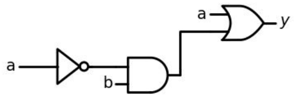
Sum-of-minterms form

Different equations may represent the same function

$y = a + b$

$y = a + a'b$



The *sameness* is not *obvious* → a **standard equation form** is desirable.

$y = a'b + ab' + ab$

see: check.py

The *standard equation* form of a function is call a *canonical form*.

Sum-of-minterms form is a canonical form of a Boolean equation such that right-side expression is a sum-of-products with each product a *unique* minterm.

- A ***minterm*** is a product term having **exactly one literal** for every function variable.
- A ***literal*** is a variable appearance, in true or complemented form, in an expression.

i.e. each function variable has exactly one representation in each product term

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Example of ***Sum-of-minterms***:
Consider a function of two variables: a, b:

because:
- ★ sum-of-products form
- ★ each product is unique
- ★ each product is a minterm

This is a valid sum-of-minterms: **y = ab + a'b + a'b'**

This is **NOT** a valid sum-of-minterms: y = ab + a'

a' is not a minterm, ***b*** does not have representation
This is a sum-of-products, but *not* sum-of-minterms

Questions to ask yourself:
sum-of-products ?
are products unique?
are products minterms?

The standard equation form of a function is call a *canonical form*.

Sum-of-minterms form is a canonical form of a Boolean equation such that right-side expression is a sum-of-products with each product a *unique* minterm.

- A ***minterm*** is a product term having **exactly one literal** for every function variable.
- A ***literal*** is a variable appearance, in true or complemented form, in an expression.

i.e. each function variable has exactly one representation in each product term

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Question:
Consider a function of three variables: a, b, c:

Is this a valid sum-of-minterms? **y = abc' + a'b'c**

The standard equation form of a function is call a *canonical form*.

<u>Questions to ask yourself</u>:
sum-of-products ?
are products unique?
are products minterms?

Sum-of-minterms form is a canonical form of a Boolean equation such that right-side expression is a sum-of-products with each product a *unique* <u>minterm</u>.

- A ***minterm*** is a product term having **exactly one literal** for every function variable.
- A ***literal*** is a variable appearance, in true or complemented form, in an expression.

i.e. each function variable has exactly one representation in each product term

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Question:
Consider a function of three variables: a, b, c:

Is this a valid sum-of-minterms? **y = ac'**

The standard equation form of a function is call a *canonical form*.

<u>Questions to ask yourself</u>:
sum-of-products ?
are products unique?
are products minterms?

Sum-of-minterms form is a canonical form of a Boolean equation such that right-side expression is a sum-of-products with each product a *unique* <u>minterm</u>.

- A ***minterm*** is a product term having **exactly one literal** for every function variable.
- A ***literal*** is a variable appearance, in true or complemented form, in an expression.

i.e. each function variable has exactly one representation in each product term

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Question:
Consider a function of three variables: a, b, c:

 Is this a valid sum-of-minterms? $\mathbf{y = abc' + abc'}$ minterm should be unique

The standard equation form of a function is call a *canonical form*.

Questions to ask yourself:
- sum-of-products ?
- are products unique?
- are products minterms?

Sum-of-minterms form is a canonical form of a Boolean equation such that right-side expression is a sum-of-products with each product a *unique* minterm.

- A ***minterm*** is a product term having **exactly one literal** for every function variable.
- A ***literal*** is a variable appearance, in true or complemented form, in an expression.

i.e. each function variable has exactly one representation in each product term

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Question:
Consider a function of three variables: a, b, c:

Is this a valid sum-of-minterms? **y = ab + ac**

c does not have representation in first product term
b does not have representation in second product term
This is a sum-of-products, but *not* sum-of-minterms

Is this a valid sum-of-minterms? **y = a(b + c)**

not in a sum-of-products form

Transforming a function to the *canonical* sum-of-minterms form.

Properties

Distributive

$$ab+ac \equiv a(b+c)$$

$$(a+b)(a+c) \equiv a+bc$$

Commutative

$$ab \equiv ba$$

$$a+b \equiv b+a$$

Complement

$$aa' \equiv 0$$

$$a+a' \equiv 1$$

Identity

$$a \cdot 1 \equiv a$$

$$a+0 \equiv a$$

Null elements

$$a \cdot 0 \equiv 0$$

$$a+1 \equiv 1$$

Idempotence

$$aa \equiv a$$

$$a+a \equiv a$$

Transforming a function to the *canonical* sum-of-minterms form.

Given the function of three variables: a,b,c. Convert $y = a'b$ to the sum-of-minterms.

c does not have representation

$$
\begin{aligned}
y &= a'b \\
&= a'b(1) && \longleftarrow \text{by Identity property } b \cdot 1 \equiv b \\
&= a'b(c+c') && \longleftarrow \text{by Complement property } c + c' \equiv 1 \\
&= a'bc + a'bc' && \longleftarrow \text{by using \_\_\_\_\_\_\_\_\_\_ property}
\end{aligned}
$$

sum-of-minterms:

- ★ sum-of-products form
- ★ each product is unique
- ★ each product is a minterm

Transforming a function to the *canonical* sum-of-minterms form.

