Logarithms

Q. what power of 2 is 16? $= 4$ $2^4 = 16$

$2^4 = 16 \iff 4 = \log_2 16$ $2^{(4)} = 16$

Q. what power of a is b? $= c$

Def: $a^c = b \iff c = \log_a b$ $a > 0$, $b > 0$, $a \neq 1$

(a: base)

Ex. $\log_{2^{(6)}} 64 = 3$ (the power of a that's b)

$\log_{a^m}(a^n) = x \iff (a^m)^x = a^n$

$a^{mx} = a^n$

$mx = n$

$x = \frac{n}{m}$

Rule 1 $\boxed{\log_{a^m}(a^n) = \frac{n}{m}}$

Ex. $\log_{36} 6 = \log_{6^2} 6^1 = \frac{1}{2}$ ($36 = 6^2$)

Ex. $\log_{\frac{1}{2}} \sqrt{2} = \log_{2^{-1}} 2^{\frac{1}{2}} = -\frac{1}{2}$

Ex. $\log_8\left((2^{24})^{\frac{3}{4}}\right) = \log_{2^3}\left(2^{18}\right) = \frac{18}{3} = 6$

Ex: $\log_a(b^c) = x \iff (a^x)^{\frac{1}{c}} = (b^c)^{\frac{1}{c}}$

$\log_a b = \frac{x}{c} \iff a^{\frac{x}{c}} = b$

$c \log_a b = x = \log_a(b^c)$

$\log_a(b^c) = c \log_a b \qquad \log_{a^c}(b) = \frac{1}{c} \log_a b$

Rule 2 $\boxed{\log_{a^m}(b^n) = \frac{n}{m} \log_a b}$

$\log_a(bc) = x \iff a^x = bc$

$\log_a b = y \iff a^y = b$

$\log_a c = z \iff a^z = c$

$a^x = a^{y+z} = bc$

$x \quad y \quad z$

Rule 3 $\boxed{\log_a(bc) = \log_a b + \log_a c}$

$\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$