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ELEN4022: Full Stack Quantum Computing
Lab 2

1.Introduction

Full adders are easily made and used in classical computers. In quantum computers they can also be created using quantum algorithms such as the Quantum Fourier Transform. Three different algorithms for quantum adders will be made, simulated and run on quantum computers and the results are compared. The first algorithm will not use a Fourier Transform, the second uses the Fourier Transform and the last uses and approximate Fourier Transform. The unitary of Fourier Transform will also be calculated

2. Circuits Used

2.1 Quantum Fourier Transform

The Quantum Fourier Transform (QFT) and its inverse for a 3-qubit circuit is given in figure 1.1 and 1.2 below [1]. The Approximate Quantum Fourier Transform can be created by using less control phase gates [2]. There are different approximations using different amount of phase gates per qubit [2]. In the QFT the maximum amount of control phase gates is used. Since the QFT being used here are for a maximum of 3 qubits the maximum amount of control phase gates used after the Hadamard gate is 2 and so the AQFT to be used will use the approximation of 1 phase gate after the Hadamard gate. The 3-qubit circuit for the AQFT and its inverse can be seen in figure 2.1 and 2.2.

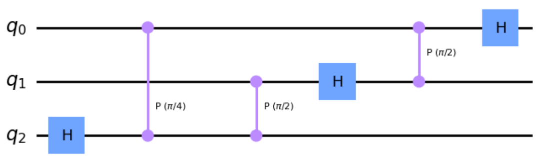

Figure 1.1: Quantum Fourier Transform Circuit

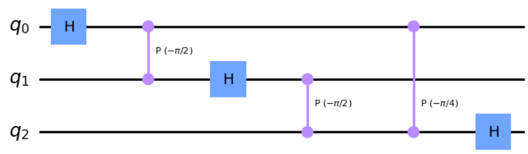

Figure 1.2: Inverse Quantum Fourier Transform circuit

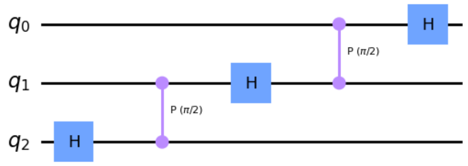

Figure2.1: Approximate Quantum Fourier Transform Circuit

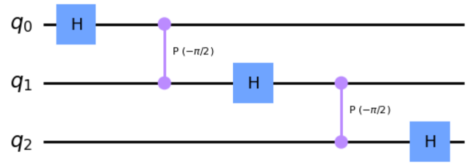

Figure 2.2: Approximate Inverse Quantum Fourier Transform Circuit

2.2 Quantum Adder Circuits

The Quantum Full Adder (QFA) circuit used is the one given in the lab brief [3]. The Quantum Fourier Transform Adder (QFTA) and the Approximate Quantum Fourier Transform Adder (AQFTA) circuits for both 2 and 3 qubits can be found in figure 3.1 and 3.2. They are the same except for the QFT and IQFT which is the circuits found in figures 1 and 2.

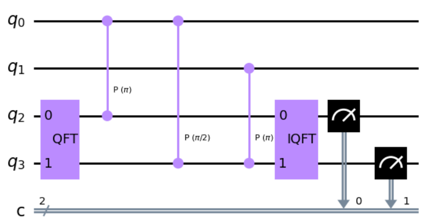

Figure 3.1: QFTA circuit for 2 qubits

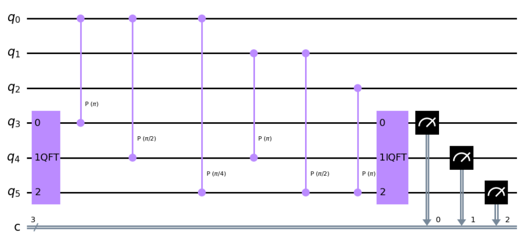

Figure 3.2: QFTA circuit for 3 qubits

3. Results

The QFA was run on the quantum computer for 2-bit adder and not the 3-bit adder as this required 10 qubits, and a quantum computer with 10 qubits is not available to the IBM student account. The 3-bit adders for the QFTA and AQFTA were done on quantum computers and results can be found below. Results of the other 2-bit adders of QFTA and AQFTA and the 3 qubit QFTA simulation are shown in Appendix A.

