Name: ______________________________ **student ID:** | S | | | | | | | |

Mechanics & Relativity
Exam M1

December 2, 2022
18:30-20:00
Aletta Jacobs Hall 1 (A1-P20)

About the Exam

1. There are 3 questions on the exam (40 points).
2. *If you tear the pages apart, write your surname and student ID at the top of each page*.
3. **Show all your work** (e.g., free-body diagrams, intermediate steps used in calculations, presumptions about problem) for full credit.
4. Write your answers *clearly* and *legibly*.
5. Use standard notation and units when appropriate.
6. You may bring a *scientific* calculator to use on the exam – one will not be provided for you.

Possibly Useful Information

$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$	$\vec{r}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{r}_0$	$\vec{v}(t) \equiv \frac{d\vec{r}}{dt}$	$\lvert\vec{v}\rvert = \sqrt{v_x^2 + v_y^2 + v_z^2}$
$M\frac{\Delta\vec{r}_{CM}}{\Delta t} = \sum m_i \vec{v}_i$	$\vec{v}(t) = \vec{a}t + \vec{v}_0$	$\vec{a}(t) \equiv \frac{d\vec{v}}{dt}$	$\lvert\vec{a}\rvert = \sqrt{a_x^2 + a_y^2 + a_z^2}$
$\sum \vec{F} = \vec{F}_{\text{net}} = m\vec{a}$	$\vec{F} = \frac{d\vec{p}}{dt}$	$\vec{p} = m\vec{v}$	$W = \int \vec{F} \cdot d\vec{r}$
$K = \frac{1}{2}m\lvert\vec{v}\rvert^2$	$dK_{\text{CM}} = \lvert\vec{F}_{\text{ext}}\rvert\lvert d\vec{r}_{\text{CM}}\rvert cos\theta$	$V_g(r) = -\frac{Gm_1m_2}{r}$	$V_g(h) = mgh$
$F_x = -\frac{dV}{dx}$	$\lvert\vec{F}_{\text{KF}}\rvert \approx \mu_K\lvert\vec{F}_{\text{N}}\rvert$ $\lvert\vec{F}_{\text{SF}}\rvert \leq \lvert\vec{F}_{\text{SF,max}}\rvert \approx \mu_S\lvert\vec{F}_{\text{N}}\rvert$	$\lvert\vec{F}_{\text{D}}\rvert \approx \frac{1}{2}C\rho A\lvert\vec{v}\rvert^2$	

Scoring

Question	Your Score	Total Points
1		16
2		10
3		14
		40

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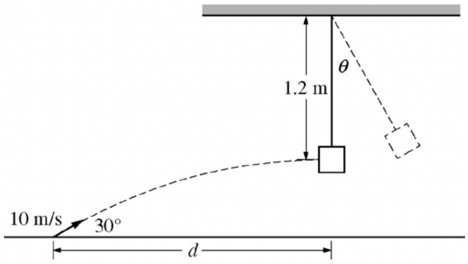
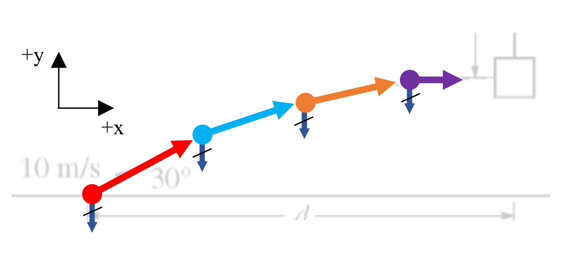
1. Dart and Hanging Block (16 points)

A small dart ($m_D = 0.020$ kg) is launched at 30° above the horizontal with an initial speed of 10.0 m/s. When the dart reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block ($m_B = 0.10$ kg) that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The dart and block then swing up until the string makes an angle θ with the vertical, as shown in the diagram. Presume air resistance is negligible.



a. (4 points) Sketch a 2D motion diagram for the path of the dart from when it is launched to just before it hits the block. Include a coordinate system and at least 4 instances of velocity and acceleration vectors. If the magnitudes of the vectors are identical, use a slash to indicate that (e.g.,).

