1. Toodud tingimustel saab trajektoori võrrand kuju

$$\frac{d^2u}{d\theta^2} + (1 - \frac{\gamma}{L^2})u = 0.$$

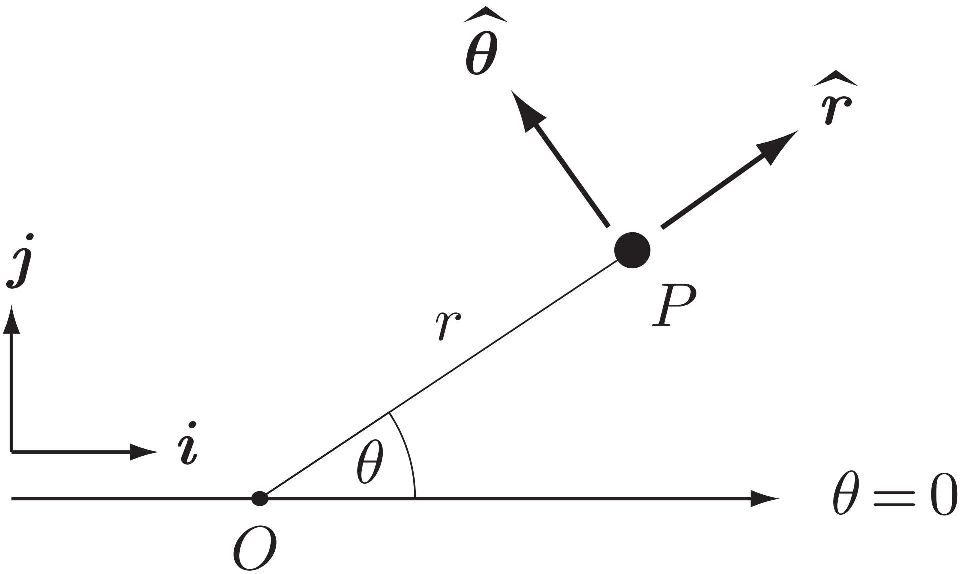
See on lineaarne homogeenne diferentsiaalvõrrand, mille üldlahendiks on

$$u = A\cos(k\theta) + B\sin(k\theta),\ k \equiv \sqrt{(1 - \frac{\gamma}{L^2})} > 0.$$

Algtingimuste fikseerimine

$$\dot{r} = \frac{d}{dt}(\frac{1}{u}) = -\frac{1}{u^2}\frac{du}{d\theta}\dot{\theta} = -r^2\dot{\theta}\frac{du}{d\theta} = -\frac{L}{m}\cdot\frac{du}{d\theta}$$

(vt neljandat loengufaili). Kui $r \to \infty$, siis $\theta \to 0$, millest saame algtingimuse $u(0) = 0$. Järelikult $A = 0$.



Meenutame kiirusvektori üldavaldist polaarkoordinaatides:

$$\overrightarrow{v} = \frac{d\overrightarrow{r}}{dt} = \dot{r}\widehat{r} + \left(r\dot{\theta}\right)\widehat{\theta}.$$

Selle avaldise mõlemad pooled sõltuvad polaarnurgast θ, kusjuures

$$\lim_{\theta\to 0}\overrightarrow{v} = -v\vec{i}\,\text{(miks?)},\ \lim_{\theta\to 0}\widehat{r} = \vec{i}\ \text{ja}\ \lim_{\theta\to 0}\left(r\dot{\theta}\right) = \lim_{r\to\infty}\frac{L}{mr} = 0.$$

Järelikult $\dot{r}(0) = -v$ ja seega, kuna $L = mvb$,

$$-v = -\frac{L}{m}\cdot\frac{du(\theta)}{d\theta}\bigg|_{\theta\to 0} = -vb\cdot\frac{du(0)}{d\theta} \to \frac{du(0)}{d\theta} = \frac{1}{b}.$$

Seega saame fikseerida konstandi B:

$$\frac{du(\theta)}{d\theta} = Bk\cos(k\theta) \to B = \frac{1}{kb},$$

nii et trajektoori võrrandiks on

$$u(\theta) = \frac{\sin(k\theta)}{kb} \to r(\theta) = \frac{kb}{\sin(k\theta)}. \tag{1}$$