God Does Not Play Dice

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In this problem, we are going to analyse an overly-idealised model of a coin toss, and predict whether the coin will flip *Heads* or *Tails*.

Problem Statement:

You are given a cylindrical coin of mass M (of homogeneous density), radius R, and negligible height. The coin initially has *Heads* on top and is completely horizontal. From a height H from the ground, you simultaneously throw and flip the coin upwards with a linear velocity V and angular velocity ω (consider this to be anti-clockwise from your side view), respectively (the axis of rotation of the coin is the diameter of the coin's face, which contains its center of mass or geometric center *(both are same here)*). The acceleration due to gravity is downwards and has a magnitude of g.

Looking up the properties of the materials the coin and ground are made of, you realise that each time the coin collides with the ground, the ground "absorbs" a fraction f of the total energy of the coin (this "absorbed" energy increases the internal energy of the ground).

We can model the behaviour of the ground as an ideal spring with spring modulus k (which will be the ratio of Stress over Strain *(length compression in this case)*). **Moreover, we can assume the ground to rigid enough that the duration of compression of the ground to be very small *(dt << 1 second).*** Also, for all cases except when the coin falls flat on the ground, assume the contact between the coin and ground to be a single point.

Hint: dt can be figured out for a given elastic modulus of the ground k. For this, neglect effects due to gravity during the compression. Consider all energy loss to occur just before the coin plunges back into the air, about to lose contact with the ground (this might be slightly inaccurate, but considering that dt << 1, it's not very bad).

Input: You are given $M, R, H, V, \omega, g, f, k$ (all are given in SI units)

Output: Find whether the coin will land *Heads* or *Tails* based on our model (The coin initially has Heads on top). Output either "Heads" or "Tails", depending on which the side the coin has on top after it lands on the ground.

[Moment of Inertia of a coin (axis through diameter): 1/4*M*R^2]