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Mechanics Exam M2

December 19, 2022

18:30-20:00

Aletta Jacobs Hall 1 (A1-P20)

About the Exam

1. There are 3 questions on the exam (60 points).
2. Write your answers *clearly* and *legibly*. Use standard notation and units when appropriate.
3. **Show all your work** (e.g., free-body diagrams, intermediate steps used in calculations, presumptions about problem) for full credit.
4. You may bring a *scientific* calculator to use on the exam – one will not be provided for you.
5. If you tear the pages apart, *write your name and student ID at the top of each page*.
6. If you use extra papers for your responses, place them **inside** the exam behind the cover page when you are done with the exam. *Make sure your name and student ID are on those papers.*

Possibly Useful Information

$\vec{v}(t) \equiv \frac{d\vec{r}}{dt}$	$\vec{r}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0t + \vec{r}_0$	$ \vec{v} = \sqrt{v_x^2 + v_y^2 + v_z^2}$	$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$	$M \frac{\Delta \vec{r}_{CM}}{\Delta t} = \sum m_i \vec{v}_i$
$\vec{a}(t) \equiv \frac{d\vec{v}}{dt}$	$\vec{v}(t) = \vec{a}t + \vec{v}_0$	$ \vec{a} = \sqrt{a_x^2 + a_y^2 + a_z^2}$	$\vec{p} = m\vec{v}$	$\vec{F} = \frac{d\vec{p}}{dt} \quad F_x = -\frac{dV}{dx}$
$K = \frac{1}{2}m \vec{v} ^2$	$dK_{CM} = \vec{F}_{ext} d\vec{r}_{CM} \cos\theta$	$V_g(r) = -\frac{Gm_1m_2}{r}$	$V_g(h) = m \vec{g} h$	$W = \int \vec{F} \cdot d\vec{r}$
$\sum \vec{F} = m\vec{a}$	$ \vec{F}_{KF} \approx \mu_K \vec{F}_N $ $ \vec{F}_{SF} \leq \vec{F}_{SF,max} \approx \mu_S \vec{F}_N $	$ \vec{F}_D \approx \frac{1}{2}C\rho A \vec{v} ^2$	$\vec{F}_T^{A(B)} = -\vec{F}_T^{B(A)}$	$m\vec{a}' = -m\vec{A} + \vec{F}_1 + \vec{F}_2 + \dots$
$ \vec{v} = \frac{2\pi R}{T}$	$ \vec{a} = \frac{ \vec{v} ^2}{R}$	$\vec{a} = \frac{d \vec{v} }{dt} \hat{v} - \frac{ \vec{v} ^2}{R} \hat{r}$	$\vec{F}_{Sp,x} = -k_s \vec{x}$	$V_{Sp}(x) = \frac{1}{2}k_s(x - x_0)^2$
$\frac{d^2x}{dt^2} = -\omega^2x$	$x(t) = A \cos(\omega t + \theta)$	$\omega \equiv \sqrt{k_s/m}$	$\omega \equiv \sqrt{ \vec{g} /L}$	$T = \frac{2\pi}{\omega} = \frac{1}{f}$

Question	Your Score	Total Points
1		22
2		18
3		20
		60

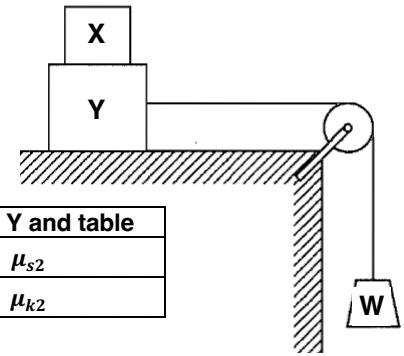
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1. Weight over Pulley (22 points)

In the figure, block X (mass m_1) is resting on top of block Y (mass m_2) resting on a rough table; a massless string connects the large block to a weight W (mass M) that passes over a frictionless and massless pulley, set at the end of the table. Presume $m_1 < m_2$.



The (non-zero) coefficients of friction between blocks X and Y, and between block Y and the table, are shown here:

	blocks X and Y	block Y and table
static	μ_{s1}	μ_{s2}
kinetic	μ_{k1}	μ_{k2}

For part a, presume mass M is small enough that it does not descend when released.

- a. (10 points) Sketch a free-body diagram (FBD) for each block and include its magnitude.
- Clearly label each force using the convention from the textbook and lectures (e.g., $\vec{F}_T^{A(B)}$ means tension force acting on object A by its interaction with object B).
 - Make sure that the magnitudes of the drawn forces are consistent **across the entire system**.
 - Use different hash marks (e.g., \longleftrightarrow , \longleftrightarrow , \longleftrightarrow , \longleftrightarrow) to indicate vectors with the same magnitude.

