Kodutöö 2 Taavi Tammaru

1) kompleks tuletis on kujul $f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$

kasutame ahela reeglit

$$\frac{\partial}{\partial x} = \frac{\partial r}{\partial x}\frac{\partial}{\partial r} + \frac{\partial x}{\partial \theta}\frac{\partial}{\partial \theta}$$

$$\frac{\partial}{\partial x} = \cos(\theta)\frac{\partial}{\partial r} - \frac{\sin(\theta)}{r}\frac{\partial}{\partial \theta}$$

arendame selle tagasi esimesse valemisse ja saame

$$f'(z) = \left(\cos(\theta)\frac{\partial u}{\partial r} - \frac{\sin(\theta)}{r}\frac{\partial u}{\partial \theta}\right) + i\left(\cos(\theta)\frac{\partial v}{\partial r} - \frac{\sin(\theta)}{r}\frac{\partial v}{\partial \theta}\right)$$

arendame cauchy-Riemanni järgi

$$f'(z) = \left(\cos(\theta)\frac{\partial u}{\partial r} + \sin(\theta)\frac{\partial v}{\partial r}\right) + i\left(\cos(\theta)\frac{\partial v}{\partial r} - \sin(\theta)\frac{\partial u}{\partial r}\right)$$

$$= \frac{\partial u}{\partial r}\left(\cos(\theta) - i\sin(\theta)\right) + \frac{\partial v}{\partial r}\left(\sin(\theta) + i\cos(\theta)\right)$$

$$= u_r e^{-i\theta} + v_r i e^{-i\theta}$$

$$= e^{-i\theta}(u_r + iv_r)$$

ja saimegi soovitud kuju

b) võtame eelmises punktis saadud valemi

$$f'(z) = \left(\cos(\theta)\frac{\partial u}{\partial r} - \frac{\sin(\theta)}{r}\frac{\partial u}{\partial \theta}\right) + i\left(\cos(\theta)\frac{\partial v}{\partial r} - \frac{\sin(\theta)}{r}\frac{\partial v}{\partial \theta}\right)$$

ja kasutame Cauchy-Riemanni võrrandeid teistpidi

$$f'(z) = \left(\frac{\cos(\theta)}{r}\frac{\partial v}{\partial \theta} - \frac{\sin}{r}\frac{\partial u}{\partial \theta}\right) + i\left(-\frac{\cos(\theta)}{r}\frac{\partial u}{\partial \theta} - \frac{\sin(\theta)}{r}\frac{\partial v}{\partial \theta}\right)$$

$$= \frac{1}{r}\frac{\partial v}{\partial \theta}\left(\cos(\theta) - i\sin(\theta)\right) + \frac{1}{r}\frac{\partial u}{\partial \theta}\left(-\sin(\theta) - i\cos(\theta)\right)$$

$$= \frac{1}{r} v_\theta\left(e^{-i\theta}\right) + \frac{1}{r} u_\theta\left(-e^{-i\theta}\right)$$

$$= \frac{1}{r} e^{-i\theta}\left(v_\theta - u_\theta\right)$$

2

$$\cosh(z) = \frac{e^{z} + e^{-z}}{2}$$

$$\frac{e^{z} + e^{-z}}{2} = 0 \qquad e^{z} + e^{-z} = 0$$

$$e^{z} + \frac{1}{e^{z}} = 0 \quad \Big| \cdot e^{z}$$

$$e^{2z} + 1 = 0 \qquad e^{2z} = -1$$

teame, ef $e^{i\pi} = -1$

elik $2z = \cancel{i\pi}\ i\pi(1+2n)$

$$z = \frac{i\pi(1+2n)}{2} \qquad n \in \mathbb{Z}$$

3

a) $(ei)^{i\pi} = -i^{i\pi}$

kenn farme $z^i = e^{i\log(z)}$

$-i^{i\pi} = e^{i\pi \log(i)}$

$\log(i) = 0 + i\left(\frac{\pi}{2} + 2\pi n\right) \quad n = \mathbb{Z}$

$-i^{i\pi} = -e^{i\pi\left(i\frac{\pi}{2} + 2\pi n i\right)}$

$Pv(-i^{i\pi}) = -e^{i^2\frac{\pi^2}{2}} \quad n = 0$

$= -e^{-\frac{\pi^2}{2}}$

b)

$\log(i^e) \qquad i^e = \cancel{\log i}\; e^{e\log(i)}$

$\log(i) = i\left(\frac{\pi}{2} + 2\pi n\right) \quad n = \mathbb{Z}$

$\log\left(e^{e\, i\left(\frac{\pi}{2} + 2\pi n\right)}\right) \quad n = \mathbb{Z}$

$e\, i\left(\frac{\pi}{2} + 2\pi n\right) \log(e)$

$= e \log(e)\, i\left(\frac{\pi}{2} + 2\pi n\right)$

$= 1{,}18 \log\left(e^{i\frac{\pi}{2}}\right)$

$e\log(e) = 1{,}18$

$i\left(\frac{\pi}{2} + 2\pi n\right) = \log\left(e^{i\frac{\pi}{2}}\right)$

c

$$\log(z) = \ln(r) + i\theta$$

$$\log(e^{i}) = \ln(1) + i$$
$$= 0 + i$$
$$= i$$

d)

$$\arcsin(ei) = \sin^{-1}(ei) = -\log\left(i\,ei + \sqrt{1-(ei)^2}\right)$$
$$= -i\log\left(-e + \sqrt{1+i^2}\right) = -i\sinh^{-1}(-e) = i\sinh^{-1}(e)$$