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ELEN4022: Full Stack Quantum Computing
Lab 1

1.Introduction

An interconnected quantum circuit using controlled-not gates is to be created and simulated using Qiskit library in python. The unitary matrix is to be simulated and calculated using linear algebra and the two results are to be compared. A 5 qubit and 10 qubit circuits are simulated and calculated in python for comparisons.

2. Circuit

The circuit is made up of 5 qubits, where every qubit is connected to every other qubit through a controlled-not gate as can be seen in figure 1 below.

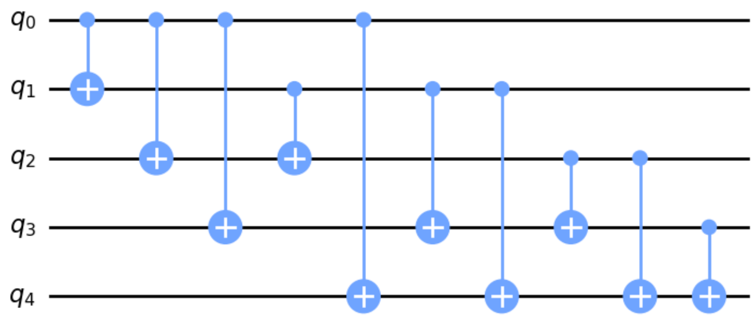

Figure 1: Quantum Circuit to be used.

3. Unitary Matrix Calculation

To calculate the unitary matrix of a single controlled-not gate on circuit with multiple qubits, we can use equation 1 below as an example where the 2^{nd} qubit(q_1) is the control qubit, and the 4^{th} qubit(q_3) is the target qubit. [1] The Identity matrix I and the X-gate are shown in equation 2 and 3 respectively.

$$I \otimes I \otimes I \otimes |0\rangle\langle 0| \otimes I + I \otimes X \otimes I \otimes |1\rangle\langle 1| \otimes I \quad \ldots (1)$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \ldots (2)$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \ldots. \ (3)$$

Equation 1 has two terms, the first is based on how the qubits would change depending on if the control qubit is $|0\rangle$, since all qubits would not change, we use the identity matrix on them. The second term is how the qubits would change if the control qubit is $|1\rangle$, The target bit would change in this case and so we use the X-gate on this qubit and the Identity matrix on the rest of the qubits. [1] Since the circuit has multiple controlled-not gates with a common control qubit we can group these gates into a single calculation step using the equation 1 above. Equations 4, 5, 6, 7 are the step equations that were written and calculated for circuit 1 above.

$$S_1 = I \otimes I \otimes I \otimes I \otimes |0\rangle\langle 0| + X \otimes X \otimes X \otimes X \otimes |1\rangle\langle 1| \quad \ldots (4)$$

$$S_2 = I \otimes I \otimes I \otimes |0\rangle\langle 0| \otimes I + X \otimes X \otimes X \otimes |1\rangle\langle 1| \otimes I \quad \ldots (5)$$

$$S_3 = I \otimes I \otimes |0\rangle\langle 0| \otimes I \otimes I + X \otimes X \otimes |1\rangle\langle 1| \otimes I \otimes I \quad \ldots (6)$$

$$S_4 = I \otimes |0\rangle\langle 0| \otimes I \otimes I \otimes I + X \otimes |1\rangle\langle 1| \otimes I \otimes I \otimes I \quad \ldots (7)$$

Lastly these equations need to be calculated and then matrix multiplied in the order from S_4 to S_1 which will give us our unitary matrix. These equations were hand calculated for a 3-qubit system while testing a function written in python using NumPy library. The 3-qubit unitary matrix was also multiplied to the input qubit states and the output states calculated were as expected therefore we can be certain the unitary matrix is correct. These calculations were not done for the 5 and 10 qubit circuits as these matrices are too large. They were however calculated using the python function that was written.

Simulations and Results

The simulations were done using the Qiskit library in python. The unitary simulation in Aer in the Qiskit library was used for the simulations of all the circuits. A function was created that takes in an integer input and the circuit with the input number of qubits is created and simulated. These simulations were then compared to the calculated matrices, and they matched exactly. The matrix in figure 2 below is the matrix simulated and calculated for the 5-qubit circuit in figure 1. The simulation and calculation for the 10-qubit circuit was done and their matrices were also matched. The matrix was a 1024 by 1024 matrix which is too large to view here.

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\end{bmatrix}
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Figure 2: unitary Matrix of the 5-qubit system.

Conclusion

The simulation and calculations done were correct and checked using hand calculations for smaller qubit systems and python functions were created to use these algorithms for higher qubit systems. The calculations for higher qubit systems were checked using the Qiskit library's simulator and they were correct. The calculations helped in learning how quantum circuits can be represented by a single matrix which can be helpful when working with quantum circuits but for bigger circuit with many qubits the unitary matrix becomes too large for practical usage.

References

[1] M. Vesely, "Stack Exchange," 22 Feb 2020. [Online]. Available: https://quantumcomputing.stackexchange.com/questions/5179/how-to-construct-matrix-of-regular-and-flipped-2-qubit-cnot. [Accessed 9 March 2023].