Name: ______________________________ student ID: | S | | | | | | | |

Mechanics Exam M3

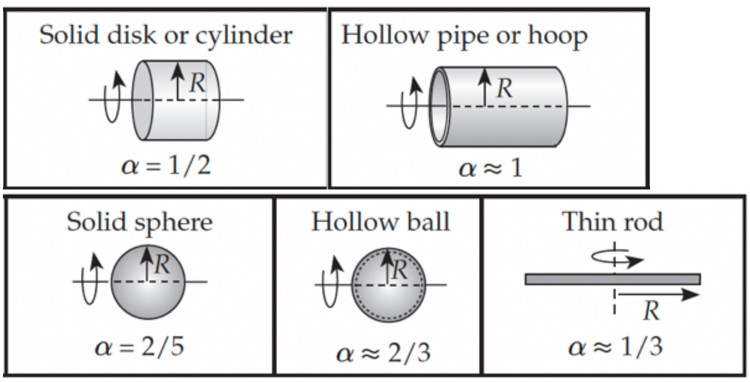
January 27, 2023
18:15-20:15
Aletta Jacobs Hall 1 (A1-P20)

About the Exam

1. There are 3 questions on the exam (60 points).
2. Write your answers *clearly* and *legibly*. Use standard notation and units when appropriate.
3. **Show all your work** (e.g., free-body diagrams, intermediate steps used in calculations, presumptions about problem) for full credit.
4. You may bring a *scientific* calculator to use on the exam – one will not be provided for you.
5. If you tear the pages apart, *write your name and student ID at the top of each page*.
6. If you use extra papers for your responses, place them ***inside*** the exam behind the cover page when you are done with the exam. *Make sure your name and student ID are on those papers.*

Possibly Useful Information

$\vec{v}(t) \equiv \frac{d\vec{r}}{dt}$	$\vec{r}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{r}_0$	$\lvert\vec{v}\rvert = \sqrt{v_x^2 + v_y^2 + v_z^2}$	$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$	$M\frac{\Delta \vec{r}_{CM}}{\Delta t} = \sum m_i \vec{v}_i$
$\vec{a}(t) \equiv \frac{d\vec{v}}{dt}$	$\vec{v}(t) = \vec{a}t + \vec{v}_0$	$\lvert\vec{a}\rvert = \sqrt{a_x^2 + a_y^2 + a_z^2}$	$\vec{p} = m\vec{v}$	$\vec{F} = \frac{d\vec{p}}{dt}$ $\quad F_x = -\frac{dV}{dx}$
$K = \frac{1}{2}m\lvert\vec{v}\rvert^2$	$dK_{\mathrm{CM}} = \lvert\vec{F}_{\mathrm{ext}}\rvert\lvert d\vec{r}_{\mathrm{CM}}\rvert cos\theta$	$V_g(r) = -\frac{Gm_1m_2}{r}$	$V_g(h) = m\lvert\vec{g}\rvert h$	$W = \int \vec{F}\cdot d\vec{r}$
$\sum \vec{F} = m\vec{a}$	$\lvert\vec{F}_{\mathrm{KF}}\rvert \approx \mu_K\lvert\vec{F}_{\mathrm{N}}\rvert$ $\lvert\vec{F}_{\mathrm{SF}}\rvert \leq \lvert\vec{F}_{\mathrm{SF,max}}\rvert \approx \mu_S\lvert\vec{F}_{\mathrm{N}}\rvert$	$\lvert\vec{F}_{\mathrm{D}}\rvert \approx \frac{1}{2}C\rho A\lvert\vec{v}\rvert^2$	$\vec{F}_{\mathrm{T}}{}^{\mathrm{A(B)}} = -\vec{F}_{\mathrm{T}}{}^{\mathrm{B(A)}}$	$m\vec{a}' = -m\vec{A} + \vec{F}_1 + \vec{F}_2 + \cdots$
$\lvert\vec{v}\rvert = \frac{2\pi R}{T}$	$\lvert\vec{a}\rvert = \frac{\lvert\vec{v}\rvert^2}{R}$	$\vec{a} = \frac{d\lvert\vec{v}\rvert}{dt}\hat{v} - \frac{\lvert\vec{v}\rvert^2}{R}\hat{r}$	$\vec{F}_{\mathrm{Sp,x}} = -k_s\vec{x}$	$V_{\mathrm{Sp}}(x) = \frac{1}{2}k_s(x - x_0)^2$
$\frac{d^2x}{dt^2} = -\omega^2 x$	$x(t) = A\cos(\omega t + \theta)$	$\omega \equiv \sqrt{k_s/m}$	$\omega \equiv \sqrt{\lvert\vec{g}\rvert/L}$	$T = \frac{2\pi}{\omega} = \frac{1}{f}$
$\vec{\tau} = \vec{r}\times\vec{F}$	$\theta \equiv \frac{s}{R} \quad \lvert\vec{\omega}\rvert = \left\lvert\frac{d\theta}{dt}\right\rvert = \frac{\lvert\vec{v}\rvert}{R}$	$\vec{L} = I\vec{\omega} = \vec{r}\times\vec{p}$	$\vec{L} = \vec{L}_{CM} + \vec{L}_{rot}$	$I = \sum m_i r_i^2 = \alpha M R^2$
$\vec{\tau} \equiv \frac{[d\vec{L}]}{dt}$	$U^{\mathrm{rot}} = \frac{1}{2}I\lvert\vec{\omega}\rvert^2$	$\lvert\vec{F}_{\mathrm{g}}\rvert = \left\lvert\frac{Gm_1m_2}{r^2}\right\rvert$	$\lvert\vec{v}\rvert = \sqrt{\frac{GM}{R}}$	$r(\theta) = \frac{b}{1 + \varepsilon\cos\theta}$
$T^2 = \frac{4\pi^2}{GM}a^3$	$a = \frac{1}{2}(r_c + r_f) = \frac{b}{1 - \varepsilon^2}$	$\frac{2E}{m} = \lvert\vec{v}\rvert^2 - \frac{2GM}{r}$	$E = \mp\frac{GMm}{2a}$	$\varepsilon^2 = 1 + \frac{1}{(GM)^2}\left(\frac{L}{m}\right)^2\left(\frac{2E}{m}\right)$



