Complex Numbers

Integers: $\ldots -2, -1, 0, 1, 2, \ldots$

Rational: $\frac{p}{q}$, p, q are integers $q \neq 0$

$\frac{1}{2}, \frac{3}{2}, \frac{-3}{9}$

Ex: Is $\sqrt{2}$ rational? NO.

Assume it is: $\sqrt{2} = \frac{p}{q}$ ← reduced

$2 = \frac{p^2}{q^2}$ $\qquad \underbrace{2q^2}_{\text{even}} = \underbrace{\overset{\text{even}}{p}{}^2}_{\text{even}}$

p is even $\Rightarrow$ $p = 2r$

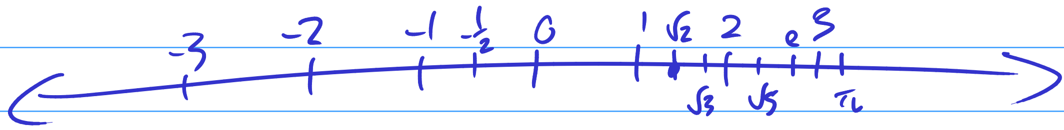
$2q^2 = (2r)^2 = 4r^2$

p even, q even,

so $\frac{p}{q}$ is not reduced X $\qquad \underbrace{\overset{\text{even}}{q}{}^2}_{\text{even}} = \underbrace{2r^2}_{\text{even}}$

Our assumption was wrong!

Irrational: $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi, e$



REAL

Ex: a, b such that $a+b=4$ $\quad ab=5$

$a = 2+\sqrt{-1} \quad b = 2-\sqrt{-1}$

$a+b = 2+\cancel{\sqrt{-1}} + 2 - \cancel{\sqrt{-1}} = 4$

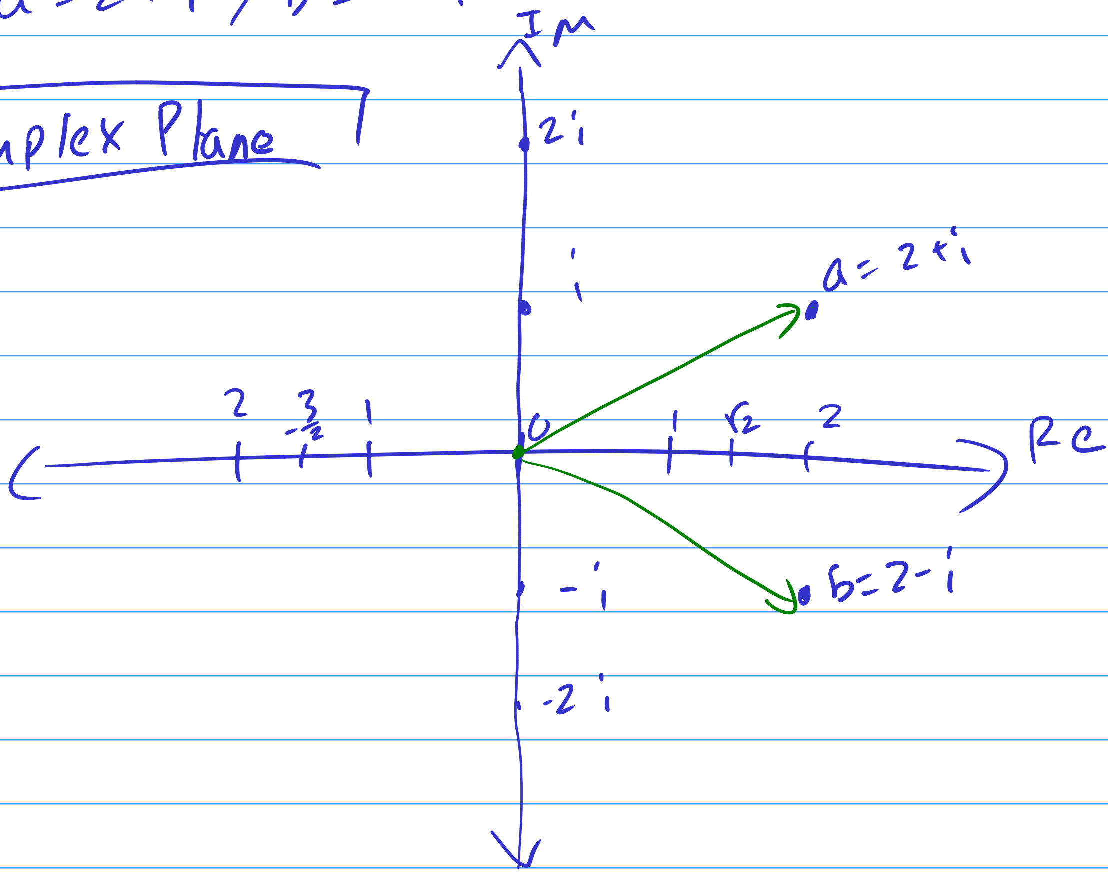
$ab = (2+\sqrt{-1})(2-\sqrt{-1})$
$= 4 + \cancel{2\sqrt{-1}} - \cancel{2\sqrt{-1}} - \underbrace{\sqrt{-1}\sqrt{-1}}_{-1}$
$= 4-(-1) = 5$

