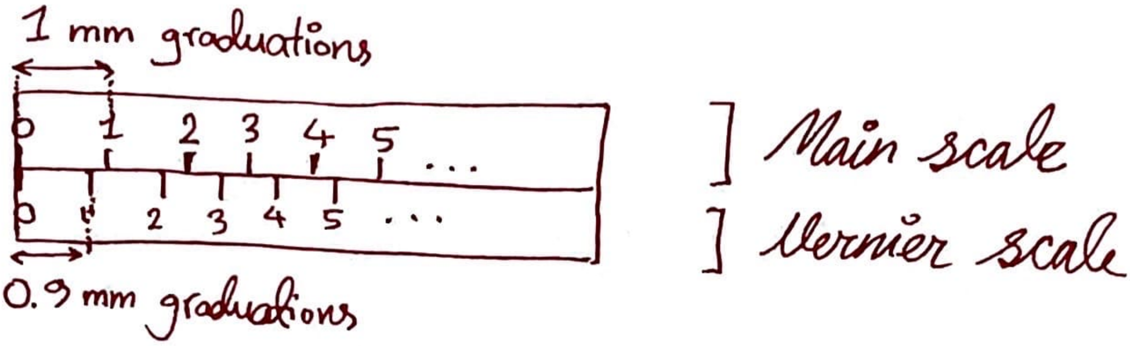
Vernier Calipers: Formula derivation

Vernier Caliper

→ **Has two scales:**

- → Main scale
- → Vernier scale



The vernier constant, v_c, is defined as:

$$v_c = (\text{1 smallest main scale division}) - (\text{1 smallest vernier scale division})$$

$$= 1\,\text{mm} - 0.9\,\text{mm}$$

$$= 0.1\,\text{mm}$$

To generalise things, let:

m = 1 smallest main scale division

V = 1 smallest vernier scale division

$$\therefore v_c = m - V$$

Now suppose, we want to find out the gap between the n^{th} main scale division and the n^{th} vernier scale division (when the 0 mark on the main scale is alligned with the 0 mark on the vernier scale):

$$gap = (n^{th} \text{ main scale reading}) - (n^{th} \text{ vernier scale reading})$$

$$= (n \cdot m) - (n \cdot v)$$

$$= n(m - v)$$

$$= n \cdot v_c$$

Therefore, the gap between the n^{th} main and n^{th} vernier scale (when the 0 mark on both scales are alligned) is:

$$g_n = n \cdot v_c$$

So, in the ~~s~~ vernier caliper where $m = 1$ mm and $v = 0.9\,m$ [So, $v_c = 0.1$ mm] :

$g_0 = 0 \times 0.1 \text{ mm} = 0 \text{ mm}$

$g_1 = 1 \times 0.1 \text{ mm} = 0.1 \text{ mm}$

$g_2 = 2 \times 0.1 \text{ mm} = 0.2 \text{ mm}$

$\vdots$

$g_{10} = 10 \times 0.1 \text{ mm} = 1 \text{ mm}$

Hence, the gap between the 10^{th} main scale division and the 10^{th} vernier scale division is 1 mm.

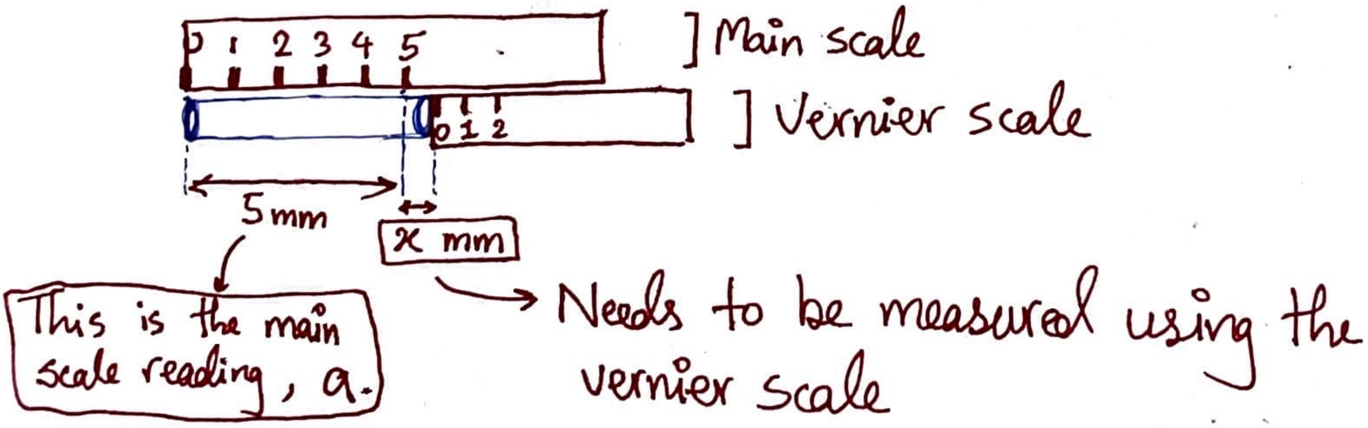
→ This means that the 10^{th} vernier scale division matches with the 9^{th} main scale division.

