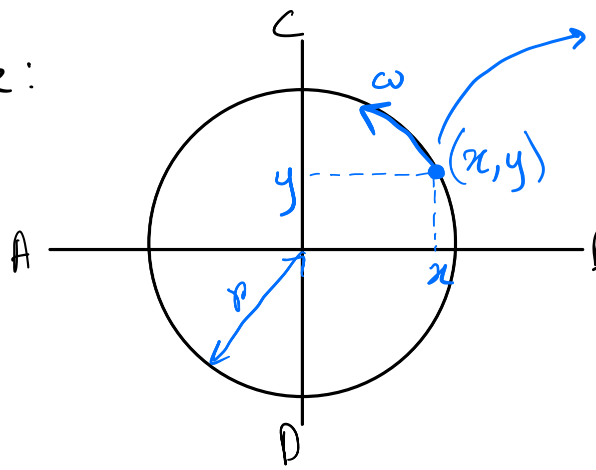
Points travelling in straight lines creating illusion of rolling circle (Numberphile inspired):

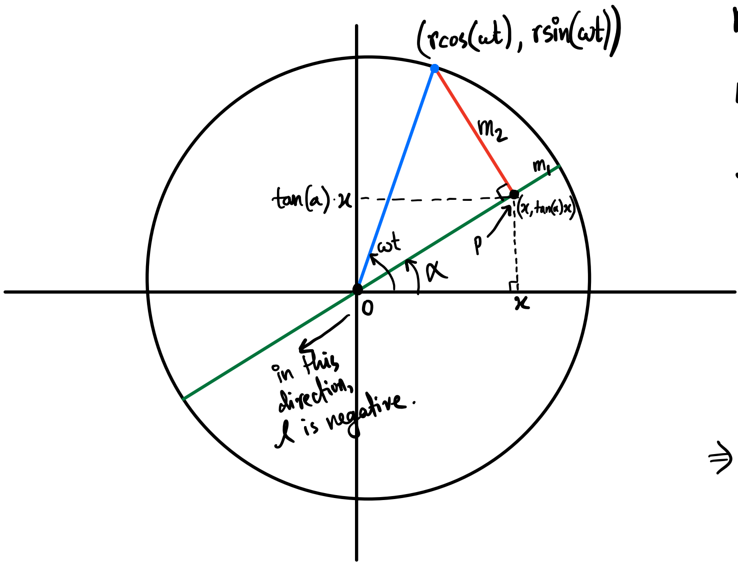
Simple case:



$$\begin{cases} x = r\cos(\omega t) \\ y = r\sin(\omega t) \end{cases}$$

The points (one on $\overline{AB}$ and one on $\overline{CD}$) follows the x and y axis projection of the rotating blue point.

More complicated case:



$m_{\text{green line}} = m_1 = \tan(\alpha)$

$m_{\text{red line}} = m_2$

Since the red and green lines are perpendicular to each other:

$$m_1 \cdot m_2 = -1$$

$$\Rightarrow m_2 = -\frac{1}{m_1} = -\frac{1}{\tan(\alpha)}$$

$$\Rightarrow \frac{\tan(\alpha)x - r\sin(\omega t)}{x - r\cos(\omega t)} = -\frac{1}{\tan(\alpha)}$$

$$\Rightarrow \tan(\alpha)x - r\sin(\omega t) = \frac{r\cos(\omega t) - x}{\tan(\alpha)}$$

$$\Rightarrow \tan^2(\alpha)x - r\sin(\omega t)\tan(\alpha) = r\cos(\omega t) - x$$

$$\Rightarrow x\left[1+\tan^2(\alpha)\right] = r\left[\cos(\omega t) + \sin(\omega t)\cdot\tan(\alpha)\right]$$

$$\therefore \boxed{x = \frac{r\left[\cos(\omega t) + \sin(\omega t)\cdot\tan(\alpha)\right]}{1+\tan^2(\alpha)}}$$

let $\overline{OP} = l$.

$$\cos(\alpha) = \frac{x}{l}$$

$$\therefore \boxed{l = \frac{r\left[\cos(\omega t) + \sin(\omega t)\cdot\tan(\alpha)\right]}{\cos(\alpha)\cdot\left[1+\tan^2(\alpha)\right]}}$$

$$\therefore \frac{dl}{dt} = \frac{r}{\cos(\alpha)\cdot\left[1+\tan^2(\alpha)\right]}\left(-\sin(\omega t)\cdot\omega + \cos(\omega t)\cdot\omega\cdot\tan(\alpha)\right)$$

$$= \frac{r\omega}{\cos(\alpha)\cdot\left[1+\tan^2(\alpha)\right]}\left(\cos(\omega t)\cdot\tan(\alpha) - \sin(\omega t)\right)$$

$$\therefore \frac{d^2 l}{dt^2} = \frac{r\omega}{\cos(\alpha)\cdot\left[1+\tan^2(\alpha)\right]}\left(-\sin(\omega t)\cdot\omega\cdot\tan(\alpha) - \cos(\omega t)\cdot\omega\right)$$

$$= -\omega^2 \cdot \frac{r\left[\cos(\omega t)+\sin(\omega t)\cdot\tan(\alpha)\right]}{1+\tan^2(\alpha)}$$

$$\therefore \boxed{a = -\omega^2 l}$$

→ SHM!

$$\therefore \boxed{l = r\cdot\cos(\omega t - \alpha)}$$

→ phase difference for line at α $(0^\circ \leq \alpha < 180^\circ)$

$$\therefore \boxed{\begin{cases} x_\alpha = r\cdot\cos(\omega t-\alpha)\cdot\cos(\alpha) \\ y_\alpha = r\cdot\cos(\omega t-\alpha)\cdot\sin(\alpha) \end{cases}}$$