criterion		points
	Includes a coordinate system.	1
	Sketches four equally spaced dots.	1
	Sketches three velocity vectors pointing right and upward (last vector points right only), with each vector decreasing in magnitude from left to right.	1
	Sketches acceleration vectors pointing downward, each equal in magnitude.	1

b. (2 points) Calculate the speed of the dart just before it hits the block.

criterion		points
Addresses horizontal component of velocity	$\|\vec{v}\| = v_x = v_0 \cos\theta$	1
Correctly substitutes values and solves for v	$\|\vec{v}\| = (10 \text{ m/s}) \cos 30° = 8.7 \text{ m/s}$	1

c. (3 points) Calculate the horizontal distance d the dart travels from its launch point to a point below the block.

criterion		points
Addresses vertical component of velocity to find t	$\vec{v} = v_y + a_y t$	1
Correctly substitutes values and solves for t	$0 = (10 \text{ m/s}) \sin 30° + (-9.8\, m/s^2)t$ $t = 0.51$ s	1
Correctly substitutes t to solve for x	$\|\vec{x}\| = v_x t = (8.7 \text{ m/s})(0.51 \text{ s}) = 4.4 \text{ m}$	1

d. (3 points) Calculate the speed of the block just after the dart hits it.

criterion		points
Indicates and shows momentum is conserved	$\vec{p}_i = \vec{p}_f \qquad m_1\vec{v}_{1,i} = (m_1 + m_2)\vec{v}_f$	2
Correctly substitutes values and solves for speed	$(0.020 \text{ kg})(8.66 \text{ m/s}) = (0.020 \text{ kg} + 0.10 \text{ kg})\vec{v}_f$ $\|\vec{v}_f\| = 1.4$ m/s	1

e. (4 points) Calculate the angle through which the dart and block will rise before they (momentarily) stop.

criterion		points
Indicates energy is conserved	$K_i + V_i = K_f + V_f \qquad \frac{1}{2}m\vec{v}_i^2 = m\vec{g}h$	1
Gives correct expression for height reached by block	$h = L - L\cos\theta = L(1 - \cos\theta)$	1
Correctly substitutes v (from part d) and h into conservation of energy equation to solve for θ	$\frac{1}{2}m\vec{v}_i^2 = m\vec{g}L(1 - \cos\theta)$ $\cos\theta = 1 - \frac{\vec{v}_i^2}{2\vec{g}L} = 1 - \frac{(1.4 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)(1.2 \text{ m})}$ $\theta = 24°$	2

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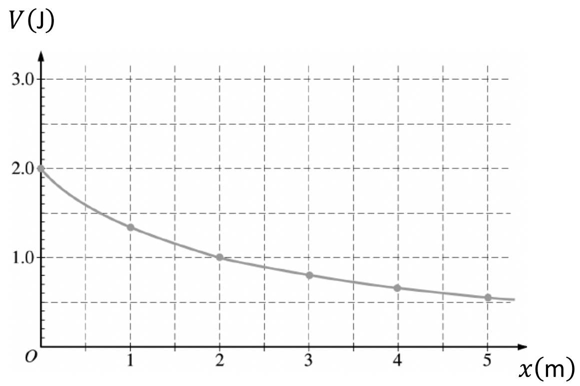
2. Potential Energy (10 points)

An object ($m = 0.5$ kg) experiences a force that is associated with the potential energy function $V(x) = \dfrac{4.0}{2.0 + x}$ where V has units of joules and x has units of meters.

a. (4 points) Using the axes below, sketch the graph of $V(x)$ versus x.

criterion		points
	Shape of curve is concave upward	1
	Shape of curve and decreases monotonically	1
	Shows y-intercept at 2.0 J (or equivalent)	1
	Shows $V(x = 5\text{ m})$ is 0.50 J ≤ V(x) ≤ 0.75 J	1

b. (3 points) Derive an expression for the force associated with this potential energy function.

