Prime Factorization

Primes: numbers with no divisors except 1 and itself

7 — 2, 3, 4, 5, 6

$\sqrt{7}$

$7 = ab$

$8 = ab$

$1 \cdot 8,\ 2 \cdot 4,\ 4 \cdot 2,\ 8 \cdot 1$

$25 = a \cdot b$

$1 \cdot 25,\ 5 \cdot 5,\ 25 \cdot 1$

You only need to check numbers up to $\sqrt{n}$

Q: Is 1 prime?

No, 1 is not prime or composite

Ex: Is 209 prime?

$209 = 11 \cdot 19$

$\sqrt{209} \approx 14$

2, 3, 5, 7, 11, 13

Ex: What's the smallest composite # that has no prime factors < 10?

$121 = 11 \cdot 11$

Prime factorization: $72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$

Def: LCM - Least Common multiple

$lcm(8, 12) = 24$

$2^3 \cdot 3^0 \quad 2^2 \cdot 3^1 \quad 2^3 \cdot 3^1$

$lcm(12, 30) = 60$

$2^2 \cdot 3^1 \cdot 5^0 \quad 2^1 \cdot 3^1 \cdot 5^1 \quad 2^2 \cdot 3^1 \cdot 5^1$

Def: GCD - Greatest Common Divisor

$gcd(12, 45) = 3$

$2^2 \cdot 3^1 \cdot 5^0 \quad 2^0 \cdot 3^2 \cdot 5^1 \quad 2^0 \cdot 3^1 \cdot 5^0$

$2^1 \cdot 3^0 \quad 2^2 \cdot 3^0 \quad 2^1 \cdot 3^1 \quad 2^2 \cdot 3^1$

$gcd(4, 6) \cdot lcm(4, 6) = 24 = 4 \cdot 6$

$gcd(8, 12) \cdot lcm(8, 12) = 96$

$gcd(6, 8) \cdot lcm(6, 8) = 48$

$2^1 \cdot 3^0 \quad 2^2 \cdot 3^1 \quad 2^2 \cdot 3^0 \cdot 2^1 \cdot 3^1$

$gcd(4, 6) \cdot lcm(4, 6) = 4 \cdot 6$

$5^1 \quad 5^1$

$$\boxed{gcd(a, b) \cdot lcm(a, b) = ab}$$

Ex: $gcd(70, n) = 10 \quad lcm(70, n) = 210$

$gcd(70, n) \cdot lcm(70, n) = 10 \cdot 210 = 70n$

$\Rightarrow n = 30$

$$\gcd(ac, bc) = c \cdot \gcd(a, b)$$
$$\operatorname{lcm}(ac, bc) = c \cdot \operatorname{lcm}(a, b)$$

$$\gcd(800, 1800) = 100 \cdot \gcd(8, 18) = \boxed{200}$$

$800 = 100 \cdot 8$, $1800 = 100 \cdot 18$, $\gcd(8, 18) = 2$