Determination of the dependence of force exerted by a permanent bar magnet on distance

→ To begin, we assume that the force exerted by a bar magnet on another magnet / magnetic material takes the form:

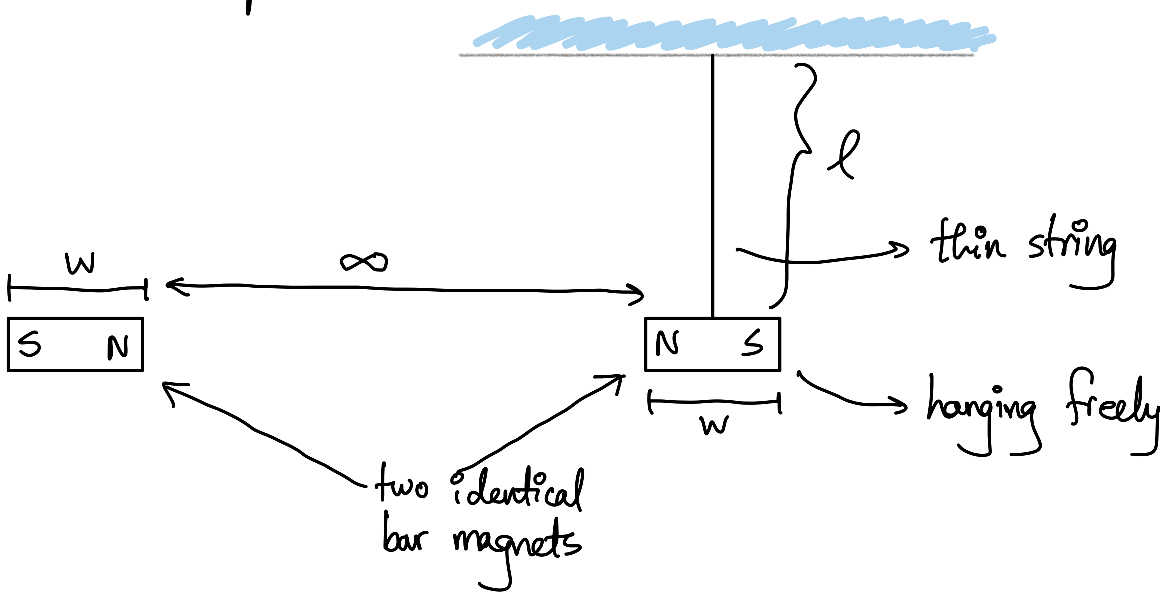
$$F_m = \frac{\alpha}{d^n}$$

, where α is a constant characterizing the intrinsic magnetic property of the bar magnet, and n is a constant to be determined.

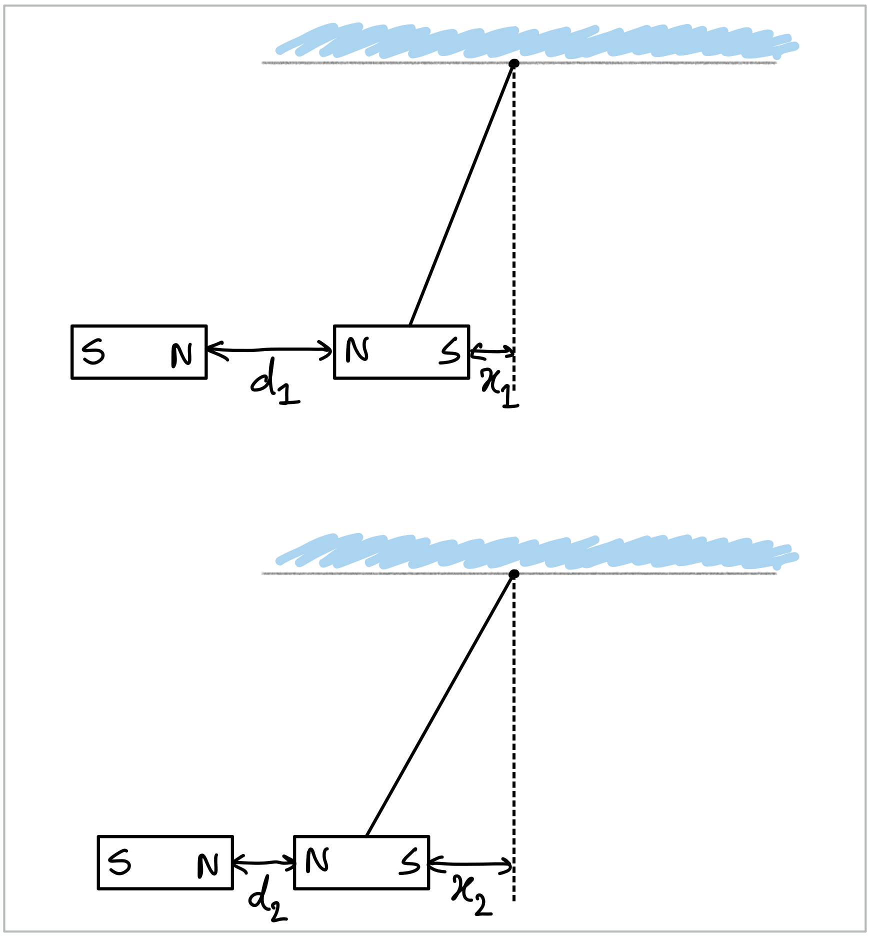
distance from the center of the bar magnet

Our goal is to work out n in an experimental manner.

