Name: Student Number:

Mechanics and Relativity: Resit R

February 14, 2023, Aletta Jacobshal

Duration: 120 mins

Before you start, read the following:

- There are 2 problems, for a total of 31 points.
- In question 1 there are 3 sub-questions (h,i and j); you have to make only one of these to get full score
- In question 2 there are 2 sub-questions (g and h); you have to make only one of these to get full score
- You can attempt all optional questions, but your finale score can never exceed a 10; if you have time to make more than 1 optional question it will only be graded / count if you have finished all non-optional questions
- Write your name and student number on all sheets.
- Make clear arguments and derivations and use correct notation. *Derive* means to start from first principles, and show all intermediate (mathematical) steps you used to get to your answer!
- Support your arguments by clear drawings where appropriate. Draw your spacetime diagrams on the provided hyperbolic paper.
- Write your answers in the boxes provided. If you need more space, use the lined drafting paper.
- Generally use drafting paper for scratch work. Don't hand this in unless you ran out of space in the answer boxes.
- Write in a readable manner, illegible handwriting will not be graded.

	Points
Problem 1:	16
Problem 2:	15
Total:	31
GRADE (1 + # Total/(31/9))	

Useful equations:

$$\Delta s^2 = \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

$$\Delta t \geq \Delta s \geq \Delta \tau$$

The Lorentz transformation equations with $\gamma \equiv (1-\beta^2)^{-1/2}$:

$$t' = \gamma(t - \beta\, x) \qquad x' = \gamma(x - \beta t) \qquad y' = y \qquad z' = z\,. \tag{1}$$

The relativistic Doppler shift formula and Einstein velocity transformations:

$$\frac{\lambda_R}{\lambda_E} = \sqrt{\frac{1+v_x}{1-v_x}} \quad v'_x = \frac{v_x - \beta}{1-\beta v_x} \quad v'_{y,z} = \gamma^{-1}\frac{v_{y,z}}{1-\beta v_x}\,.$$

Possibly relevant equations:

$$F = G\frac{Mm}{r^2}; \qquad F = ma; \qquad PV \propto k_b T; \qquad F = \frac{dp}{dt}$$

Possibly relevant numbers:

$$h = 6.626\times10^{-34}\ \mathrm{m^2\ kg\ s^{-1}} \qquad c = 299792458\ \mathrm{m/s} \qquad 1\ \mathrm{eV} = 1.602176565\times10^{-19}\ \mathrm{J}$$

$$M_{\rm sun} = 1.989\times10^{30}\ \mathrm{kg} \qquad L_{\rm sun} = 3.83\times10^{26}\ \mathrm{kg\ m^2\ s^{-3}} \qquad M_{\rm gal} \sim \mathcal{O}(10^9 M_{\rm sun})$$

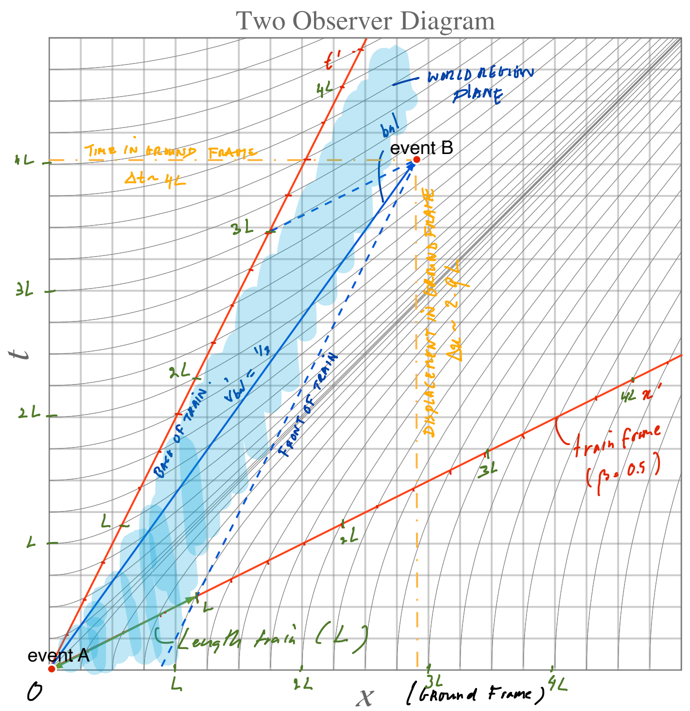
Question 1: In flight entertainment (16 pts)

