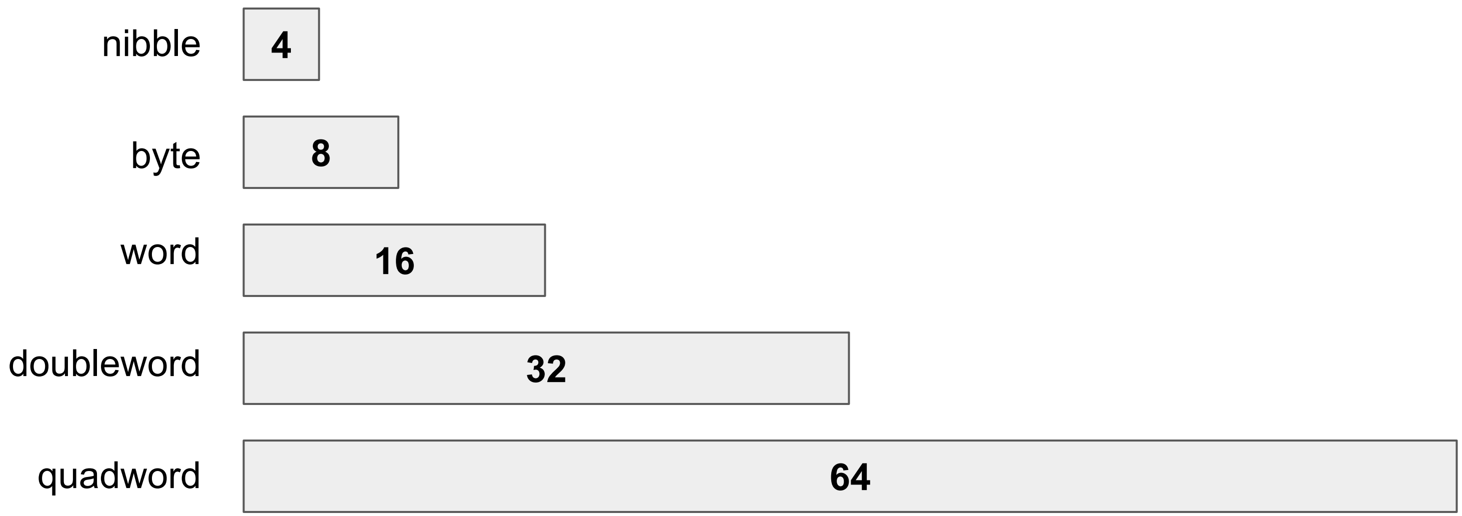
cs260
lecture 1

Basic storage size

bit

Computer stores instructions and data in memory as collection of charges, represented as bits. Each binary digit is called a **bit** is either 0 or 1.



- Most computers handle and store binary data and information in groups of eight bits.

  1 byte = 8 bits

  - A byte can represent numerous types of data/information, e.g. char.

Weighted-Positional number systems:

System Name	Base	Alphabet of possible symbols
Binary	2	0 1
Octal	8	0 1 2 3 4 5 6 7
Decimal	10	0 1 2 3 4 5 6 7 8 9
Hexadecimal	16	0 1 2 3 4 5 6 7 8 9 A B C D E F

Number systems that are commonly used in computer science

- We must develop a certain fluency with number formats/systems i.e quickly translate numbers from one format to another

Note:

Base:

The base can the thought of as the **size** of the alphabet of possible symbols.

