Test 1: Mechanics and Relativity

Tuesday, September 21, 2021, Aletta Jacobshal

Duration: 1 hour

Before you start, read the following:
There are 3 problems for a total of 90 points
Write your name and student number on each sheet of paper
Make clear arguments and derivations and use correct notation
Support your arguments by clear drawings where appropriate
Write in a readable manner, illegible handwriting will not be graded
Draw your space-time diagrams on the provided graph paper and put them inside your written solutions when handing in

Name: ....
Student Number: S - ....

	Points
Problem 1:	
Problem 2:	
Problem 3:	
Total:	
GRADE (1 + # Total/10)	

Name: Student Number:

Problem 1: Basics (25 pt)

Write **TRUE** if the statement is true; write **FALSE** if the statement is false.
If you don't know the answer, **LEAVE THE ANSWER BLANK!** Points are deducted for wrong answers.

1. False (2 pt) Newton's laws are invariant under the Maxwell transformations.
   Newton's laws are invariant under the Galilean transformations. Since some students were confused that the Maxwell transformations do not exist, nobody gets negative points for this question.

2. False (2 pt) An object must have the same kinetic energy in all inertial frames.

3. True (2 pt) An object must have the same gravitational potential energy in all inertial frames. (Hint: Recall E=mgh)
   Everyone gets 2 points for this question since the answer actually changes if we take the Lorentz transformations into account, although this test didn't cover the Lorentz transformations.

4. True (2 pt) In Newtonian relativity, observers in both inertial frames agree about an object's acceleration at a given time.

5. False (2 pt) Suppose there are two reference frames designated by S and S′ such that the co-ordinate axes are parallel to each other. In S, we have the co-ordinates $\{t, x, y, z\}$ and in S′ we have the co-ordinates $\{t', x', y', z'\}$. S′ is moving with respect to S with velocity $\vec{\beta}$ in the positive x direction. The Galilean transformations which relate the primed and unprimed coordinates are given as

$$
\begin{aligned}
t' &= t\,, \\
x' &= x + |\vec{\beta}|\,t\,, \\
y' &= y\,, \\
z' &= z\,.
\end{aligned}
$$

6. False (2 pt) According to the conventions established in this course, the slope of the worldline for a particle represents its speed on a spacetime diagram.
   Everyone gets 2 points for this question since the usage of the word 'represent' caused ambiguity.

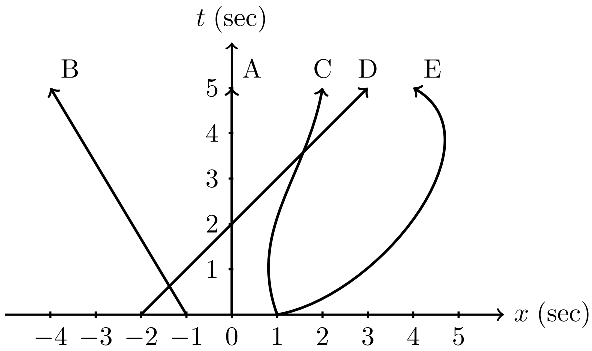
7. True (2 pt) The distances and time intervals are invariant under the Galilean transformations.

8. False (2 pt) X-rays travel faster than microwaves.

9. True (2 pt) Sunlight reflected from the Moon's surface takes around 1.3 seconds to travel to the Earth's surface. Hence, the sunlight travelled roughly 350,000 to 400,000 km.

10. False (2 pt) A dog named Laika runs with a constant speed of 10km/hr relative to a spaceship for an hour. The speed of the spaceship is 36,000 km/hr with respect to observer Bob. According to Bob, Laika travels approximately 12 milliseconds in SR units.

11. The spacetime diagram below shows the worldlines of various objects.
    False (1 pt) object E is the only one accelerating
    True (1 pt) object D could be light
    False (1 pt) in a frame attached to object A objects C, D and E always move in the positive x direction
    False (1 pt) object B has the largest speed at time $t = 4s$
    False (1 pt) except for object A, all objects are always in motion



Problem 1: Space Kidnapping

An extraterrestrial space ship *Pandik* is headed for earth at a constant velocity of $2/3$ and is located at $(t, x) = (0\,\text{min}, 4\,\text{min})$. (Take the earth to be at the spatial origin of the coordinate system, $x = 0$.) The space-defence radar system (SDRS), located on earth, sends out a radar signal at $t = 1$ min (event A) and finds out about *Pandik* some time later (event B).

