The Ninth Grade Math Competition Class
Quadratic Equations and Vieta
Anthony Wang

1. Let a and b denote the solutions of $18x^2 + 3x - 28 = 0$, find the value of $(a-1)(b-1)$.

2. Let a and b be the roots of the equation $x^2 - mx + 2 = 0$. Suppose that $a + \frac{1}{b}$ and $b + \frac{1}{a}$ are roots of the equation $x^2 - px + q = 0$, find q.

3. Let p, q and r be constants. One sulution to the equation $(x-p)(x-q) = (r-p)(r-q)$ is $x = r$. Find the other solution in terms of p, q and r.

4. If m and n are the roots of $x^2 + mx + n = 0$, where $m \neq 0$ and $n \neq 0$, then what number does $m + n$ equal?

5. For what values of k does the equation $\frac{x-1}{x-2} = \frac{x-k}{x-6}$ have no solution for x?

6. Find all solutions to $2w^4 - 5w^2 + 2 = 0$.

7. Find the value of $\sqrt{90+\sqrt{90+\sqrt{90+\cdots}}}$.

8. Let a and b be the roots of $x^2 - 3x - 1 = 0$. Try to solve the following problems without finding a and b, it will be easier that way, anyway.

- Find a quadratic equations whose roots are a^2 and b^2.
- Compute $\frac{1}{a+1} + \frac{1}{b+1}$.

$x^2 - 3x - 1 = 0$

a, b roots

$a+b = 3$

$ab = -1$

$y^2 + cy + d = 0$

a^2, b^2 roots

$a^2 + b^2 = -c$

$a^2 b^2 = d$

$a^2 b^2 = 1$

$\Rightarrow d = 1$

$(a+b)^2 = a^2 + 2ab + b^2 = 9$

$\Rightarrow a^2 + b^2 = 11 = -c$ $\Rightarrow c = -11$

$y^2 - 11y + 1 = 0$

$\frac{1}{a+1} + \frac{1}{b+1} = \frac{b+1+a+1}{(a+1)(b+1)} = \frac{a+b+2}{ab+a+b+1}$

$= \frac{3+2}{-1+3+1} = \frac{5}{3}$

9. For some integer a, the equation $1988x^2 + ax + 8891 = 0$, and $8891x^2 + ax + 1988 = 0$ share a common root. Find a.

r

$1988x^2 + ax + 8891 = 0$

$8891x^2 + ax + 1988 = 0$

$10879x^2 + 2ax + 10879 = 0$

$6903x^2 - 6903 = 0$

$6903(x^2 - 1) = 0$

$6903(x-1)(x+1) = 0$

$(x-r)$? $r = 1$ $(x-r)$? $r = -1$

$x = 1 \Rightarrow 10879 + 2a + 10879 = 0$

$a = -10879$

$x = -1 \Rightarrow 10879 - 2a + 10879 = 0$

$a = 10879$

$\Rightarrow a = \pm 10879$

10. The product of the roots of the quadratic $6x^2 + cx + 4$ is 2 greater than the sum of the roots, and c is a constant. What is c?

$6x^2 + cx + 4 = 0$

Prod. of the roots: $\frac{4}{6} = \frac{2}{3}$

Sum of the roots: $-\frac{c}{6}$

$\frac{2}{3} = 2 + \left(-\frac{c}{6}\right)$

$\frac{c}{6} = 2 - \frac{2}{3} \Rightarrow \boxed{c = 8}$

11. Let a,b, and c be the roots of $x^3 - 3x^2 + 1$.

- Find a polynomial whose roots are $a + 3$, $b + 3$ and $c + 3$.
- Find a polynomial whose roots are $\frac{1}{a+3}$, $\frac{1}{b+3}$, and $\frac{1}{c+3}$.
- Compute $\frac{1}{a+3} + \frac{1}{b+3} + \frac{1}{c+3}$.
- Find a polynomial whose roots are a^2, b^2 and c^2.
- Find a recurrence relation for $x_n = a^n + b^n + c^n$, and use it to compute $a^5 + b^5 + c^5$.