A plane of proper length L is moving with respect to the ground with a speed $v_{\text{plane}} = 1/2$. Inside the back of the plane, some clown throws a ball towards the front of the plane with a speed of $v'_{\text{ball}} = 1/3$. For clarity, adopt unprimed coordinates (x) for the ground frame, primed coordinates (x') for the frame comoving with the plane and double primed (x'') coordinates for observers comoving with the ball.

(a) **(5 pts)** Draw a space-time diagram including the following:

- The spacetime coordinates of the ground frame
- The spacetime coordinates of the frame comoving with the plane
- The worldline of the ball
- The worldregion of the plane

Label the axis in units of L, the proper length of the plane. Assume the ground frame is the HOME frame (as defined in Moore) and the frame comoving with the plane as the OTHER frame. Assume the back of the plane is located at the origin at time $t = t' = 0$. Label the spacetime coordinate event when the clown throws the ball as **event** A and the spacetime coordinate event when it reaches the front of the plane as **event** B. Use the two observer diagram paper provided at the end of this exam.



(0.5 pt): ground frame axis
(0.5 pt): plane frame axis
(1 pt): worldline ball
(1 pt): world region plane
(1 pt): ticks using L
(1 (0.5 each) pt): labeling events A and B

(b) **(1 pt)** Based on your space-time diagram, as observed from the ground (HOME) frame, read off

- the time it takes for the ball to reach the front of the plane in units of L, i.e. the time elapsed between events A and B as measured on a clock at rest
- the distance the ball covers in that time in units of L, i.e. the distance between events A and B as measured on a ruler at rest

If your space-time diagram is correct, you should be able to obtain the answer with $\sim 10\%$ accuracy.

From the diagram we read off the time interval measured in the ground frame to be $\Delta t_{\text{ground}} \simeq 4L$ **(0.5 pts)**. Similarly we find $\Delta x_{\text{ground}} \sim 2.9L$ **(0.5 pts)**.

(c) **(1 pt)** Answer the same question but now for the frame comoving with the plane (OTHER).

The distance the ball covers is obviously just the length of the plane L as observed in the frame comoving with the plane **(0.5 pts)**. The time it takes can be read off, but also simply computed as **(0.5 pts)**

$$\Delta t'_{\text{plane}} = L/v'_{\text{ball}} = 3L. \tag{2}$$

(d) **(3 pts)** To check your answer of question (b) (HOME frame), compute the distance and time covered using velocity transformations.

We first need to obtain $v_{\rm ball}$ (the speed of the ball in the HOME frame). From the cover page we have the Einstein velocity transformations. However, we should replace the $-$ with $+$ since we go from OTHER to HOME **(0.5 pts)**. We have $v'_{\rm ball} = 1/3$ and $\beta = 0.5$

$$v_{\rm ball} = \frac{v'_{\rm ball} + \beta}{1 + \beta v'_{\rm ball}} = \frac{1/3 + 1/2}{1 + 1/6} = \frac{5}{7} \, (\mathbf{0.5 \ pts}). \tag{3}$$

Next, we need to obtain the length of the plane as observed from the ground. As observed from the ground, the plane should be contracted via **(0.5 pts)**

$$L_{\rm ground} = L/\gamma_{1/2} = L\sqrt{1-\beta^2} = L\sqrt{1-(1/2)^2} = \sqrt{3}L/2. \tag{4}$$

Since the plane is moving with a speed 1/2, as the ball moves towards the front of the plane, the front of the plane is also moving forward ($v_{\rm plane} \equiv \beta = 0.5$) as observed in the ground frame. We should therefore account for this by noting that the relative speed of the ball with respect to the front of the plane is given by $v_{\rm rel} = v_{\rm ball} - v_{\rm plane} = 5/7 - 1/2 = 3/14$ **(0.5 pts)**. The time it takes for the ball to reach the front of the plane as observed from the ground is therefore **(0.5 pts)**:

$$\Delta t_{\rm ground} = L_{\rm ground}/v_{\rm rel} = \sqrt{3}L/2 * 14/3 = 7L/\sqrt{3} \simeq 4L. \tag{5}$$