Question	Your Score	Total Points
1		20
2		20
3		20
		60

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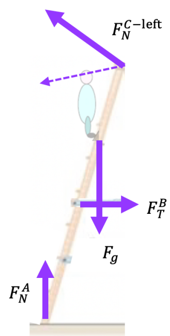
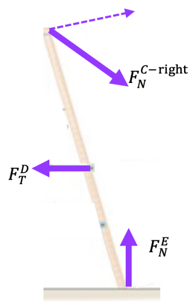
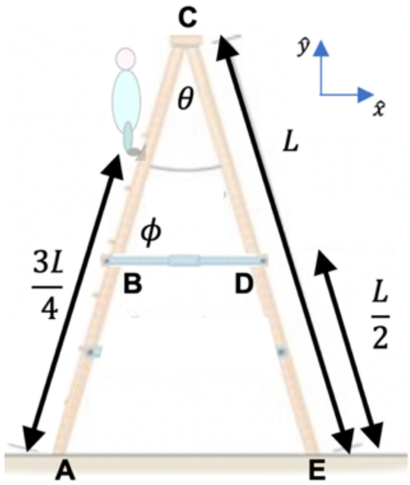
1. Stepladder (20 points)

A stepladder ($L = 4\ \mathrm{m}$ on each side) is connected by a hinge (C) at the top and a crossbar ($\mathrm{BD} = 1.25\ \mathrm{m}$) in the middle. Presume the stepladder (including hinge and crossbar) is massless and stands on a horizontal frictionless surface (AE).

