School of Electrical and Information Engineering
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ELEN4022A: Full-Stack Quantum Computing | Lab 01

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1. Introduction

To understand using Qiskit circuit model simulation it is encouraged to explore the linear algebraic solution in conjunction to grasp a better understanding of the complexities of quantum circuits. This is done using Python where the unitary matrix of a five and ten qubit sized system is simulated using Qiskit and then compared to its linear algebra counterpart.

2. The Systems

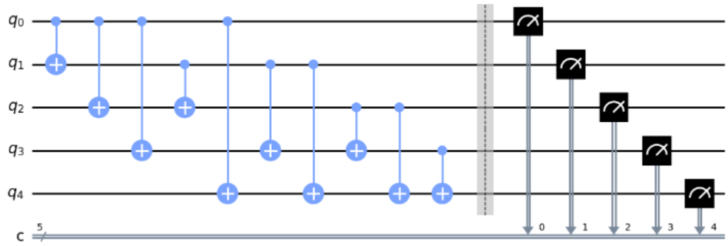
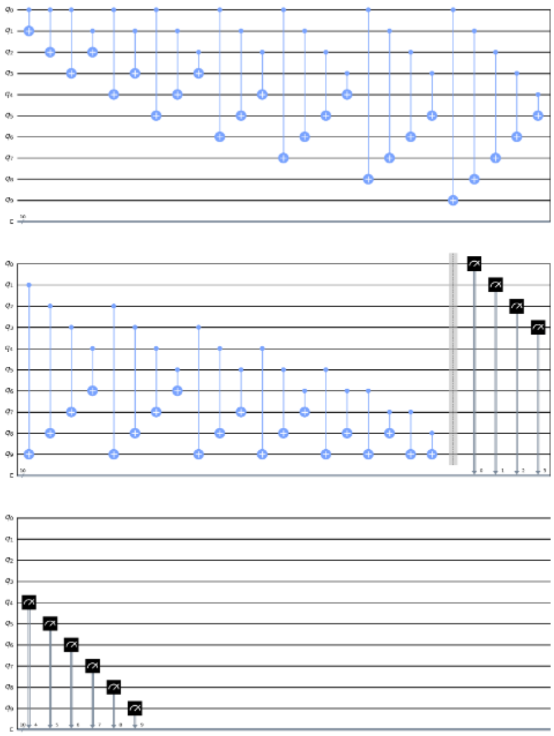


Figure 1: Quantum Circuit containing a chain of CNOT gates for five qubits.



These systems were visualised using the Qiskit visualisation library. Their unitary matrices were computed using built-in functionality and stored in variables to later be compared to the respective unitary matrices derived through the linear algebra present in the code.

The simulation of the ten qubit system's unitary matrix is not visualised as it is much too large to display.

Each system is described as having a CNOT gate from the control qubit to every other qubit below the control qubit. Each qubit becomes a control qubit at some stage of the circuits lifetime.

Figure 2: Quantum Circuit containing a chain of CNOT gates for ten qubits.

3. The Linear Algebra of the Code

To generate the unitary matrix of each system we first consider the number of input qubits. This number determines the number of unique binary combinations (Generated by 'bin_combos(num)'). Each bit then takes its turn acting as the control qubit targeting the remaining qubits below it to generate new matrices which have the same unique bit combinations but in a different order (achieved by 'flip(combinations, control, target)'). Converting both the original combination matrix and the new order matrices to decimal give sets of coordinates in the zeros matrix to be converted into ones. The original matrix can be thought of as the y coordinate and any of the new order matrices as the x coordinate (achieved by 'index_to_matrix(control_row, control_col, shape)'). Each of these generated matrices acts as a stage in the circuit. Finally, the dot product of each stage is computed in the reverse order because the multiplications are not communitative. This dot product yields the unitary matrix.

4. Results

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\end{bmatrix}$$

Figure 3: Unitary matrix for a five-qubit system, generated by Qiskit and the linear algebra computations.

The result of the linear algebra correctly matches that of the Qiskit simulations suggesting the code for the linear algebra calculations is correct for at least the five-qubit sized system. The ten-qubit sized system was simulated using Qiskit but its unitary matrix is much too large to display as its dimensions are $2^{10} \times 2^{10}$.

5. Conclusion

The lab was partially successful as the five-qubit sized system was simulated and calculated. However, I was unable to correct the code to work for a system of any number of qubits. In essence a much clearer understanding of multi-qubit gates was obtained through comparing the Qiskit simulations to my own. It was observed that an entire quantum circuit can be represented by a single 'gate' (being the unitary matrix) sharing the similarity of a transfer function.