

Binary to Octal, and Hexadecimal

Binary	Octal
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Binary to Octal, and Hexadecimal

Going from octal or hexadecimal to binary is a straightforward process. We just replace the octal or hexadecimal digit with its binary equivalent.

$$\begin{aligned}347646_8 &\Rightarrow 011\ 100\ 111\ 110\ 100\ 110_2 \\B1FF74_{16} &\Rightarrow 1011\ 0001\ 1111\ 1111\ 0111 \\&0100_2\end{aligned}$$

When going from binary to octal or hexadecimal, we have to ensure the number of bits is a multiple of the base. If it is not, we pad the number until it is.

$$\begin{aligned}1\ 101\ 011\ 110\ 000\ 110_2 &= \textcolor{teal}{11}\ 101\ 011\ 110\ 000\ 110_2 \Rightarrow 753606_8 \\101\ 0101\ 0001\ 0111\ 0100_2 &= \textcolor{teal}{1101}\ 0101\ 0001\ 0111\ 0100_2 = D5174_{16}\end{aligned}$$

Signed Binary Multiplication

For the multiplication of two binary numbers we use Booth's algorithm.

Before starting, ensure both numbers are of the same length. The length of the numbers we call n and the length of the product will be at most $2n$.

Create 4 variables:

- $A = 0\dots$ (padded to a bit length of n)
- M = Multiplicand (first number)
- Q = Multiplier (second number)
- $Q_{-1} = 0$, (bit length of 1)

We will also be referring to a variable Q_0 that represents the least significant bit or the right most bit of the binary number in Q .

Let variables separated by commas be considered a concatenation of the variables: A, Q, Q_{-1} (length of $2n + 1$) and Q_0, Q_{-1} (length of 2)

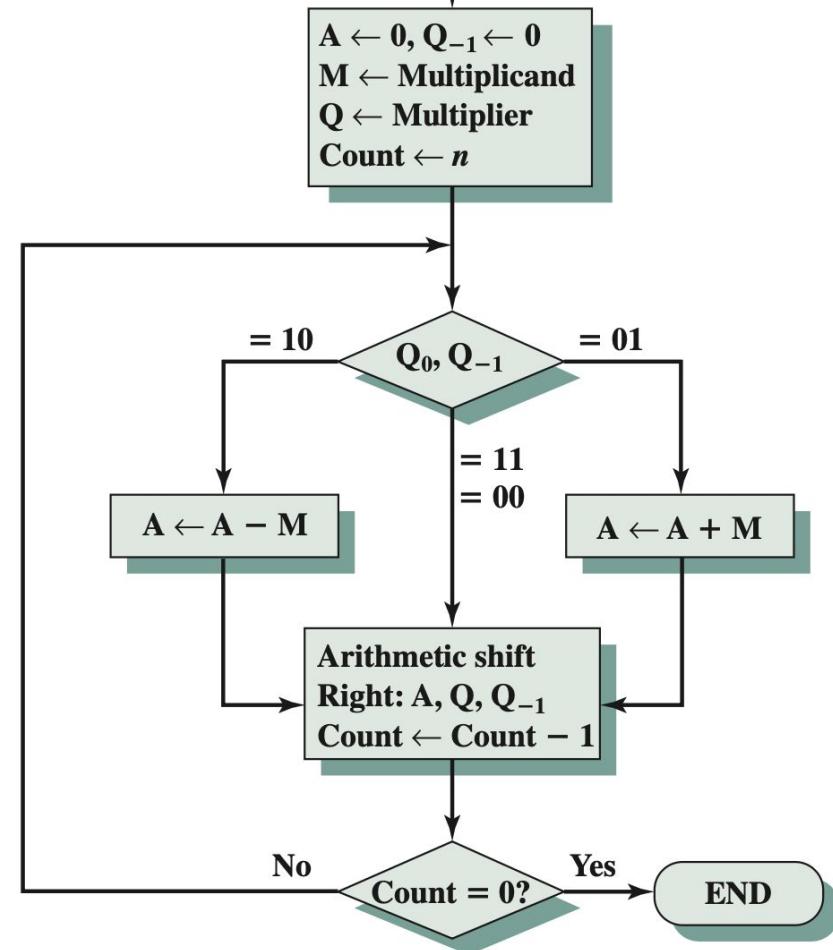
Signed Binary Multiplication

Booth's Algorithm

For n times:

1. If $Q_0, Q_{-1} = 01$: $A \leftarrow A + M$
Else if $Q_0, Q_{-1} = 10$: $A \leftarrow A - M$
2. $A, Q, Q_{-1} >> 1$ (arithmetic shift)

product = $A, Q,$



Signed Binary Multiplication

Lets test **0111 x 0011**. We know $0111 = 7$ and $0011 = 3$. Since $7 \times 3 = 21$, our binary product should be **00010101** untrimmed.