(a) Construct the space-time diagram. Draw and label the worldlines of the earth and *Pandik*. For the latter, draw the worldline until it reaches the earth.

See the spacetime diagram below. Since Pandik moves at a speed of $2/3$ in the negative x direction, the slope of the worldline should be $-3/2$. Pandik reaches earth at $(6\,\text{min}, 0\,\text{min})$. Since we take the earth stationary, its worldline is vertically up at $x = 0\,\text{min}$.

- Correct labels axes: 0.5pt.
- Correct numbers/ticks axes: 0.5pt.
- Worldline earth: 2pt.
- Worldline Pandik: 2pt.
- Label worldline earth: 1pt.
- Label worldline Pandik: 1pt.

(b) Draw and label the wordline of the radar signal and denote events A and B in the space-time diagram. When will SDRS find out about *Pandik*? Is this before or after *Pandik* reaches earth?

See the spacetime diagram below. The radar signal moves at the speed of light and hence its worldline has a slope of 1. Event A is located at $(1\,\text{min}, 0\,\text{min})$. The signal reflects off Pandik at $(3\,\text{min}, 2\,\text{min})$, moving back to earth and arriving at $(5\,\text{min}, 0\,\text{min})$ (event B). This is *before* Pandik reaches earth.

- Worldine light: 2pt.
- Label events A and B: 2pt.
- Find $t = 5$ min as moment when SDRS finds out about Pandik: 2pt.
- Conclusion, before Pandik reaches earth: 1 pt.

Pandik manages to arrive on earth and kidnap Tion Miscom, the director of SDRS. It leaves earth at a constant velocity in the direction it came from. The arrival, kidnapping and departure can be considered a single event (event C). *Pandik*'s departing worldline crosses $(t, x) = (11\text{ min}, 1\text{ min})$, at which *Pandik*'s steering wheel breaks and it cannot change trajectory anymore (Event D).

(c) Algebraically find out the speed of *Pandik* after it leaves earth.

Use that the departing worldline of Pandik crosses $(6\,\text{min}, 0\,\text{min})$ and $(11\,\text{min}, 1\,\text{min})$. One can then find the speed as:

$$v_{\text{Pandik}} = \frac{1-0}{11-6} = \frac{1}{5}. \tag{1}$$

Alternatively, once could explicitly setup the equation for the Pandik's departing worldline. Because of the constant velocity, the equation will be of the form $x(t) = at + b$. Using the coordinates we find two equations:

$$0 = 6a + b, \qquad 1 = 11a + b, \tag{2}$$

which are solved to give $a = 1/5$ and $b = -6/5$. Then the speed is $v_{\text{Pandik}} = dx/dt = a = 1/5$.

- Find two coordinates on the departing worldline of Pandik: 2×2pt.
- Calculating the velocity: 3pt.

(d) Draw the departing worldline of *Pandik* and label events C and D.

See spacetime diagram below.

- Correct departing worldline Pandik: 2pt.
- Correct labelling events C and D: 2×1pt.

To intercept *Pandik*, the space-defence agency proposes to use freight ship *Polluter*. It moves at a constant speed of 1/2 towards earth and was located at $x = 5$ min when the kidnapping of Tion Miscom took place. However, because of space-pollution regulations, the *Polluter* is not allowed to come closer to earth than 2.5 minutes.

(e) What is the latest moment in time the *Polluter* can break in order to be able to intersect *Pandik* and comply to the regulations? You may assume the breaking is instantaneous and can be considered an event.

Since we assume the breaking is instantaneous and the Polluter is not allowed in the domain $x < 2.5\,\text{min}$, it should break at $x = 2.5\,\text{min}$. To find out the corresponding time, we use the fact that the Polluter's worldline crosses $(6\,\text{min}, 5\,\text{min})$ (since it is located at $x = 5\,\text{min}$ when Tion Miscom was kidnapped (event C) at $t_C = 6\,\text{min}$) and its speed is 1/2. Based on this information one can choose to draw the Polluter's worldline and find that $x = 2.5\,\text{min}$ is reached at $t = 11\,\text{min}$. So that the latest time at which the Polluter can break is $t = 11\,\text{min}$. Equivalently, one can set up the equation for the Pollutor's worldline: $x_{\text{Polluter}}(t) = -t/2 + 8$. To find the latest time of breaking, we solve $2.5 = -t/2 + 8$, which gives $t = 11\,\text{min}$.

- Break latest at $x = 2.5$ min: 2pt.
- Construct (equation for) worldline of polluter: 2pt.
- Fill in x to find out the corresponding latest time for breaking is $t = 11$ min: 2pt.

