

The Ninth Grade Math Competition Class
Unit Digit
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1. What is the units digit of $(3^{13})^3$?

$$(3^{13})^3 = 3^{39}$$

$$\begin{array}{l} 3^1 = 3 \\ 3^2 = 9 \\ 3^3 = \dots 7 \\ 3^4 = \dots 1 \\ 3^5 = \dots 3 \end{array} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\}$$

2. Find the units digit of n given that $m \cdot n = 71^6$ and m has a units digit of 7.

7
7, 3

3. (a:) Find the units digit of the sum

$$1! + 2! + 3! + \dots + 2006!$$

(b:) Find the units digit of the above sum when it is expressed in base 7.

$$\begin{array}{ccccccc}
 1! & + & 2! & + & 3! & + & 4! + 5! + 6! + 7! + \dots \\
 \hline 1 & & 2 & & 6 & & 24 \quad 120 \quad 720 \quad 5040 \\
 & & & & & & \hline
 & & & & & & + \dots + 2006! \\
 & & & & & & \hline
 \end{array}$$

$$1 + 2 + 6 + 4 = 13$$

$$\begin{array}{ccccccc}
 1! & + & 2! & + & 3! & + & 4! + 5! + 6! + 7! + 8! + \dots \\
 \hline 1 & & 2 & & 6 & & 24 \quad 120 \quad 720 \quad 5040 \\
 & & & & & & \hline
 \end{array}$$

$$1 + 2 + 6 + 3 + 1 + 5 = 21 = 30_7$$

$$\begin{aligned}
 n = abc_d \equiv 7 &= 7^3a + 7^2b + 7c + d \\
 n &= 7(7^2a + 7b + c) + d
 \end{aligned}$$

4. A positive two-digit integer is divisible by n and its units digit is n . What is the greatest possible value of n ?

9

99



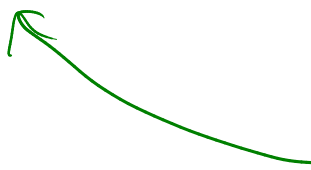
5. Find the units digit of $3^{2016} - 2^{2016}$.

$3^{2016} - 2^{2016} = (-5) = 5$

$3^1 = 3$
 $3^2 = 9$
 $3^3 = 7$
 $3^4 = 1$
 $3^5 = 3$

$2^1 = 2$
 $2^2 = 4$
 $2^3 = 8$
 $2^4 = 6$
 $2^5 = 2$

6. The cube of the 3-digit natural number $A7B$ is 108531333. What is $A + B$?

$$\begin{aligned} (\underline{A7B})^3 &= 108,531,333 \\ B &= 7 \\ &\approx 108 \cdot 10^6 \\ A7B &= \sqrt[3]{108} \cdot 10^2 \\ &\quad \underbrace{4 \leq \leq 5}_{400 \leq \leq 500} \end{aligned}$$


 How many of the positive divisors of 6^{2006} have a units digit of 6?

8. (a) Convert 1599 to base 16.

$63F_{16}$

(b) Find all possible units digits of perfect fourth powers when written in base 16.

0 or 1

(c) Determine all non-negative integral solutions $(n_1, n_2, \dots, n_{14})$ if any, of the Diophantine equation.

$$n_1^4 + n_2^4 + \dots + n_{14}^4 = 1599.$$

(A Diophantine equation is an equation in which only integer solutions are allowed.)

$$\begin{array}{r} 99 R 15 \\ 16 \overline{) 1599} \\ \underline{-144} \\ 159 \\ \underline{-144} \\ 15 \end{array}$$

$$0^4 = 0$$

$$1^4 = 1$$

$$2^4 = 16 \quad 0$$

$$3^4 = 81 \quad 1$$

$$4^4 = 256 \quad 0$$

$$\begin{array}{r} 6 R 3 \\ 16 \overline{) 99} \\ \underline{96} \\ 3 \end{array}$$

$$n_1^4 + n_2^4 + \dots + n_{14}^4 = 1599$$

$\underbrace{0,1} \quad \underbrace{0,1} \quad \dots \quad \underbrace{0,1} \quad \underbrace{15}$

NO solutions