A	Q	Q_{-1}	M	Initial values	
0000	0011	0	0111		
1001	0011	0	0111	$A \leftarrow A - M$	First cycle
1100	1001	1	0111	Shift	
1110	0100	1	0111	Shift	Second cycle
0101	0100	1	0111	$A \leftarrow A + M$	
0010	1010	0	0111	Shift	Third cycle
0001	0101	0	0111	Shift	

Signed Binary Multiplication

Lets try: $01011 \times 1011\dots 11 \times -5 = -55\dots 1001001$

A	Q	Q_{-1}	M		
00000	11011	0	01011	Initial values	
10101	11011	0	01011	$A = A - M$	First Cycle
11010	11101	1	01011	Shift	
11101	01110	1	01011	Shift	Second Cycle
01000	01110	1	01011	$A = A + M$	Third Cycle
00100	00111	0	01011	Shift	
11001	00111	0	01011	$A = A - M$	Fourth Cycle
11100	10011	1	01011	Shift	
11110	01001	1	01011	Shift	Fifth Cycle

Signed Binary Multiplication

Lets try: $1010 \times 1010\dots -6 \times -6 = 36\dots 0100100$

A	Q	Q_{-1}	M		
0000	1010	0	1010	Initial values	
					First Cycle
					Second Cycle
					Third Cycle
					Fourth Cycle

Signed Binary Multiplication

Lets try: $01111 \times 01010\dots$ $15 \times 10 = 150\dots$ 0010010110

A	Q	Q_{-1}	M		
00000	01010	0	01111	Initial values	
00000	00101	0	01111	Shift	First Cycle
10001	00101	0	01111	$A = A - M$	Second Cycle
11000	10010	1	01111	Shift	
00111	10010	1	01111	$A = A + M$	Third Cycle
00011	11001	0	01111	Shift	
10100	11001	0	01111	$A = A - M$	Fourth Cycle
11010	01100	1	01111	Shift	
01001	01100	1	01111	$A = A + M$	Fifth Cycle
00100	10110	0	01111	Shift	

Signed Binary Division

For signed binary division, we need to treat the operands as positive. This requires us to convert negative numbers to their two's complement. We must also keep track of the signs of the original numbers. The result of our division algorithm will produce the quotient as well as the remainder. The sign of the quotient and remainder will depend on the signs of the original numbers.

Let S_D = sign of the dividend

Let S_V = sign of the divisor

If $S_D \neq S_V$: the sign of the quotient is negative (two's complement)

If $S_D < 0$: the sign of the quotient is negative (two's complement)

$$D \div V = Q, R \quad D \div -V = -Q, R \quad -D \div V = -Q, -R \quad -D \div -V = Q, -R$$

Signed Binary Division

Before starting, ensure the length of the dividend is greater than or equal to the length of the divisor. The length of the dividend we call n . The length of the divisor we call m .

Create 3 variables:

- $A = 0\dots$ (padded to a bit length m)
- $Q =$ Dividend (first number)
- $M =$ Divisor (second number)

We will also be referring to a variable Q_0 that represents the least significant bit or the right most bit of the binary number in Q .

Let variables separated by commas be considered a concatenation of the variables: A, Q (length of $m + n$)

Signed Binary Division

For n times:

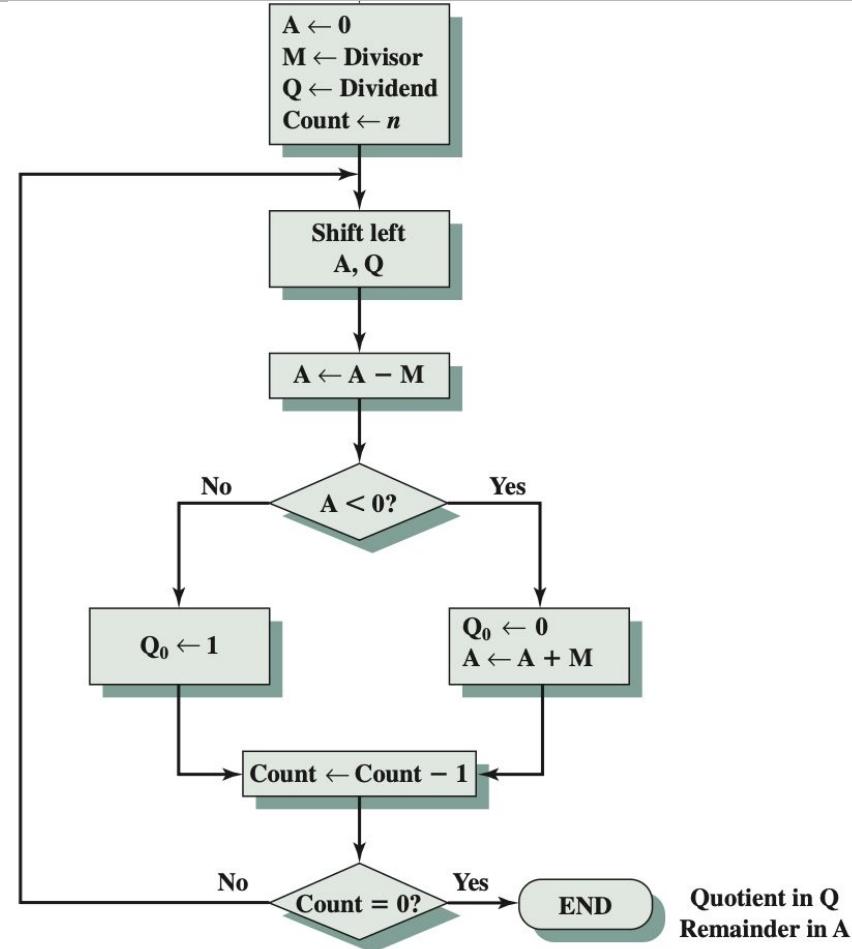
1. $A, Q \ll 1$ (logical shift)
2. $A \leq A - M$
3. If A is negative: $Q_0 \leq 0$

