1

lahendame $\frac{d^2u}{d\theta} + u = \frac{\gamma}{L^2}u$ et saada $u(\theta)$ nagu soovitud

karakteristlik võrrand

$$r^2 + (1 - \frac{\gamma}{L^2}) = 0$$

arendame $1 - \frac{\gamma}{L^2} = a$

$$r^2 = -a \quad |\sqrt{\ } \qquad r = \sqrt{-a}$$

ehk karakteristlikul võrrandil tulevad kompleks juured

$$\cancel{r = \pm\sqrt{a} + i} \quad r_{1,2} = \pm\sqrt{a}\,i$$

$$\cancel{r(\theta)\ u(\theta) = c_1\cos(\sqrt{a}\theta) + c_2\sin(}$$

$$\cancel{r(\theta)} \cdot u(\theta) = c_1\cos(\sqrt{a}\theta) + c_2\sin(\sqrt{a}\theta)$$

$$u'(\theta) = -\sqrt{a}c_1\sin(\sqrt{a}\theta) + \sqrt{a}c_2\cos(\sqrt{a}\theta)$$

$$u(0) = 0 = c_1$$

$$u'(0) = \frac{1}{b} = \sqrt{a}c_2 \qquad c_2 = \frac{1}{b\sqrt{a}} = \frac{1}{b\sqrt{(1-\frac{\gamma}{L^2})}}$$

$$\cancel{u(\theta) = \frac{1}{b(1-\frac{\gamma}{L^2}}} \qquad u(\theta) = \frac{1}{b(1-\frac{\gamma}{L^2})}\sin(\sqrt{1-\frac{\gamma}{L^2}}\,\theta)$$

$$K\theta \subset (0, \pi)$$

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$$u(\theta) = \frac{1}{b\sqrt{1-\frac{\gamma_t}{L^2}}} \sin\left(\sqrt{1-\frac{\gamma_t}{L^2}}\,\theta\right)$$

arendame kontandi $\sqrt{1-\frac{\gamma_t}{L^2}} = a$

$$u(\theta) = \frac{\sin(a\theta)}{ab}$$

kuna $u = \frac{1}{r}$

$$u(\theta)^{-1} = r(\theta) = \frac{ab}{\sin(a\theta)}$$

2. θ määramis~~muutumis~~piirkond

$r(\theta) = \frac{ab}{\sin(a\theta)}$ kaugus r peab olema positiivne

seega $a\theta$ muutumispiirkond on $[0; \pi]$

seega $\theta \Rightarrow [0; \frac{\pi}{a}]$

3. tehke kindlaks vähim kaugus tsentrist ja suurim kiirus

vähim kaugus:

$$r'(\theta) = -\frac{\cos(a\theta)\,a^2 b}{\sin(a\theta)^2}$$

$$-\frac{\cos(a\theta)\,a^2 b}{\sin(a\theta)^2} = 0 \qquad -\cos(a\theta)a^2 b = 0$$

$$-\cos(a\theta) = 0$$

$$a\theta = \frac{\pi}{2} \qquad \theta = \frac{\pi}{2a}$$

$$r\left(\frac{\pi}{2a}\right) = \frac{ab}{\sin\left(\frac{\pi a}{2a}\right)} = \frac{ab}{\sin\left(\frac{\pi}{2}\right)} = ab$$

vähim kaugus tsentrist on ab

suurim kiirus: on vähima kaugusega punktis

energia jäävuse seadus

$$E_{k_1} + E_{p_1} = E_{k_2} + E_{p_2}$$

$$\frac{mv^2}{2} + ? = \frac{mv_1^2}{2} + ?$$

$$U(r) = -\int F\,dr$$

$$U = -\int -\frac{y}{mr^3}\,dr = +\frac{y}{m}\int \frac{1}{r^3}\,dr = \frac{y}{m}\int r^{-3}\,dr$$

$$= \frac{y}{m}\,\frac{r^{-2}}{-2} = \frac{y}{-m2r^2}$$

$$\frac{mv^2}{2} = \frac{mv_1^2}{2} + \frac{y}{-2mr^2}$$

$$v_1^2 = \frac{2}{m}\left(\frac{mv^2}{2} + \frac{y}{2m(1-\frac{y}{Z})b^2}\right)$$