This is exactly what we found from reading this off in the spacetime diagram. The distance can be obtained by multiplying the time with the speed of the ball **(0.5 pts)**:

$$\Delta x_{\rm ground} = v_{\rm ball} * \Delta t_{\rm ground} = 5/7 * 7L/\sqrt{3} = 5L/\sqrt{3} \simeq 2.9L. \tag{6}$$

Again this is consistent with what we read off directly from the diagram.

(e) **(1.5 pts)** Now try to obtain the answer of question (*b*) and (*d*) by using the Lorentz transformation to go from the frame comoving with the plane to the ground frame.

The Lorentz transformations are given on the cover page, but since we go from the OTHER frame to the HOME frame we should replace $\beta \to -\beta$ **(0.5 pts)**. From question (*c*) we got $\Delta t'_{\rm plane} = 3L$ and $\Delta x'_{\rm plane} = L$. Applying the Lorentz transformations thus gives **(1 pt, 0.5 each)**:

$$\Delta t_{\rm ground} = \gamma_{1/2}(\Delta t'_{\rm plane} + \beta \Delta x'_{\rm plane}) = \frac{2}{\sqrt{3}}(3L + L/2) = 7L/\sqrt{3} \tag{7}$$

$$\Delta x_{\rm ground} = \gamma_{1/2}(\Delta x'_{\rm plane} + \beta \Delta t'_{\rm plane}) = \frac{2}{\sqrt{3}}(L + 3L/2) = 5L/\sqrt{3}. \tag{8}$$

This agrees with our answers in (*b*) and (*d*).

(f) **(2 pts)** What is the distance and time between A and B for a ruler and a clock comoving with the ball?

In a frame comoving with the ball, no distance is covered, since in that frame the ball is at rest, i.e. $\Delta x''_{\rm ball} = 0$ **(0.5 pts)**. The time passed can be computed by the observed length of the plane as viewed from the frame comoving with the ball, and the speed at which the plane is passing the ball. The first is obtained by applying a length contraction, i.e. $L'' = L/\gamma_{1/3} = 2\sqrt{2}L/3$ **(0.5 pts)**. The speed at which the plane is passing by is the same as the speed at which the ball is moving through the plane, as observed from the frame comoving with the plane, i.e. $v''_{\rm plane} = v'_{\rm ball} = 1/3$ **(0.5 pts)**. Thus the time to reach the end of the plane is given by **(0.5 pts)**:

$$\Delta t''_{\rm ball} = L''/v''_{\rm plane} = 2\sqrt{2}L. \tag{9}$$

(g) **(1.5 pts)** Confirm that the spacetime interval between events A and B, Δs^2, is frame independent (i.e. is the same in the ball, plane and ground frames).

This is just a matter of putting in the derived space and time intervals **(1.5 pts, 0.5 each)**:

$$\Delta s''_{\rm ball}{}^2 = \Delta t''_{\rm ball}{}^2 - \Delta x''_{\rm ball}{}^2 = (2\sqrt{2}L)^2 - 0 = 8L^2, \tag{10}$$

$$\Delta s'_{\rm plane}{}^2 = \Delta t'_{\rm plane}{}^2 - \Delta x'_{\rm plane}{}^2 = (3L)^2 - L^2 = 8L^2, \tag{11}$$

$$\Delta s^2_{\rm ground} = \Delta t^2_{\rm ground} - \Delta x^2_{\rm ground} = (7L/\sqrt{3})^2 - (5L\sqrt{3})^2 = (49-25)L^2/3 = 8L^2. \tag{12}$$

So that seems to be the case!

(h) **(1 pt, option 1)** Show that the time interval between events A and B as measured from the ground is related to the time interval observed from the ball with the appropriate Lorentz factor γ.