Consider the setup:



Initially, the seperation of the two magnets is ∞. Then, we slowly begin to bring the non-hanging magnet closer to the hanging magnet and take the two sets of readings (d_1, x_1) and (d_2, x_2), where $d_1 > d_2$ [See next page].



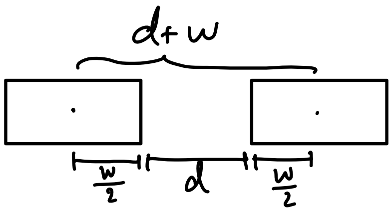
(notably, the experiment is more accurate if $l \gg x_1, x_2$)

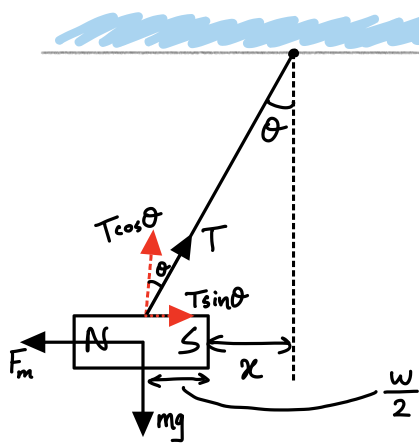
With these information, we can figure out n:

We consider a generic case where the horizontal displacement of the hanging magnet is $\boxed{x}$ and the seperation between the rear ends of the two magnets is $\boxed{d}$. (note that $l \gg x$).

Now, the free-body diagram of the hanging magnet, which is in static equilibrium, is:

Seperation between the centers of the two magnets:





(dotted red arrows represent vector components)

We know,

$$\sum F_y = 0 \Rightarrow T\cos\theta = mg \Rightarrow \boxed{T = \frac{mg}{\cos\theta}}$$

Also, $\sum F_x = 0 \Rightarrow T\sin\theta = F_m$

$$\Rightarrow \frac{mg}{\cos\theta} \cdot \sin\theta = \frac{\alpha}{(d+w)^n}$$

$$\Rightarrow mg\tan\theta = \frac{\alpha}{(d+w)^n}$$

$$\Rightarrow mg\left(\frac{x+\frac{w}{2}}{l}\right) = \frac{\alpha}{(d+w)^n} = \alpha(d+w)^{-n}$$

$$\Rightarrow \log\left(\frac{mg}{l}\cdot\left(x+\frac{w}{2}\right)\right) = \log\left(\alpha(d+w)^{-n}\right) \quad \left[\text{any base works}\right]$$

$$\Rightarrow \log\left(\frac{mg}{l}\right) + \log\left(x+\frac{w}{2}\right) = -n\cdot\left[\log(\alpha) + \log(d+w)\right]$$

$$= -n\cdot\log(\alpha) - n\cdot\log(d+w)$$

<u>This is true for both of our readings, so:</u>

$$\rightarrow \log\left(\frac{mg}{l}\right) + \log\left(x_1+\frac{w}{2}\right) = -n\cdot\log(\alpha) - n\cdot\log(d_1+w)$$

$$\rightarrow \log\left(\frac{mg}{l}\right) + \log\left(x_2+\frac{w}{2}\right) = -n\cdot\log(\alpha) - n\cdot\log(d_2+w)$$

Subtracting the second equation from the first, we get:

$$\log\left(x_1+\frac{w}{2}\right) - \log\left(x_2+\frac{w}{2}\right) = -n\cdot\log(d_1+w) + n\cdot\log(d_2+w)$$

$$\Rightarrow \log\left(\frac{x_1+w/2}{x_2+w/2}\right) = n\left(\log(d_2+w) - \log(d_1+w)\right)$$

$$= n\cdot\log\left(\frac{d_2+w}{d_1+w}\right)$$

$$\therefore\ n = \frac{\log\left(\dfrac{x_1 + w/2}{x_2 + w/2}\right)}{\log\left(\dfrac{d_2 + w}{d_1 + w}\right)}$$

From experimental data :

$$\begin{cases} d_1 = 3.5\text{ cm} \\ x_1 = 1\text{ cm} \\ d_2 = 3\text{ cm} \\ x_2 = 2\text{ cm} \\ w = 10\text{ cm} \end{cases}$$

This gives us $\boxed{n = 4.08 \approx 4}$.

$\therefore$ Magnetic force, $F_m \propto \dfrac{1}{d^4}$.