Test 1: Mechanics and Relativity

Tuesday, September 21, 2021, Aletta Jacobshal

Duration: 1 hour

Before you start, read the following:
There are 3 problems, for a total of 90 points.
Write your name and student number on each sheet of paper.
Make clear arguments and derivations and use correct notation.
Support your arguments by clear drawings where appropriate.
Write in a readable manner, illegible handwriting will not be graded.
Draw your spacetime diagrams on the provided graph paper and put them inside your written solutions when handing in.

Name: .....
Student Number: S - .....

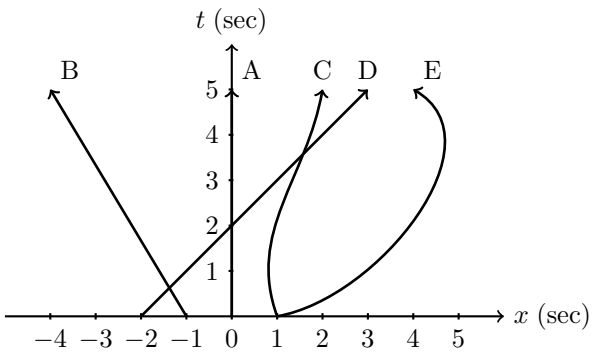
	Points
Problem 1:	
Problem 2:	
Problem 3:	
Total:	
GRADE (1 + # Total/10)	

Name: Student Number:

Problem 1: Basics (25 pt)

Write **TRUE** if the statement is true; write **FALSE** if the statement is false.
If you don't know the answer, **LEAVE THE ANSWER BLANK!** Points are deducted for wrong answers.

1. _____ (2 pt) Newton's laws are invariant under the Maxwell transformations.
2. _____ (2 pt) An object must have the same kinetic energy in all inertial frames.
3. _____ (2 pt) An object must have the same gravitational potential energy in all inertial frames. (Hint: Recall E=mgh)
4. _____ (2 pt) In Newtonian relativity, observers in both inertial frames agree about an object's acceleration at a given time.
5. _____ (2 pt) Suppose there are two reference frames designated by S and S$'$ such that the co-ordinate axes are parallel to each other. In S, we have the co-ordinates $\{t, x, y, z\}$ and in S$'$ we have the co-ordinates $\{t', x', y', z'\}$. S$'$ is moving with respect to S with velocity $\vec{\beta}$ in the positive x direction. The Galilean transformations which relate the primed and unprimed coordinates are given as
$$\begin{aligned} t' &= t\,, \\ x' &= x + |\vec{\beta}|\, t\,, \\ y' &= y\,, \\ z' &= z\,. \end{aligned}$$
6. _____ (2 pt) According to the conventions established in this course, the slope of the worldline for a particle represents its speed on a spacetime diagram.
7. _____ (2 pt) The distances and time intervals are invariant under the Galilean transformations.
8. _____ (2 pt) X-rays travel faster than microwaves.
9. _____ (2 pt) Sunlight reflected from the Moon's surface takes around 1.3 seconds to travel to the Earth's surface. Hence, the sunlight travelled roughly 350,000 to 400,000 km.
10. _____ (2 pt) A dog named Laika runs with a constant speed of 10km/hr relative to a spaceship for an hour. The speed of the spaceship is 36,000 km/hr with respect to observer Bob. According to Bob, Laika travels approximately 12 milliseconds in SR units.
11. The spacetime diagram below shows the worldlines of various objects.
    _____ (1 pt) object E is the only one accelerating
    _____ (1 pt) object D could be light
    _____ (1 pt) in a frame attached to object A objects C, D and E always move in the positive x direction
    _____ (1 pt) object B has the largest speed at time $t = 4s$
    _____ (1 pt) except for object A, all objects are always in motion



Problem 2: Space Kidnapping (37 pt)

An alien spaceship, *Pandik*, is located at $(t, x) = (0 \text{ min}, 4 \text{ min})$ and heads for Earth at a constant velocity of 2/3. (Presume Earth is at the spatial origin of the coordinate system, $x = 0$.) The space-defense radar system (SDRS), located on Earth, sends out a radar signal at $t = 1$ min (event A), which is reflected on *Pandik* and received by the SDRS at some later time (event B).

(a) Construct a quantitatively accurate[1] spacetime diagram for this scenario.[2] Draw and label the worldlines for Earth and *Pandik*. For the latter, draw the worldline until it reaches Earth. (7 pt)

(b) Draw and label the worldline of the radar signal and events A and B in the spacetime diagram. Determine the time that SDRS will find out about *Pandik*. Does this occur before or after *Pandik* reaches Earth? (7 pt)

Pandik arrives on Earth and kidnaps Tion Miscom, the director of SDRS. *Pandik* then leaves Earth at a constant velocity in the direction it came from. Presume the arrival, kidnapping, and departure are a single event (event C). *Pandik*'s departing worldline crosses at $(t, x) = (11 \text{ min}, 1 \text{ min})$, which is when *Pandik*'s steering wheel breaks and cannot change trajectory anymore (Event D).

(c) Algebraically determine the speed of *Pandik* after it leaves Earth. (7 pt)

(d) Draw the departing worldline of *Pandik* and label events C and D. Do this in the same spacetime diagram you created for (a) and (b). (4 pt)

Freight ship *Polluter* was sent to intercept *Pandik*. *Polluter* moves at a constant speed of 1/2 towards Earth and was located at $x = 5$ min when Tion Miscom was kidnapped. However, the *Polluter* cannot come closer than 2.5 minutes to Earth (regulation).

(e) Determine the latest moment in time that *Polluter* can brake in order to intersect with *Pandik* and comply with the regulation. Presume braking is instantaneous and consider it as a single event. (6 pt)

(f) Presuming it brakes at the latest possible moment, algebraically determine the moment in time that *Polluter* intercepts *Pandik*. (6 pt)

Problem 3: Relativity of Simultaneity (28 pt)

A spaceship hovers over observer M on Mars. Treat both Mars and the spaceship as separate inertial frames for this problem. The speed of the spaceship is v. When the spaceship is overhead, a light signal is emitted from observer C (at the center of the spaceship). Subsequently it is detected by observer A (at point A in the front of the spaceship) and observer B (at point B in the rear of the spaceship). Both observers measure their distance from the center of the spaceship to be d'.

(a) Draw a spacetime diagram of the process as observed by observer C. Identify and label all events. (8 pt)

(b) Explain why observers A and B agree that the light signal reaches them simultaneously. Please write your response in 5 sentences or less. (8 pt)

(c) According to observer M, the light signal reaches observer B before observer A. The time difference between the two events is Δt. Moreover, observer M measures the distance from the center of the spaceship to be d, where $d < d'$. Taking these events into account, draw a spacetime diagram of the process as observed by observer M; include the worldline of observer C. Identify and label all events. (8 pt)

(d) If the distance d is in SR units, what is this distance in SI units? (4 pt)

[1] All slopes, coordinates, and labels should be numerically correct, no sketch!

[2] Hint: To not run out of space, put the origin of your spacetime diagram in the lower left corner of the graph paper.

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