$$v_1^2 = v^2 + \frac{y}{m^2(1-\frac{y}{Z^2})b^2}$$

kiirus lähimas punktis on

$$v_1 = \sqrt{v^2 + \frac{y}{m^2(1+\frac{y}{Z^2})b^2}}$$

4. näita et trajektoor ei ole perioodiline kui

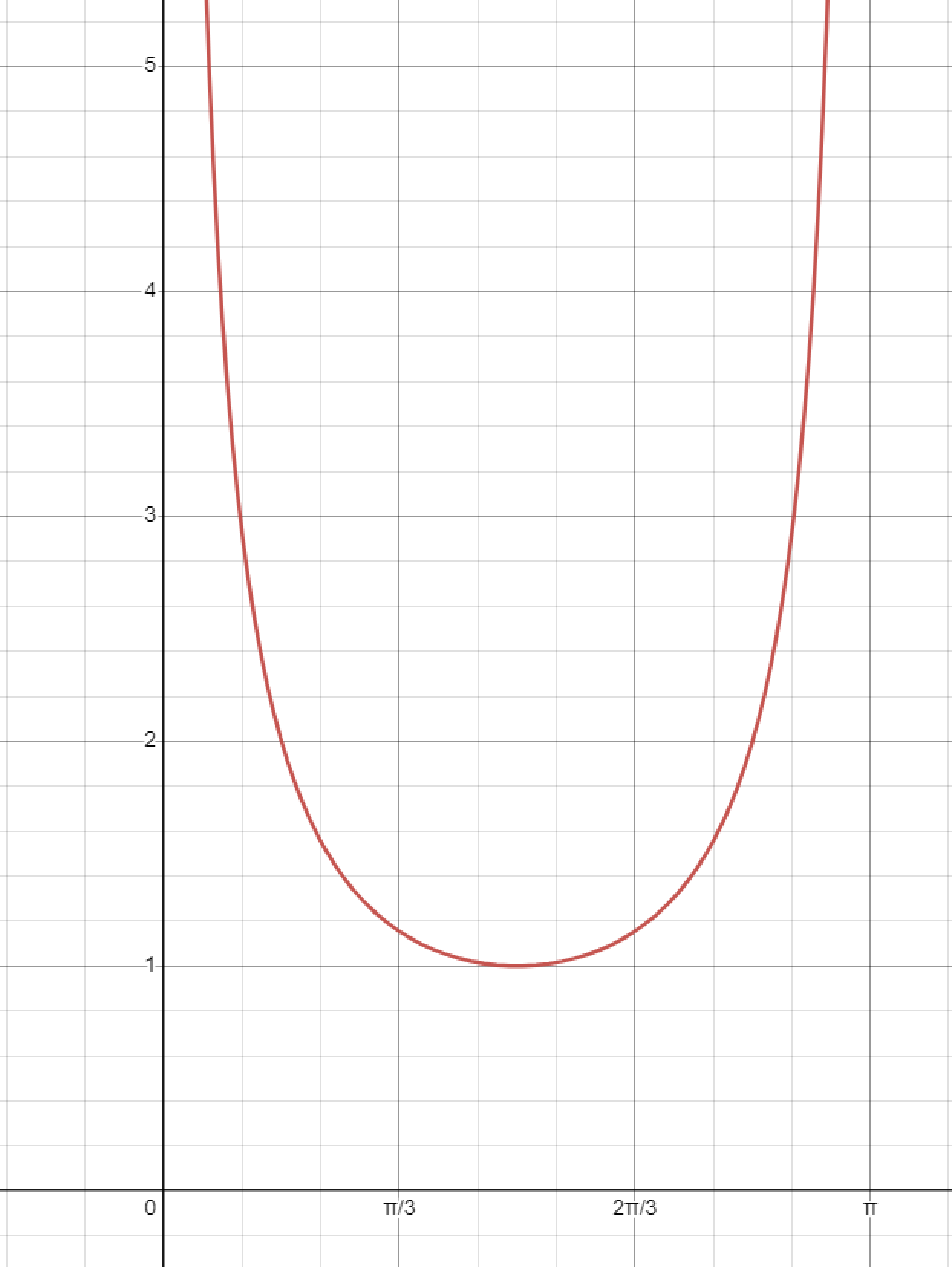
$$\frac{y}{L^2} = \frac{3}{4}$$

seega $1 - \frac{3}{4} = \frac{1}{4}$ $\sqrt{\frac{1}{4}} = \sqrt{0{,}25} = 0{,}5 = a$

~~seto~~ $r(\theta) = \frac{a\,b}{\sin(a\theta)} = \frac{b}{2\sin(\frac{\theta}{2})}$

sellel funktsioonil on asümptoodid punktides kus $\frac{\theta}{2} = k\pi$

Ja funktsioon vende asümptootide vahel on kombinatsioon funktsioonidest $\frac{1}{x}$ ja $\sin(x)$ mis on mõlemad ühtsed funktsioonid elik ei lõika ennast.



5
4
3
2
1
0
π/3
2π/3
π