

Homework Assignment 3 Answered

Signed Binary to Decimal

Convert each **signed binary number** into decimal. **DO NOT USE** two's complement. See task 3 of programming assignment 3 for instructions. Please show your work.

$$1. \quad 0110 \Rightarrow -(0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$$

$$\Rightarrow 0 + 4 + 2 + 0$$

$$\Rightarrow 6$$

$$2. \quad 1110 \Rightarrow -(1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$$

$$\Rightarrow -8 + 4 + 2 + 0$$

$$\Rightarrow -2$$

$$3. \quad 001011 \Rightarrow -(0 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$

$$\Rightarrow 0 + 0 + 8 + 0 + 2 + 1$$

$$\Rightarrow 11$$

$$4. \quad 111111 \Rightarrow -(1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$

$$\Rightarrow -32 + 16 + 8 + 4 + 2 + 1$$

$$\Rightarrow -1$$

$$5. \quad 01111111 \Rightarrow -(0 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4)$$

$$+ (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$

$$\Rightarrow 0 + 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1$$

$$\Rightarrow 255$$

$$6. \quad 10000000 \Rightarrow -(1 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (0 \times 2^4)$$

$$+ (0 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)$$

$$\Rightarrow -256 + 0 + 0 + 0 + 0 + 0 + 0 + 0$$

$$\Rightarrow -256$$

$$\begin{aligned}
 7. \quad 000000000101 &\Rightarrow -(0 \times 2^{11}) + (0 \times 2^{10}) + (0 \times 2^9) + (0 \times 2^8) + (0 \times 2^7) + (0 \times 2^6) \\
 &\quad + (0 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\
 &\Rightarrow 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 4 + 0 + 1 \\
 &\Rightarrow 5
 \end{aligned}$$

$$\begin{aligned}
 8. \quad 111111111111 &\Rightarrow -(1 \times 2^{11}) + (1 \times 2^{10}) + (1 \times 2^9) + (1 \times 2^8) + (1 \times 2^7) + (1 \times 2^6) \\
 &\quad + (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\
 &\Rightarrow -2048 + 1024 + 512 + 256 + 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 \\
 &\Rightarrow -1
 \end{aligned}$$

$$\begin{aligned}
 9. \quad 101010 &\Rightarrow -(1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) \\
 &\Rightarrow -32 + 0 + 8 + 0 + 2 + 0 \\
 &\Rightarrow -22
 \end{aligned}$$

$$\begin{aligned}
 10. \quad 0101 &\Rightarrow -(0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\
 &\Rightarrow 0 + 4 + 0 + 1 \\
 &\Rightarrow 5
 \end{aligned}$$

Unsigned Hexadecimal to Binary

Convert each **unsigned hexadecimal number** to binary. No work needed.

Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

1. A \Rightarrow 1010
2. 2F \Rightarrow 0010 1111
3. 0 \Rightarrow 0000
4. 7FF \Rightarrow 0111 1111 1111
5. 1C \Rightarrow 0001 1100
6. 9B \Rightarrow 1001 1011
7. FF \Rightarrow 1111 1111
8. 123 \Rightarrow 0001 0010 0011
9. 5 \Rightarrow 0101
10. ABCD \Rightarrow 1010 1011 1100 1101

Signed Binary Subtraction

Find the difference for each of the following signed binary subtractions. Subtraction is just addition with the subtrahend inverted. $10 - 5 = 10 + (-5)$. Please show your work.

1. $0101 - 111$

One's Complement: $111 \Rightarrow 000$

$$\begin{array}{r} \text{Carry Bits} \quad 0 \ 0 \\ 0 \ 0 \ 0 \\ + \quad 0 \ 0 \ 1 \\ \hline \text{Sum} \quad 0 \ 0 \ 1 \end{array}$$

Two's Complement: 001

$$\begin{array}{r} \text{Carry Bits} \quad 0 \ 0 \ 1 \\ 0 \ 1 \ 0 \ 1 \\ + \quad 0 \ 0 \ 0 \ 1 \\ \hline \text{Sum} \quad 0 \ 1 \ 1 \ 0 \end{array}$$

Answer = 0110

Both signed binary operands are positive. The final answer is also positive. There is no overflow.

2. $1110 - 100101$

One's Complement: $100101 \Rightarrow 011010$

$$\begin{array}{r} \text{Carry Bits} \quad 0 \ 0 \ 0 \ 0 \ 0 \\ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \\ + \quad 0 \ 0 \ 0 \ 0 \ 0 \ 1 \\ \hline \text{Sum} \quad 0 \ 1 \ 1 \ 0 \ 1 \ 1 \end{array}$$

Two's Complement: 011011

$$\begin{array}{r} \text{Carry Bits} \quad 1 \ 1 \ 1 \ 1 \ 0 \\ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \\ + \quad 0 \ 1 \ 1 \ 0 \ 1 \ 1 \\ \hline \text{Sum} \quad 0 \ 1 \ 1 \ 0 \ 0 \ 0 \end{array}$$

Answer = 011000

The signed binary operands have different sign bits. Therefore, we don't need to check for overflow.

3. $001011 - 101$

One's Complement: $101 \Rightarrow 010$

$$\begin{array}{r}
 \text{Carry Bits} \quad 0 \ 0 \\
 \quad 0 \ 1 \ 0 \\
 + \quad 0 \ 0 \ 1 \\
 \hline
 \text{Sum} \quad 0 \ 1 \ 1
 \end{array}$$

Two's Complement: 011

$$\begin{array}{r}
 \text{Carry Bits} \quad 0 \ 0 \ 0 \ 1 \ 1 \\
 \quad 0 \ 0 \ 1 \ 0 \ 1 \ 1 \\
 + \quad 0 \ 0 \ 0 \ 0 \ 1 \ 1 \\
 \hline
 \text{Sum} \quad 0 \ 0 \ 1 \ 1 \ 1 \ 0
 \end{array}$$

Answer = 001110

Both signed binary operands are positive. The final answer is also positive. There is no overflow.