From question (b) we know that $v_{\rm ball} = 5/7$. We compute $\gamma_{5/7}$ **(0.5 pts)**:

$$\gamma_{5/7} = \frac{1}{\sqrt{1-(5/7)^2}} = \frac{7}{2\sqrt{6}}. \tag{13}$$

The time related should be given by $\Delta t_{\rm ground} = \Delta t''_{\rm ball}\gamma_{5/7}$ **(0.5 pts)**:

$$\Delta t_{\rm ground} = 7L/\sqrt{3} = \Delta t''_{\rm ball}\gamma_{5/7} = 2\sqrt{2}L\frac{7}{2\sqrt{6}} = 7L/\sqrt{3}. \tag{14}$$

So again this makes sense.

(i) **(1 pt, option 2)** Show that the time interval between events A and B as measured from the plane is also related to the time interval observed from the ball with the appropriate Lorentz factor γ

We know that $v'_{\text{ball}} = 1/3$. We compute $\gamma_{1/3}$ **(0.5 pts)**:

$$\gamma_{1/3} = \frac{1}{\sqrt{1-(1/3)^2}} = \frac{3}{2\sqrt{2}}. \tag{15}$$

The time related should be given by $\Delta t'_{\text{plane}} = \Delta t''_{\text{ball}}\gamma_{1/3}$ **(0.5 pts)**:

$$\Delta t'_{\text{plane}} = 3L = \Delta t''_{\text{ball}}\gamma_{1/3} = 2\sqrt{2}L\frac{3}{2\sqrt{2}} = 3L. \tag{16}$$

Makes sense again.

(j) **(1 pt, option 3)** Finally, is the time interval between events A and B in the ground frame and in the frame comoving with the place related by a Lorentz factor? Why or why not? Explain your answer.

If we follow the same steps as before we should take $\gamma_{1/2}$ **(0.5 pts)**:

$$\gamma_{1/2} = \frac{1}{\sqrt{1-(1/2)^2}} = \frac{2}{\sqrt{3}}. \tag{17}$$

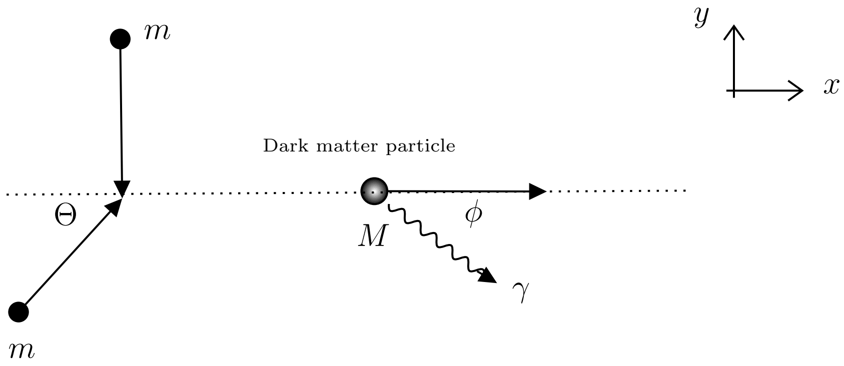
Then by logic of the previous question, we compute **(0.5 pts), including explanation, otherwise partial points**

$$\Delta t_{\text{ground}} = 7L/\sqrt{3} \neq \Delta t'_{\text{plane}}\gamma_{1/2} = 3L\frac{2}{\sqrt{3}} = 2\sqrt{3}L. \tag{18}$$

The times measured are NOT related by a Lorentz factor. The reason is that the time dilation factor we obtain which relates different time intervals in two frames, is only true if the event in at least one of the frames is at the same place, i.e. in this instance, in the frame comoving with the ball we have $\Delta x''_{\text{ball}} = 0$, hence when applying the Lorentz transformation rules it is evident you relate the time intervals simply by a factor of γ. For the frame comoving with the plane and the ground frame, the space interval is not 0 (the ball is moving w.r.t. the plane and w.r.t. the ground).

