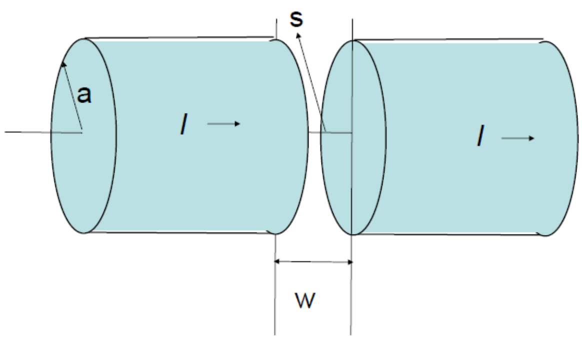
Electricity and Magnetism, Resit Part II July 10 2023 18:15-21:15

Write your name and student number on each answer sheet. Use of a calculator is allowed. You may make use of 1 A4 (double sided) with handwritten notes and of the provided formula sheet. The same notation is used as in the book, i.e. a bold-face $\vec{\mathbf{A}}$ is a vector, $\hat{\mathbf{x}}$ is the unit vector in the x-direction. **In your handwritten answers, remember to indicate vectors (unit vectors) with an arrow (hat) above the symbol**.

5 questions, 53 points

General remark: the exam is mainly composed of tutorial/conceptual problems so the students should be familiar with most of them

Question 1 (9 points)



A fat wire, with radius a, carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor, as shown in the figure.

A. Calculate in the quasistatic approximation the time-derivative of the electric field in the gap at a distance $s \ll a$ from the axis and show that

$$\frac{\partial \vec{\mathbf{E}}}{\partial t} = \frac{I}{\epsilon_0 \pi a^2}\hat{\mathbf{z}} \qquad \text{(2 points)}$$

B. Calculate the magnetic field in the gap at a distance $s \ll a$ from the axis (3 points)

C. Calculate the ratio of the magnitude of the displacement current density $\vec{\mathbf{J}}_d \equiv \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$ over the current density. Does your result make any sense? Explain why. (2 points)

D. The solutions above rely on the fact that the current is constant. Let's consider a more realistic scenario where at time $t = 0$ two outer ends of the wire are connected to a power supply (internal resistance r) which provides the potential difference V. For how long time will your solutions still be valid? The qualitative explanation will do but the formulas are also ok. (2 points)

Answer to Question 1 Chapter 7 Electrodynamics

(Problem 7.34 modified)

A. Charges are accumulated at the interface between the plates, making the electric field change. Under the assumption that the plates of the capacitor are very close to each other $w \ll a$, we can write the electric field between the two plates as:

$$\vec{\mathbf{E}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{z}} = \frac{1}{\epsilon_0}\frac{Q}{A}\hat{\mathbf{z}} = \frac{1}{\epsilon_0}\frac{Q}{\pi a^2}\hat{\mathbf{z}} \qquad \text{(1 point)}$$

where Q is the charge and A is the area of the wire cross-section.

$$\frac{\partial \vec{\mathbf{E}}}{\partial t} = \frac{1}{\epsilon_0}\frac{1}{\pi a^2}\frac{\partial Q}{\partial t}\hat{\mathbf{z}} = \frac{I}{\epsilon_0 \pi a^2}\hat{\mathbf{z}} \qquad \text{(1 point)}$$

B. *Variant 1:*

$$\text{Faraday's equation: } \nabla \times \vec{\mathbf{B}} = \mu_0 \frac{I}{\pi a^2}\hat{\mathbf{z}} \quad \text{(1 point)}$$

i.e. there is only the $\hat{\mathbf{z}}$-component of the curl operator. Making use of the expression of the curl operator in cylindrical coordinates (see the formula sheet) and the radial symmetry of the system under consideration ($B_s = 0$):

$$\frac{1}{s}\frac{\partial}{\partial s}(sB_\varphi) = \mu_0 \frac{I}{\pi a^2}; \; \frac{\partial}{\partial s}(sB_\varphi) = s\,\mu_0 \frac{I}{\pi a^2} \quad \text{(1 point)}$$

$$sB_\varphi = \frac{s^2}{2}\,\mu_0 \frac{I}{\pi a^2}$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I s}{2\pi a^2}\hat{\boldsymbol{\varphi}} \qquad \text{(1 point)}$$