Radix Point:

In other number systems the '**decimal point'** is called a **radix point**

Weighted-Positional number systems:

System Name	Base	Alphabet of possible symbols
Binary	2	0 1
Octal	8	0 1 2 3 4 5 6 7
Decimal	10	0 1 2 3 4 5 6 7 8 9
Hexadecimal	16	0 1 2 3 4 5 6 7 8 9 A B C D E F

Number systems that are commonly used in computer science

Notation: $(\text{number})_{\text{base}}$

$(101)_2$	0b101
$(17)_8$	0o17
$(31)_{10}$	31
$(A1)_{16}$	0xA1

Weighted-Positional number systems:

Reminder: decimal numbers system, base = 10

Weighted-Positional number systems:

Example: Consider 1052, decimal number (base-10 number)

Most Significant Digit (MSD) Least Significant Digit (LSD)

$$(\ 1\ 0\ 5\ 2\ )_{10}$$

10^3 10^2 10^1 10^0 ← *weights*

- starting with lowest weight of 10^0, moving from right-to-left
- the **weight** of each digit, increases by a factor of 10

The value of a number is weighted sum of its digits:

$$(\ 1\ 0\ 5\ 2\ )_{10} = 1 \cdot 10^3 + 0 \cdot 10^2 + 5 \cdot 10^1 + 2 \cdot 10^0$$

Weighted-Positional number systems:

<u>Another example</u>: Consider 537**.**74, decimal number (base-10 number)

$$(\,5\;3\;7\,.\,7\,4\,)_{10}$$

10^2 10^1 10^0 10^{-1} 10^{-2} ← *weights*

Moving <u>left</u> from the **decimal point:** **weight** of each digit increases by a factor of 10, starting weight is 10^0

Moving <u>right</u> from the **decimal point:** The weight of each digit decreases by a factor of 10

The value of a number is weighted sum of its digits:

$$(\,537.74\,)_{10} = 5\cdot10^2 + 3\cdot10^1 + 7\cdot10^0 + 7\cdot10^{-1} + 4\cdot10^{-2}$$

more: https://www.mathsisfun.com/decimals.html

Weighted-Positional number systems:

translating to decimal number system
(from binary number system, from base = 2)

Weighted-Positional number systems:

Example: Consider 1011, binary number (base-2 number)

Most Significant Digit (MSD) Least Significant Digit (LSD)

$$(\mathbf{1\ 0\ 1\ 1})_2$$

$$2^3 \quad 2^2 \quad 2^1 \quad 2^0 \quad \leftarrow \textit{weights}$$

- starting with lowest weight of 2^0, moving from right-to-left
- the **weight** of each bit, increases by a factor of 2

The value of a number is weighted sum of its digits:

$$(1\ 0\ 1\ 1)_2 = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

Weighted-Positional number systems:

Another example: Consider 111.11, binary number (base-2 number)

$$(\,1\ 1\ 1.1\ 1\,)_2$$

$$2^2 \quad 2^1 \quad 2^0 \quad 2^{-1} \quad 2^{-2} \leftarrow \textit{weights}$$

Moving left from the **decimal point:** **weight** of each bit increases by a factor of 2, starting weight is 2^0

Moving right from the **decimal point:** The weight of each bit decreases by a factor of 2

The value of a number is weighted sum of its digits:

$$(\ 111.11\ )_2 = 1\cdot2^2 + 1\cdot2^1 + 1\cdot2^0 + 1\cdot2^{-1} + 1\cdot2^{-2}$$

more: https://www.mathsisfun.com/decimals.html

<u>Weighted-Positional number systems</u>: binary

- Bits in a byte a numbered sequentially started at zero on the right side and increasing towards the left.

- **MSB** and **LSB**
  bit on the left side is called the **M**ost **S**ignificant **B**it
  bit on the right side is called the **L**east **S**ignificant **B**it

	MSB							**LSB**
	1	0	1	1	0	1	0	1
bit number	**7**	**6**	**5**	**4**	**3**	**2**	**1**	**0**
weights	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Weighted-Positional number systems: binary

- **For unsigned binary integers:**
  Starting from bit #0 with starting weight of 2^0, each bit has a weight associated with it that is an increasing power of 2.

