} \cdot 5^{\frac{1}{4}}$$

$$4 = 2 \quad 3 \quad 5^{\frac{1}{4}}$$

$$(a+1)(b+1)(c+1) = 12$$

$$4 \quad 1 \quad 3 \quad 4$$

$$4 \quad 1 \quad 3 \quad 4$$

$$4 \quad 1 \quad 3 \quad 4$$

$$7 \quad 3 \quad 4 \quad 4$$

$$7 \quad 3 \quad 2 \quad 4$$

$$7 \quad 2 \quad 3 \quad 2$$

$$7 \quad 2 \quad 3 \quad 3$$

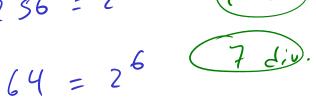
t(m)=10 t(n)=6

7. If m has 10 positive divisors, n has 6 positive divisors, and gcd(m,n)=1, how many positive divisors does mn have?

$$\ell(mn) = \ell(m) \ell(n) = 60$$

**8.** If n has exactly 7 positive divisors, how many positive divisors does  $n^2$  have?



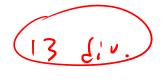


 $64^2 = (2^6)^2 = 2^{12}$  (13)



 $n = a \xrightarrow{6} b f$ . =  $a \xrightarrow{6} f$   $f(a) = (e+1)(f+1) \cdot ... = 7$ 

 $n^2 = (a^6)^2 = a^{12}$ 



**9.** How many of the positive divisors of 168 are even?

$$|68 = 84 = 2^{2} \cdot 3' \cdot 7'$$

$$(241)(141)(141) = 12$$

10. Show that any positive perfect square has an odd number of positive divisors?

$$N = P_{1} P_{2}^{2} e_{2} \dots P_{K}^{e_{K}}$$

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$$E(n^{2}) = (2e_{1} t_{1})(2e_{2} t_{1}) \dots (e_{K} t_{1})$$

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