Factorials

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

$$\boxed{n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1}$$

6 distinct books, arrange them on a bookshelf

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6! = 720$

Q: $6! + 5!$ largest prime factor

$\underbrace{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} + \underbrace{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 (6+1) = \boxed{7}\, 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$6! = 6 \cdot 5! \rightarrow \boxed{n! = n \cdot (n-1)!}$

$6 \cdot 5! + 5! = 5!(6+1) = 7 \cdot 5!$

Q. what is the largest pow. of 2 that divides 20!?

$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

1 3 1 2 1

$20! = 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

2 1 4 1 2 1 3 1 2 1

10
5
2
1

10
5
2
1

at least 1 : $\left\lfloor \frac{20}{2} \right\rfloor = 10$

at least 2 : $\left\lfloor \frac{20}{4} \right\rfloor = 5$

at least 3 : $\left\lfloor \frac{20}{8} \right\rfloor = 2$

at least 4 : $\left\lfloor \frac{20}{16} \right\rfloor = 1$

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Q. What is it's 200!?

$$\left\lfloor \frac{200}{2} \right\rfloor \Rightarrow \left\lfloor \frac{100}{2} \right\rfloor \Rightarrow \left\lfloor \frac{50}{2} \right\rfloor \Rightarrow \left\lfloor \frac{25}{2} \right\rfloor \Rightarrow \left\lfloor \frac{12}{2} \right\rfloor \Rightarrow \left\lfloor \frac{6}{2} \right\rfloor \Rightarrow \left\lfloor \frac{3}{2} \right\rfloor \Rightarrow \left\lfloor \frac{1}{2} \right\rfloor = 199$$

$$\left\lfloor \frac{200}{4} \right\rfloor = \left\lfloor \frac{\left\lfloor \frac{200}{2} \right\rfloor}{2} \right\rfloor$$

Q. 8! product of divisors

$$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$
$$= 2^3 \cdot 7 \cdot 2 \cdot 3 \cdot 5 \cdot 2^2 \cdot 3 \cdot 2$$
$$= 2^7 \cdot 3^2 \cdot 5^1 \cdot 7^1$$

$$t(8!) = 8 \cdot 3 \cdot 2 \cdot 2 = 96$$

$$P(8!) = (8!)^{\frac{96}{2}} = \boxed{40320^{48}}$$