Block X		Block Y	
criterion			points
Both blocks	Correct block names are indicated by the first letter in the superscripts.		0.5
	Correct interacting objects are indicated in parenthetical part of superscripts.		1
	Correct types of force are indicated in subscripts.		1
	Indicates $\vec{F}_N^{X(Y)}$ and $\vec{F}_N^{Y(X)}$ have the same magnitude.		0.5
Block X	Marks normal and gravitational forces as equal in magnitude and opposite in direction vertically.		1
Block Y	Marks static friction force and tension as equal in magnitude and opposite in direction horizontally.		1
	Indicates normal force with table is opposite in direction to $\vec{F}_N^{Y(X)}$ and $\vec{F}_g^{Y(Earth)}$ and that its magnitude is the sum of the magnitudes of $\vec{F}_N^{Y(X)}$ and $\vec{F}_g^{Y(Earth)}$.		1

For each force listed above, indicate its magnitude (e.g., $|\vec{F}_g^{A(B)}| \rightarrow m|\vec{g}|$).

criterion		points
Block X	$ \vec{F}_N^{X(Y)} = m_1 \vec{g} $	0.5
	$ \vec{F}_g^{X(Earth)} = m_1 \vec{g} $	0.5
Block Y	$ \vec{F}_N^{Y(table)} = \vec{F}_N^{Y(X)} + \vec{F}_g^{Y(Earth)} = m_1 \vec{g} + m_2 \vec{g} = (m_1 + m_2) \vec{g} $	1
	$ \vec{F}_N^{Y(X)} = \vec{F}_N^{X(Y)} = m_1 \vec{g} $	0.5
	$ \vec{F}_g^{Y(Earth)} = m_2 \vec{g} $	0.5
	$ \vec{F}_{SF}^{Y(table)} = \mu_{s2} \vec{F}_N^{Y(table)} = \mu_{s2}(m_1 + m_2) \vec{g} $	0.5
	$ \vec{F}_T^{Y(string)} = M \vec{g} $	0.5

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For parts b-d, use the given variables (m_1 , m_2 , M , μ_{s1} , μ_{k1} , μ_{s2} , μ_{k2}) and fundamental constants as needed.

- b. (3 points) Derive an expression for the largest value of mass M such that blocks X and Y remain at rest.

critrion	points
Writes expression for maximum frictional force.	$ \vec{F}_{SF}^{Y(\text{table})} = \mu_{s2} \vec{F}_N^{Y(\text{table})} = \mu_{s2} (m_1 + m_2) \vec{g} $ 1
Equates frictional force to tension.	$\mu_{s2} (m_1 + m_2) \vec{g} = \vec{F}_T^{Y(\text{string})} = M \vec{g} $ 1
Correctly solves for M.	$M = \mu_{s2} (m_1 + m_2)$ 1

For parts c and d, mass M is large enough so that it descends when released.

- c. (5 points) Presume blocks X and Y **move together as one unit** (they do not slip). Derive an expression for the tension in the string.

	critrion	points
Correctly applies Newton's second law to hanging weight.	$\sum \vec{F} = m\vec{a}$ $M \vec{g} - \vec{F}_T = M \vec{a} $	1
Correctly applies Newton's second law to two-block system.	$\sum \vec{F} = (m_1 + m_2)\vec{a}$ $ \vec{F}_T - \vec{F}_f^{Y(\text{table})} = (m_1 + m_2) \vec{a} $	1
Use previous equations to solve for acceleration.	$ \vec{F}_T = M \vec{g} - M \vec{a} $ $ \vec{F}_T = \vec{F}_f^{Y(\text{table})} + (m_1 + m_2) \vec{a} $ $ \vec{F}_f^{Y(\text{table})} = \mu_{k2}(m_1 + m_2) \vec{g} $ $\mu_{k2}(m_1 + m_2) \vec{g} + (m_1 + m_2) \vec{a} = M \vec{g} - M \vec{a} $ $M \vec{a} + (m_1 + m_2) \vec{a} = M \vec{g} - \mu_{k2}(m_1 + m_2) \vec{g} $ $ \vec{a} (M + m_1 + m_2) = \vec{g} [M - \mu_{k2}(m_1 + m_2)]$ $ \vec{a} = \frac{ \vec{g} [M - \mu_{k2}(m_1 + m_2)]}{(M + m_1 + m_2)}$	2.5
Correctly solves for tension.	$ \vec{F}_T = M(\vec{g} - \vec{a}) = M\left(\vec{g} - \frac{ \vec{g} [M - \mu_{k2}(m_1 + m_2)]}{(M + m_1 + m_2)}\right)$ $= M \vec{g} \left(1 - \frac{[M - \mu_{k2}(m_1 + m_2)]}{(M + m_1 + m_2)}\right)$	0.5