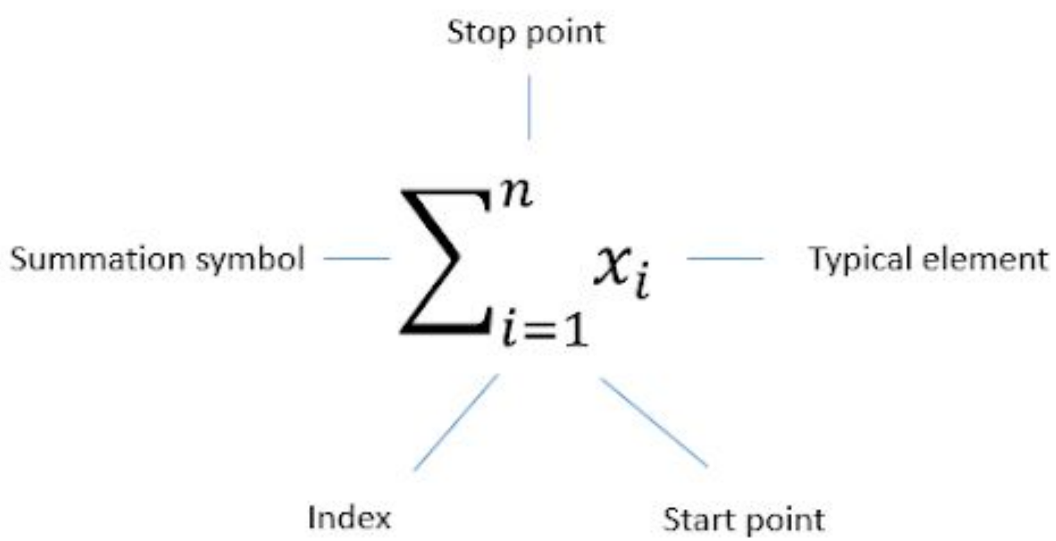
MSB							**LSB**
1	0	1	1	0	1	0	1

weights 2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

Binary Bit Position **Weights** (values)

2^n	Decimal Value	...	2^n	Decimal Value
2^0	1		2^8	256
2^1	2		2^9	512
2^2	4		2^{10}	1024
2^3	8		2^{11}	2048
2^4	16		2^{12}	4096
2^5	32		2^{13}	8192
2^6	64		2^{14}	16384
2^7	128		2^{15}	32768

Reminder: sigma sum notation



$$\sum_{i=1}^{4} x_i = x_1 + x_2 + x_3 + x_4$$

more: https://www.mathsisfun.com/algebra/sigma-notation.html

Weighted-Positional number systems: binary

A convenient way to calculate the **decimal value** of unsigned binary number.

MSB							LSB
1	0	1	1	0	1	0	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

The decimal value of a binary number is weighted sum of its binary digits:

- ***We need the following information:***
  - ***weights*** for each bit position
  - Formula

Let n = length(binary_number) - 1

$$(\text{decimal\_value})_{10} = \sum_{i=0}^{n} \text{bit}_{i}\,(\,\text{base}^{i}\,)$$

pure math

$$\begin{aligned}(\text{decimal\_value})_{10} &= (1\cdot 2^0) + \cancel{(0\cdot 2^1)} + (1\cdot 2^2) + \cancel{(0\cdot 2^3)} + (1\cdot 2^4) + (1\cdot 2^5) + \cancel{(0\cdot 2^6)} + (1\cdot 2^7) \\ &= 1 + 4 + 16 + 32 + 128 \\ &= (181)_{10}\end{aligned}$$

Weighted-Positional number systems: binary

- A **word** is a group of bits that represents a certain unit of information.

  Word size is usually specific to processor design or architecture .

MSB															LSB
1	1	1	0	1	1	0	0	0	1	1	0	1	0	1	0
15															**0**

Find the decimal value of: $(1110110001101010)_2$

To do.

- ***We need the following information:***
  - ***weights*** for each bit position
  - Formula

- The double-dabble method avoids addition of large numbers:
  - Write down the left-most 1 in the binary number.
  - Double it and add the next bit to the right.
  - Write down the result under the next bit.
  - Continue with steps 2 and 3 until finished with the binary number.