Variant 2: Making use the radial symmetry of the system under consideration ($B_s = 0$) and drawing the Ampèrian circular loop with the radius s and centered at the axis:

$$B \cdot 2\pi s = \mu_0 \epsilon_0 \frac{\partial}{\partial t}\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}} = \mu_0 \epsilon_0 \frac{I}{\epsilon_0 \pi a^2}\pi s^2 \qquad \text{(2 points)}$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I s}{2\pi a^2}\hat{\boldsymbol{\varphi}} \qquad \text{(1 point)}$$

C. $$\frac{|\vec{\mathbf{J}}_d|}{J} \equiv \frac{\epsilon_0 \frac{I}{\epsilon_0 \pi a^2}}{I \pi a^2} = 1 \quad \text{(1 point)}$$

It makes sense: the displacement current density is equal the current density so that the total current is constant through the wire and the gap (1 point)

Alternative: The result makes sense because one could have chosen a different surface with the same boundary that extends cylindrically into the wire (similarly to problem 7.35c in Griffiths; see also the changing capacitor paradox); J in the integral would be the physical current from the wire. Because the choice of the surface is irrelevant (by Stoke's theorem), the current densities should agree in order to give the same result.

D. In this case, the capacitor will begin to charge with a characteristic time $\tau = Cr = \epsilon_0 \frac{\pi a^2}{d} r$. During the much shorter times, the current can be considered constant and equals V/r; after that it will decrease exponentially. (2 points)

Note 1: the equations are not necessary here; (correct) qualitative explanations will do.

Note 2: other creative (but correct!) answers are also accepted

Question 2 (11 points)

Monochromatic plane electromagnetic wave with frequency ω, intensity I, phase $\delta = 0$ and linearly polarized in the y direction, is traveling in the x direction.

A. Explain why the electric and magnetic fields of such a wave are expressed as

$$\vec{\mathbf{E}}(x,t) = E_0 \cos(kx - \omega t)\,\hat{\mathbf{y}} = \sqrt{\frac{2I}{c\epsilon_0}} \cos(kx - \omega t)\,\hat{\mathbf{y}}$$

$$\vec{\mathbf{B}}(x,t) = \frac{E_0}{c} \cos(kx - \omega t)\,\hat{\mathbf{z}} = \frac{1}{c}\sqrt{\frac{2I}{c\epsilon_0}} \cos(kx - \omega t)\,\hat{\mathbf{z}} \quad \text{(3 points)}$$

B. Show that the Maxwell stress tensor $\overleftrightarrow{\mathbf{T}}$ of this wave can be expressed as

$$\overleftrightarrow{\mathbf{T}} = -\frac{2I}{c}\cos^2(kx - \omega t)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{(4 points)}$$

Tip: the components of Maxwell's stress tensor read as

$$T_{ij} \equiv \epsilon_0\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)$$

C. From (B), what is the momentum current density and in which direction? Does it make sense that only $T_{xx} \neq 0$? Explain why (2 points)

Tip: to remind you, the momentum current density is $-\overleftrightarrow{\mathbf{T}}$

D. How is the momentum flux density related to the energy density in this case? (2 points)

Model answers (Chapters 8 and 9, Conservation laws and EM waves)

(Problem 9.13 modified)

A. $I = \frac{c\epsilon_0 E_0^2}{2}$; $E_0 = \sqrt{\frac{2I}{c\epsilon_0}}$ (1/2 point)

As the wave is polarized into the y direction, the electric field should have the same component. (1/2 point)

The wave propagates into the x-direction so the (part of) argument should be kx (1/2 point)

The wave is monochromatic which gives the cos dependence on $kx - \omega t$ (1/2 point)

$$\vec{\mathbf{E}}(x,t) = E_0 \cos(kx - \omega t)\,\hat{\mathbf{y}} = \sqrt{\frac{2I}{c\epsilon_0}} \cos(kx - \omega t)\,\hat{\mathbf{y}}$$

The wave propagates into the x direction and polarized into the y direction so that the magnetic field has the z-direction (1/2 point)

