

The Ninth Grade Math Competition Class
Base Numbers 1
Anthony Wang

1. What is the largest base 10 number that can be expressed as a three-digit base 5 number?

$$444_5 = 4 \cdot 5^2 + 4 \cdot 5^1 + 4 = 124_{10}$$

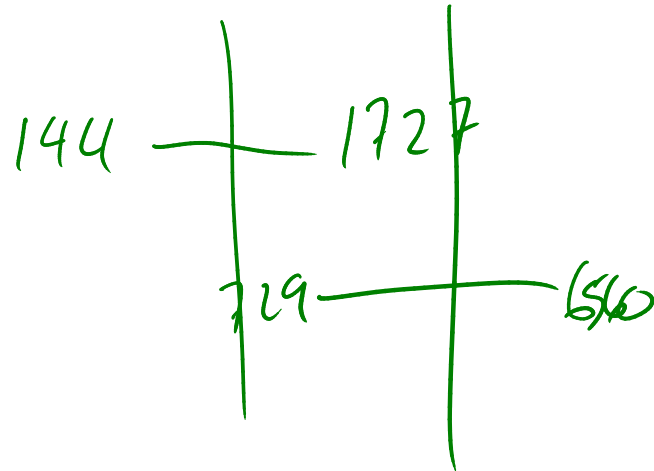
2. How many natural numbers require 3 digits when written in base 12, but require 4 digits when written in base 9?

$$100_{12} = 144$$

$$BBB_{12} = 1727$$

$$1000_9 = 729$$

$$8888_9 = 6560$$



$$\begin{array}{r} 1727 \\ - 729 \\ \hline 998 \end{array}$$

999

3. Given $9^6 = 531441$, how would you represent 531440 in base 9?

$$1,000,000_9$$

$$9^6 - 1 = 888888_9 \text{ in base } 9$$

$$1,000,000$$

$$10^6 - 1 = 999999 \text{ in base } 10$$

$$\begin{array}{r} 888888_9 \\ \times 000001_9 \\ \hline \end{array}$$

$$888888_9$$

4. How many integers from 1 to 1992 inclusive have a base-three representation that does not contain the digit 2?

$$1992_{10} = 2201210_3$$

$$\begin{array}{ccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ , & , & , & , & , & , & , \end{array}$$

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 128$$

$$127$$

5. When written in base 3, a positive integer n has two terminal zeros. When written in base 4 or base 5, this same integer has one terminal zero. In how many other positive integral bases greater than 1 must the representation of this integer have at least one terminal zero?

$$n = \dots abcd00_3 = 0 \cdot 3^0 + 0 \cdot 3^1 + d \cdot 3^2 + c \cdot 3^3 + \dots$$

$$= 3^2 (d + c \cdot 3 + \dots)$$

$$\Rightarrow 9 \mid n, 27 \nmid n \quad n = 3^2 m$$

$$n = \dots ba0_4 = 0 \cdot 4^0 + a \cdot 4^1 + b \cdot 4^2 + \dots$$

$$= 4(a + 4b + \dots)$$

$$\Rightarrow 4 \mid n, 16 \nmid n$$

$$5 \mid n, 25 \nmid n$$

$$n = 3^2 \cdot 4^1 \cdot 5^1$$

$$= 2^2 \cdot 3^2 \cdot 5^1$$

$$3 \cdot 3 \cdot 2 = 18$$

$$18 - 3 = 15 - 1 = 14$$

6. Find the 100th smallest positive integer that can be written using only the digits 1, 3, and 5 in base 7.

1st 1
2nd 3
3rd 5
4th 11
5th 13
6th 15
7th 31
8th 33
9th 35
10th 51
11th 53
12th 55
13th 111
14th 113
15th 115
16th 131
17th 133
18th 135

111
113
115
311
333
355
511
533
555

100
- 39
61
- 27
34
- 27
7

1 5111
2 5113
3 5115
4 5131
5 5133
6 5135
7 5151

7. A number N has three digits when expressed in base 7. When N is expressed in base 9, the digits are reversed. Find the middle digit in either representation of N .

$$N = \overset{1-b}{\underset{-}{a}} \overset{0-b}{\underset{-}{b}} \overset{1-b}{\underset{-}{c}}_7 = 49a + 7b + c$$

$$N = \underset{-}{c} \underset{-}{b} \underset{-}{a}_9 = 81c + 9b + a$$

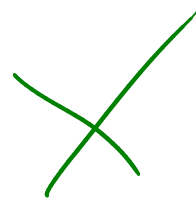
$$49a + 7b + c = 81c + 9b + a$$

$$48a = 80c + 2b$$

$$24a = 40c + b$$

$$c=1 \quad 24a = 40 + b$$

$$24, 48, \dots$$



$$c=2 \quad 24a = 80 + b$$

$$24, 48, 72, \dots$$

$$\textcircled{c=3} \quad 24a = 120 + b$$

$$24, 48, 72, 96, 120, \dots$$



$$\boxed{a=5, b=0}$$

$$b=0$$