4. $111111 - 11010$

One's Complement: $11010 \Rightarrow 00101$

$$\begin{array}{r}
 \text{Carry Bits} \quad 0 \ 0 \ 0 \ 0 \ 1 \\
 \quad 0 \ 0 \ 0 \ 1 \ 0 \ 1 \\
 + \quad 0 \ 0 \ 0 \ 0 \ 0 \ 1 \\
 \hline
 \text{Sum} \quad 0 \ 0 \ 0 \ 1 \ 1 \ 0
 \end{array}$$

Two's Complement: 00110

$$\begin{array}{r}
 \text{Carry Bits} \quad 1 \ 1 \ 1 \ 1 \ 0 \\
 \quad 1 \ 1 \ 1 \ 1 \ 1 \ 1 \\
 + \quad 0 \ 0 \ 0 \ 1 \ 1 \ 0 \\
 \hline
 \text{Sum} \quad 0 \ 0 \ 0 \ 1 \ 0 \ 1
 \end{array}$$

Answer = 000101

The signed binary operands have different sign bits. Therefore, we don't need to check for overflow.

5. $01111111 - 1010010$

One's Complement: $1010010 \Rightarrow 0101101$

Carry Bits		0	0	0	0	1
		0	1	0	1	1
	+	0	0	0	0	0
Sum		0	1	0	1	1

Two's Complement: 0101110

Carry Bits		0	1	1	1	1	1	0
		0	1	1	1	1	1	1
	+	0	1	0	1	1	1	0
Sum		0	1	1	0	1	1	0

Answer = 01101101

Both signed binary operands are positive. However, the answer is negative. There is overflow. A positive sign bit was prepended to the final answer to indicate that is positive.

6. $10000000 - 11111111$

One's Complement: $11111111 \Rightarrow 00000000$

Carry Bits		0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	+	0	0	0	0	0	0	0	1
Sum		0	0	0	0	0	0	0	1

Two's Complement: 00000001

Carry Bits		0	0	0	0	0	0	0	0
		1	1	0	0	0	0	0	0
	+	0	0	0	0	0	0	0	1
Sum		1	1	0	0	0	0	0	1

Answer = 11000001

The signed binary operands have different sign bits. Therefore, we don't need to check for overflow.

7. 000000000101 – 110

One's Complement: 110 => 001

$$\begin{array}{r}
 \text{Carry Bits} \quad 0 \ 1 \\
 \quad 0 \ 0 \ 1 \\
 + \quad 0 \ 0 \ 1 \\
 \hline
 \text{Sum} \quad 0 \ 1 \ 0
 \end{array}$$

Two's Complement: 010

$$\begin{array}{r}
 \text{Carry Bits} \quad 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\
 \quad 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \\
 + \quad 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \\
 \hline
 \text{Sum} \quad 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1
 \end{array}$$

Answer = 000000000111

Both signed binary operands are positive. The final answer is also positive. There is no overflow.

8. 111111111111 – 10101

One's Complement: 10101 => 01010

$$\begin{array}{r}
 \text{Carry Bits} \quad 0 \ 0 \ 0 \ 0 \\
 \quad 0 \ 1 \ 0 \ 1 \ 0 \\
 + \quad 0 \ 0 \ 0 \ 0 \ 1 \\
 \hline
 \text{Sum} \quad 0 \ 1 \ 0 \ 1 \ 1
 \end{array}$$

Two's Complement: 01011

$$\begin{array}{r}
 \text{Carry Bits} \quad 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \\
 \quad 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \\
 + \quad 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1 \\
 \hline
 \text{Sum} \quad 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0
 \end{array}$$

The signed binary operands have different sign bits. Therefore, we don't need to check for overflow.

9. $101010 - 111$

One's Complement: $111 \Rightarrow 000$

$$\begin{array}{r}
 \text{Carry Bits} \quad 0 \ 0 \\
 \quad 0 \ 0 \ 0 \\
 + \quad 0 \ 0 \ 1 \\
 \hline
 \text{Sum} \quad 0 \ 0 \ 1
 \end{array}$$

Two's Complement: 001

$$\begin{array}{r}
 \text{Carry Bits} \quad 0 \ 0 \ 0 \ 0 \ 0 \\
 \quad 1 \ 0 \ 1 \ 0 \ 1 \ 0 \\
 + \quad 0 \ 0 \ 0 \ 0 \ 0 \ 1 \\
 \hline
 \text{Sum} \quad 1 \ 0 \ 1 \ 0 \ 1 \ 1
 \end{array}$$

Answer = 101011

The signed binary operands have different sign bits. Therefore, we don't need to check for overflow.

10. $0101 - 1111001$

One's Complement: $1111001 \Rightarrow 0000110$

$$\begin{array}{r}
 \text{Carry Bits} \quad 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\
 \quad 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \\
 + \quad 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \\
 \hline
 \text{Sum} \quad 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1
 \end{array}$$

Two's Complement: 0000111

$$\begin{array}{r}
 \text{Carry Bits} \quad 0 \ 0 \ 0 \ 1 \ 1 \ 1 \\
 \quad 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \\
 + \quad 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \\
 \hline
 \text{Sum} \quad 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0
 \end{array}$$

Answer = 0001100

Both signed binary operands are positive. The final answer is also positive. There is no overflow.