Quadratic Formula

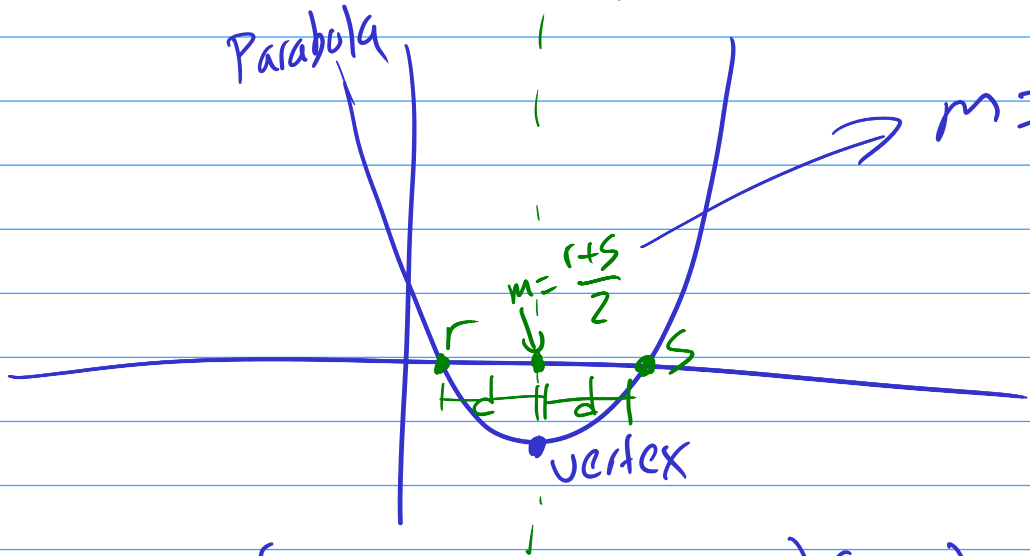
$ax^2+bx+c=0$

r, s roots

Vieta's form.

$rs = \frac{c}{a}$

$r+s = \frac{-b}{a}$



$m = \frac{r+s}{2} = \frac{-b}{2a}$

$r = m-d$
$s = m+d$

$rs = (m-d)(m+d) = \frac{c}{a}$

$m^2 - d^2 = \frac{c}{a}$

$m^2 - \frac{c}{a} = d^2$

$\sqrt{m^2-\frac{c}{a}} = d$

$r = m - \sqrt{m^2 - \frac{c}{a}}$

$s = m + \sqrt{m^2 - \frac{c}{a}}$

$$\boxed{x = m \pm \sqrt{m^2-\frac{c}{a}}, \quad m = \frac{-b}{2a}}$$

$$x = -\frac{b}{2a} \pm \sqrt{\left(\frac{-b}{2a}\right)^2 - \frac{c}{a}} = -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$$

$$= -\frac{b}{2a} \pm \sqrt{\frac{b^2-4ac}{4a^2}} = \boxed{\frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

$$ax^2+bx+c=0$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$\frac{b}{a}=-2m \Rightarrow m=\frac{-b}{2a}$$

$$\frac{c}{a}=m^2+n \Rightarrow n=\frac{c}{a}-\frac{b^2}{4a^2}$$

$$(x-m)^2+n=0$$

$$x^2-2mx+m^2+n=0$$

$$\boxed{\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}=0}$$

vertex: $\left(-\frac{b}{2a}, \frac{c}{a}-\frac{b^2}{4a^2}\right)$

$$x^2+3x+1=0$$

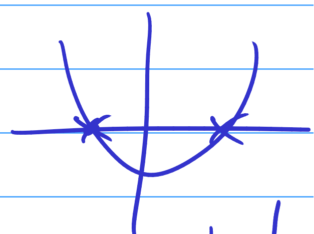
$$\left(x+\frac{3}{2}\right)^2+1-\frac{9}{4}=0$$

$$\left(x+\frac{3}{2}\right)^2-\frac{5}{4}=0$$

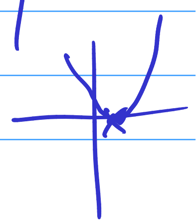
vertex $\left(\frac{3}{2}, \frac{5}{4}\right)$

Discriminant

$b^2-4ac>0 \Rightarrow$ 2 real root



$b^2-4ac=0 \Rightarrow$ 1 double real root



$b^2-4ac<0 \Rightarrow$ 0 real roots, 2 complex roots