The Ninth Grade Math Competition Class
Divisors
Anthony Wang

1. Find the product of the positive divisors of 2400 that are multiples of 6.

$\frac{x}{6}$ $6x$

$$2400 = 2^5 \cdot 3^1 \cdot 5^2$$

$$n = 400 = 2^4 \cdot 3^0 \cdot 5^2$$

15 div.

$$n^{\frac{t(n)}{2}} = 400^{\frac{15}{2}} = 20^{15}$$

$$\boxed{20^{15} \cdot 6^{15}}$$

2. Find the product of the divisors of 3200 that are perfect squares.

$\sqrt{x}$ 16 x^2 4

$3200 = 2^7 \cdot 5^2$

$\sqrt{3200} = 2^{\frac{7}{2}} \cdot 5^1$

$40 = 2^3 \cdot 5^1$

$4 \cdot 2 = 8$ div

$n^{\frac{t(n)}{2}} = 40^{\frac{8}{2}} = 40^4$

$(40^4)^2 = \boxed{40^8}$

3. A proper divisor of a number is a divisor of the number that is not the number itself. What is the smallest positive integer that is less than the sum of its positive proper divisors?

$8 = 2^3$

$1, 2, 4, \cancel{8}$

$1+2+4 = 7$

$7 =$

$1, \cancel{7}$

1

$4, 6, 8, 9, 10,$

$\boxed{12} = 2^2 \cdot 3^1$

$1, 2, 3, 4, 6, \cancel{12}$

$1+2+3+4+6 = 16$

4. How many positive cubes divide $3! \cdot 5! \cdot 7!$?
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

$$3! \cdot 5! \cdot 7! = (3 \cdot 2 \cdot 1) \cdot (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) \cdot (7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)$$

$$= 2^8 \cdot 3^4 \cdot 5^2 \cdot 7^1$$

2^0 3^0 5^0 7^0

2^3 3^3

2^6

$$3 \cdot 2 \cdot 1 \cdot 1 = \boxed{6}$$

5. How many of positive divisors of 3200 are not multiples of any perfect square greater than 1?

$$3200 = 32 \cdot 100$$

$$2^5 \cdot 10^2$$

$$2^5 \cdot 2^2 \cdot 5^2$$

$$2^7 \cdot 5^2$$

$$2^0 \quad 5^0$$

$$2^1 \quad 5^1$$

$$2 \cdot 2 = \boxed{4}$$

6. How many positive integers have exactly three proper divisors, each of which is less than 50?

4 div.

1 prime $n = p^3$ $\boxed{2^3 \quad 3^3 \quad 5^3 \quad 7^3}$ $\cancel{11^3}$

1, 7, 49 $\quad$ 1, 11, 121

$n = p^x q^y$

2 primes $n = p^1 q^1$ $\quad$ 1, p, q

$(1+1)(1+1) = 4$ ✓

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31

37, 41, 43, 47 $\quad$ 15 primes

$\frac{15 \cdot 14}{2} = 105$ $\quad$ $2 \cdot 3$ $\quad$ $3 \cdot 2$

$105 + 4 = 109$

7. Jan is thinking of a positive integer. Her integer has exactly 16 positive divisors, two of which are 12 and 15. What is Jan's number?

$12 = 2^2 \cdot 3^1$

$15 = 3^1 \cdot 5^1$

$\underset{3 \geq}{(x+1)}\,\underset{2\geq}{(y+1)}\,\underset{2\geq}{(z+1)} = 16$

$4 \cdot 2 \cdot 2$

$n = 2^x \cdot 3^y \cdot 5^z$

$= 2^3 \cdot 3^1 \cdot 5^1 = \boxed{120}$

8. What is the sum of all positive integers less than 100 that have exactly 12 divisors?

9. Dentoe p_k be the k^{th} prime number. Show that $p_1 p_2 \cdots p_n + 1$ cannot be the perfect square of an integer.

10. Prove that it is impossible for three consecutive squares to sum to another perfect squares.

11. A positive integer n is nice if there is a positive integer m with exactly four positive divisors (including 1 and m) such that the sum of the four divisors is equal to n. How many numbers in the set $\{2010, 2011, 2012, \cdots, 2019\}$ are nice?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5 .