- Binary numbers verify the double-dabble method:

$$
\begin{array}{llllll}
\text{Given:} & 1 & 1 & 0 & 1 & 1_2 \\
\text{Results:} & 1 \times 2 = 2 & & & & \\
 & \underline{+\,1} & & & & \\
 & & 3 \times 2 = 6 & & & \\
 & & \underline{+\,0} & & & \\
 & & & 6 \times 2 = 12 & & \\
 & & & \underline{+1} & & \\
 & & & & 13 \times 2 = 26 & \\
 & & & & \underline{+1} & \\
 & & & & & 27_{10}
\end{array}
$$

convert **from** decimal **to** other number systems

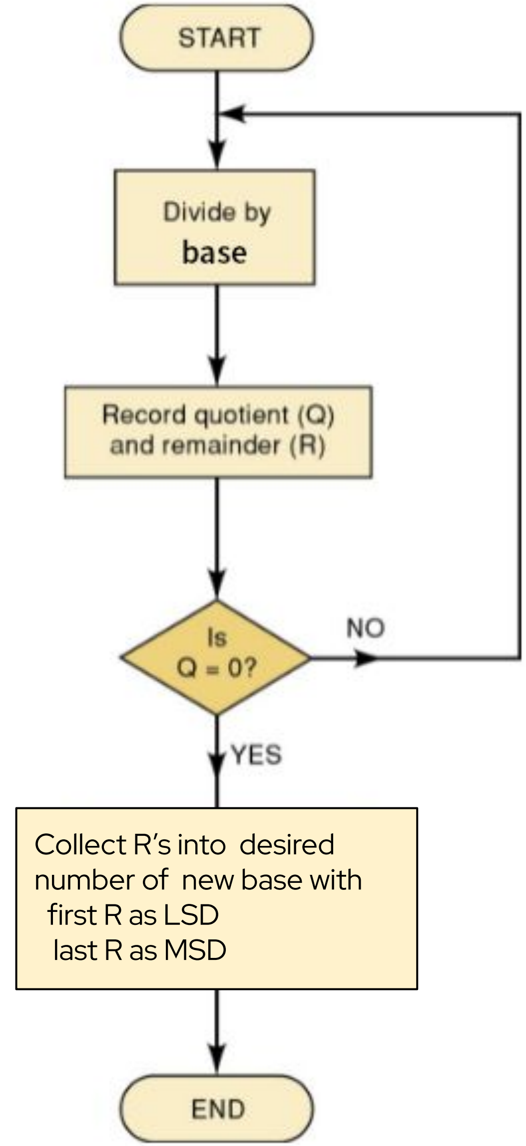
Repeated Division

This flowchart describes the algorithm that can be used to convert **from decimal** to **any** other number system.

decimal → binary

decimal → octal

decimal → hexadecimal



Digital Systems: Principles and Applications, 11/e
Ronald J. Tocci, Neal S. Widmer, Gregory L. Moss

convert **from** decimal **to** binary $(25)_{10} \rightarrow (?)_2$

Repeated Division

Divide the given decimal number by base=2.

Write the remainder after each division until a quotient of zero is obtained.

Division	Quotient	Remainder	
25/2	12	1	**LSB**
12/2	6	0	
6/2	3	0	
3/2	1	1	
1/2	0	1	**MSB**

We read the answer from the remainder column, bottom-up.