The amplitude of the magnetic field scales as E_0/c (1/2 point)

$$\vec{\mathbf{B}}(x,t) = \frac{E_0}{c}\cos(kx-\omega t)\,\hat{\mathbf{z}} = \frac{1}{c}\sqrt{\frac{2I}{c\epsilon_0}}\cos(kx-\omega t)\,\hat{\mathbf{z}}$$

Alternative answer: (3 points in total)

2) GENERAL MONOCHROMATIC PLANE WAVE

$\vec{E}(\vec{r},t) = E_0 \cos(\vec{k}\cdot\vec{r}-\omega t+\delta)\,\hat{n}$

$\vec{B}(\vec{r},t) = \frac{1}{c}E_0 \cos(\vec{k}\cdot\vec{r}-\omega t+\delta)\,(\hat{k}\times\hat{n})$

$\hat{n}$ IS POLARIZATION DIRECTION. IN THIS CASE

$\hat{n}=\hat{y}$, $\vec{k}=\vec{x} \Rightarrow \hat{k}\times\hat{n}=\hat{z}$

$\delta=0$

EQ. (9.63) $\Rightarrow I = \frac{1}{2}c\epsilon_0 E_0^2$ SO

$E_0 = \sqrt{\frac{2I}{c\epsilon_0}}$

B. $\vec{\mathbf{E}}$ has only a y-component, and $\vec{\mathbf{B}}$ has only a z-component, so that all the "off-diagonal" $(i \neq j)$ terms in $\overleftrightarrow{\mathbf{T}}$ are zero (1 point)

The "diagonal" elements:

$$T_{xx} = \epsilon_0\left[-\frac{1}{2}(E_y^2)\right] + \frac{1}{\mu_0}\left[-\frac{1}{2}(B_z^2)\right] = -\frac{\epsilon_0}{2}E_y^2 - \frac{1}{2\mu_0}B_z^2 = -\frac{1}{2}\left[\epsilon_0 E_y^2 + \frac{1}{\mu_0}\frac{1}{c^2}E_y^2\right]$$

$$= -\frac{1}{2}\left[\epsilon_0 E_y^2 + \frac{\epsilon_0\mu_0}{\mu_0}E_y^2\right] = -\epsilon_0 E_y^2 = -\frac{2I}{c}\cos^2(kx-\omega t) \quad \text{(1 point)}$$

$$T_{yy} = \epsilon_0\left[E_y\ E_y\ -\frac{1}{2}(E_y^2)\right] + \frac{1}{\mu_0}\left[-\frac{1}{2}(B_z^2)\right] = \frac{\epsilon_0}{2}E_y^2 - \frac{1}{2\mu_0}B_z^2 = 0 \quad \text{(1 point)}$$

because of the flipped sign in front of $\frac{\epsilon_0}{2}$ as compared to T_{xx}.

$$T_{zz} = \epsilon_0\left[-\frac{1}{2}(E_y^2)\right] + \frac{1}{\mu_0}\left[B_z\ B_z\ -\frac{1}{2}(B_z^2)\right] = -\frac{\epsilon_0}{2}E_y^2 + \frac{1}{2\mu_0}B_z^2 = 0 \quad \text{(1 point)}$$

because of the flipped sign in front of $\frac{1}{2\mu_0}$ as compared to T_{xx}.

Summarizing,

$$\overleftrightarrow{\mathbf{T}} = -\frac{2I}{c}\cos^2(kx-\omega t)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

C. The momentum current density into the x direction (electromagnetic pressure) is

$\frac{2I}{c}\cos^2(kx-\omega t)$ (1 point)

The momentum of the wave is being transported only into the x direction, so yes, it does make sense that T_{xx} is the only nonzero element in T_{ij} (1 point)

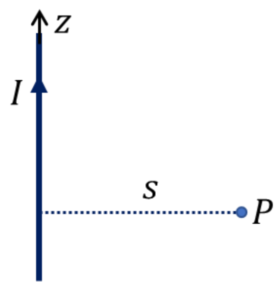
D. The energy density is $u = \epsilon_0 E^2 = \frac{2I}{c} cos^2(kx-\omega t)$ (1 point)

momentum flux density = energy density (1 point)