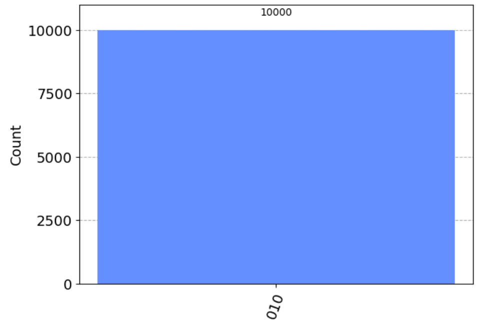

Figure 4.1: Result of simulation QFA of "01" and "01"

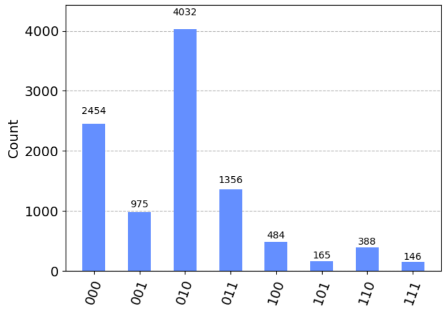

Figure 4.2: Result of QFA on Quantum computer of "01" and "01"

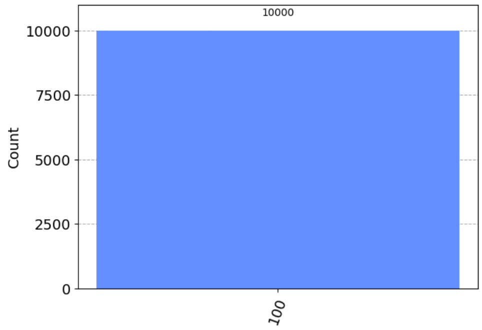

Figure 5.1: Result of simulation QFTA of "010" and "010"

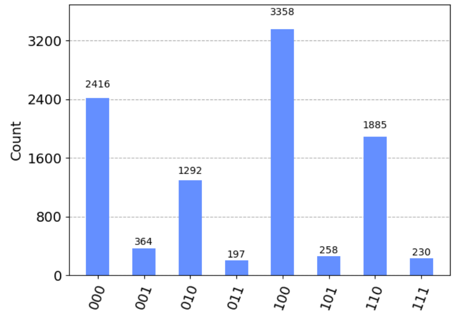

Figure 5.2: Result of QFTA on quantum computer of "010" and "010"

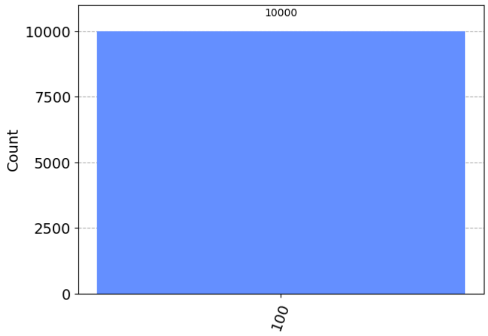

Figure 6.1: Result of simulation AQFTA of "010" and "010"

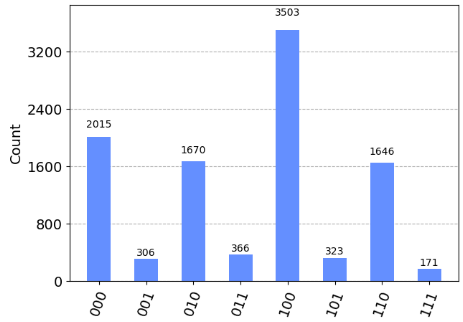

Figure 6.2: Result of AQFTA on quantum computer of "010" and "010"

4. Comparisons

4.1 Quantum vs Simulation on classical

As can be seen from the results the quantum computers are not always giving correct answers from the adder circuits, they are mostly correct and so circuits need to be run many times to find the correct answers. There can be many reasons for this such as the noise experienced by the quantum computers. Since the simulations don't have noise and is also not affected by the natural randomness of quantum computers, they always give the correct answers unlike the outcomes of the actual quantum computers.

4.2 Comparison of algorithms

From the results it seems like the QFA is the best in terms of the most probable to give the correct answer, yet this method uses many more qubits to run. The AQFTA seems to be the second best in terms of probability of the correct answers. This is unusual as the approximate would be expected to do worse. More testing on more summing of different numbers will need to be done to get more accurate results on this but wait times in queues to use quantum computers hasn't allowed for this. The AQFT is also faster as the is less gates compared to the QFT.

