Soft Cube Falls and Bounces off the Ground

Introduction

In this mini project, we simulate a soft cube falling and bouncing off the ground using the Taichi programming language. We implement the Neo-Hookean and Saint Venant-Kirchhoff material models, along with the Explicit Euler and RK4 numerical integration methods. To avoid unnecessary mistake, we use auto diff from taichi to compute force on each vertexes.

Modelling

Cube

we model the cube's parameter using the following quantities:

- N: number of elements per edge
- E: Young's modulus
- ν: Poisson's ratio
- ρ: density of an element
- pos: initial position
- length: length of edge
- angle: initial rotation angle

Having such modeling, we can compute Lamé constants λ and μ using the folowing formula

$$\mu = \frac{E}{2(1+\nu)}$$
$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

Saint Venant-Kirchhoff Material

Saint Venant-Kirchhoff model is a constitutive model commonly used to describe the behavior of deformable materials. It has the following strain energy density function:

$$W(\mathbf{E}) = \frac{\lambda}{2}[\mathrm{tr}(\mathbf{E})]^2 + \mu \mathrm{tr}\left(\mathbf{E}^2\right)$$
$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - I)$$

where $\mathbf{F}$ is the deformation gradient and $\mathbf{E}$ is the Green–St Venant strain.

Neo-Hookean Material

The Neo-Hookean material model is another constitutive model used to describe the behavior of elastic materials. It has the following strain energy density function:

$$W = \frac{\mu}{2}(\mathrm{tr}(\mathbf{F}^T\mathbf{F}) - 2 - 2\ln J) + \frac{\lambda}{2}(J-1)^2$$
$$J = \det \mathbf{F}$$

where $\mathbf{F}$ is the deformation gradient.

Implementation

Numerical Methods

We use both Explicit Euler and RK4 as integration methods.

Autodiff

To get forces on each vertexes, we use auto diff from taichi instead of explicitly compute the first Piola-Kirchhoff stress tensor $\mathbf{P}$.

Bouncing

To simulate bouncing, we detect the status of the cube at each timestep. If any vertex is out of the screen and having outward speed, we set its speed to 0.

Result

Change `Using_RK4` and `Using_NeoHookean` flag to switch between different methods and models.

Cube is more unstable and elastic under Explicit Euler than RK4.

Saint Venant-Kirchhoff Material seems to have invertible deformation than Neo-Hookean material.