$\sqrt{-1} \cdot \sqrt{-1} = (\sqrt{-1})^2 = -1$

Def: $i = \sqrt{-1}$

$a = 2+i$, $b = 2-i$

Complex Plane



Def: Complex: $a+bi$ $\quad$ a, b are real

$x^2 = -1$ $\quad$ $x = \pm\sqrt{-1} = i$ $\quad$ imaginary unit

$i^2 = -1$

$i^3 = i^2 \cdot i = -1 \cdot i = -i$

$i^4 = i^3 \cdot i = -i \cdot i = 1$

$i^5 = i$

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

$i^9 = i$

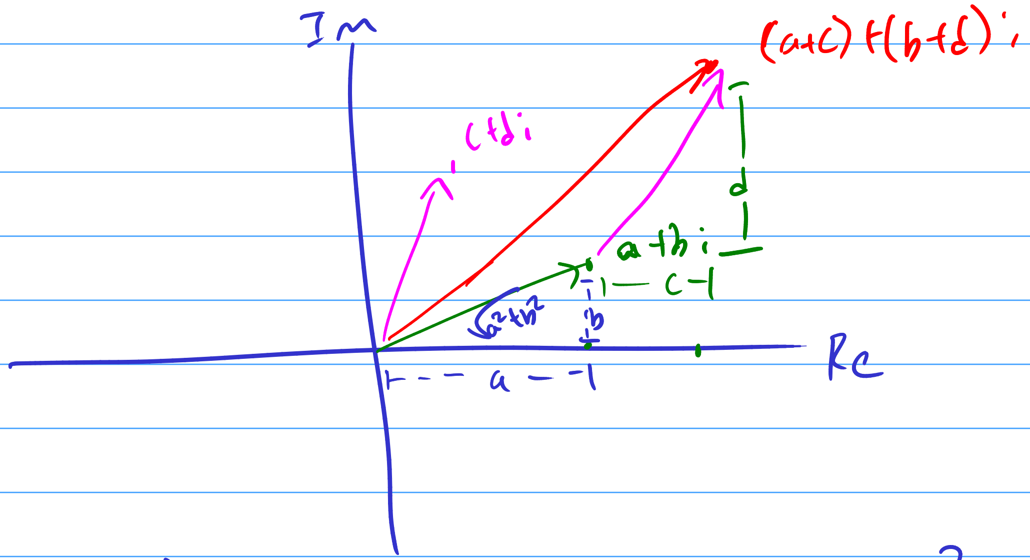
Ex: $i^{2020} = 1$

Ex: $i^6 + i^{16} + i^{26}$

$-1 \quad 1 \quad -1 = -1$

Addition: $(a+bi)+(c+di)=(a+c)+(b+d)i$

$(a+bi)-(c+di)=(a-c)+(b-d)i$



Multi: $(a+bi)(c+di)=ac+bci+adi+bd\underbrace{i^2}_{-1}$

$=(ac-bd)+(bc+ad)i$

Division: $\dfrac{1}{1+i}\,\dfrac{(1-i)}{(1-i)}=\dfrac{1-i}{1\cancel{+i}\cancel{-i}+1}=\dfrac{1-i}{2}=\dfrac{1}{2}-\dfrac{1}{2}i$

$1+i \qquad 1-i$

Conjugates

$z=a+bi \qquad \bar{z}=a-bi \qquad \bar{\bar{z}}=a+bi=z$

$\bar{\bar{z}}=z$

$z=7+2i \qquad \bar{z}=7-2i$

$z\bar{z}=(a+bi)(a-bi)=a^2+\cancel{abi}-\cancel{abi}-b^2i^2=a^2+b^2$

$\|z\|=\sqrt{a^2+b^2}$

$\boxed{z\bar{z}=a^2+b^2=\|z\|^2}$

"norm"

"magnitude"

"length of the arrow"

Ex: $$\frac{(3-i)}{(-2+5i)}\frac{(-2-5i)}{(-2-5i)} = \frac{-6+2i-15i-5}{4+25} = \frac{-11-13i}{29}$$

$$= \frac{-11}{29} - \frac{13}{29}i$$