A student ($m = 43\ \mathrm{kg}$) climbs up one side of the stepladder to $3\ \mathrm{m}$.

a. (5 points) In the boxes below, sketch and ***clearly*** label each force acting on the side of the stepladder indicated by that column.

left side of stepladder	right side of stepladder
	



criteria		points
left side	at base of ladder, normal force points straight up	0.5
	at crossbar, tension force points straight to the right	0.5
	at point where person stands, gravitational force points straight down	0.5
	at hinge, normal force points *left upward* OR *left downward* (not directly left)	1
right side	at base of ladder, normal force points straight up	0.5
	at crossbar, tension force points straight to the left	0.5
	at hinge, normal force points *right upward* OR *right downward* (not directly right) – AND equal in magnitude, opposite in direction to left normal force for hinge	1.5
NOTE: full notation not needed if type of force (e.g., tension) is shown; one point off for any extra forces		

b. (7 points) Below each diagram, analyze and write down the relationships between the forces related to that side of the stepladder.

criteria			points
left side	force	$\sum \vec{F}_x = 0 = \lvert\vec{F}_T^B\rvert - \lvert\vec{F}_{N,x}^{C-\text{left}}\rvert \qquad \sum \vec{F}_y = 0 = \lvert\vec{F}_N^A\rvert + \lvert\vec{F}_{N,y}^{C-\text{left}}\rvert - \lvert\vec{F}_g\rvert$	2
	torque	$\sum \vec{\tau} = 0 = -\left(\vec{r}_A \times \vec{F}_N^A \sin\frac{\theta}{2}\right) + \left(\vec{r}_{\text{student}} \times \vec{F}_g \sin\frac{\theta}{2}\right) + \left(\vec{r}_B \times \vec{F}_T^B \sin\phi\right)$ $= -\left(L\vec{F}_N^A \sin\frac{\theta}{2}\right) + \left(\frac{L}{4}\vec{F}_g \sin\frac{\theta}{2}\right) + \left(\frac{L}{2}\vec{F}_T^B \sin\phi\right)$ $= L\left(-\vec{F}_N^A \sin\frac{\theta}{2} + \frac{1}{4}\vec{F}_g \sin\frac{\theta}{2} + \frac{1}{2}\vec{F}_T^B \sin\phi\right)$	1.5
right side	force	$\sum \vec{F}_x = 0 = -\lvert\vec{F}_T^D\rvert + \lvert\vec{F}_{N,x}^{C-\text{right}}\rvert \qquad \sum \vec{F}_y = 0 = \lvert\vec{F}_N^E\rvert - \lvert\vec{F}_{N,y}^{C-\text{right}}\rvert$	2
	torque	$\sum \vec{\tau} = 0 = \left(\vec{r}_E \times \vec{F}_N^E \sin\frac{\theta}{2}\right) - \left(\vec{r}_D \times \vec{F}_T^D \sin\phi\right) = L\left(\vec{F}_N^E \sin\theta - \frac{1}{2}\vec{F}_T^D \sin\phi\right)$	1.5
Note: numbers may be used instead of variables			

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c. (6 points) Determine the magnitude of the forces exerted by the stepladder on the floor **at points A and E**. Make sure to explicitly indicate which point on the diagram you chose to use as your pivot point.