(f) Find out algebraically at what moment in time the *Polluter* intercepts *Pandik* assuming it breaks at the latest possible moment.

After breaking (at the latest possible moment), the Polluter is stationary with respect to earth at $x_{\text{Polluter}} = 2.5\,\text{min}$. The departing worldline of Pandik is given by $x_{\text{Pandik}}(t) = t/5 + b$ as the velocity was found in (c) to be 1/5. Using coordinate pair $(6\,\text{min}, 0\,\text{min})$ or $(11\,\text{min}, 1\,\text{min})$ we find $b = -6/5$. Equating $x_{\text{Polluter}} = x_{\text{Pandik}}(t)$ gives:

$$t/5 - 6/5 = 5/2, \tag{3}$$

which is solved to give $t = 18.5\,\text{min}$ as the time of interception.

- Find out that $x_{\text{Polluter}} = 2.5\,\text{min}$ after breaking: 1pt.
- Construct equation for worldline Pandik: 2pt.
- Equate equations for worldline Pandik and Polluter: 2pt.
- Solve the equation to find out the moment of interception: 1pt.

Name: Student Number:

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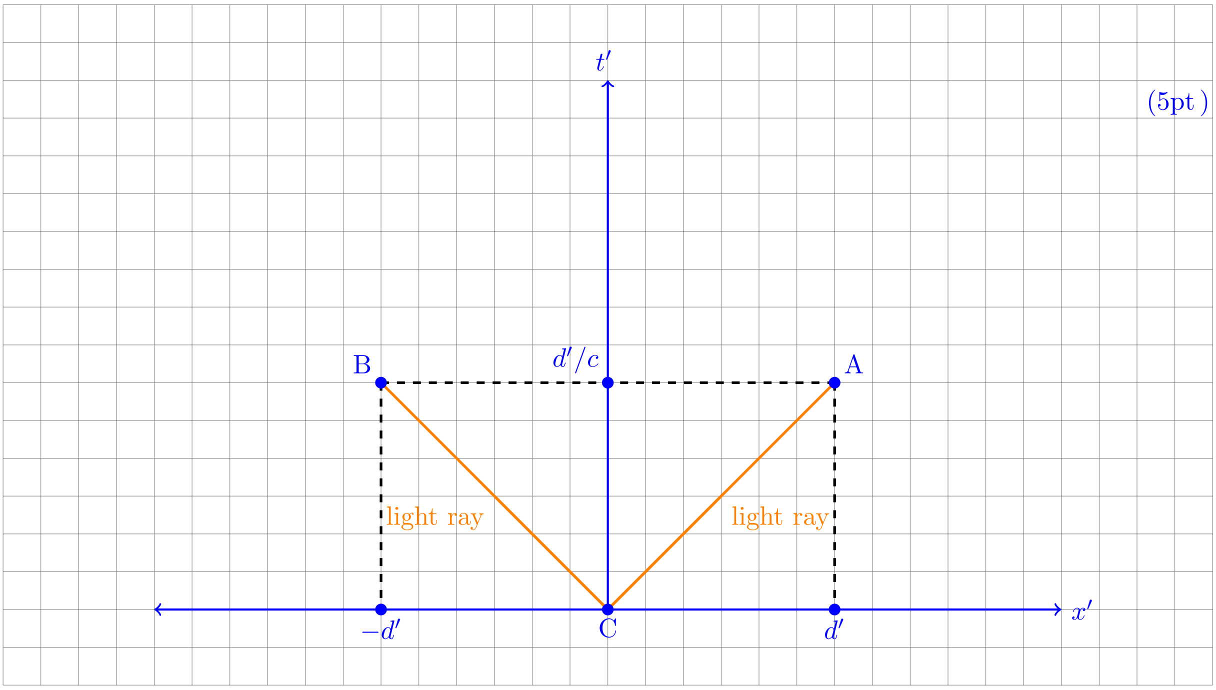
Problem 3: Relativity of Simultaneity (28 pts)

A spaceship hovers over observer M on Mars. Treat both Mars and the spaceship as separate inertial frames for this problem. The speed of the spaceship is v. When the spaceship is overhead, a light signal is emitted from observer C (at the center of the spaceship). Subsequently it is detected by observer A (at point A in the front of the spaceship) and observer B (at point B in the rear of the spaceship). Both observers measure their distance from the center of the spaceship to be d'.

