$$z = e^{i\theta} \qquad z = \frac{1+ia}{1-ia}$$

$$e^{i\theta} = \frac{1+ia}{1-ia} \qquad (1-ia)e^{i\theta} = 1+ia$$

$$e^{i\theta} - e^{i\theta} ia - 1 - ia = 0$$

leiame funktsiooni a-st θ suhtes

$$a(-e^{i\theta} i - i) = e^{i\theta} - 1$$

$$a = \frac{e^{i\theta}-1}{(-e^{i\theta} i - i)} = \frac{e^{i\theta}-1}{i(-e^{i\theta}-1)}$$

korrutame murru lugeja ja nimetaja i-ga läbi

$$a = \frac{i(e^{i\theta}-1)}{e^{i\theta}+1}$$

a ei ole defineeritud kui

$$e^{i\theta}+1=0$$

$\cancel{e^{i\theta}+1=0}$

see on tuntud kui euleri identiteet

$e^{i\pi}+1 = 0$ ehk $\theta = \pi$

aga muidugi on vastus perioodiline

$\theta = n\pi$ kus $n \in \mathbb{Z}$

seega ei kehti valem (1) juhul kui

$$e^{i\theta} = -1$$

$$z = -1$$

1 teisendame kõigepealt

a) $\left(\frac{1-i}{\sqrt{2}}\right)^{25} + \frac{i}{\sqrt{2}} = \frac{(1-i)^{25}}{\sqrt{2}^{25}} + \frac{i}{\sqrt{2}} = \frac{(1-i)^{25}}{2^{12}\sqrt{2}} + \frac{i}{\sqrt{2}}$

$$= \frac{(1-i)^{25} + 2^{12} i}{2^{12}\sqrt{2}}$$

$$\frac{\left(\frac{1-i}{\sqrt{2}}\right)^{25} + \frac{i}{\sqrt{2}}}{\left(\frac{1+i}{\sqrt{2}}\right)^{25} - \frac{i}{\sqrt{2}}} = \frac{(1-i)^{25} + 2^{12} i}{2^{12}\sqrt{2}} \cdot \frac{2^{12}\sqrt{2}}{(1+i)^{25} - 2^{12} i}$$

$$= \cancel{(1-i)^{25} + 4\ldots}$$

$$= \frac{(1-i)^{25} + 2^{12} i}{(1+i)^{25} - 2^{12} i}$$

$$= \frac{(1-i)^{25} + 4096i}{(1+i)^{25} - 4096i}$$

~~b)~~ c)

$$(\sqrt{3}-i)^6$$

$$= 2\left(\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right)^6$$

Kasutame de Moivre teoreemi

$$= 2^6\left(\cos\left(6\cdot\frac{11\pi}{6}\right) + i\sin\left(6\cdot\frac{11\pi}{6}\right)\right)$$

$$= 64\left(\cos(11\pi) + i\sin(11\pi)\right)$$

$$= 64$$

b)

$$\cancel{\left(\frac{2}{2-\sqrt{3}+i}\right)^{-24} = \tan(75^\circ)}$$

$$\left(\frac{2-\sqrt{3}+i}{2}\right)^{24} = \left(\frac{2-\sqrt{3}}{2} + \frac{i}{2}\right)^{24} \Rightarrow$$

~~kr~~ kirjutame selle kompleksarvu polaarkoordinaatides.

$$r = \sqrt{a^2+b^2} = \sqrt{\left(\frac{2-\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 0{,}518$$

$$\theta = \arctan\left(\frac{b}{a}\right) = 1{,}3 \text{ rad}$$

$$\Rightarrow \quad \left(0{,}518\, e^{i\,1{,}3}\right)^{24} = 1{,}3\cdot 10^{-7}\, e^{i\,31{,}2}$$

3

a) $(-1)^{1/4} \rightarrow z^4 = -1 \qquad z^4 = e^{i(\pi + 2k\pi)} \Big| ^{1/4}$

$z = e^{\frac{i(\pi + 2k\pi)}{4}} \qquad k = 0 \ldots n-1$

$z_1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \qquad z_2 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$

$z_3 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i \qquad z_4 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$

Vastus on kooskõlas fundamentaalse algebra teoreem

b) $-i^{\frac{1}{3}} = a + bi \;\Big|^3 \qquad -i = (a+bi)^3 \qquad -i = a^3 + 3a^2bi + 3ab^2i^2 +$

$-i = a^3 + 3a^2bi - 3ab^2 - b^3 i \qquad -i = (a^3 - 3ab^2) + (3a^2b - b^3)i$

ehk $\begin{cases} 3a^2b - b^3 = -1 \\ a^3 - 3ab^2 = 0 \end{cases}$ $\quad a^2 = 3b^2 \quad 9b^3 - b^3 = -1 \quad 8b^3 = -1$ $\quad b = -0{,}5 \quad a^2 = 3(-0{,}5)^2 \quad a = \pm\frac{\sqrt{3}}{2}$

$z_1 = \frac{\sqrt{3}}{2} - 0{,}5$

$z_2 = -\frac{\sqrt{3}}{2} - 0{,}5$

$z_3 = i$

4. Tõestame kolmnurga võrratuse

$$|z_1 + z_2| \leq |z_1| + |z_2|$$

$$
\begin{aligned}
|z_1 + z_2|^2 &= (z_1 + z_2)\overline{(z_1 + z_2)} \\
&= (z_1 + z_2)(\bar{z}_1 + \bar{z}_2) \\
&= z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2 \\
&= |z_1|^2 + 2\,\mathrm{Re}(z_1\bar{z}_2) + |z_2|^2 \\
&\leq |z_1|^2 + 2|z_1\bar{z}_2| + |z_2|^2 \\
&= |z_1|^2 + 2|z_1||\bar{z}_2| + |z_2|^2 \\
&= |z_1|^2 + 2|z_1||z_2| + |z_2|^2 \\
&= (|z_1| + |z_2|)^2
\end{aligned}
$$

ehk saame

$$|z_1 + z_2|^2 \leq (|z_1| + |z_2|)^2 \quad | \sqrt{}$$

$$|z_1 + z_2| \leq |z_1| + |z_2|$$

5 $z = (\bar{z})^3$

$a + bi = (a - bi)^3$

$a + bi = a^3 - 3a^2bi + 3a(bi)^2 - (bi)^2$

$a + bi = a^3 - 3a^2bi - 3ab^2 + b^3i$

reaalosad ja imaginaarosad peavad võrrandi pooltel olema võrdsed.

$$\begin{cases} a = a^3 - 3ab \\ b = -3a^2b + b^3 \end{cases}$$

$$\begin{cases} a(a^2 - 3b - 1) = 0 \\ b(-3a^2 + b^2 - 1) = 0 \end{cases}$$

~~lahendid~~

kui $b = 0$

$a(a^2 - 1) = 0$

$a = 0$ või $a = \pm 1$

kui $a = 0$

$b(b^2 - 1) = 0$

$b = 0$ või $b^2 = 1$ $b = \pm 1$

siit saame lahendid

$z_1 = 0$ $z_2 = 0 + i$

$z_5 = 0 - i$

siit saame veel ~~kaks~~ lahendit

$z_3 = 1 + 0i$

$z_4 = -1 + 0i$

Vastused: $z_1 = 0$ $z_2 = i$ $z_3 = -i$ $z_4 = 1$ $z_5 = -$