criteria		points
Chooses C as pivot so there is no torque acting on it.		0.5
Calculates angles.	$\phi = \cos^{-1}\left(\frac{0.625\text{ m}}{2\text{ m}}\right) = 71.8°$ $\quad \frac{\theta}{2} = 90° - \phi = 18.2°$	0.5
Addresses vertical forces from both sides of ladder.	$\lvert\vec{F}_{N,y}^{C\text{-left}}\rvert = \lvert\vec{F}_g\rvert - \lvert\vec{F}_N^A\rvert \quad \lvert\vec{F}_{N,y}^{C\text{-right}}\rvert = \lvert\vec{F}_N^E\rvert$ $\lvert\vec{F}_N^E\rvert = \lvert\vec{F}_g\rvert - \lvert\vec{F}_N^A\rvert$	1.5
Uses torque equation (right side) to find tension at D.	$\vec{r}_E\lvert\vec{F}_N^E\rvert\sin\frac{\theta}{2} = \vec{r}_D\lvert\vec{F}_T^D\rvert\sin\phi \quad L\lvert\vec{F}_N^E\rvert\sin\frac{\theta}{2} = \frac{L}{2}\lvert\vec{F}_T^D\rvert\sin\phi$ $\lvert\vec{F}_T^D\rvert = \lvert\vec{F}_T^B\rvert = \frac{L\lvert\vec{F}_N^E\rvert\sin\frac{\theta}{2}}{\frac{L}{2}\sin\phi} = \frac{2\lvert\vec{F}_N^E\rvert\sin\frac{\theta}{2}}{\sin\phi} = 0.658\lvert\vec{F}_N^E\rvert$	1
Combines torque equation (left side) with previous steps to find normal force at E.	$\vec{r}_B\lvert\vec{F}_T^B\rvert\sin\phi = \vec{r}_A\lvert\vec{F}_N^A\rvert\sin\frac{\theta}{2} - \vec{r}_{\text{student}}\lvert\vec{F}_g\rvert\sin\frac{\theta}{2}$ $\frac{L}{2}\lvert\vec{F}_T^B\rvert\sin\phi = L\lvert\vec{F}_N^A\rvert\sin\frac{\theta}{2} - \frac{L}{4}\lvert\vec{F}_g\rvert\sin\frac{\theta}{2}$ $\frac{1}{2}(0.658)\lvert\vec{F}_N^E\rvert\sin\phi = L(\lvert\vec{F}_g\rvert - \lvert\vec{F}_N^E\rvert)\sin\frac{\theta}{2} - \frac{L}{4}\lvert\vec{F}_g\rvert\sin\frac{\theta}{2}$ $\lvert\vec{F}_N^E\rvert\left(0.658\frac{L}{2}\sin\phi + L\sin\frac{\theta}{2}\right) = \lvert\vec{F}_g\rvert\sin\frac{\theta}{2}\left(L - \frac{L}{4}\right)$ $\lvert\vec{F}_N^E\rvert = \frac{\left(\frac{3L}{4}\right)m\lvert\vec{g}\rvert\sin\frac{\theta}{2}}{0.658\frac{L}{2}\sin\phi + L\sin\frac{\theta}{2}} = \frac{(3)m\lvert\vec{g}\rvert\sin\frac{\theta}{2}}{0.658(2)\sin\phi + 4\sin\frac{\theta}{2}} = 158\text{ N}$	2
Calculates normal force at A.	$\lvert\vec{F}_N^A\rvert = \lvert\vec{F}_g\rvert - \lvert\vec{F}_N^E\rvert = 421\text{ N} - 158\text{ N} = 263\text{ N}$	0.5

d. (2 points) Determine the components of the force that each side of the stepladder exerts on each other at point C ***and*** address the consistency of these values with the force components sketched for point C in part a.

criteria		points
Calculates normal force components at point C.	$\lvert\vec{F}_{N,x}^{C\text{-left}}\rvert = \lvert\vec{F}_T^B\rvert = 104\text{ N} \quad \lvert\vec{F}_{N,x}^{C\text{-right}}\rvert = \lvert\vec{F}_T^D\rvert = 104\text{ N}$ $\lvert\vec{F}_{N,y}^{C\text{-left}}\rvert = \lvert\vec{F}_g\rvert - \lvert\vec{F}_N^A\rvert = 158\text{ N} \quad \lvert\vec{F}_{N,y}^{C\text{-right}}\rvert = \lvert\vec{F}_N^E\rvert = 158\text{ N}$	1
May address consistency by talking about how well the horizontal and vertical values align (or not) with the magnitudes of the force vector they sketched. The idea is that any Newton's third law pair of vectors could have been sketched, and "corrected" by calculations (if needed).		1

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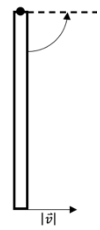
2. Swinging Rod (20 points)

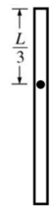


A rod (mass M, length L) with uniform density hangs vertically by a pivot (negligible friction) at one end. Express your responses in terms of the given variables and fundamental constants.

a. (5 points) Derive an expression for the horizontal speed $|\vec{v}|$ that must be applied to the bottom of the rod such that it swings **up to** the horizontal position (without going any higher).