- d. (4 points) Presume blocks X and Y **do not** move together as one unit (they slip). Derive an expression for the magnitude of the acceleration for block Y.

	critrion	points
Correctly applies Newton's second law to hanging weight.	$\sum \vec{F} = m\vec{a} \quad M \vec{g} - \vec{F}_T = M \vec{a} $	0.5
Correctly applies Newton's second law to block Y.	$\sum \vec{F} = m_2\vec{a} \quad \vec{F}_T - \vec{F}_f^{Y(\text{table})} - \vec{F}_f^{Y(X)} = m_2 \vec{a} $	1.5
Correctly solves for magnitude of acceleration for block Y.	$\begin{aligned} \vec{F}_T &= M \vec{g} - M \vec{a} & \vec{F}_T &= m_2 \vec{a} + \vec{F}_f^{Y(\text{table})} + \vec{F}_f^{Y(X)} \\ M \vec{g} - M \vec{a} &= m_2 \vec{a} + \vec{F}_f^{Y(\text{table})} + \vec{F}_f^{Y(X)} \\ \vec{a} (m_2 + M) &= M \vec{g} - \vec{F}_f^{Y(\text{table})} - \vec{F}_f^{Y(X)} \\ \vec{F}_f^{Y(X)} &= \mu_{k1}m_1 \vec{g} & \vec{F}_f^{Y(\text{table})} &= \mu_{k2}(m_1 + m_2) \vec{g} \\ \vec{a} (m_2 + M) &= M \vec{g} - \mu_{k2}(m_1 + m_2) \vec{g} - \mu_{k1}m_1 \vec{g} \\ \vec{a} &= \frac{M - \mu_{k2}(m_1 + m_2) - \mu_{k1}m_1}{(m_2 + M)} \vec{g} \end{aligned}$	2

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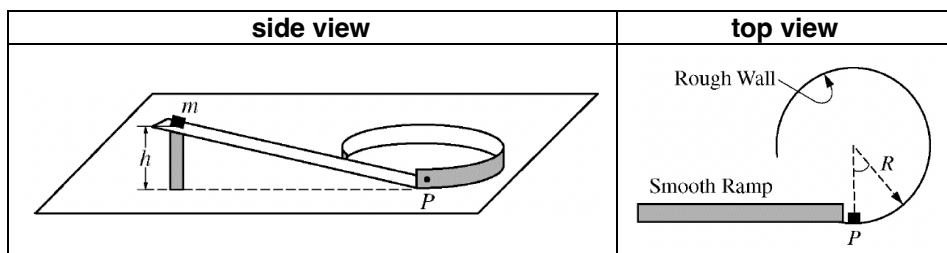
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2. Block Sliding on Ramp (18 points)

A block (**mass m**) starts from rest at the top of a frictionless ramp (**height h**) above a horizontal table (see side view). This block slides down the smooth ramp until it reaches point P, at the bottom of the ramp, with **speed v_0** .

At point P, the block moves along the rough circular vertical wall (**radius R , coefficient of kinetic friction μ_k**) and slides on the frictionless table (see top view).



In your responses, use the given variables (m, h, v_0, R, μ_k) and fundamental constants as needed.

- a. (3 points) Derive an expression for the height of the ramp (h).

criteria	points
Indicates energy is conserved.	$V_i + K_i = V_f + K_f$ 1
Correct substitutions for potential and kinetic energies	$m \vec{g} h_i = \frac{1}{2}m \vec{v}_f ^2$ 1
Correctly solves for height.	$h_i = \frac{v_0^2}{2 \vec{g} }$ 1

Alternate solutions possible with kinematics and dynamics equations.

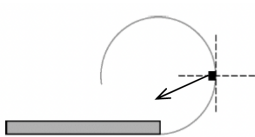
- b. (2 points) After passing point P, the block remains in contact with the wall and moves with a **speed v** . The vertical component of the net force on the moving block is _____. Choose one:

_____ upward _____ zero _____ downward

Justify the response you chose.

criteria	points
Selected "zero."	1
Provides correct explanation that vertical component of net force is not accelerating. If incorrect choice was selected, then no point was awarded for the explanation.	1

- c. (3 points) Sketch an arrow starting on block shown in the figure below to indicate the **horizontal component of the net force** on the moving block in the position shown. Justify the direction of the arrow you sketched.

