Full Stack Quantum Computing Lab 1

Shehwar Faisal Finger

School of Electrical & Information Engineering, University of the Witwatersrand, Private Bag 3, 2050, Johannesburg, South Africa

1. INTRODUCTION

Lab 1 of full stack Quantum Computing required the design of a quantum circuit with CNOT gates extending from the first wires down to the last. Quantum circuits are integral to Quantum computing as the architecture of a quantum computer is based on these very circuits. Quantum circuits are based on a set of wires with qubits passing through them. These qubits can exist in either the |0> or the |1> states.

1.1. Requirements:

- Quantum circuits from 2-10 wires automatically generated.
- Matrix representation of the respective quantum circuit generated.

2. METHOD

2.1. Background

Before proceeding to the design of the circuit and the linear algebra it is important to first discuss matrix representation of various things.

A qubit in the state |0> has the matrix representation of:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

A qubit in the state |1> has the matrix representation of:

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

A CNOT gate has the matrix representation of:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

A X gate has the matrix representation of:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

An Identity matrix has the matrix representation of:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2.1.1. CNOT gate

The Controlled NOT (CNOT) gate is a quantum gate that operates on two different qubits. It takes in a control and a target qubit. It works according to the following *truth table 1*.

Table 1: CNOT gate Truth Table

Control bit Before	Target Bit Before	Control Bit After	Target Bit After
|0>	|0>	|0>	|0>
|0>	|1>	|0>	|1>
|1>	|0>	|1>	|1>
|1>	|1>	|1>	|0>

The CNOT gate most is importantly used to entangle qubits. If the target qubit is in the |1> state it will flip, to the |0> state and essentially entangling the qubits. If the qubit is in the |0> state, however, the gate will not entangle it and will let it pass through as is.

There are many methods a CNOT gate may be implemented either by ion traps or photonic circuits etc.

2.2. Generation of the Quantum circuits

The quantum circuit was required to have CNOT gates extending from every wire of the circuit down to each consecutive wire. Meaning, if a circuit contains n wires the first qubit will have *n-1* wires going down to each consecutive qubit.

This was implemented using the qiskit simulator in python. There were various libraries used such as, *numpy*, *qiskit*, *sympy* and *math*. The circuit was created using two for loops, the first controlling the index of the control bit and the second controlling the entangled or target qubit.

2.3. Calculating the matrix representation of the circuit

Before progressing to the linear algebra of the circuit it is important to understand the form of the input (qubit). The qubit exists in a *braket* form. The bra is |0> or |1> and the ket is <0| or <1|. Hence, *braket* is represented by |0><0| or |1><1|. To calculate this, you must multiply |0> or |1> matrices by themselves to generate a 4x4 matrix.

To calculate the matrix representation of a 2-wire circuit with a single CNOT gate the algorithmic method is employed. This method follows the following formula:

$$|0><0| \otimes I + |1><1| \otimes X$$

The algorithmic method follows a simple system of “if the qubit is a |0> multiply by the identity matrix and if qubit is a |1> multiply it by a X gate (matrix)”. The Kroenke product is taken between each matrix. Following that same method, we can extrapolate this method to systems bigger than 2 wires. For understanding, the matrix calculation for a 4-wire matrix was conducted manually as opposed to using the in-built qiskit function. Using this same method along with the fact that spare wires with the |0><0| state will lead to a Kroenke product with the identity matrix and for the |1><1| state a Kroenke product with the X gate we can calculate the matrix. Also, to make the calculations easier each set of gates extending from the same control bit were grouped together in states. These states were individually calculated and then thereafter multiplied from right to left to calculate the final matrix.

For example, when calculating the second state of a 4-wire quantum circuit we would have the equation:

$$State2 = I \otimes I \otimes |0 >< 0| \otimes I + X \otimes X \otimes |1 >< 1| \otimes X$$

The order of the Kroenke multiplication is dependent on where the gate is taken from. For state 2 the CNOT gate is taken from qubit 1 and since this equation is read from right to left (meaning in this equation I is q0, then |0><0| is q1, then I is q2 and I is q3) the position of the state needs to be exactly where it is to prevent an incorrect calculation. The same rule applies to the |1><1| state calculation. Once all these states are calculated they need to be multiplied from right to left (meaning State 3 x State 2…). Order is very important for accuracy.

3. RESULTS

3.1. 4-wire Circuit and Matrix representation

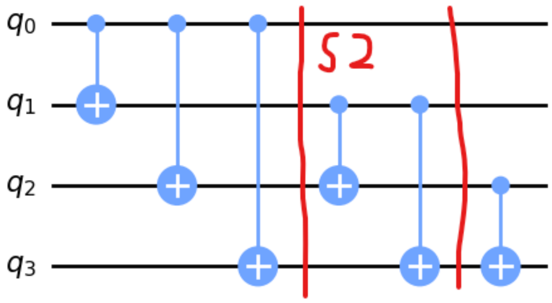

Figure 1: Circuit of 4 wires with state 2 marked.

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Figure 2: Matrix Representation of circuit using in-built tools.

```
array([[1.00000000000000, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1.00000000000000],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1.00000000000000, 0],
       [0, 1.00000000000000, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1.00000000000000, 0, 0, 0],
       [0, 0, 0, 1.00000000000000, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 1.00000000000000, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1.00000000000000, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 1.00000000000000, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 1.00000000000000, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 1.00000000000000, 0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 1.00000000000000, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 1.00000000000000, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1.00000000000000, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1.00000000000000, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 1.00000000000000, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]],
      dtype=object)
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Figure 3: Matrix Representation calculated by hand.

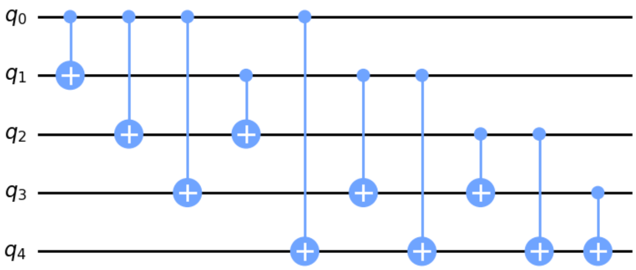

Figure 4:Circuit of 5 wires.

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
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\end{bmatrix}$$

Figure 5:Matrix Representation of a 5-wire circuit

4. CONCLUSION

As can be seen from the figures above the 4-wire circuit and its matrix representation matches the circuit from the brief and the hand calculated matrix. Hence, it can be assumed that the hand calculations were done correctly. The 5-wire quantum circuit and matrix representation are also shown above as required in the brief. The 10-wire quantum circuit and matrix representation are not shown as it was too large to fit but it can be found in the code.