Question 3 (13 points)

An infinite, electrically-neutral straight wire carries a constant current I_0 which is turned on abruptly at $t = 0$:

$$I(t) = \begin{cases} 0, \text{for } t < 0 \\ I_0, \text{for } t \geq 0 \end{cases}$$



For the observation point P laying at a distance s from the wire:

A. Calculate the vector potential and show that

$$\vec{\mathbf{A}}(s,t) = \begin{cases} \frac{\mu_0 I_0}{2\pi} ln\left(\frac{ct+\sqrt{(ct)^2-s^2}}{s}\right) \hat{\mathbf{z}} & \text{for } s \leq ct \\ 0 & \text{for } s > ct \end{cases} \quad \text{(4 points)}$$

Tip 1: as a reminder, you might find the following expression useful:

$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}',t_r)}{\mathscr{r}} d\tau'$, where t_r is the retarded time

Tip 2: you might find this integral useful $\int \frac{1}{\sqrt{s^2+z^2}} dz = ln\left(\sqrt{s^2+z^2}+z\right)$

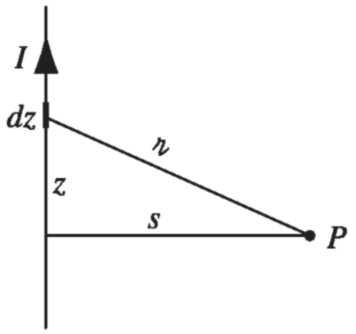
B. Calculate the resulting electric field (2 points)

C. Calculate the resulted magnetic field (3 points)

D. Now your observation point is very close to the wire ($s \ll ct$). What kind of temporal behavior (i.e. their dependence on time) of both fields would you expect? Explain your answer. (4 points)

Tip: You don't need to use the results of (B) and (C) for that in case you didn't make them; simply use your general knowledge of electrodynamics. However, if you did of (B) and (C), you could use their results, too

Model Answers (Chapter 10, Potentials and Fields)



(Example 10.2 modified; also a concept question for tutorials)

A.

$$\vec{\mathbf{A}}(s,t) = \frac{\mu_0}{4\pi}\hat{\mathbf{z}}\int_{-\infty}^{+\infty}\frac{I(t_r)}{\mathscr{r}}dz$$

For $t < s/c$, $\vec{\mathbf{A}} = 0$ as the news of the current has not reached point P yet. (1 point)

For $t > s/c$, only the segment $|z| \leq \sqrt{(ct)^2 - s^2}$ contributes to the integral. Noting the symmetry of the problem,

$$\vec{\mathbf{A}}(s,t) = \frac{\mu_0 I_0}{4\pi}\hat{\mathbf{z}}\,2\int_0^{\sqrt{(ct)^2-s^2}}\frac{1}{\sqrt{s^2+z^2}}dz = \frac{\mu_0 I_0}{2\pi}\hat{\mathbf{z}}\,ln\left(\sqrt{s^2+z^2}+z\right)\Big|_0^{\sqrt{(ct)^2-s^2}}$$

$$= \frac{\mu_0 I_0}{2\pi}ln\left(\frac{ct+\sqrt{(ct)^2-s^2}}{s}\right)\hat{\mathbf{z}} \quad \text{(3points)}$$

B. As there are no charges, $V = 0$ (1 point)

The electric field

$$\vec{\mathbf{E}}(s,t) = -\frac{\partial\vec{\mathbf{A}}}{\partial t} = -\frac{\mu_0 I_0}{2\pi}\frac{c+\frac{2ct\,c}{2\sqrt{(ct)^2-s^2}}}{ct+\sqrt{(ct)^2-s^2}}\hat{\mathbf{z}} = -\frac{\mu_0 I_0 c}{2\pi}\frac{\frac{\sqrt{(ct)^2-s^2}+ct}{\sqrt{(ct)^2-s^2}}}{ct+\sqrt{(ct)^2-s^2}}\hat{\mathbf{z}} =$$

$$= -\frac{\mu_0 I_0 c}{2\pi}\frac{1}{\sqrt{(ct)^2-s^2}}\hat{\mathbf{z}} \quad \text{(1 point)}$$

