The Ninth Grade Math Competition Class
Factorials
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$8^n = (2^3)^n = 2^{3n}$

1. Find the largest integer value of n for which 8^n evenly divides $100!$.

$10!$

$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ $\quad 8^2$

$\left\lfloor \frac{10}{8} \right\rfloor \Rightarrow \left\lfloor \frac{1}{8} \right\rfloor$

97

$\left\lfloor \frac{100}{2} \right\rfloor \Rightarrow \left\lfloor \frac{50}{2} \right\rfloor + \left\lfloor \frac{25}{2} \right\rfloor \Rightarrow \left\lfloor \frac{12}{2} \right\rfloor + \left\lfloor \frac{6}{2} \right\rfloor + \left\lfloor \frac{3}{2} \right\rfloor + \left\lfloor \frac{1}{2} \right\rfloor$

$\left\lfloor \frac{97}{3} \right\rfloor = 32$

$2^{97} \mid 100!$

$2^{3 \cdot \frac{97}{3}} \mid 100!$

2. Find the prime factorization of $10!$.

$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$2 \cdot 5 \quad 3^2 \cdot 2^3 \cdot 7 \cdot 2 \cdot 3 \cdot 5 \cdot 2^2 \cdot 3 \cdot 2$

$\boxed{2^8 \cdot 3^4 \cdot 5^2 \cdot 7}$

3. What is the product of the positive divisors of 7!.

4. How many positive cubes divide $3!5!7!$.

5. For how many positive integers n less than or equal to 24 is $n!$ evenly divible by $1 + 2 + \cdots + n$?

6. In how many zeros does the decimal expansion of $100^{100} - 100!$ end?

7. Let P be the product of the first 100 positive odd integers. Find the largest integer k such that P is divisible by 3^k.