criteria		points
Expresses energy conservation with rotational kinetic energy at start and gravitational potential energy (of center of mass) at end.	$\sum E_i = \sum E_f \quad \frac{1}{2}I\omega^2 = mg\left(\frac{L}{2}\right)$	2
Determines moment of inertia for rod swinging at one end, with λ = linear density.	$I = \int_0^L r^2\, dm = \int_0^L x^2\,(\rho)dx$ $= \frac{\lambda}{3}x^3\Big\vert_0^L = \frac{M}{L}\frac{1}{3}L^3 = \frac{1}{3}ML^2$	1
Substitutes moment of inertia into energy equation to solve for angular speed.	$\frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 = mg\left(\frac{L}{2}\right) \quad \omega = \sqrt{\frac{3g}{L}}$	1
Converts angular speed into linear speed.	$\omega = \frac{v}{L} \quad \left(\frac{v}{L}\right)^2 = \frac{3g}{L} \quad v = \sqrt{3gL}$	1

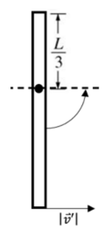
Presume the pivot is now shifted down to a distance $(L/3)$ from the end of the rod.



b. (4 points) Derive an expression for the moment of inertia of the rod about this pivot.

criteria		points
Uses integral expression for moment of inertia.	$I = \int r^2\, dm$	0.5
Indicates correct change of variables.	$dm = \frac{M}{L}dr$	0.5
Uses correct limits of integration.	$I = \int_{-L/3}^{2L/3} \frac{M}{L} r^2\, dr$	1
Sets up integral correctly.	$I = \frac{M}{3L}\left[\left(\frac{2L}{3}\right)^3 - \left(-\frac{L}{3}\right)^3\right]$	1
Calculates correctly.	$I = \frac{1}{9}ML^2$	1
Note: may also use parallel axis theorem to solve for moment of inertia		

c. (3 points) A horizontal speed $|\vec{v}'|$ is applied to the bottom of this rod so it swings **up to** the horizontal position.



i. Compared to speed $|\vec{v}|$ from part a, this speed $|\vec{v}'|$ is (choose one):

_______ less _______ same _______ greater

ii. Justify your choice.

criteria	points
Selects "less"	1
Indicates that if moment of inertia decreases, the rotational speed is greater.	2

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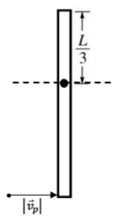
The rod is brought to rest in the vertical position and hangs freely before the bottom of the rod is hit by a small projectile (mass m) with a horizontal speed $|\vec{v}_p|$.



d. (5 points) Derive an expression for the angular speed of the rod after the projectile embeds itself (instantaneously) into the rod. Explicitly state reasonable presumptions.

criteria		points
Expresses conservation of angular momentum.	$\sum \vec{L}_i = \sum \vec{L}_f$	0.5
Indicates initial momentum is only due to projectile (as rod is at rest)	$\vec{L}_i = r \times m\vec{v}_p = \frac{2L}{3} m\lvert\vec{v}_p\rvert$	1
Indicates final momentum is due to rod and projectile, which are moving with same angular velocity…so moments of inertia can be summed.	$\vec{L}_f = I\vec{\omega} = \left[\frac{1}{9}ML^2 + m\left(\frac{2}{3}L\right)^2\right]\vec{\omega}$	2
Substitutes initial and final momenta into conservation equation to solve for angular speed.	$\frac{2L}{3} m\lvert\vec{v}_p\rvert = \left[\frac{1}{9}ML^2 + m\left(\frac{2}{3}L\right)^2\right]\lvert\vec{\omega}\rvert$ $\frac{2}{3} m\lvert\vec{v}_p\rvert = \left(\frac{M}{9} + \frac{4m}{9}\right) L\lvert\vec{\omega}\rvert$ $\lvert\vec{\omega}\rvert = \frac{2m\lvert\vec{v}_p\rvert}{\left(\frac{M}{3} + \frac{4m}{3}\right)L} = \frac{6m\lvert\vec{v}_p\rvert}{(M+4m)L}$	1.5

e. (3 points) Derive an expression for the fraction of the system's initial kinetic energy that was converted into internal energy.