criteria	points
	1
Sketched single arrow pointing downward and to the left.	1
Indicates left direction is due to centripetal force.	1
Indicates downward direction is due to friction (slows forward motion).	1

- d. (2 points) Determine an expression for the magnitude of the normal force (\vec{F}_N) exerted on the block by the circular wall as a function of v .

criteria	points
Indicates Newton's second law applies to scenario.	$\sum \vec{F} = m\vec{a}$ 1
Correctly substitutes normal force for net force and linear acceleration with centripetal acceleration.	$ \vec{F}_N = \frac{m \vec{v} ^2}{R}$ 1

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- e. (3 points) Derive an expression for the magnitude of the tangential acceleration of the block at the instant the block attains a speed of v .

criterion		points
Applies Newton's second law to correctly substitute relevant forces.	$- \vec{F}_f = -\mu_k \vec{F}_N = m \vec{a}_t $	1
Equates tangential acceleration with part d.	$-\mu_k \vec{F}_N = -\mu_k \frac{m \vec{v} ^2}{R} = m \vec{a}_t $	1
Correctly solves for tangential acceleration of the block.	$ \vec{a}_t = -\mu_k \frac{ \vec{v} ^2}{R}$	1

- f. (5 points) Derive an expression for $v(t)$, the speed of the block as a function of time t after passing point P.

criterion		points
May substitute $\frac{d \vec{v} }{dt}$ for \vec{a}_t or use $m \frac{d v }{dt} = - \vec{F}_f $.	$\frac{d \vec{v} }{dt} = -\mu_k \frac{ \vec{v} ^2}{R} \quad OR \quad m \frac{d \vec{v} }{dt} = - \vec{F}_f $	1
Correctly completes separation of variables.	$\frac{1}{ \vec{v} ^2} dv = -\frac{\mu_k}{R} dt$	1
Uses correct limits.	$\int_{v_0}^v \frac{1}{ \vec{v} ^2} dv = \int_0^t -\frac{\mu_k}{R} dt$	1
Correctly integrates to solve for speed.	$\left[-\frac{1}{ \vec{v} } \right]_{v_0}^v = \left[-\frac{\mu_k t}{R} \right]_0^t \quad \frac{1}{ \vec{v} } - \frac{1}{v_0} = \frac{\mu_k t}{R}$ $ \vec{v} = \frac{R v_0}{R + \mu_k v_0 t} \quad OR \quad \vec{v} = \frac{v_0}{1 + \left(\frac{\mu_k v_0 t}{R} \right)}$	2

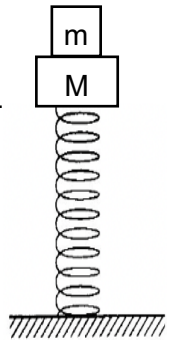
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3. Oscillations and Frames (20 points)

The figure on the right shows a system that consists of a small block ($m = 2.0 \text{ kg}$) resting on top of a large block ($M = 4.5 \text{ kg}$); this large block is connected to the top of a massless spring ($k = 820 \text{ N/m}$).



Both blocks are currently at rest and the system is not oscillating.

- a. (2 points) Determine the distance the spring is compressed from its original length.

criterion		points
Uses Hooke's law.	$\vec{F}_{\text{Sp},x} = -k_s \vec{x} $	1
Correctly solves for \vec{x} .	$\vec{x} = \frac{(m+M) \vec{g} }{k_s} = \frac{(2.0 \text{ kg} + 4.5 \text{ kg})(9.8 \text{ m/s}^2)}{820 \text{ N/m}} = 0.078 \text{ m}$	1

For parts b and c, the blocks are pushed down and released so that the system oscillates.

- b. (2 points) Determine the frequency of oscillation.

criterion		points
Identifies frequency relationships.	$\frac{2\pi}{\omega} = \frac{1}{f} \quad \omega \equiv \sqrt{k_s/m}$	1
Correctly solves for frequency. Point also awarded for $\omega = 11 \text{ s}^{-1}$.	$f = \frac{\omega}{2\pi} = \frac{\sqrt{k_s/m}}{2\pi} = \frac{\sqrt{820 \text{ N/m} / (2.0 \text{ kg} + 4.5 \text{ kg})}}{2\pi} = 1.8 \text{ s}^{-1}$	1

- c. (3 points) Determine the magnitude of the *maximum acceleration* that both blocks can attain **and** remain in contact with each other.

criterion		points
Indicates that if large block compresses spring with acceleration greater than $ \vec{g} $, the small block will lose contact with it... concludes $ \vec{a}_{\text{max}} $ of large block must be $ \vec{g} = 9.8 \text{ m/s}^2$.		3
OR		
Uses Newton's second law to mathematically determine maximum acceleration of small block. $\sum \vec{F} = m\vec{a} \qquad m \vec{g} - \vec{F}_N = m \vec{a} $ When $ \vec{F}_N = 0$, small block loses contact with large block. Thus, $ \vec{a}_{\text{max}} = \vec{g} = 9.8 \text{ m/s}^2$.		