Question 2: Dark matter creation (15 pts)

In this question we will look into the hypothetical process of creating a dark matter particle via the annihilation of two standard matter particles. In the process, a dark matter (DM) particle and a photon are created as shown below



Here γ presents the photon (not to be confused with the Lorentz factor), M is the mass of the created dark matter particle, the two standard model particles both have mass m. One of the particles moves along the vertical axis (y) and the other particle is moving with an angle Θ w.r.t. the horizontal axis (x). The produced DM particle moves along the horizontal axis. The photon moves with some angle ϕ w.r.t. to the horizontal axis. **We assume that the incoming particles move at the same speed and that $0 \leq \Theta, \phi \leq \pi/2$.**

(a) **(3 pts)** Write down the components of the four vectors of the particles (including the photon). You should write the components in terms of energy E and 3-momentum $\vec{p}$. Label these components and make sure you explain what the labels represent. For example, the produced DM particle has four-vector:

$$\mathbf{p}_M = (E_M, p_M, 0, 0). \tag{19}$$

Here E_M is the energy of the DM particle and p_M is the DM particle momentum in the $+x$ direction. You can use 'box' notation for the four-vectors, i.e. $\mathbf{p} = \boxed{p}$.

We write **(3 pts, 1 for each, this includes explanation of labeling below)**

$$\mathbf{p}_1 = (E, 0, -p, 0) \tag{20}$$
$$\mathbf{p}_2 = (E, p\cos\Theta, p\sin\Theta, 0) \tag{21}$$
$$\mathbf{p}_\gamma = (E_\gamma, E_\gamma\cos\phi, -E_\gamma\sin\phi, 0) \tag{22}$$

Here we labeled the particle of mass m coming in along the vertical axis as particle (1) and the other incoming particle as particle (2). First, since we assume that the particles are moving at the same speed, their energies and the magnitude of their 3-momenta should be the same. We call these $E_1 = E_2 = E$ and $|\vec{p}_1| = |\vec{p}_2| = p$ Since the first particle is moving 'down', the momentum component should be negative (recall, these are vector components and they should tell us what the arrow head is doing). We have projected the x and y components using the cos and sin. The outgoing photon has an opposite direction to the incoming standard model particle in the y direction and therefore should have a $-$ sign for the y component. Because a photon is massless ($m^2 = \sqrt{E^2 - \vec{p}^2} = 0$) we have that $|p| = E$. Hence we can write the momentum components in terms of the energy of the photon E_γ.

(b) **(1.5 pts)** Using the conservation of 4-momentum find one expression for p_M (1) and another one for E_M (2) in terms of the 3-momentum amplitude and energy E of the incoming particles p, the energy of the photon E_γ, Θ and ϕ. Find a third (3) equation relating p and Θ to E_γ and ϕ.

From the coservation of energy we find **(0.5 pts)**:

$$2E = E_M + E_\gamma \to E_M = 2E - E_\gamma. \tag{23}$$

From conservation of momentum in the x direction we obtain **(0.5 pts)**:

$$p\cos\Theta = p_M + E_\gamma\cos\phi \to p_M = p\cos\Theta - E_\gamma\cos\phi. \tag{24}$$

Finally, from conservation of momentum in the y direction we find **(0.5 pts)**

$$p\sin\Theta - p = -E_\gamma\sin\phi \to p(1 - \sin\Theta) = E_\gamma\sin\phi. \tag{25}$$

(c) **(1.5 pts)** Now consider $\Theta = \pi/2$. From the derived equations, explain the two possible scenarios that can happen.

From the last equation above, when $\Theta = \pi/2 \to E_\gamma \sin\phi = 0$ **(0.5 pts)**. We can then consider two cases. Either $E_\gamma = 0$ **(0.5 pts)** (no photon is produced) or $\phi = 0$. In the second scenario we find $p_M = -E_\gamma \cos\phi = -E_\gamma$, i.e. the produced DM particle will have to move to the left. The reason is that $\Theta = \pi/2$ implies the two annihilated particle have net zero momentum in the x direction. For any momentum of the resulting DM particle in the x direction, we need to move the particle to the left (since by def of the angle ϕ the photon moves to the right, or straight down)**(0.5 pts)**.

(d) **(1.5 pts)** Next consider $\Theta = 0$. Explain what happens when either $\phi = 0$ or $\phi = \pi/2$.

