Divisors

Q: Product of all divisors 24?

$$1, 2, 3, 4, 6, 8, 12, 24$$

$$1 \cdot 2 \cdot 3 \cdot 4 \cdot 6 \cdot 8 \cdot 12 \cdot 24 = 24^4$$

$$1 + 2 + 3 + 4 + 5 + 6$$

Q: Product of all div. of n?

$$n^{\frac{t(n)}{2}}$$

↑
product of
each pair

Ex. Find the prod. of the div. of 450
that are multiples of 3

1. $450 = 2 \cdot 3^2 \cdot 5^2$

$$150 = 2^1 \cdot 3^1 \cdot 5^2$$

$$t(150) = 12$$

$$150^{\frac{12}{2}} = 150^6$$

$$\boxed{150^6 \cdot 3^{12}} = 1350^6$$

2. Complement

$$450 = 2^1 \cdot 3^2 \cdot 5^2$$

$$50 = 2 \cdot 5^2$$

$$t(50) = 6$$

$$50^{\frac{6}{2}} = 50^3 \leftarrow \text{Prod. of Divisors that aren't multi. of 3}$$

$$\boxed{450}^a \qquad t(450) = 18$$

↑ prod. of all divisors

$$\frac{450^a}{50^3} =$$

Q: Sum of the divisors of 24?

1, 2, 3, 4, 6, 8, 12, 24

$$24 = 2^3 \cdot 3^1$$

	2^0	2^1	2^2	2^3
3^0	1	2	4	8
3^1	3	6	12	24

$$\underbrace{(2^0 + 2^1 + 2^2 + 2^3)}_{15} \underbrace{(3^0 + 3^1)}_{4} = 60$$

$$x = 2^0 + 2^1 + 2^2 + 2^3$$

$$2x = 2^1 + 2^2 + 2^3 + 2^4$$

$$(2-1)x = 2^4 - 2^0$$

$$x = \frac{2^4 - 2^0}{2-1}$$

$$n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$$

$$S(n) = \left(\frac{p_1^{e_1+1} - 1}{p_1 - 1} \right) \left(\frac{p_2^{e_2+1} - 1}{p_2} \right) \cdots \left(\frac{p_k^{e_k+1} - 1}{p_k} \right)$$