C. The magnetic field $\vec{\mathbf{B}}(s,t) = \vec{\nabla}\times\vec{\mathbf{A}}$. As the vector potential has only z-coordinate and depends explicitly on s only (and not ϕ), all terms of curl in the cylindrical system are zeros except the following one

$$\vec{\mathbf{B}}(s,t) = -\frac{\partial A_z}{\partial s}\hat{\boldsymbol{\phi}} = \quad \text{(1 point)}$$

$$\frac{\mu_0 I_0}{2\pi}\hat{\boldsymbol{\phi}}\frac{s}{ct+\sqrt{(ct)^2-s^2}}\frac{ct+\sqrt{(ct)^2-s^2}-s\frac{-2s}{2\sqrt{(ct)^2-s^2}}}{s^2}$$

$$= \frac{\mu_0 I_0}{2\pi}\hat{\boldsymbol{\phi}}\frac{1}{ct+\sqrt{(ct)^2-s^2}}\frac{ct\sqrt{(ct)^2-s^2}+(ct)^2-s^2+s^2}{s\sqrt{(ct)^2-s^2}}$$

$$= \frac{\mu_0 I_0}{2\pi s}\hat{\boldsymbol{\phi}}\frac{ct}{ct+\sqrt{(ct)^2-s^2}}\frac{\sqrt{(ct)^2-s^2}+ct}{\sqrt{(ct)^2-s^2}}$$

$$= \frac{\mu_0 I_0}{2\pi s}\frac{ct}{\sqrt{(ct)^2-s^2}}\hat{\boldsymbol{\phi}} \quad \text{(2 points)}$$

D. For the observation point close to the wire, the magnetic field is zero at $t < 0$ and is established very quickly as if the current were always on. (1 point)

Hence, the magnetic filed is given by a familiar formula which could be deduced (if you don't remember it) from Ampère's law as

$$B \cdot 2\pi s = \mu_0 I_0; \ \vec{\mathbf{B}}(s,t) = \frac{\mu_0 I_0}{2\pi s}\hat{\boldsymbol{\phi}} \text{ for } t \gtrsim 0 \text{ and } 0 \text{ for } t < 0 \quad (1 \text{ point})$$

As $\vec{\mathbf{B}} = const$ at $t \gtrsim 0$, $\vec{\mathbf{E}} = \mathbf{0}$ (1 point)

However, according Faraday's law $\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$ the discontinuity of the magnetic field at $t = 0$ should lead to an infinitely short burst of an infinitely large amplitude of the electric field (=a derivative of the step function). (1 point)

Let's check up these conclusions:

If you have solved (B) and (C), putting $s \ll ct$ into the electric field expression yields

$$\vec{\mathbf{E}}(s,t) = -\frac{\mu_0 I_0}{2\pi t}\,\hat{\mathbf{z}}$$

i.e. infinite at $t = 0$ and quickly decaying at longer times – exactly as we already deduced.

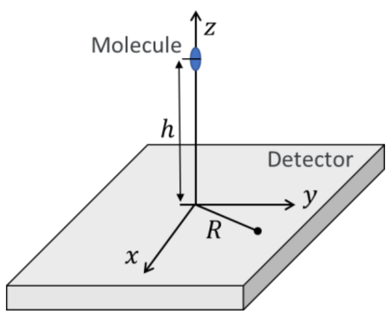
From (C), the magnetic field becomes

$$\vec{\mathbf{B}}(s,t) = \frac{\mu_0 I_0}{2\pi s}\hat{\boldsymbol{\phi}} \text{ for small } s \text{ and } t > 0$$

Again, as we have already deduced from general considerations

<u>Question 4 (10 points)</u>

You would like to study the radiation diagram of a small molecule of a size of ~1 nm which dipole moment oscillates in the z direction. You detect the emitted radiation at the wavelength of $\lambda = 500$ nm with a two-dimensional detector (e.g., a photographic plate or a CCD matrix) which is positioned in the xy plane (see the figure for the experimental arrangement). The distance from the molecule to the detector is $h = 1$ mm.