answer: $(11001)_2$

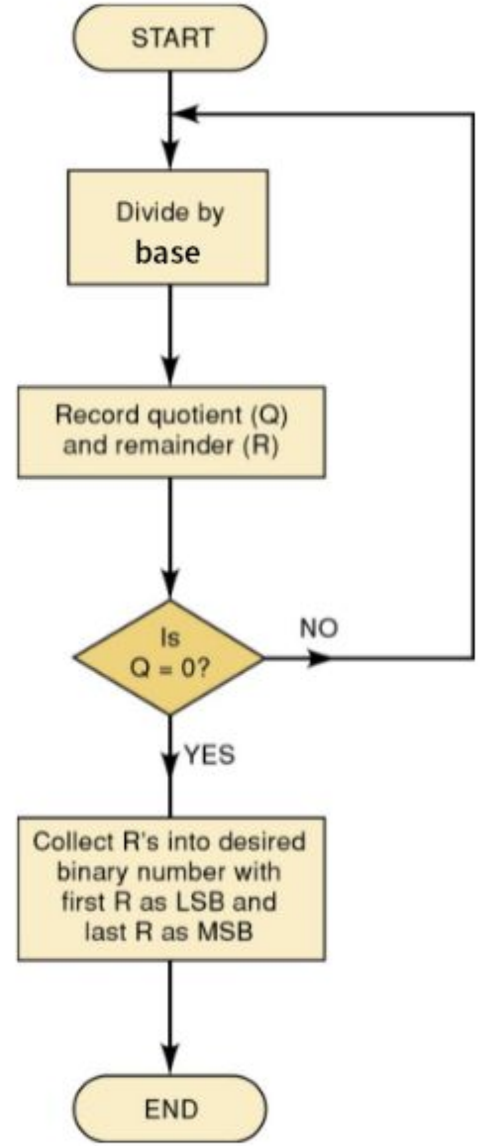
convert **from** decimal **to** binary $(37)_{10} \rightarrow (?)_2$

Repeated Division Algorithm

Division	Quotient	Remainder
37/2	18	1
18/2	9	0
9/2	4	1
4/2	2	0
2/2	1	0
1/2	0	1

LSB

MSB



$(100101)_2$

Digital Systems: Principles and Applications, 11/e
Ronald J. Tocci, Neal S. Widmer, Gregory L. Moss

Weighted-Positional number systems:

translating to decimal number system
(from octal number system, from base = 8)

Weighted-Positional number systems: base-8 to decimal

	MSD			LSD	
	$($ 1	2	3	7 $)_8$	
weights	8^3	8^2	8^1	8^0	

The decimal value of an octal base-8 number is weighted sum of its digits:

- ***We need the following information:***
  - ***weights*** for each digit position
  - Formula

Let n = length(base_8_number) - 1

$$(\text{decimal\_value})_{10} = \sum_{i=0}^{n} \text{digit}_i (\text{ base}^i)$$

$$\begin{aligned}(\text{decimal\_value})_{10} &= (7 \cdot 8^0) + (3 \cdot 8^1) + (2 \cdot 8^2) + (1 \cdot 8^3) \\ &= 7 + 24 + 128 + 512 \\ &= (671)_{10}\end{aligned}$$

convert **from** decimal **to** octal $(781)_{10} \rightarrow (?)_8$

Repeated Division

Divide the given decimal number by base=8.

Write the remainder after each division until a quotient of zero is obtained.

Division	Quotient	Remainder	
781/8	97	5	**LSD**
97/8	12	1	
12/8	1	4	
1/8	0	1	**MSD**

We read the answer from the remainder column, bottom-up.

