The Ninth Grade Math Competition Class
Unit Digit
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1. What is the units digit of $(\cancel{12}3^{13})^3$?

$$(3^{13})^3 = 3^{39}$$

$$3^1 = 3$$

$$3^2 = 9$$

$$3^3 = ..7$$

$$3^4 = ..1$$

$$3^5 = ..3$$

7

2. Find the units digit of n given that $mn = 21^6$ and m has a units digit of 7.

7, 3

3. **(a:)** Find the units digit of the sum

$$1! + 2! + 3! + \cdots + 2006!$$

(b:) Find the units digit of the above sum when it is expressed in base 7.

$\underbrace{1!}_{1} + \underbrace{2!}_{2} + \underbrace{3!}_{6} + \underbrace{4!}_{24} + \underbrace{5!}_{120} + \underbrace{6!}_{720} + \underbrace{7!}_{5040} + \cdots$

$+ \cdots + \underbrace{2006!}_{\ldots 0}$

$1 + 2 + 6 + 4 = 13$

$\underbrace{1!}_{1} + \underbrace{2!}_{2} + \underbrace{3!}_{6} + \underbrace{4!}_{3} + \underbrace{5!}_{1} + \underbrace{6!}_{5} + \underbrace{7!}_{0} + \underbrace{8!}_{0} + \cdots$

$1 + 2 + 6 + 3 + 1 + 5 = 21 = 30_7$

$n = \underline{abcd}_7 = 7^3a + 7^2b + 7c + d$

$n = 7(\underbrace{7^2a + 7b + c}) + d$

4. A positive two-digit integer is divisible by n and its units digit is n. What is the greatest possible value of n?

9 99

5. Find the units digit of $3^{2016} - 2^{2016}$.

$1 - 6 = -5 = 5$

3^{2016}

$3^1 = 3$

$3^2 = 9$

$3^3 = \ldots 7$

$3^4 = \ldots 1$

$3^5 = \ldots 3$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 6$

$2^5 = 2$

6. The cube of the 3-digit natural number $A7B$ is 108531333. What is $A + B$?

$$(\underline{A7B})^3 = 108{,}531{,}333$$

$B = 7$

$$\approx 108 \cdot 10^6$$

$$A7B = \underbrace{\sqrt[3]{108} \cdot 10^2}_{400 \leq \quad \leq 500}$$

$4 \leq \quad \leq 5$

7. How many of the positive divisors of 6^{2006} have a units digit of 6?

8. **(a)** Convert 1599 to base 16. $63F_{16}$

(b) Find all possible units digits of perfect fourth powers when written in base 16. 0 or 1

(c) Determine all non-negative integral soulations $(n_1, n_2, \ldots, n_{14})$ if any, of the Diophantine equation.

$$n_1^4 + n_2^4 + \cdots + n_{14}^4 = 1599.$$

(A Diophantine equation is an equation in which only integer solutions are allowed.)

99 R15

$16 \overline{)1599}$
-144
159
-144
15

$16 \overline{)99}$ 6 R3
96

$0^4 = 0$

$1^4 = 1$

$2^4 = 16$ 0

$3^4 = 81$ 1

$4^4 = 256$ 0

$\underbrace{n_1^4}_{0,1} + \underbrace{n_2^4}_{0,1} + \cdots \underbrace{n_{14}^4}_{0,1} = \underbrace{1599}_{15}$

NO solutions