A. Show that the intensity of the radiation hitting the detector is

$$I(R) = \frac{3\langle P\rangle}{8\pi}\frac{R^2 h}{(R^2 + h^2)^{5/2}}$$

where R is the distance from the projection of the molecule onto the detector, and $\langle P\rangle$ is the total radiated power (measured is a separate experiment). (5 points)

Tip 1. Don't forget that you need to calculate the power per unit area *of the detector*.

Tip 2. As a reminder, the (time-averaged) Poynting vector $\langle\vec{\mathbf{S}}\rangle$ and radiated power $\langle P\rangle$ of a dipole are given as

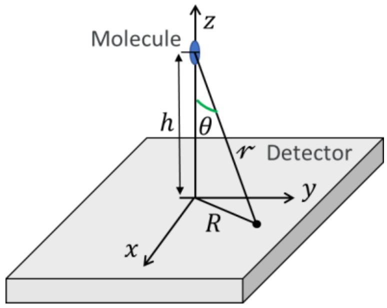
$$\langle\vec{\mathbf{S}}\rangle = \frac{\mu_0 p_0^2\omega^4}{32\pi^2 c}\left(\frac{sin^2\theta}{r^2}\right)\hat{\mathbf{r}};\ \langle P\rangle = \frac{\mu_0 p_0^2\omega^4}{12\pi c}$$

B. At which distance R_{Max} is the intensity maximal? (3 points)

C. Sketch the graph of dependence of detected intensity on the distance R from the projection of the dipole onto the detector. (2 points)

Answer to Question 4 (Chapter 11, Radiation)
(Problems 11.22 &11.23 Griffiths modified)

A. Power per unit area of the detector



$$I = \langle\mathbf{S}\rangle\cdot\hat{\mathbf{z}} = \frac{\mu_0 q^2 d^2\omega^4}{32\pi^2 c}\left(\frac{sin^2\theta}{r^2}\right)cos\theta \text{ (1 point)}$$

$$sin\theta = \frac{R}{r}; cos\theta = \frac{h}{r};\ r^2 = R^2 + h^2 \text{ (2 points)}$$

$$I = \frac{\mu_0 p_0^2\omega^4}{32\pi^2 c}\frac{R^2}{R^2+h^2}\left(\frac{1}{R^2+h^2}\right)\frac{h}{\sqrt{R^2+h^2}}$$

$$= \frac{\mu_0 p_0^2\omega^4}{32\pi^2 c}\frac{R^2 h}{(R^2+h^2)^{5/2}} \text{ (1 point)}$$

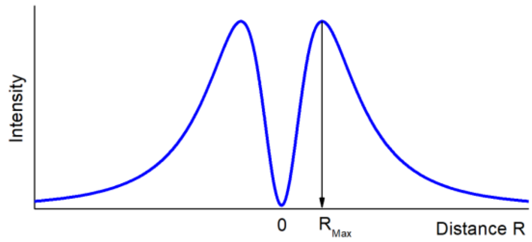
$$\langle P\rangle = \frac{\mu_0 p_0^2\omega^4}{12\pi c} \text{ so that } I = \frac{3\langle P\rangle}{8\pi}\frac{R^2 h}{(R^2+h^2)^{5/2}} \text{ (1 point)}$$

B. $$\frac{dI}{dR} = \frac{3\langle P\rangle h}{8\pi}\frac{2R(R^2+h^2)^{5/2} - 5/2\,(R^2+h^2)^{3/2}2RR^2}{(R^2+h^2)^5} = 0 \text{ (1 point)}$$

$$R_{Max}^2 + h^2 - 5/2\,R_{Max}^2 = 0;\ 3/2\,R_{Max}^2 = h^2 \text{ (1 point)}$$

