Cricket on Rooftop!

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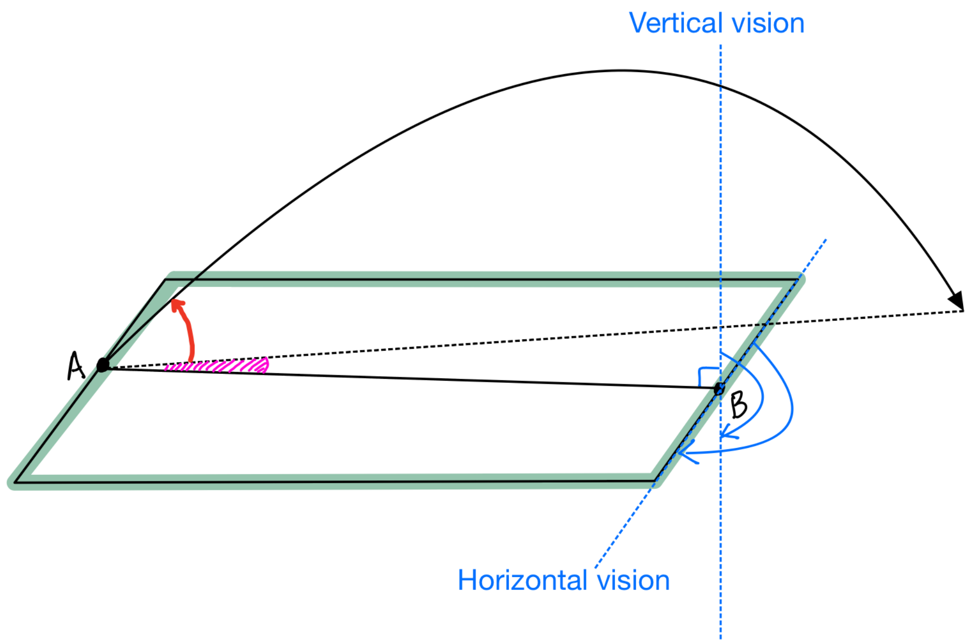

Figure 1

(Neglect air resistance throughout the problem)

As a teenager living in urban Dhaka, the capital of Bangladesh, you have become fed up trying to find unoccupied fields to play cricket with your friends. Having no other choices, you have decided to play cricket in your rooftop! However, there is one major problem; hitting sixes. Obviously, the ball will just fly off the roof if a six is hit!

To solve the problem, you steal Mark Rober's engineering equipments and make a claw attached to an infinite amount of massless, inextensible string, all of which is loaded and set up at point B in *Figure 1*. This setup can launch the claw at a fixed velocity of u. In addition to that, you have also set up an ultrasonic distance measurement camera (which has 180º vision both vertically and horizontally) at that same point B.

The length AB = d.

In *Figure 1*, the green outlined plane is actually your roof. Point A and B are directly opposite to each other. The bat hits the ball at point A, making the magenta angle (measured anticlockwise) with line AB and the red angle with the plane of the roof.

The moment the flying ball comes into the vision of the ultrasonic camera, the camera orients itself accordingly (making an angle of λ_1 with line AB (measured anticlockwise) and ϕ_1 with the plane of the roof (upwards is positive)) and makes the following measurements:

1. Sends an ultrasonic sound wave (having a velocity of v >> *velocity of ball* (so the ball can be assumed stationary during that time)) towards the ball and records the duration between emission and reception of reflected wave (T_1).

2. After some time t (small enough), the camera orients itself accordingly again (now the positioning angles are λ_2 and ϕ_2) and makes the same measurement (now the value is T_2).

3. After some time t again, the camera orients itself accordingly again (now the positioning angles are λ_3 and ϕ_3) and makes the same measurement (now the value is T_3).

Just after this three measurements, the claw is launched in the objective of grabbing and capturing the ball.

You are given the acceleration due to gravity g, $(\lambda_1, \phi_1), (\lambda_2, \phi_2), (\lambda_3, \phi_3)$, T_1, T_2, T_3, u, v, d.

Determine the angle of elevation with the plane, θ and angle (measured anticlockwise) with line AB, α, that the claw must be launched at in order to capture the ball (the string attached to the claw is infinite in length).