(a) Draw a spacetime diagram of the process as observed by observer C. Identify and label all events. (8 pt)
(1pt) Event C: observer C sends a light signal at $(0, 0)$,
(1pt) Event A: observer A receives a light signal at $(d'/c, d')$
(1pt) Event B: observer B receives a light signal at $(d'/c, -d')$

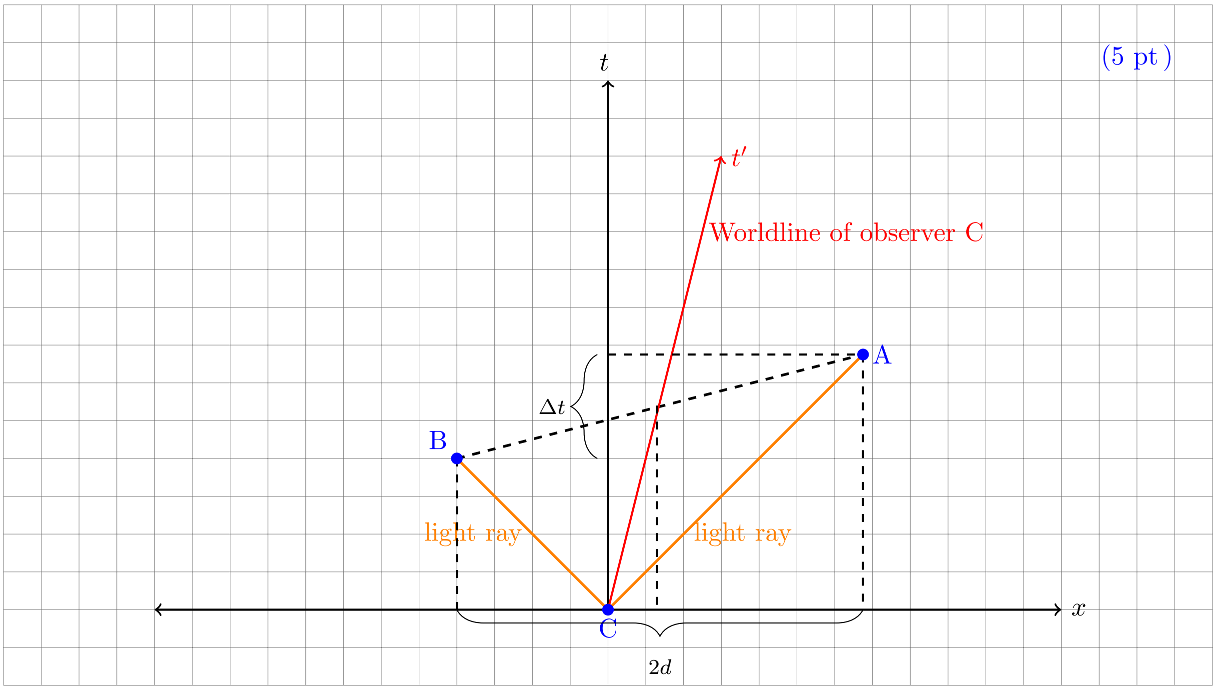


- (1 pt) Label coordinate axes
- (1 pt) Draw and Label light rays, Identify the slope of the light ray
- (3 × 1 pt) Label each events (3 × 1 pt)

(b) In no more than 3 sentences, explain why observers A and B agree that the light signal reaches them simultaneously. (8 pt)
Since observer A and observer B moving with the same velocity, they are relatively stationary. Therefore, they are actually the same inertial reference frame. That's why they receive the signals at the same time.
The time it takes for light to reach them d'/c.

(c) According to observer M, the light signal reaches observer B before observer A. The time difference between the two events is Δt. Moreover, observer M measures the distance from the center of the spaceship to be d, where $d < d'$. Taking these events into account, draw a spacetime diagram of the process as observed by observer M; include the worldline of observer C. Identify and label all events. (8 pt)
(1pt) Event C: observer C sends a light signal at $(0, 0)$,

(1pt) Event A: observer A receives a light signal at $((b + \Delta t)/c,\, 2d - b)$
(1pt) Event B: observer B receives a light signal at $(b/c, -b)$



- (0.5 pt) Label coordinate axes
- (0.5 pt) Draw and label light rays, identify the slope of a light ray $1/c$
- (1 pt) Draw and label worldline of observer C, identify the slope of the red line to be $1/v$
- (3×1 pt) Label each events

(d) If the distance d is in SR units, what is this distance in SI units? (4 pt)

(4 pt) The distance d in SR units is $d.c$ in SI units .