The Ninth Grade Math Competition Class

Exponents

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1. $a^m \cdot a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $a^{-n} = \frac{1}{a^n}$
4. $(a^m)^n = a^{mn}$

1. Find $5^{-3}5^{5}5^{1}$.

$5^{-3+5+1} = 5^3 = 125$

2. Find $\frac{3^4 3^{-2}}{3^5 3^{-1}}$.

$$3^{4+(-2)-5-(-1)} = 3^{-2} = \frac{1}{9}$$

3. Find 4^{x+1} if 2^x is 9.

$$2^x = 9 \qquad 4^{x+1} = ?$$

$$(2^x)^2 = 81 \qquad (2^2)^{x+1} = 2^{2x+2}$$

$$4 \cdot 2^{2x} = 81 \cdot 4$$

$$2^{2x+2} = \boxed{324}$$

4. If $8^x = 27$, what is 4^{2x-3}.

$$8^x = 27$$

$$(2^3)^x = 27$$

$$(2^{3x})^{\frac{1}{3}} = 27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}}$$

$$2^x = 3$$

$$(2^2)^{2x-3} = 2^{4x-6}$$

$$2^{4x} = 81$$

$$2^{4x-6} = \frac{81}{2^6} = \frac{81}{64}$$

5. Find all values of x such that $25^{-2} = \frac{5^{\frac{48}{x}}}{5^{\frac{26}{x}} 25^{\frac{17}{x}}}$.

5^2

$$(5^2)^{-2} = 5^{\frac{48}{x} - \frac{26}{x} - \frac{34}{x}}$$

$$5^{-4} = 5^{-\frac{12}{x}}$$

$$-4 = -\frac{12}{x} \Rightarrow x = 3$$

$$(-2)^{-2} \neq -2^{-2}$$

6. Simplify the expression $81^{-2^{-2}}$.

$$\left(3^4\right)^{-2^{-2}} = \left(3^4\right)^{-\frac{1}{4}} = 3^{4 \cdot -\frac{1}{4}} = \frac{1}{3}$$

7. Find x if $2^{16^x} = 16^{2^x}$.

$$2^{16^x} = (2^4)^{2^x}$$

$$2^{16^x} = 2^{4 \cdot 2^x}$$

$$16^x = 4 \cdot 2^x$$

$$2^{4x} = 2^{2+x}$$

$$4x = 2+x$$

$$\Rightarrow x = \frac{2}{3}$$

8. Solve for n: $\sqrt{1+\sqrt{2+\sqrt{n}}} = 2$.

$$1+\sqrt{2+\sqrt{n}} = 4$$

$$\sqrt{2+\sqrt{n}} = 3$$

$$2+\sqrt{n} = 9$$

$$\sqrt{n} = 7$$

$$n = 49$$

9. Find, with a rational common denominator, the sum

$$(\frac{1}{2})^{-\frac{1}{2}} + (\frac{3}{2})^{-\frac{3}{2}} + (\frac{5}{2})^{-\frac{5}{2}}$$

$$\left(\frac{2}{1}\right)^{\frac{1}{2}} = \frac{2^{\frac{1}{2}}}{1^{\frac{1}{2}}}$$
$$= \sqrt{2}$$

$$\frac{4\sqrt{10}}{125} + \frac{2\sqrt{6}}{9} + \sqrt{2}$$

$$\frac{9 \cdot 4\sqrt{10} + 125 \cdot 2\sqrt{6} + 1125\sqrt{2}}{1125}$$

$$\frac{36\sqrt{10} + 250\sqrt{6} + 1125\sqrt{2}}{1125}$$

10. What is the difference between $x^2 = 9$ and $x = \sqrt{9}$?

$x^2 = 9$

$x = 3$

$x = -3$

$x = 3$

11. Suppose that $y = \frac{3}{4}x$ and $x^y = y^x$, the quantity $x + y$ can be expressed as a rational number $\frac{r}{s}$, where r and s are relatively prime positive integers. Find $r + s$.

$$\left(x^{\frac{3}{4}x}\right)^{\frac{1}{x}} = \left(\left(\frac{3}{4}x\right)^{x}\right)^{\frac{1}{x}}$$

$$\left(x^{\frac{3}{4}}\right)^{4} = \left(\frac{3}{4}x\right)^{4}$$

$$\frac{x^3}{\cancel{x^3}} = \left(\frac{3}{4}\right)^4 \frac{x^4}{\cancel{x^3}}$$

$$1 = \left(\frac{3}{4}\right)^4 x$$

$$\left(\frac{4}{3}\right)^4 = x = \frac{256}{81}$$

$$y = \left(\frac{3}{4}\right)\left(\frac{4}{3}\right)^4 = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$$

$$\frac{256}{81} + \frac{64}{27} = \frac{448}{81}$$

r, s

$$\boxed{r+s = 529}$$

12. The formula $N = 8 * 10^8 * x^{-\frac{3}{2}}$ gives, for a certain group, the number of individuals whose income exceeds x dollars. What is the smallest possible value of the lowest income of the wealthiest 800 individuals?

$$800 = 8 \cdot 10^8 \cdot x^{-\frac{3}{2}}$$

$$\cancel{8} \cdot 10^2 = \cancel{8} \cdot 10^8 \cdot x^{-\frac{3}{2}}$$

$$\left(10^{-6}\right)^{-\frac{2}{3}} = \left(x^{-\frac{3}{2}}\right)^{-\frac{2}{3}}$$

$$\boxed{10^4 = x}$$

13. Solve for x in the equation $2^{333x-2} + 2^{111x+2} = 2^{222x+1}$.

$$\frac{1}{4} \cdot 2^{333x} + 4 \cdot 2^{111x} = 2 \cdot 2^{222x}$$

$$\frac{1}{4} (2^{111x})^3 + 4 \cdot 2^{111x} = 2 \cdot (2^{111x})^2$$

$$y = 2^{111x}$$

$$\frac{1}{4} y^3 + 4y = 2y^2$$

$$\frac{1}{4} y^2 + 4 = 2y$$

$$y^2 - 8y + 16 = 0 \qquad (y-4)^2 = 0$$

$$y = 4 = 2^{111x}$$

$$2^2 = 2^{111x} \qquad 2 = 111x \qquad x = \frac{2}{111}$$