The Ninth Grade Math Competition Class
Base Numbers 1
Anthony Wang

1. What is the largest base 10 number that can be expressed as a three-digit base 5 number?

$444_5 = 4 \cdot 5^2 + 4 \cdot 5^1 + 4 = 124_{10}$

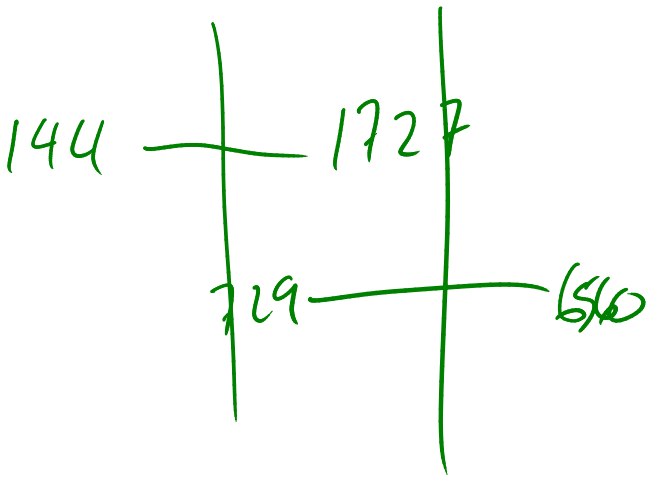
2. How many natural numbers require 3 digits when written in base 12, but require 4 digits when written in base 9?

$100_{12} = 144$

$BBB_{12} = 1727$

$1000_9 = 729$

$8888_9 = 6560$



$1727 - 729 = 998$

999

3. Given $9^6 = 531441$, how would you represent 531440 in base 9?

$1,000000_9$

$9^6 - 1 = 888888_9$ in base 9

$1,000000$

$10^6 - 1 = 999999$ in base 10

$$\begin{array}{r} 8\,8\,8\,8\,8\,8_9 \\ \times\ 0\,0\,0\,0\,0\,0_9 \\ \hline 1 \\ - \\ \hline 8\,8\,8\,8\,8\,8_9 \end{array}$$

4. How many integers from 1 to 1992 inclusive have a base-three representation that does not contain the digit 2?

$1992_{10} = 2201210_3$

$1\,1\,1\,1\,1\,1\,1$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 128$

$128 - 1$

127

5. When written in base 3, a positive integer has two terminal zeros. When written in base 4 or base 5, this same integer has one terminal zero. In how many other positive integral bases greater than 1 must the representation of this integer have at least one terminal zero?

6. Find the 100^{th} smallest positive integer that can be written using only the digits 1, 3, and 5 in base 7.

7. A number N has three digits when expressed in base 7. When N is expressed in base 9, the digits are reversed. Find the middle digit in either representation of N.