5. Unitary Calculations for Fourier Transforms

5.1 One Qubit

Single qubit is just a Hadamard gate therefore:

$$U = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

5.2 Two-Qubit

Different steps will first be showed and then all will be multiplied together to get the final unitary.

$$S_1 = H \otimes I$$

$$S_2 = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes R_{\frac{\pi}{2}}$$

$$S_3 = I \otimes H$$

$$U = S_3 \times S_2 \times S_1$$

$$U = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \\ 1 & -i & -1 & i \end{bmatrix}$$

5.3 Three-Qubit

$$S_1 = H \otimes I \otimes I$$

$$S_2 = |0\rangle\langle 0| \otimes I \otimes I + |1\rangle\langle 1| \otimes R_{\frac{\pi}{2}} \otimes R_{\frac{\pi}{4}}$$

$$S_3 = I \otimes H \otimes I$$

$$S_4 = I \otimes |0\rangle\langle 0| \otimes I + I \otimes |1\rangle\langle 1| \otimes R_{\frac{\pi}{2}}$$

$$S_5 = I \otimes I \otimes H$$

$$U = S_3 \times S_2 \times S_3 \times S_2 \times S_1$$

$$U = \frac{1}{4}\begin{bmatrix} \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -\sqrt{2} & \sqrt{2} & -\sqrt{2} & \sqrt{2} & -\sqrt{2} & \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2}i & -\sqrt{2} & -\sqrt{2}i & \sqrt{2} & \sqrt{2}i & -\sqrt{2} & -\sqrt{2}i \\ \sqrt{2} & -\sqrt{2}i & -\sqrt{2} & \sqrt{2}i & \sqrt{2} & -\sqrt{2}i & -\sqrt{2} & \sqrt{2}i \\ \sqrt{2} & 1+i & \sqrt{2}i & -1+1 & -\sqrt{2} & -1-i & -\sqrt{2} & 1-i \\ \sqrt{2} & -1-i & \sqrt{2}i & 1-i & -\sqrt{2} & 1+i & -\sqrt{2} & -1+i \\ \sqrt{2} & -1+i & -\sqrt{2}i & 1+i & -\sqrt{2} & 1+i & \sqrt{2}i & -1-i \\ \sqrt{2} & 1-i & -\sqrt{2}i & -1-i & -\sqrt{2} & -1-i & \sqrt{2}i & 1+i \end{bmatrix}$$

Conclusion

The circuits simulations and running on quantum computers were successful and the adder circuits were working well although they are not always accurate in quantum computers due to noise and quantum randomness. The unitary matrices of Fourier Transforms were calculated using linear algebra. The different adder algorithms were compared, and all have their advantages and disadvantages.

References

[1] "Qiskit," [Online]. Available: https://learn.qiskit.org/course/ch-algorithms/quantum-fourier-transform. [Accessed 27 April 2023].

[2] A. Prokopenya, "Approximate Quantum Fourier Transform and Quantum Algorithm for Phase Estimation," 2015 DOI: 10.1007/978-3-319-24021-3_29.

[3] T. Surtee, "ELEN4022 - Full Stack Quantum Computing Laboratory Exercise 2," 2023.

Appendix A

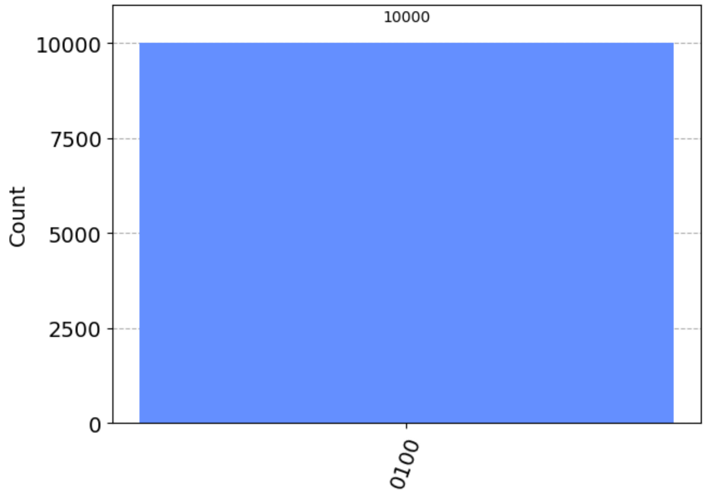

Figure 7: Result of simulation QFA of "010" and "010"

