The Ninth Grade Math Competition Class
Factorials and Palindrome
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1. What is the largest 4-digit palindrome that is the sum of 2 different 3-digit palindromes?

$$\begin{array}{r} 999 \\ +\ 666 \\ \hline 1665 \end{array}$$

$$\begin{array}{r} 1\,A\,A\,1 \\ 555 \\ +\ 666 \\ \hline \boxed{1221} \end{array}$$

2. Find the largest n for which 12^n evenely divides $20!$. ≤ 18

$$(12)^n = (2^2 \cdot 3)^n = 2^{2n} \cdot 3^n$$

≤ 8

$2n \leq 18$

$n \leq 8$

$$\left\lfloor \frac{20}{3} \right\rfloor = \left\lfloor \frac{6}{3} \right\rfloor = \left\lfloor \frac{2}{3} \right\rfloor = 8$$

$$\left\lfloor \frac{20}{2} \right\rfloor = \left\lfloor \frac{10}{2} \right\rfloor = \left\lfloor \frac{5}{2} \right\rfloor = \left\lfloor \frac{2}{2} \right\rfloor = \left\lfloor \frac{1}{2} \right\rfloor = 18$$

3. What is the first year after 2018 that is a palindrome?

2 1 1 2

4. What is the product of the largest 3 digit palindrome and the least 3 digit palindrome?

999 101

$$
\begin{array}{r}
999 \\
\cdot\ 101 \\
\hline
999 \\
999 \\
100899
\end{array}
$$

5. How many 5-digit palindromes are there?

$$\underset{9}{\underline{A}}\ \underset{10}{\underline{B}}\ \underset{10}{\underline{C}}\ \underset{1}{\underline{B}}\ \underset{1}{\underline{A}} = \boxed{900}$$

6. Find the sum of all 3-digit plaindromes.

$1+2+3+4+5+6 = 7 \cdot 3 = 21$

$\underbrace{\underbrace{\underbrace{3+4}_{7}}}$ 7 7

1100

101, 111, 121, 131, . . . 979, 989, 999

1100

1100

$\underline{A}\ \underline{B}\ \underline{A}$

9 10 1 = 90

$\underbrace{1100}_{\text{sum of pair}} \cdot \underbrace{\frac{90}{2}}_{\text{\# of pairs}} = \boxed{49500}$

7. Palindromic primes are numbers that are both palindromic and prime. Find the greatest 3-digit palindromic prime?

$999 = 9 \cdot 111 = 3 \cdot 333$

$989 = 23 \cdot 43$

$979 = 11 \cdot 89$

$969 = 3 \cdot 323$

$959 = 7 \cdot 137$

$949 = 13 \cdot 73$

$939 = 3 \cdot 313$

929

$10001, 10101, \ldots, 99899, 99999$

$S = 110000 \cdot \frac{900}{2} = 49500\ldots$

$\boxed{18}$

8. A five-digit palindrome is a positive integer with respective digits $abcba$, where a is non-zero. Let S be the sum of all five-digit palindromes. What is the sum of the digits of S?

9. h There are unique integers $a_2, a_3, a_4, \ldots, a_7$ such that

$$\frac{5}{7} = \frac{a_2}{2!} + \frac{a_3}{3!} + \frac{a_4}{4!} + \frac{a_5}{5!} + \frac{a_6}{6!} + \frac{a_7}{7!},$$

$1 \quad 1 \quad 1 \; 0 \; 4 \; 2 = 9$

with $0 \le a_i \le i$, for $i = 2, 3, \ldots, 7$. Find $a_2 + a_3 + \ldots + a_7$.

$0 \le a_7 < 7$

$5 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 a_2 + 7 \cdot 6 \cdot 5 \cdot 4 a_3 + 7 \cdot 6 \cdot 5 a_4 + 7 \cdot 6 a_5 + 7 a_6 + a_7$

$514 \text{ R } 2$

$7 \overline{)3600}$

$= 7(\underbrace{\quad\quad}_{514}) + \underbrace{a_7}_{2}$

$514 = 6 \cdot 5 \cdot 4 \cdot 3 a_2 + 6 \cdot 5 \cdot 4 a_3 + 6 \cdot 5 a_4 + 6 a_5 + a_6$

$0 \le a_6 < 6$

$85 \text{ R}4$

$6 \overline{)514}$

$514 = 6(\underbrace{\quad\quad}_{85}) + \underbrace{a_6}_{4}$