$$A \leq A + M$$

Else: $Q_0 \leq 1$

Quotient = Q

Remainder = A



Signed Binary Division

Lets try:

$$01010 \div 010 = 00000101$$

$$10 \div 2 = 5$$

Quotient = 00101

Remainder = 000

$S_D == S_V$: no change

$S_D >= 0$: no change

A	Q	M		
000	01010	010	Initial values	
000	10100	010	A,Q << 1 A = A - M $Q_0 = 0$ A = A + M	1st Cycle
110	10100	010		
110	10100	010		
000	10100	010		
001	01000	010	A,Q << 1 A = A - M $Q_0 = 0$ A = A + M	2nd Cycle
111	01000	010		
111	01000	010		
001	01000	010		
010	10000	010	A,Q << 1 A = A - M $Q_0 = 1$	3rd Cycle
000	10000	010		
000	10001	010		
001	00010	010	A,Q << 1 A = A - M $Q_0 = 0$ A = A + M	4th Cycle
111	00010	010		
111	00010	010		
001	00010	010		
010	00100	010	A,Q << 1 A = A - M $Q_0 = 1$	5th Cycle
000	00100	010		
000	00101	010		

Signed Binary Division

Lets try:

$$0111 \div 101 = 0011110$$

$$7 \div 3 = 2 \text{ remainder } 1$$

$$Q = 0010$$

$$A = 001$$

$S_D \neq S_V$: two's comp Q

$S_D \geq 0$: no change

$$\text{Quotient} = 1110$$

$$\text{Remainder} = 001$$

A	Q	M		
000	0111	011	Initial values	
000	1110	011	A,Q << 1 A = A - M $Q_0 = 0$ A = A + M	1st Cycle
101	1110	011		
101	1110	011		
000	1110	011		
001	1100	011	A,Q << 1 A = A - M $Q_0 = 0$ A = A + M	2nd Cycle
110	1100	011		
110	1100	011		
001	1100	011		
011	1000	011	A,Q << 1 A = A - M $Q_0 = 1$	3rd Cycle
000	1000	011		
000	1001	011		
001	0010	011	A,Q << 1 A = A - M $Q_0 = 0$ A = A + M	4th Cycle
110	0010	011		
110	0010	011		
001	0010	011		

Signed Binary Division

Lets try:

$$1100 \div 011 = 1111111$$

$$-4 \div 3 = -1 \text{ remainder } 1$$

$$Q = 0001$$

$$A = 001$$

$S_D \neq S_V$: two's comp Q

$S_D < 0$: two's comp A

Quotient = 1111

Remainder = 111

A	Q	M		
000	0100	011	Initial values	
000	1000	011	A,Q << 1	1st Cycle
101	1000	011	A = A - M	
101	1000	011	$Q_0 = 0$	
000	1000	011	A = A + M	
001	0000	011	A,Q << 1	2nd Cycle
110	0000	011	A = A - M	
110	0000	011	$Q_0 = 0$	
001	0000	011	A = A + M	
010	0000	011	A,Q << 1	3rd Cycle
111	0000	011	A = A - M	
111	0000	011	$Q_0 = 0$	
010	0000	011	A = A + M	
100	0000	011	A,Q << 1	4th Cycle
1 001	0000	011	A = A - M	
1 001	0001	011	$Q_0 = 1$	

Signed Binary Division

Lets try:

$$1000 \div 0101 = 11011111$$

$$-8 \div 5 = -1 \text{ remainder } -3$$

Quotient = 0001

Remainder = 0011

$S_D \neq S_V$: two's comp Q

$S_D < 0$: two's comp A

Quotient = 1111

Remainder = 1101

A	Q	M		
0000	1000	0101	Initial values	
		0101 0101 0101 0101		1st Cycle
		0101 0101 0101 0101		2nd Cycle
		0101 0101 0101 0101		3rd Cycle
		0101 0101 0101		4th Cycle