If $\Theta = 0$ we find that $-p = E_\gamma \sin\phi$ and $p_M = p - E_\gamma \cos\phi$ **(0.5 pts)**. Let us consider the two cases. First, if $\phi = 0 \to p = 0$. The annihilating particles are at rest (no momentum) because the photon and DM particle only have momentum in the x direction (in other words, no momentum in the y direction is permitted and hance p has to be zero). For the produced photon and DM particle to have momentum in the x direction, we thus find $p_M = -E_\gamma$, i.e. the move in opposite directions (Note that if the students say this is not allowed because the DM particle moves to the right in the figure, this should be considered correct) **(0.5 pts)**. The second case when $\phi = \pi/2$ implies that $p = E_\gamma$ and $p_M = p$. In other words, all particles would have the same net momentum $|\vec{p}|$ **(0.5 pts)**.

(e) **(1.5 pts)** When does the resulting DM particle have the largest mass? Explain your answer. Show that this mass is given by

$$M = 2\gamma_m m, \tag{26}$$

where γ_m is the Lorentz factor of the two annihilating particles.

Intuitively, the largest mass would be generated if there is no mass conversion into any kinetic energy, i.e. the DM particle should not move and there should be no generation of a photon **(0.5 pts)**. In addition, the two annihilating particles should maximally convert their kinetic energy into mass, i.e. since the first particle is moving along the y direction, so should the second particle to have exact momentum cancellation **(0.25 pts)**. In this case there is no net x or y momentum and energy conservation implies $E_M = 2E = 2\gamma_m m$ **(0.25 pts)**. Since the DM is not moving, its Lorentz factor is 1, and we find that $M = 2\gamma_m m$ **(0.5 pts)**.

(f) **(2.5 pts)** Using dimensional analysis, derive a relation between the energy E_γ, Planck's constant h and the frequency ν of a photon . Use L, M and T as fundamental dimensions. Use information provided on the cover page.

From the cover page $[h] = ML^2T^{-1}$ **(0.2 pts)** and frequency $[\nu] = T^{-1}$ **(0.2 pts)**. The energy (from for example $E = \frac{1}{2}mv^2$) has dimensions $[E] = ML^2T{-}2$ **(0.3 pts)**. We can thus write the following relation **(0.5 pts)**

$$\begin{aligned} E &= h^\alpha \nu^\beta && (27) \\ ML^2T{-}2 &= (ML^2T^{-1})^\alpha (T^{-1})^\beta. && (28) \end{aligned}$$

This implies **(0.6 pts, 0.2 for each)**:

$$\begin{aligned} M &: \quad 1 = \alpha && (29) \\ L &: \quad 2 = 2\alpha && (30) \\ T &: \quad -2 = -\alpha - \beta && (31) \end{aligned}$$

We thus find $\alpha = 1$ **(0.2 pts)** and $\beta = 1$ **(0.2 pts)**, and the relation is give by **(0.3 pts)**

$$E = h\nu. \tag{32}$$

(g) **(3.5 pts, option 1)** In our universe, the ratio of DM to ordinary matter is roughly 5 to 1 (i.e. there is about 5 times as much DM as there is ordinary matter). In the center of our galaxy, processes occur that accelerate standard model particles to 0.9998(c). Observations of the center galaxy show an excess of gamma-ray radiation with a mean frequency of $\nu \sim 10^{20}$ Hz. Assuming that the annihilating particles have a mass of ~ 1 GeV, guesstimate the mass of the produced DM particle.