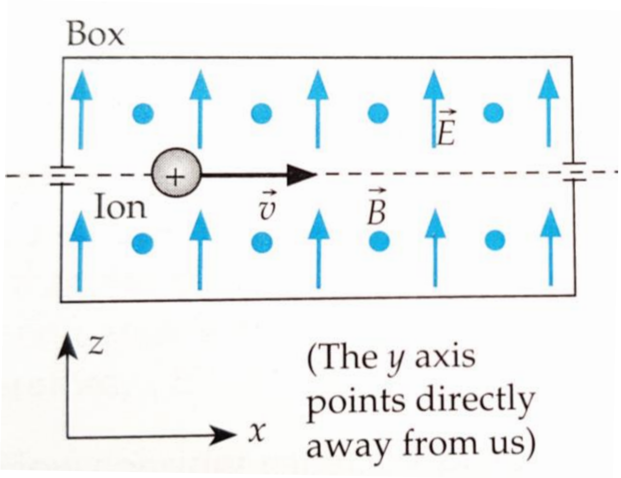
$$R_{Max} = h\sqrt{2/3} \text{ (1 point)}$$

C. (2 points)



Question 5 (10 points)

The figure shows a schematic diagram of velocity selector, a device that allows a charged particle to pass through two coaxial (i.e. laying at the same axis) openings only if it has a specific velocity v which vector is also coaxial with the two openings. In the laboratory (box) system of coordinates, electric and magnetic fields are uniform inside the box and point into $+z$ and $-y$ directions, respectively. An ion with positive charge moves through the box in the +x direction with speed v. Gravity can be neglected.



A. Find electric and magnetic fields in the system of coordinates moving together with the charge. (4 points)

B. In the charge system of coordinates (i.e. if the observer sits on the ion), what are the components (electric or/and magnetic) of the Lorentz force that are exerted upon the charge? (1 points)

C. In the system of coordinates that moves to the right with the speed of v, what are requirements to the electric and magnetic fields to ensure the *straightforward* movement of the charge? (2 points)

D. Now we are back to the laboratory system. What are the components (electric or/and magnetic) of the Lorentz force that are exerted upon the charge? (1 point)

E. In the laboratory system, what are requirements to the electric and magnetic fields to ensure the *straightforward* movement of the charge? Explain why your answer makes sense. (2 points)

Tip: you might find these transformations of the fields useful:

$$\bar{E}_x = E_x; \bar{E}_y = \gamma(E_y - \mathrm{v}B_z); \quad \bar{E}_z = \gamma(E_z + \mathrm{v}B_y)$$

$$\bar{B}_x = B_x; \ \bar{B}_y = \gamma\left(B_y + \frac{\mathrm{v}}{c^2}E_z\right); \ \bar{B}_z = \gamma\left(B_z - \frac{\mathrm{v}}{c^2}E_y\right)$$

Answers to Question 5

A. $\vec{\mathbf{E}} = (0,0,E_z)$; $\vec{\mathbf{B}} = (0,-B_y,0)$

$$\bar{E}_x = 0_x; \ \bar{E}_y = \gamma(0_y - \mathrm{v}0_z); \quad \bar{E}_z = \gamma(E_z - \mathrm{v}B_y)$$

$$\bar{B}_x = 0_x; \ \bar{B}_y = \gamma\left(-B_y + \frac{\mathrm{v}}{c^2}E_z\right); \ \bar{B}_z = \gamma\left(B_z - \frac{\mathrm{v}}{c^2}0_y\right)$$

$$\vec{\bar{\mathbf{E}}} = \left(0,0,\gamma(E_z - \mathrm{v}B_y)\right); \ \vec{\bar{\mathbf{B}}} = \left(0,-\gamma B_y + \gamma\frac{\mathrm{v}}{c^2}E_z,0\right)$$

(4 points in total)

B. $\vec{\bar{\mathbf{F}}} = q\left(\vec{\bar{\mathbf{E}}} + \vec{\bar{\mathbf{v}}} \times \vec{\bar{\mathbf{B}}}\right)$

In the charge system, velocity is equal to zero $\vec{\mathbf{v}} = 0$ so that only electric field component is exerted. (1 point)

C. For the straightforward movement wrt that system coordinate, the total force must be equal zero so that

$\vec{\mathbf{E}} = \mathbf{0}$, or $E_z - \mathrm{v}B_y = 0$, or $E_z = \mathrm{v}B_y$ (2 points)

D. $\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$ Both components are exerted (1 point)

E. The total force must be equal zero so that $E_z = \mathrm{v}B_y$ (1 point)

Despite different forces exerted on the charge for two observers, they come to the same conclusion. (1 point)