Product-of-sums form and maxterms

Product-of-sums form

A ***sum term*** is an ORing of (one or more) variables

a $(a + b')$ $(a + b' + c')$

$a'+cd$

sum term must only have OR, **not AND**

product-of-sums (POS) form consists solely of an ANDing of <u>sum terms</u>:

a $(a+b')$ $(a'+c)(b+d)$ $(a + b' + c)(a + b)$

$a'+cd$

$(ab') + (a'c)$

POS

maxterm and Product-of-maxterms form

A ***maxterm*** is a sum term that has all of the function variables exactly once in either true or complemented form.

For a function of three variables: a, b, and c

$(a' + b' + c)$ $(a' + b + c')$ $(a' + b' + c')$

$(a' + b')$ → c representation is missing

$(a' + b' + bc)$ → not a sum term

Product-of-maxterms form is a **canonical** form of a Boolean equation where the right-side expression is a product-of-sum with each sum consisting only of maxterms.

For a function of three variables: a, b, and c

$$y = (a' + b' + c')$$

Product-of-maxterms

maxterm and Product-of-maxterms form

A ***maxterm*** is a <u>sum term</u> that has all of the function variables <u>exactly once</u> in either true or complemented form.

For a function of three variables: a, b, and c

$(a' + b' + c)$ $(a' + b + c')$ $(a' + b' + c')$

$(a' + b')$ $\rightarrow$ c representation is missing

$(a' + b' + bc)$ $\rightarrow$ not a sum term

Product-of-maxterms form is a **canonical** form of a Boolean equation where the right-side expression is a product-of-sum with each sum consisting only of <u>maxterms</u>.

For a function of three variables: a, b, and c

$$y = (a + b + c)(a' + b' + c')$$

Product-of-maxterms

maxterm and Product-of-maxterms form

A ***maxterm*** is a <u>sum term</u> that has all of the function variables <u>exactly once</u> in either true or complemented form.

For a function of three variables: a, b, and c

$(a' + b' + c)$ $(a' + b + c')$ $(a' + b' + c')$

$(a' + b')$ $\rightarrow$ c representation is missing

$(a' + b' + bc)$ $\rightarrow$ not a sum term

Product-of-maxterms form is a **canonical** form of a Boolean equation where the right-side expression is a product-of-sum with each sum consisting only of <u>maxterms</u>.

For a function of three variables: a, b, and c

$y = (a' + b)(a' + b' + c')$

Product-of-maxterms

Truth Tables → Product-of-maxterms form

inputs

output

		a	b	f
M_0	$a + b$	0	0	0
M_1	$a + b'$	0	1	1
M_2	$a' + b$	1	0	1
M_3	$a' + b'$	1	1	0

→ $f(a,b) = (a+b)(a'+b')$

Each row of the truth table corresponds to a *maxterm*, denoted by Mx

Any input that has value of **0** → variable is in the true from for our *maxterm*.

Any input that has value of **1** → variable is in the complemented from for our *maxterm*.

For each row where output is **0:**

convert truth table to a product-of-maxterms form by ANDing the *maxterms*

converting from product-of-sums to sum-of-products

Converting from POS to SOP:

$y = (a + b)(a' + b')$	Product-of-sums form
$y = a(a' + b') + b(a' + b')$	Distributive
$y = aa' + ab' + a'b + bb'$	Distributive
$y = 0 + ab' + a'b + 0$	Complement (AND)
$y = ab' + a'b$	Sum-of-products form

				Minterms		Maxterms
X	Y	Z		Product Terms		Sum Terms
0	0	0		$m_0 = \overline{X} \cdot \overline{Y} \cdot \overline{Z} = \min\left(\overline{X}, \overline{Y}, \overline{Z}\right)$		$M_0 = X + Y + Z = \max\left(X, Y, Z\right)$
0	0	1		$m_1 = \overline{X} \cdot \overline{Y} \cdot Z = \min\left(\overline{X}, \overline{Y}, Z\right)$		$M_1 = X + Y + \overline{Z} = \max\left(X, Y, \overline{Z}\right)$
0	1	0		$m_2 = \overline{X} \cdot Y \cdot \overline{Z} = \min\left(\overline{X}, Y, \overline{Z}\right)$		$M_2 = X + \overline{Y} + Z = \max\left(X, \overline{Y}, Z\right)$
0	1	1		$m_3 = \overline{X} \cdot Y \cdot Z = \min\left(\overline{X}, Y, Z\right)$		$M_3 = X + \overline{Y} + \overline{Z} = \max\left(X, \overline{Y}, \overline{Z}\right)$
1	0	0		$m_4 = X \cdot \overline{Y} \cdot \overline{Z} = \min\left(X, \overline{Y}, \overline{Z}\right)$		$M_4 = \overline{X} + Y + Z = \max\left(\overline{X}, Y, Z\right)$
1	0	1		$m_5 = X \cdot \overline{Y} \cdot Z = \min\left(X, \overline{Y}, Z\right)$		$M_5 = \overline{X} + Y + \overline{Z} = \max\left(\overline{X}, Y, \overline{Z}\right)$
1	1	0		$m_6 = X \cdot Y \cdot \overline{Z} = \min\left(X, Y, \overline{Z}\right)$		$M_6 = \overline{X} + \overline{Y} + Z = \max\left(\overline{X}, \overline{Y}, Z\right)$
1	1	1		$m_7 = X \cdot Y \cdot Z = \min\left(X, Y, Z\right)$		$M_7 = \overline{X} + \overline{Y} + \overline{Z} = \max\left(\overline{X}, \overline{Y}, \overline{Z}\right)$

src: https://www.geeksforgeeks.org/canonical-and-standard-form/