

Binary to Octal, and Hexadecimal

Binary	Octal
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Binary to Octal, and Hexadecimal

Going from octal or hexadecimal to binary is a straightforward process. We just replace the octal or hexadecimal digit with its binary equivalent.

$$\begin{aligned}347646_8 &\Rightarrow 011\ 100\ 111\ 110\ 100\ 110_2 \\B1FF74_{16} &\Rightarrow 1011\ 0001\ 1111\ 1111\ 0111 \\&0100_2\end{aligned}$$

When going from binary to octal or hexadecimal, we have to ensure the number of bits is a multiple of the base. If it is not, we pad the number until it is.

$$\begin{aligned}1\ 101\ 011\ 110\ 000\ 110_2 &= \textcolor{teal}{11}\ 101\ 011\ 110\ 000\ 110_2 \Rightarrow 753606_8 \\101\ 0101\ 0001\ 0111\ 0100_2 &= \textcolor{teal}{1101}\ 0101\ 0001\ 0111\ 0100_2 = D5174_{16}\end{aligned}$$

Signed Binary Multiplication

For the multiplication of two binary numbers we use Booth's algorithm.

Before starting, ensure both numbers are of the same length. The length of the numbers we call n and the length of the product will be at most $2n$.

Create 4 variables:

- $A = 0\dots$ (padded to a bit length of n)
- $M = \text{Multiplicand}$ (first number)
- $Q = \text{Multiplier}$ (second number)
- $Q_{-1} = 0$, (bit length of 1)

We will also be referring to a variable Q_0 that represents the least significant bit or the right most bit of the binary number in Q .

Let variables separated by commas be considered a concatenation of the variables: A, Q, Q_{-1} (length of $2n + 1$) and Q_0, Q_{-1} (length of 2)

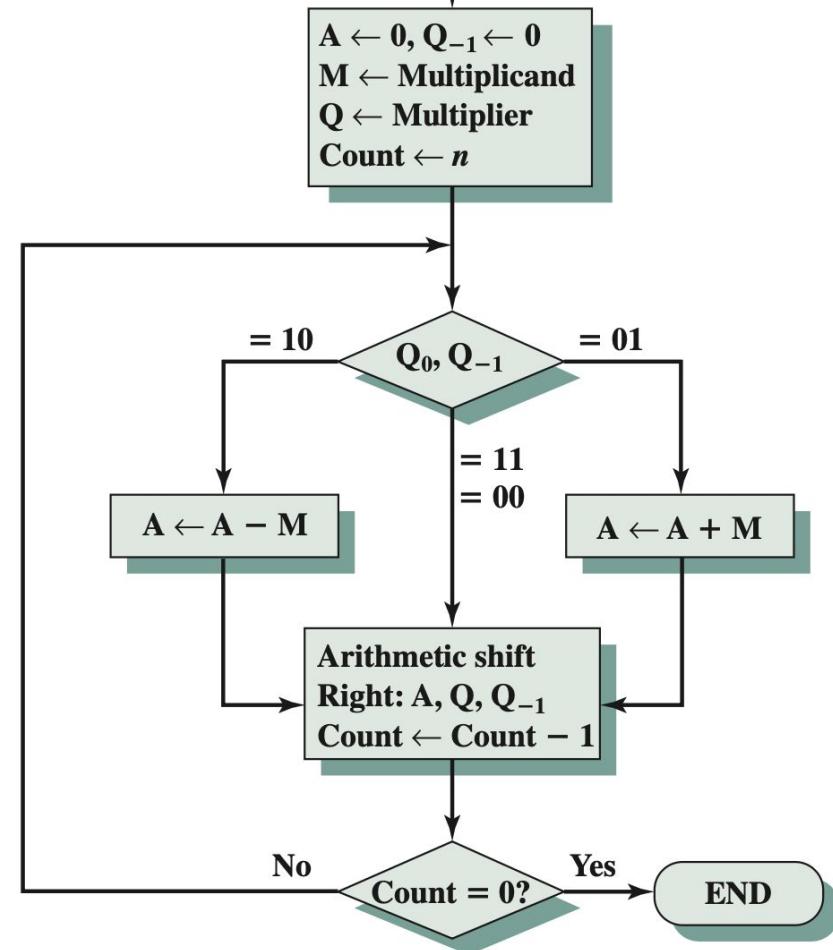
Signed Binary Multiplication

Booth's Algorithm

For n times:

1. If $Q_0, Q_{-1} = 01$: $A \leftarrow A + M$
Else if $Q_0, Q_{-1} = 10$: $A \leftarrow A - M$
2. $A, Q, Q_{-1} >> 1$ (arithmetic shift)

product = $A, Q,$



Signed Binary Multiplication

Lets test **0111 x 0011**. We know $0111 = 7$ and $0011 = 3$. Since $7 \times 3 = 21$, our binary product should be **00010101** untrimmed.

A	Q	Q_{-1}	M	Initial values	
0000	0011	0	0111		
1001	0011	0	0111	$A \leftarrow A - M$	First cycle
1100	1001	1	0111	Shift	
1110	0100	1	0111	Shift	Second cycle
0101	0100	1	0111	$A \leftarrow A + M$	
0010	1010	0	0111	Shift	Third cycle
0001	0101	0	0111	Shift	

Signed Binary Multiplication

Lets try: $01011 \times 1011\dots 11 \times -5 = -55\dots 1001001$

A	Q	Q_{-1}	M		
00000	11011	0	01011	Initial values	
10101	11011	0	01011	$A \Leftarrow A - M$	First Cycle
11010	11101	1	01011	Shift	
11101	01110	1	01011	Shift	Second Cycle
01000	01110	1	01011	$A \Leftarrow A + M$	Third Cycle
00100	00111	0	01011	Shift	
11001	00111	0	01011	$A \Leftarrow A - M$	Fourth Cycle
11100	10011	1	01011	Shift	
11110	01001	1	01011	Shift	Fifth Cycle

Signed Binary Multiplication

Lets try: $1010 \times 1010\dots -6 \times -6 = 36\dots 0100100$

A	Q	Q_{-1}	M		
0000	1010	0	1010	Initial values	
					First Cycle
					Second Cycle
					Third Cycle
					Fourth Cycle