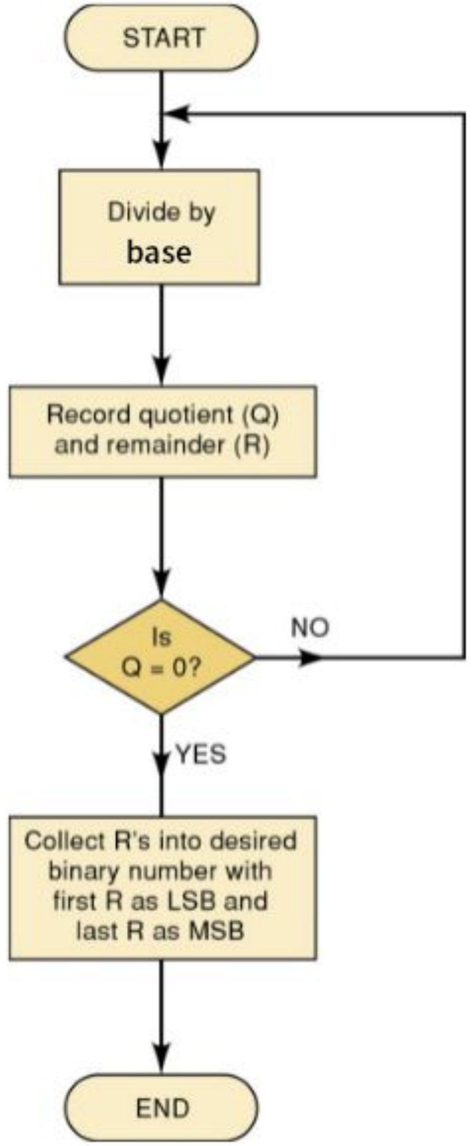
answer: $(1415)_8$

convert **from decimal to octal** $(143{,}529)_{10} \rightarrow (?)_8$

Repeated Division Algorithm

Division	Quotient	Remainder	
143,529/8	17,941	1	**LSD**
17,941/8	2,242	5	
2,242/8	280	2	
280/8	35	0	
35/8	4	3	
4/8	0	4	**MSD**

$(430251)_8$



Digital Systems: Principles and Applications, 11/e
Ronald J. Tocci, Neal S. Widmer, Gregory L. Moss

Weighted-Positional number systems:

translating to decimal number system
(from hexadecimal number system, from base = 16)

Weighted-Positional number systems: base-16 to decimal

1. We need to have a way to convert individual digit from hexadecimal alphabet to decimal

hexadecimal	decimal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
A	10
B	11
C	12
D	13
E	14
F	15

For example:

lookup_table("B") → 11

lookup_table(hex) → dec:

hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
dec	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Weighted-Positional number systems: base-16 to decimal

MSD LSD

$$(\ 1\ \ D\ \ E\ \ 8\ )_{16}$$

weights	16^3	16^2	16^1	16^0

The decimal value of a hexadecimal base-16 number is weighted sum of its digits:

- ***We need the following information:***
  - ***weights*** for each digit position
  - Formula

Let n = length(base_16_number) - 1

$$(\text{decimal\_value})_{10} = \sum_{\boldsymbol{i}=0}^{n} \text{digit}_{\boldsymbol{i}} (\ \text{base}^{\boldsymbol{i}}\ )$$

$$(\text{decimal\_value})_{10} = (8 \cdot 16^0) + (14 \cdot 16^1) + (13 \cdot 16^2) + (1 \cdot 16^3)$$
$$= 8 + 224 + 3{,}328 + 4{,}096$$
$$= (7{,}656)_{10}$$

convert **from** decimal **to** hexadecimal $(781)_{10} \rightarrow (?)_{16}$

Repeated Division

Divide the given decimal number by base=16.

Write the remainder after each division until a quotient of zero is obtained.

Division	Quotient	Remainder	
781/16	48	13 → d	**LSD**
48/16	3	0	
3/16	0	3	**MSD**

answer: $(30d)_{16}$

Because we are converting to the hexadecimal base any value in the remainder column that is greater than or equal to ten has to be converted to a hex digit.

dec	0	1	2	3	4	5	6	7	8	9	**10**	**11**	**12**	**13**	**14**	**15**
hex	'0'	'1'	'2'	'3'	'4'	'5'	'6'	'7'	'8'	'9'	**'a'**	**'b'**	**'c'**	**'d'**	**'e'**	**'f'**

convert **from** decimal **to** hexadecimal $(423)_{10} \rightarrow (?)_{16}$

Repeated Division

Divide the given decimal number by base=16.

Write the remainder after each division until a quotient of zero is obtained.

Division	Quotient	Remainder	
			LSD
			MSD

answer: $(\; ? \;)_{16}$

Because we are converting to the hexadecimal base any value in the remainder column that is greater than or equal to ten has to be converted to a hex digit.

dec	0	1	2	3	4	5	6	7	8	9	**10**	**11**	**12**	**13**	**14**	**15**
hex	'0'	'1'	'2'	'3'	'4'	'5'	'6'	'7'	'8'	'9'	**'a'**	**'b'**	**'c'**	**'d'**	**'e'**	**'f'**