We know that $2E = E_M + E_\gamma$ **(0.2 pts)**. First we need to estimate the energy of the photon in eV. This can done via the conversion factor on the cover page **(0.3 pts)**:

$$E_\gamma = h\nu \simeq 6 \times^{-34} \times 10^{20} \text{ Joules} \tag{33}$$

We divide the LHS and RHS by 1 eV in Joules, i.e. **(0.3 pts)**

$$\frac{E}{1 \text{ J/eV}} = \frac{6 \times 10^{-14}}{1.6 \times 10^{-19}} \simeq 3 \times 10^6 \text{eV} = 3 \text{ MeV} \tag{34}$$

In other words, the energy of this photon is negligible compared to the energy of the annihilating particles **(0.2 pts)**. All energy will therefore be converted into the mass energy of the DM particle, i.e. $2E \simeq E_M$. We then use $E = \gamma m$ to find **(0.4 pts)**:

$$M = \frac{2\gamma_m m}{\gamma_M} \simeq 100 \text{ GeV}/\gamma_M \tag{35}$$

Here we used that $\gamma_{0.9998} \simeq 50$ **(0.3 pts)**. Since the produced photons have minimal energy compared to the energy of DM and annihilating particles, this means that to have a net momentum of the DM only in the x direction, requires that $\Theta \sim \pi/2$ **(0.3 pts)**. More heuristically, the net y momentum of the annihilating particles would have to cancel against the momentum of the photon, i.e. $p(1 - \sin\Theta) = E_\gamma \sin\phi$. The maximum value on the RHS is when $\phi = \pi/2$ **(0.3 pts)**. In that case (using the numbers above for $m = 1\text{GeV}$, $v = 0.9998 \simeq 1$, E_γ and $\gamma_{0.9998}$) **(0.3 pts)**

$$1 - \sin\Theta \simeq 10^{-3} \tag{36}$$

Therefore we conclude that $\Theta \sim \pi/2$ **(0.2 pts)**. In that case, there will also be limited net momentum in the x direction (i.e. $\gamma_M \simeq 1$) **(0.2 pts)** and we guess the mass of the DM particle to be ~ 100 GeV **(0.5 pts)**.

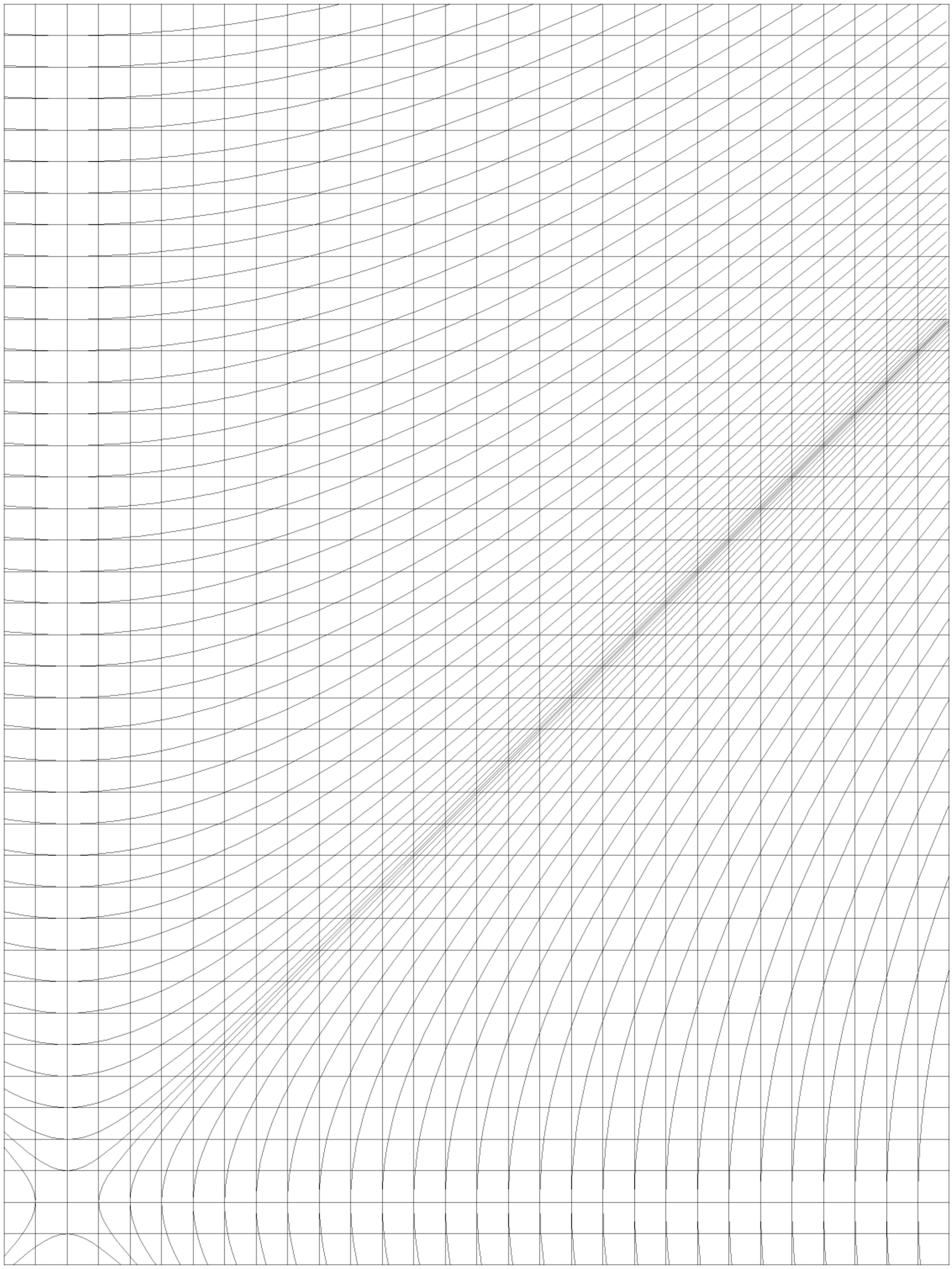
(h) **(3.5 pts, option 2)** Suppose the gamma-ray excess observed from the center of the galaxy equates to the luminosity of 10^7 suns, guesstimate the time it takes for all gas in the center of our Galaxy to be converted to dark matter (**assume that the gas is about 10% of the mass of ordinary matter, and that most matter in our galaxy is at the center of the galaxy**). How does this compare to the current age of the universe? You can assume a DM particle of ~ 100 GeV.

