① partition (A, p, q)

when $x \leftarrow A[P]$

$i \leftarrow P$

for $j = p+1$ to r

if $A[j] <= x$

$i = i+1$.

exchange $A[i]$ with $A[j]$.

exchange $A[i]$ with $A[P]$

return i.

②. Inversions is that a sorted array has no inversions in it. since every element should be smaller than everything coming after it and larger than everything coming before it.

$$T(n) = 1\% \cdot \Theta(n^2) + 99\% \, O(n^2)$$

$$= \Theta(n^2).$$

(3). best-case: iq sort.

$O(n) + O(n\log n) + O(n-r)$

$= O(n)$

worst-case.

$O(n) + O(r^2) + O((n-r)^2)$

$= O(n^2)$

average-case.

$O(n) + O(n\log n) + O((n-r)^2)$

$= O(n^2)$

(4)

best case. when the data goes up, $O(n)$.

worst-case.

$$T(n) = \max_{1 \le r \le n} \{ T(r-1) + O(n-r)^2 + O(n) \}$$

$T(n) = O(n^2) \le Cn^2$ when $C > 0$.

$$T(n) = \max_{1 \le r \le n} \{ T(r-1) + a(n-r)^2 + an \}$$

$$\le \max_{1 \le r \le n} \{ C(r-1)^2 + a(n-r)^2 + an \}$$

$$= \max\{ a(n-1)^2 + an,\ C(n-1)^2 + an \}$$

$$= C(n-1)^2 + an$$

$$= Cn^2 - 2Cn + 1 + an$$

$$= Cn^2$$

Avgerr case. $T(n) = \frac{1}{n} \sum_{r=1}^{n} \{ T(r-1) + O(n-r)^2 + O(n) \}$

$T(n) = O(n^2) \le Cn^2$ for $C > 0$

$$T_{Avg}(n) = \frac{1}{n} \sum_{r=1}^{n} \{ T(r-1) + a(n-r)^2 + an \}$$

$$= \frac{1}{n} \sum_{r=1}^{n} \{ C(r-1)^2 + a(n-r)^2 + an \}$$

$$= \frac{1}{n} \{ \sum_{r=1}^{n} C(r-1)^2 + \sum_{r=1}^{n} a(n-r)^2 + Cn^2 \}$$

$$= \frac{1}{n} \{ (a+c) \int_0^n x^2 dx + an^2 \}$$

$$= \frac{1}{n} \{ (a+c) \frac{n^3}{3} + an^2 \}$$

$$= \frac{1}{n}\left[\frac{an^3}{3} + \frac{cn^3}{3} + an^2 \right]$$

$$= \frac{an^2}{3} + \frac{cn^2}{3} + an \Big]$$

$$= O(n^2)$$