In general, whenever g_n is a whole number, the n^{th} vernier scale division MATCHES with a main scale division. [Example: When $n = 0$, $n = 10$, etc.]

Now, say we have an object with a length of :

L = 5.4 mm

If we place one end of the object alligned to the 0 mark of the main scale, and another end alligned to the 0 mark of the vernier scale :



[In our case, $x = 0.4$ mm]

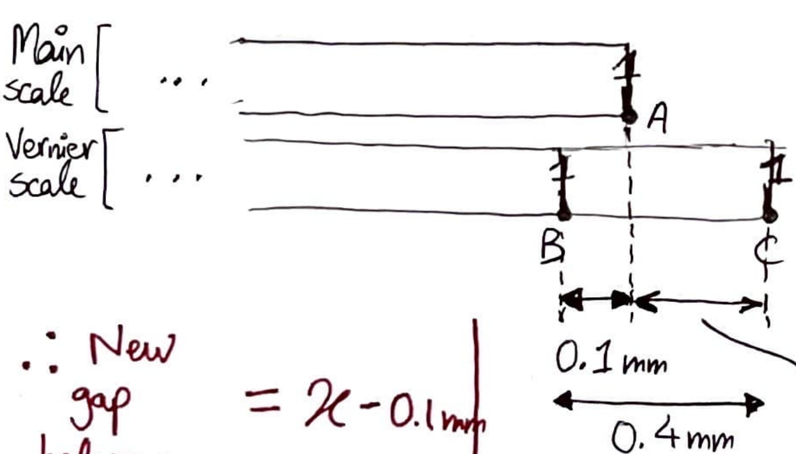
To measure x using the vernier scale, at first, let's consider the 5mm mark on the main scale as our BASE MARK. [We will consider it as the 0 mark]

Coming back to the vernier scale :

The 1st mark ~~after~~ on the vernier scale previously had a gap of $\boxed{1 \times 0.1\,\text{mm} = 0.1\,\text{mm}}$ with the 1st mark (after the base ~~to p~~ mark) of the main scale.

→ But now, the 0 mark in the ~~main~~ vernier scale ADVANCED by $\boxed{x = 0.4\,\text{mm}}$ from the base mark of the main scale.

So, the 1st mark on the vernier scale ALSO advanced by $\boxed{x = 0.4\,\text{mm}}$.



∴ New gap between 1st ~~vern~~ main scale division (after base mark) and 1st vernier scale division :

$= x - 0.1\,\text{mm}$

$= 0.4\,\text{mm} - 0.1\,\text{mm}$

$= 0.3\,\text{mm}$

So, this gap is :

$0.4\,\text{mm} - 0.1\,\text{mm}$

$= 0.3\,\text{mm}$

B : Previous 1st mark on vernier scale

C : 1st mark on vernier scale after advancement by $x = 0.4\,\text{mm}$

A : 1st mark on main scale (after the base mark)

So, NOW, the n^{th} gap [gap between the n^{th} division in main scale (after base mark) and n^{th} division in vernier scale] is :

$$(g_n)_{new} = x - g_n$$

$$\therefore (g_n)_{new} = x - (n \cdot v_c)$$

As said before, the n^{th} vernier scale division will match with a main scale division when $(g_n)_{new}$ is ~~the~~ a whole number.

→ So, when $(g_n)_{new}$ is equal to 0 [The least whole number], the n^{th} vernier scale division will match with some main scale division.

→ Let, the k^{th} vernier scale division matches with a main scale division FOR THE FIRST TIME.

$\therefore$ The gap (new) will be 0.

$$\therefore x - (k \cdot v_c) = 0$$

$$\Rightarrow \boxed{x = v_c \cdot k}$$

Hence:

$$L = \left(\begin{matrix}\text{Main scale} \\ \text{reading}\end{matrix}\right) + x$$

$$\therefore \boxed{L = a + (v_c \cdot k)}$$

[a : Main scale reading
v_c : Vernier constant
k : The number of the vernier scale division which matches with some ~~the~~ main scale division for the first time]