From the cover page we have **(0.3 pts)**

$$L_{\text{sun}} = 3.8 \times 10^{26} \text{ Watts} \simeq 2 \times 10^{45} \text{ eV/sec} = 2 \times 10^{39} \text{ MeV/sec.} \tag{37}$$

This implies that, given one photon has an energy of $\simeq 3$ MeV, we have about $\sim 10^{39}$ annihilation per second to produce the luminosity of a single sun-like star **(0.2 pts)**, and $\sim 10^{46}$ annihilation for 10 million suns **(0.2 pts)**. During each annihilation, 2 particles of 1 GeV are lost **(0.1 pts)**. Most (ordinary) mass of our galaxy is in the center, assuming 10^9 solar masses **(0.2 pts)**, and about 10% of that in gas, yields 10^8 $\mathrm{M_{sun}}$ **(0.2 pts)**. From the cover page, we have $M_{\text{sun}} \simeq 2 \times 10^{30}$ kg **(0.1 pts)**. We have to convert this to eV for easy computation. $E = mc^2 \simeq 10^{17}$ Joules $\simeq 0.5 \times 10^{36}$ eV **(0.3 pts)**. So one solar mass is $M_{\text{sun}} \simeq 10^{66}$ eV **(0.2 pts)**. This means there is $\sim 10^{74}$ eV $= 10^{65}$ GeV in gas **(0.3 pts)**. Per second, the annihilation in the center of our galaxy converts 2 GeV $\times\ 10^{46}$ annihilations/sec $= 2 \times 10^{46}$ GeV/sec **(0.3 pts)**. Hence it would take (total energy)/(energy per second) $= 10^{65}/(2 \times 10^{46}) = 0.5 \times 10^{19}$ seconds **(0.2 pts)**. There are $60 \times 60 \times 24 \times 365 \simeq 3 \times 10^7$ seconds in a year **(0.2 pts)**. Assuming the universe is about 14×10^9 years old, or $\sim 0.5 \times 10^{18}$ seconds **(0.2 pts)**. So we would have to wait another 10 times **(0.5 pts)** the current age of the universe for all the mass in the center to be annihilated. **answers with correct logic that are off by an order of magnitude should get full points. answers that have the logic off should receive only partial credit.**

Name: Student Number:



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