criterion		points
Indicates that force is a derivative of potential with respect to position	$F_x = -\dfrac{dV}{dx}$	1
Correctly substitutes potential energy function into derivative	$F_x = -\dfrac{d}{dx}\left(\dfrac{4.0}{2.0 + x}\right)$	1
Correctly evaluates the derivative	$F_x = \dfrac{4.0}{(x + 2.0)^2}$	1

c. (3 points) Suppose the object is **released from rest at the origin**. Determine the object's speed at $x = 2.0$ m.

criterion		points
Indicates energy is conserved	$K_i + V_i = K_f + V_f$	1
Correctly substitutes and solves for speed	$V_i - V_f = \frac{1}{2}mv_f^2 \qquad \lvert v_f \rvert = \sqrt{\dfrac{2(V_i - V_f)}{m}} = \sqrt{\dfrac{2(2\text{ J} - 1\text{ J})}{0.5\text{ kg}}} = 2.0\text{ m/s}$	2

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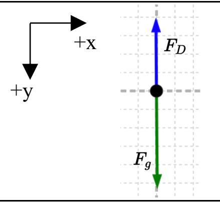
3. Falling Ball (14 points)

A rubber ball (mass m) dropped over a cliff's edge experiences a drag force with a magnitude of bv^2 (b is a constant that is directly proportional to the cross-sectional area of the ball and density of the air; v is the instantaneous speed). As the ball is falling, it approaches terminal speed.

a. (4 points) In the box below, **draw and label** all the forces on the ball at some instant before it reaches terminal speed. Include a coordinate system.

	criterion	points
	Includes a coordinate system	1
	Force vector due to gravitational field points downward	1
	Force vector due to drag points upward	1
	Magnitude of gravitational force is greater than magnitude of drag force	1
	For *any extra forces* sketched in the diagram, 1 point was deducted.	

b. (3 points) As the ball approaches terminal speed, ***explain*** whether the magnitude of its acceleration will **decrease, remain the same,** or **increase**.

criterion	points
Indicates that acceleration only decreases	1
An explanation: As ball approaches terminal speed, its velocity increases so the drag force also increases and approaches the magnitude of the gravitational force. The net force (difference between gravitational and drag forces) becomes smaller and since force is proportional to acceleration, acceleration decreases.	2
For explanation that only states definition of terminal velocity (e.g., since terminal velocity is constant, acceleration must decrease from 9.8 m/s^2 to zero), 1 point was awarded.	

c. (2 points) Derive an expression for the acceleration of the falling ball with respect to the given variables and constants.

criterion		points
Correctly expresses sum of all forces (consistent with coordinate system)	$\sum \vec{F} = m\vec{g} - b\vec{v}^2$	1
Applies Newton's second law for net force	$m\vec{a} = m\vec{g} - b\vec{v}^2$	0.5
Correctly derives expression for acceleration	$\vec{a} = \vec{g} - \frac{b}{m}\vec{v}^2$	0.5

d. (2 points) Derive an expression for the terminal speed with respect to the given variables and constants.

criterion		points
Indicates acceleration is zero at terminal speed	$\sum \vec{F} = m\vec{a} = 0$	1
Indicates drag force is equal to gravitational force	$m\vec{g} = b\vec{v}^2$	0.5
Correctly derives expression for terminal speed	$\lvert\vec{v}_f\rvert = \sqrt{m\vec{g}/b}$	0.5

The ball is released at height h and reaches its terminal speed before hitting the ground.

e. (3 points) Derive an expression for the energy dissipated by the drag force during the fall with respect to the given variables and constants.

criterion		points
Uses work-energy theorem to indicate energy dissipated by drag force is equal to change in energy		1
Correctly describes change in energy	$\Delta E = m\vec{g}h - \frac{1}{2}m\vec{v}_f{}^2$	1
Correctly substitutes speed from d into energy equation	$\Delta E = m\vec{g}h - \frac{1}{2}m(m\vec{g}/b) = m\vec{g}\left(h - \frac{m}{2b}\right)$	1
Alternate scoring: Instead of substituting terminal speed, may also state that $\Delta E = W = \int \vec{F}_D \cdot d\vec{r} = \int b\vec{v}^2\, dy$. If student ***incorrectly*** substitutes b with $\frac{1}{2}C\rho A$ (and everything else is correct), deduct 0.5 points; otherwise, award full credit.		