Given a function with three variables: a, b, c. Convert y = a(b + bc') to sum-of-minterms.

not a sum-of-product form

$$
\begin{aligned}
y &= a(b + bc') \\
&= ab + abc' && \leftarrow \text{by distributive} \\
&= ab(1) + abc' && \leftarrow \text{by identity property } b \cdot 1 \equiv b \\
&= ab(c + c') + abc' && \leftarrow \text{Complement property } c + c' \equiv 1 \\
&= abc + abc' + abc' && \leftarrow \text{identity distributive} \\
&= abc + abc' && \leftarrow \text{by idempotent}
\end{aligned}
$$

sum-of-minterms:

★ sum-of-products form

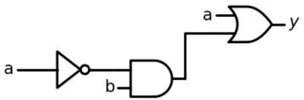
★ each product is unique

★ each product is a minterm

Different equations may represent the same function

$y = a + b$

$y = a + a'b$



The *sameness* is not *obvious* $\longrightarrow$ a **standard equation form** is desirable.

$y = a + b$

$y = a(1) + b(1)$

$y = a(b + b') + b(a + a')$

$y = ab + ab' + ba + ba'$

$y = ab + ab' + \cancel{ab} + a'b$

$y = ab + ab' + a'b$

$y = a'b + ab' + ab$

$y = a + a'b$

$y = a(1) + a'b$

$y = a(b + b') + a'b$

$y = ab + ab' + a'b$

$y = a'b + ab' + ab$

canonical sum-of-minterms form

Compact function notation: Minterm numbers

Compact function notation: Minterm numbers

A compact function notation represents each minterm by a number.

Suppose that we have the following minterm: a’b’c $\rightarrow$ 001 $\rightarrow$ 1 $\rightarrow$ M_1

Suppose that we have the following minterm: a’bc $\rightarrow$ 011 $\rightarrow$ 3 $\rightarrow$ M_3

Suppose that we have the following minterm: ab’c $\rightarrow$ 101 $\rightarrow$ 5 $\rightarrow$ M_5

Suppose that we have the following minterm: a’b’c’ $\rightarrow$ 000 $\rightarrow$ 0 $\rightarrow$ M_0

f(a,b,c) = a'b'c' + ab'c + abc can be written compactly as f(a,b,c) = $M_0 + M_5 + M_7$

alternative - more compact - notation is f(a,b,c) = $\sum(0, 5, 7)$

Compact function notation: Minterm numbers

$f(a,b,c) = abc + abc'$

111	110
7	6

$f(a,b,c) = M_7 + M_6$

Or in the alternative notation:
$f(a,b,c) = \sum(7, 6)$

$f(a,b) = a'b + ab' + ab$

01	10	11
1	2	3

$f(a,b) = M_1 + M_2 + M_3$

Or in the alternative notation:
$f(a,b) = \sum(1, 2, 3)$

$f(a, b, c) = a'bc + ab'c$

Truth tables: Recall

inputs

output

a	b	y
0	0	1
0	1	0
1	0	0
1	1	1

a	b	f(a, b)
0	0	1
0	1	0
1	0	0
1	1	1

A *truth table* lists all possible variable value combinations on the left, and lists the function's value for each combination on the right.

A function with N variables will have a truth table with 2^N rows:

- 2 variables yields $2^2 = 4$ rows
- 3 variables yields $2^3 = 8$ rows
- 4 variables yields $2^4 = 16$ rows
- (And so on)

Minterm numbers and Truth tables

		a	b	f(a, b)
M_0	a'b'	0	0	1
M_1	a'b	0	1	0
M_2	ab'	1	0	0
M_3	ab	1	1	1

We can write a Truth table such that each row corresponds to a *possible* minterm.
Generating all combinations is done by counting up in binary.

Minterm numbers and Truth tables

$f(a,b) = a'b' + ab$

		a	b	f(a, b)
M_0	a'b'	0	0	1
M_1	a'b	0	1	0
M_2	ab'	1	0	0
M_3	ab	1	1	1

We can write a Truth table such that each row corresponds to a *possible* minterm.

Q: How do we find the actual minterms for our function?
A: The minterms are the rows that have 1 in the ***output*** column, f(a,b).

Minterm numbers and Truth tables

We just saw that Minterms are sometimes written as M_0, M_1,M_2, ...,

		a b c	f(a, b,c)
M_0	a’b’c’	0 0 0	1
M_1	a’b’c	0 0 1	0
M_2	a’bc’	0 1 0	0
M_3	a’bc	0 1 1	0
M_4	ab’c’	1 0 0	1
M_5	ab’c	1 0 1	0
M_6	abc’	1 1 0	1
M_7	abc	1 1 1	**0**

Minterm numbers and Truth tables

$$f(a,b,c) = a'b'c' + ab'c' + abc'$$

		a b c	f(a, b, c)
M_0	$a'b'c'$	0 0 0	1
M_1	$a'b'c$	0 0 1	0
M_2	$a'bc'$	0 1 0	0
M_3	$a'bc$	0 1 1	0
M_4	$ab'c'$	1 0 0	1
M_5	$ab'c$	1 0 1	0
M_6	abc'	1 1 0	1
M_7	abc	1 1 1	0

Q: How do we find the actual minterms for our function?
A: The minterms are the rows that have 1 in the result column, f(a,b,c).

Truth tables → Equation → Circuit Diagram

Truth table

a b c	f
0 0 0	0
0 0 1	0
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	1
1 1 1	0

Equation

$$f = a'bc + abc'$$

Circuit