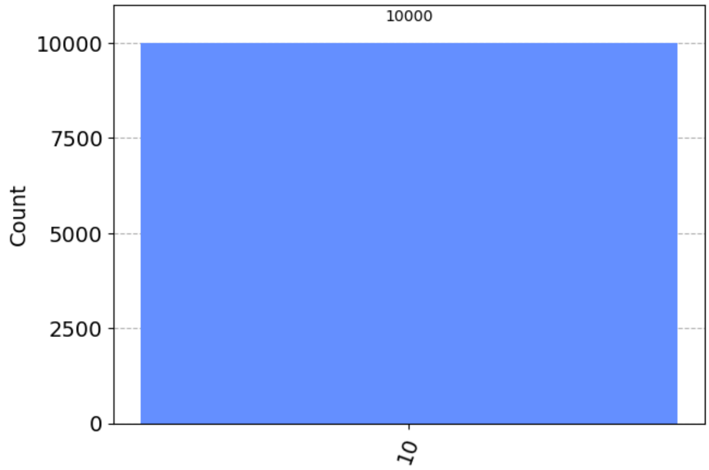

Figure 7: Result of simulation QFTA of "01" and "01"

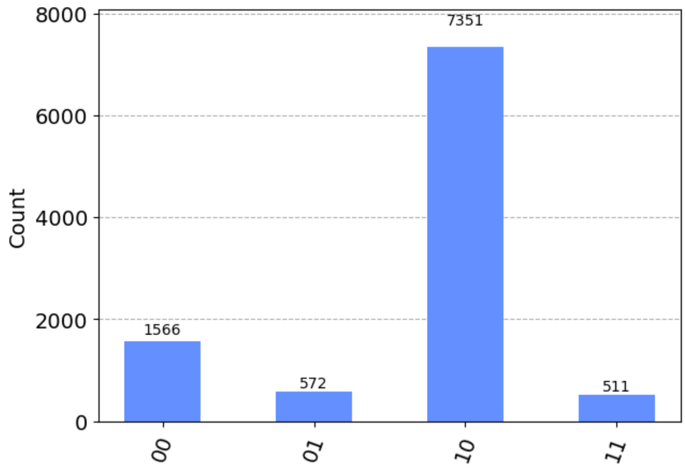

Figure 7: Result of QFTA on quantum computer of "01" and "01"

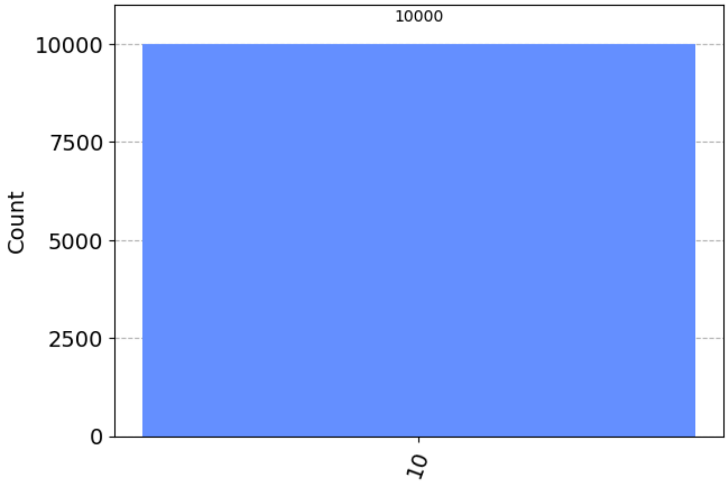

Figure 7: Result of simulation AQFTA of "01" and "01"

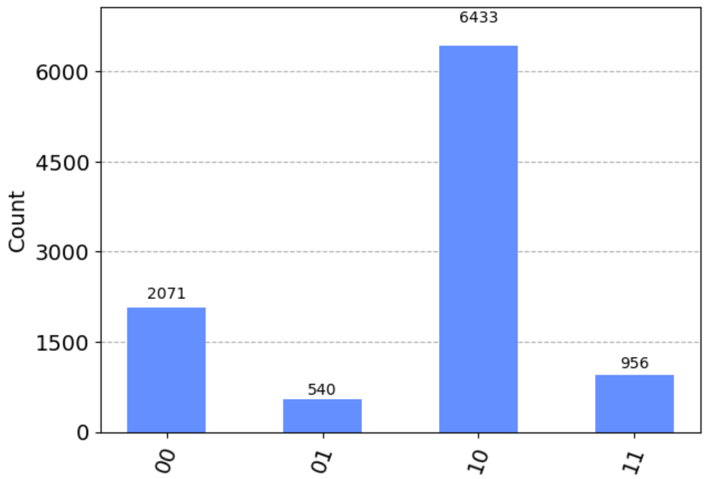

Figure 7: Result of AQFTA on quantum computer of "01" and "01"