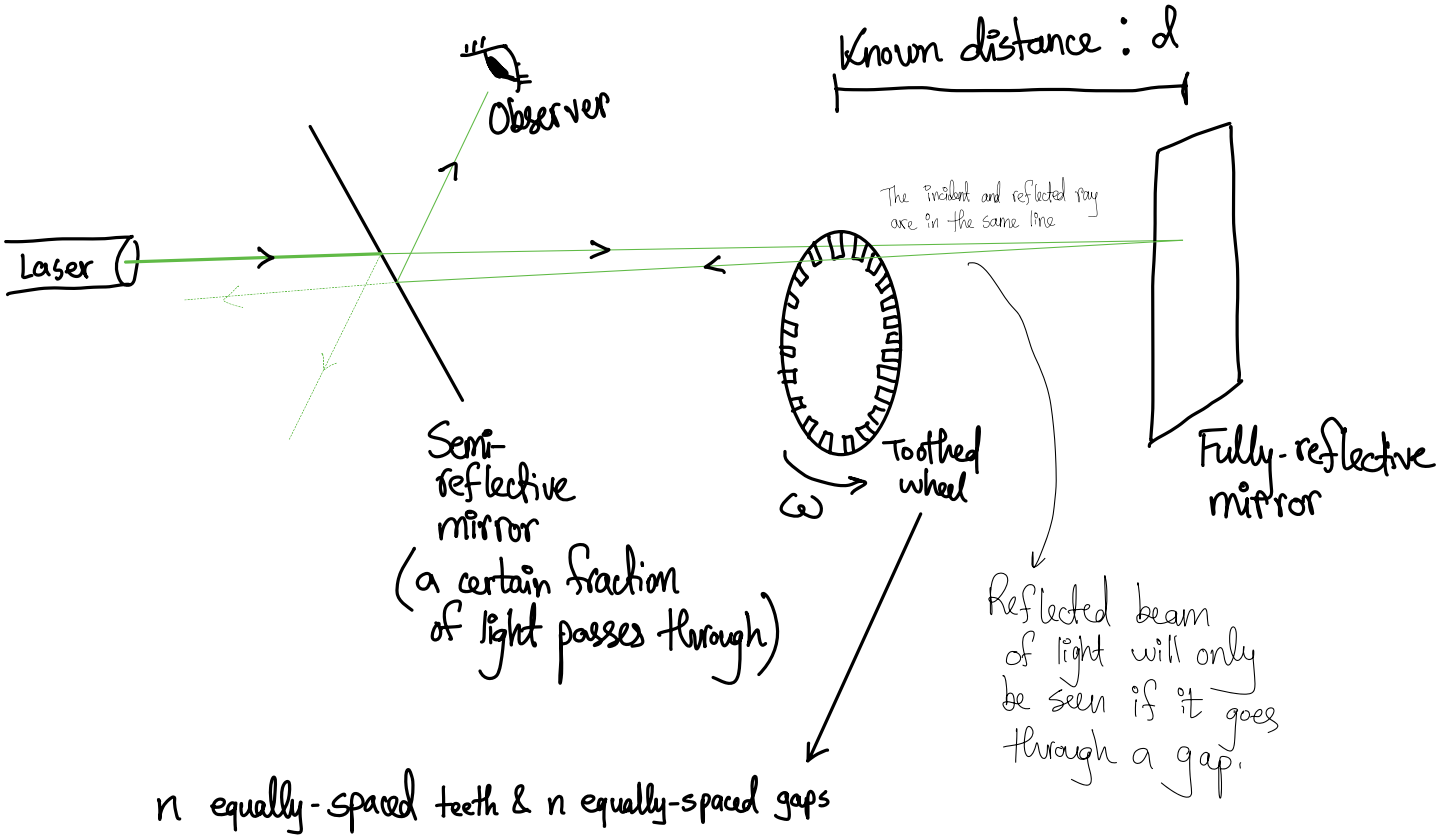
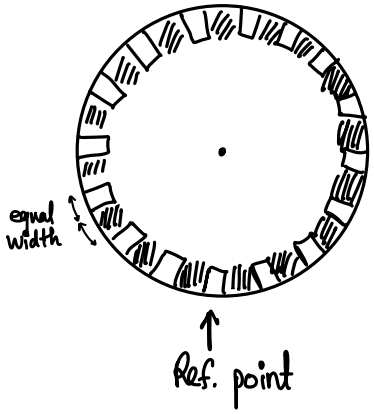
Fizeau's Toothed Wheel Experiment



→ Observing the toothed wheel :

□ : Gap
▨ : Tooth



At an angular velocity of ω :

$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}$$

In time T, n gaps and n teeth passes over the Ref. point :

$T \longrightarrow 2n$ teeth widths (since gap and teeth have equal widths)

$\therefore$ 1 teeth width $\longrightarrow \frac{T}{2n}$ → Time between two successive teeth & gap. passing over Ref. point, which we will call t_1.

So,

$$t_1 = \frac{T}{2n} = \frac{2\pi}{\omega} \times \frac{1}{2n}$$

$$\therefore \boxed{t_1 = \frac{\pi}{\omega n}}$$

Now, when ω is quite small, t_1 is quite large. As a result, light has enough time to cover the 2d distance (two-way trip) and then, slip in through the SAME gap through which it went towards the fully-reflective mirror.

As a result, for small-enough ω, the observer can see flickers of light (Due to some light beams slipping in through the SAME gap, as mentioned. However, some light beams might not slip in because it entered a certain gap at a time delay and hence get blocked by a tooth on the return trip. This is the reason behind flickers seen by the observer).

→ However, at a certain (very fast) ω, no light can pass through, since by the time a passed beam of light travels $2d$, t_1 time has already elapsed and the reflected beam is blocked by a tooth.

$\therefore (t_1)$ = Time taken for light to cover $2d$ distance

$$\Rightarrow \frac{\pi}{\omega n} = \frac{2d}{c} \quad \Rightarrow \quad \frac{c}{2d} = \frac{\omega n}{\pi}$$

$$\therefore c = \frac{2d\omega n}{\pi}$$

The critical angular speed

So, we have:

$$c = \left(\frac{2n}{\pi}\right) \times \omega_{critical} \times d$$