1 KT3 Taavi Tammaru

proovime minimeerida massikeskme kõrgust funktsionaali mis võtab argumendiks ahela kuju funktsiooni.

paneme tähele, et suurus nimetajas on ahela kogumass ja seejuures konstant. ehk proovime minimeerida ainult lugejat.

$$Y[y(x)] = \rho \int_{x_1}^{x_2} y(x)\sqrt{1+y'(x)^2}\,dx$$

kasutame Beltrami samasust

$$F - y'(x)\frac{\partial F}{\partial y'} = C$$

$$y(x)\sqrt{1+y'(x)^2} - \frac{y'(x)^2 y(x)}{\sqrt{1+y'(x)^2}} = C$$

$$\frac{y(x)(1+y'(x)^2) - y'(x)^2 y(x)}{\sqrt{1+y'(x)^2}} = \frac{y(x)}{\sqrt{1+y'(x)^2}} = C$$

$$y(x) = c\sqrt{1+y'(x)^2} \quad \Big| \frac{d^2}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx} c\sqrt{1+y'(x)^2}$$

$$\frac{d^2y}{dx^2} = \frac{1}{c}\sqrt{1+\left(\frac{dx}{dy}\right)^2}$$

kasutades Euler-Lagrange võrrandit

$$\frac{\partial F}{\partial y} = \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)$$

$$\sqrt{1-y'(x)^2} = \frac{d}{dx}\left(\frac{y(x)'\, y(x)}{\sqrt{1-y'(x)^2}}\right)$$

bel ttami samaaegselt teame et

$$k = \frac{y}{\sqrt{1-y'(x)^2}}$$

$$\sqrt{1-y'(x)^2} = \frac{d}{dx}\left(y(x)'\, k\right)$$

$$\sqrt{1-y'(x)^2} = y''(x)\, k$$

$$\frac{d^2y}{dx^2} = \frac{1}{k}\sqrt{1-\frac{dy}{dx}^2}$$