- d. (4 points) Determine the additional distance the spring can be compressed from its *original compression* in part a without exceeding the *maximum acceleration* calculated in part c.

criterion		points
Indicates maximum acceleration occurs when compression of spring is at the amplitude (A).	$\vec{a}_{\text{max}} = \frac{d^2x}{dt^2} = -\omega^2x = -\omega^2A$	2
Correctly substitutes $ \vec{a}_{\text{max}} $ from part c to solve for amplitude.	$\vec{a}_{\text{max}} = - \vec{g} = -\omega^2A$ $A = \frac{ \vec{g} }{\omega^2} = \frac{9.8 \text{ m/s}^2}{(11 \text{ s}^{-1})^2} = 0.081 \text{ m}$	2

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The small block is removed. The large block and spring system is suspended from the ceiling of an elevator car. The elevator car moves upward with a constant acceleration of 3.4 m/s^2 . Choose the upward direction as positive.

- e. (4 points) Determine the amplitude of the resulting oscillation.

	criterion	points
Uses Newton's second law and Hooke's law to determine distance spring is stretched when at rest.	$\sum \vec{F} = 0 \quad \vec{F}_{\text{Sp},x} = -k_s \vec{x} \quad m \vec{g} - k_s \vec{x} = 0$ $ \vec{x}_{\text{rest}} = \frac{m \vec{g} }{k_s} = \frac{(4.5 \text{ kg})(9.8 \text{ m/s}^2)}{820 \frac{\text{N}}{\text{m}}} = 0.054 \text{ m}$	1.5
Correctly determines effective gravity inside elevator.	$ \vec{g}_{\text{eff}} = \vec{g} + \vec{a}_{\text{elevator}} = 9.8 \frac{\text{m}}{\text{s}^2} + 3.4 \frac{\text{m}}{\text{s}^2} = 13 \frac{\text{m}}{\text{s}^2}$	0.5
Uses Newton's second law and Hooke's law to determine distance spring is stretched when elevator is accelerating.	$\sum \vec{F} = 0 \quad \vec{F}_{\text{Sp},x} = -k_s \vec{x} \quad m \vec{g}_{\text{eff}} - k_s \vec{x} = 0$ $ \vec{x}_{\text{acc}} = \frac{m \vec{g}_{\text{eff}} }{k_s} = \frac{(4.5 \text{ kg})(13 \text{ m/s}^2)}{820 \frac{\text{N}}{\text{m}}} = 0.071 \text{ m}$	1.5
Correct subtraction to find amplitude.	$ \vec{x}_{\text{acc}} - \vec{x}_{\text{rest}} = 0.017 \text{ m}$	0.5

- f. (2 points) From a person's perspective outside the building, how does the frequency of oscillation appear compared to the actual frequency of oscillation inside the elevator? Choose one:

_____ smaller

_____ same

_____ larger

Justify the response you chose.

	criterion	points
Selects same.		1
Indicates frequency is only dependent on spring force constant and mass.		1

- g. (3 points) Presume the elevator is now filled with a fluid that causes the system to experience a drag force of $\vec{F}_D = -0.7v$ in the direction of motion. Write the equation of motion for this scenario with respect to position as a function of time. ~~Your answer should have this form: $x(t) = \dots$~~

	criterion	points
Uses Newton's second law and correctly substitutes relevant forces.	$\sum \vec{F} = m\vec{a}$ $\vec{F}_g + \vec{F}_D + \vec{F}_{\text{sp}} = m\vec{a}$	1
Substitutes previous relations to set up second order differential equation.	$m \vec{g} - k_s \vec{x} - 0.7 \frac{dx}{dt} = m \frac{d^2x}{dt^2}$ $k_s \vec{x}_{\text{rest}} - k_s \vec{x} - 0.7 \frac{dx}{dt} = m \frac{d^2x}{dt^2}$ $-k_s (\vec{x} - \vec{x}_{\text{rest}}) - 0.7 \frac{dx}{dt} = m \frac{d^2x}{dt^2}$ $(4.5 \text{ kg}) \frac{d^2x}{dt^2} + 0.7 \frac{dx}{dt} + \left(820 \frac{\text{N}}{\text{m}}\right) (\vec{x} - 0.054 \text{ m}) = 0$	2