criteria		points
Indicates initial kinetic energy is only due to projectile and final kinetic energy is rotational.	$K_i = \frac{1}{2} m\lvert\vec{v}_p\rvert^2 \qquad K_f = \frac{1}{2} I\lvert\vec{\omega}\rvert^2$	1
Takes the difference between initial and final kinetic energy and determines the fraction of initial kinetic energy represented by this difference.	$\Delta U = K_i - K_f$ $\frac{\Delta U}{K_i} = \frac{\frac{1}{2} m\lvert\vec{v}_p\rvert^2 - \frac{1}{2} I\lvert\vec{\omega}\rvert^2}{\frac{1}{2} m\lvert\vec{v}_p\rvert^2} = 1 - \frac{I\lvert\vec{\omega}\rvert^2}{m\lvert\vec{v}_p\rvert^2}$ $= 1 - \frac{\left[\frac{1}{9}ML^2 + m\left(\frac{2}{3}L\right)^2\right]\left[\frac{6m\lvert\vec{v}_p\rvert}{(M+4m)L}\right]^2}{m\lvert\vec{v}_p\rvert^2}$ $= 1 - \frac{4m}{M+4m} = \frac{M}{M+4m}$	2

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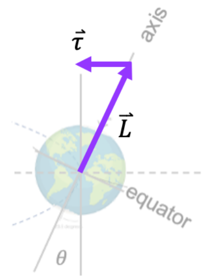
3. Outer Space (20 points)

The Earth bulges slightly outward at the equator, which contributes to the Earth's precession as it orbits the Sun. The Earth's axis makes an angle ($\theta = 23.5°$) to the direction that is perpendicular to the orbital plane (xy-plane).



criteria	points
Sketches angular momentum vector right and upward, *along Earth's axis.*	1
Sketches torque vector to the left towards the Sun.	1

a. (2 points) In the diagram (not drawn to scale), sketch and label the following vectors:

   i. angular momentum for Earth's northern hemisphere ($\vec{L}$)

   ii. torque applied to Earth's axis ($\vec{\tau}$)

b. (3 points) Explain where the applied torque comes from and how it contributes to Earth's precession. Make sure to address the role of angular momentum in your explanation.

criteria	points
Because Earth rotates about its axis, it has angular momentum. The Earth precesses (in a plane above and parallel to the orbital plane) in a direction that is out of the page; this is a change in angular momentum over time ($d\vec{L}/dt$). Since torque is defined as $d\vec{L}/dt$ and $\vec{L} = \vec{r} \times \vec{p}$ ($\vec{r}$ is with respect to the orbital plane), the direction of the torque is to the left. The torque comes from forces caused by the gravitational influences of the Sun (and Earth's moon) acting on the bulge around Earth's equator to try and align the bulge with the orbital plane.	3

The Earth ($m_E = 5.97 \times 10^{24}$ kg, $r_E = 6.38 \times 10^6$ m) and its moon ($m_M = 7.34 \times 10^{22}$ kg, $r_M = 1.74 \times 10^6$ m) are closest to each other at 3.65×10^8 m and furthest from each other at 4.07×10^8 m. ($G = 6.67 \times 10^{-11}$ Nm^2/kg^2)

c. (3 points) Determine the speed of the moon (relative to Earth) at its point of closest approach.

criteria		points
Indicates that major axis is the sum of the closest and farthest distances.	$2a = (r_c + r_f)$ $a = \frac{1}{2}[(3.65 \times 10^8\ \mathrm{m} + r_E + r_M) + (4.07 \times 10^8\ \mathrm{m} + r_E + r_M)]$ $= 3.94 \times 10^8\ \mathrm{m}$	1
Expresses or derives total energy.	$E = \frac{1}{2}m\lvert\vec{v}\rvert^2 - \frac{GMm}{r}$ $\quad \frac{\boldsymbol{2E}}{\boldsymbol{m}} = \lvert\vec{\boldsymbol{v}}_{\boldsymbol{c}}\rvert^2 - \frac{\boldsymbol{2GM}}{\boldsymbol{r_c}}$	0.5
Indicates (correct sign) energy for elliptical orbit.	$E = -\frac{GMm}{2a}$ $\quad a = -\frac{GMm}{2E}$	0.5
Correctly solves for speed after substitutions.	$a = -\frac{GM}{(2E/m)} = -\frac{GM}{\lvert\vec{v}_c\rvert^2 - (2GM/r_c)}$ $\quad \lvert\vec{v}_c\rvert^2 - \frac{2GM}{r_c} = -\frac{GM}{a}$ $\lvert\vec{v}_c\rvert = \sqrt{\frac{2GM}{r_c} - \frac{GM}{a}} = 1.06 \times 10^3\ \frac{\mathrm{m}}{\mathrm{s}}$	1

A small asteroid has a speed of 8.7×10^4 m/s at its closest to the Sun ($m_S = 2.0 \times 10^{30}$ kg, $r_S = 6.95 \times 10^8$ m), which is 3.83×10^7 m above the Sun's surface. ($G = 6.67 \times 10^{-11}$ Nm^2/kg^2)

d. (4 points) Determine the eccentricity of its orbit ***and*** shape (circle, ellipse, parabola, or hyperbola).

criteria		points
Expresses equation for eccentricity.	$\varepsilon^2 = 1 + \frac{1}{(GM)^2}\left(\frac{L}{m}\right)^2\left(\frac{2E}{m}\right)$	0.5
Addresses angular momentum.	$L = rm\lvert\vec{v}\rvert$ $\quad \frac{\boldsymbol{L}}{\boldsymbol{m}} = r_c\lvert\vec{v}_c\rvert$	0.5
Addresses energy.	$E = \frac{1}{2}m\lvert\vec{v}\rvert^2 - \frac{GMm}{r}$ $\quad \frac{\boldsymbol{2E}}{\boldsymbol{m}} = \lvert\vec{v}_c\rvert^2 - \frac{2GM}{r_c}$	0.5
Correctly solves for eccentricity after substitutions.	$\varepsilon = \sqrt{1 + \frac{(r_c\lvert\vec{v}_c\rvert)^2}{(GM)^2}\left(\lvert\vec{v}_c\rvert^2 - \frac{2GM}{r_c}\right)} = 0.96$	2
Indicates shape of the orbit is an ellipse.		0.5

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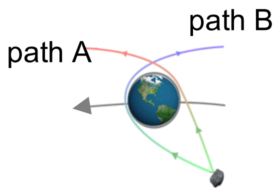
The same asteroid approaches Earth at 1.6×10^4 m/s and slows down as it moves away from Earth.

e. (3 points) This described asteroid's path is best represented by (choose one):



_______ path A _______ path B

Justify your choice.

criteria	points
Selects "path B"	1
Indicates that asteroid must travel in front of the Earth to decelerate because as the asteroid passes in front of Earth, the gravitational force due to Earth starts to pull it in a direction opposite of Earth's trajectory, deflecting the asteroid from its original trajectory.	2

f. (5 points) Determine the speed of the asteroid after it moves away from Earth. Presume that Earth moves at a speed of 2.98×10^4 m/s. Explicitly state reasonable presumptions. ($G = 6.67 \times 10^{-11}$ Nm^2/kg^2)

criteria		points
Presumes mass of asteroid (m) is much smaller than mass of Earth (M), asteroid approaches Earth perpendicular and leaves parallel, and/or effects of other planetary objects (e.g., Sun) are negligible in this interaction.		1
Expresses relations due to conservation of momentum.	$\sum \vec{p}_i = \sum \vec{p}_f \quad \begin{bmatrix}0\\ m\lvert\vec{v}_i\rvert\end{bmatrix} - \begin{bmatrix}M\lvert\vec{V}_i\rvert\\ 0\end{bmatrix} = \begin{bmatrix}m\lvert\vec{v}_f\rvert\\ 0\end{bmatrix} + \begin{bmatrix}MV_x\\ MV_y\end{bmatrix}$ $V_x = -\lvert\vec{V}_i\rvert - \frac{m}{M}\lvert\vec{v}_f\rvert \qquad V_y = \frac{m}{M}\lvert\vec{v}_i\rvert$	1
Expresses relations due to conservation of energy.	$\sum E_i = \sum E_f \quad \frac{1}{2}m\lvert\vec{v}_i\rvert^2 + \frac{1}{2}M\lvert\vec{V}_i\rvert^2 = \frac{1}{2}m\lvert\vec{v}_f\rvert^2 + \frac{1}{2}M\lvert\vec{V}\rvert^2$ $\frac{m}{M}\lvert\vec{v}_i\rvert^2 + \lvert\vec{V}_i\rvert^2 = \frac{m}{M}\lvert\vec{v}_f\rvert^2 + V_x^2 + V_y^2$	1
Correctly solves for asteroid's speed after substitutions, quadratic solution, and presumption that asteroid's mass is much less than Earth's mass.	$\frac{m}{M}\lvert\vec{v}_i\rvert^2 + \lvert\vec{V}_i\rvert^2 = \frac{m}{M}\lvert\vec{v}_f\rvert^2 + \left(-\lvert\vec{V}_i\rvert - \frac{m}{M}\lvert\vec{v}_f\rvert\right)^2 + \left(\frac{m}{M}\lvert\vec{v}_i\rvert\right)^2$ $\frac{m}{M}\lvert\vec{v}_i\rvert^2 + \lvert\vec{V}_i\rvert^2 = \frac{m}{M}\lvert\vec{v}_f\rvert^2 + \left(\lvert\vec{V}_i\rvert^2 + 2\frac{m}{M}\lvert\vec{V}_i\rvert\lvert\vec{v}_f\rvert + \frac{m^2}{M^2}\lvert\vec{v}_f\rvert^2\right) + \left(\frac{m^2}{M^2}\lvert\vec{v}_i\rvert^2\right)$ $0 = \frac{m}{M}\lvert\vec{v}_f\rvert^2 + \frac{m^2}{M^2}\lvert\vec{v}_f\rvert^2 + 2\frac{m}{M}\lvert\vec{V}_i\rvert\lvert\vec{v}_f\rvert + \frac{m^2}{M^2}\lvert\vec{v}_i\rvert^2 - \frac{m}{M}\lvert\vec{v}_i\rvert^2$ $0 = \lvert\vec{v}_f\rvert^2 + \frac{m}{M}\lvert\vec{v}_f\rvert^2 + 2\lvert\vec{V}_i\rvert\lvert\vec{v}_f\rvert + \frac{m}{M}\lvert\vec{v}_i\rvert^2 - \lvert\vec{v}_i\rvert^2$ $0 = \left(1 + \frac{m}{M}\right)\lvert\vec{v}_f\rvert^2 + 2\lvert\vec{V}_i\rvert\lvert\vec{v}_f\rvert - \left(1 - \frac{m}{M}\right)\lvert\vec{v}_i\rvert^2$ $\lvert\vec{v}_f\rvert = \frac{-2\lvert\vec{V}_i\rvert \pm \sqrt{\left(-2\lvert\vec{V}_i\rvert\right)^2 + 4\left(1+\frac{m}{M}\right)\left(1-\frac{m}{M}\right)\lvert\vec{v}_i\rvert^2}}{2\left(1+\frac{m}{M}\right)}$ $\lvert\vec{v}_f\rvert = \frac{-\lvert\vec{V}_i\rvert \pm \sqrt{\lvert\vec{V}_i\rvert^2 + \left(1-\frac{m^2}{M^2}\right)\lvert\vec{v}_i\rvert^2}}{\left(1+\frac{m}{M}\right)}$ $\approx -\lvert\vec{V}_i\rvert \pm \sqrt{\lvert\vec{V}_i\rvert^2 + \lvert\vec{v}_i\rvert^2} = 4.02 \times 10^3 \,\frac{\text{m}}{\text{s}}$	2