1 Taavi Tammaru

Inertsmomendi tuletamine

$$I = \int m r^2 \, dr$$

kus on ühtlane massijaotus

vaatame pulka kui kahe võrdse osana

$$I = 2m\int_0^{l/2} r^2 \, dr = 2m \frac{r^3}{3}\Bigg|_0^{l/2}$$

$$= 2m \frac{l^2}{24}$$

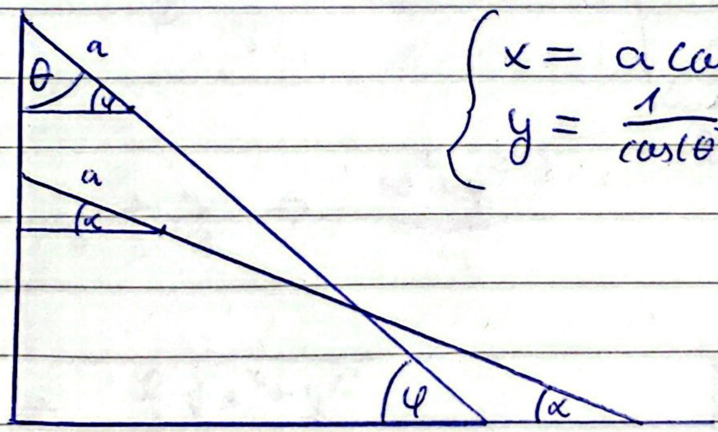
$$= \frac{ml^2}{12}$$

**2**

suvalise punkti trajektoori võrrand xy-koordinaadistikus.

kuna ülesandes ei ole aeg tähtis, siis ülesanne taandub täielikult matemaatikale.

vaatame varrast algasendis ja varrast pärast mingit hetke



$$\begin{cases} x = a\cos(\alpha) \\ y = \dfrac{1}{\cos(\theta)} \cdot \sin(\alpha) - \frac{a}{1}\sin(\alpha)a \end{cases}$$

$\cos(\alpha) = \dfrac{x}{a}$ $\qquad \sin^2 = 1 - \cos^2$

$$\sin(\alpha) = \sqrt{1 - \frac{x^2}{a^2}}$$

arendame $\sin(\alpha)$ $y$-i võrrandisse

$$y = \sin(\alpha)\left(\frac{1}{\cos(\theta)} - a\right)$$

$$y = \sqrt{1 - \frac{x^2}{a^2}}\left(\frac{1}{\cos(\theta)} - a\right)$$

$a$ ja $\theta$ on konstandid.

3

massikeskme liikumise kiirus on

$v = \frac{1}{2} l \dot{\varphi}$

$T_1 = \frac{1}{2} m \left(\frac{1}{4} l^2 \dot{\varphi}^2\right) = \frac{1}{8} m l^2 \dot{\varphi}^2$

varda inertsmoment on $I = \frac{1}{12} m l^2$

$T_2 = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \frac{1}{12} m l^2 \dot{\varphi}^2 = \frac{1}{24} m l^2 \dot{\varphi}^2$

$T = \left(\frac{1}{24} + \frac{1}{8}\right)\left(m l^2 \dot{\varphi}^2\right) = \frac{1}{6} m l^2 \dot{\varphi}^2$

lagrange funktsioon on

$L = \frac{1}{6} m l^2 \dot{\varphi}^2 - \frac{1}{2} m g l \sin(\varphi)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = \frac{\partial L}{\partial \varphi}$ $\qquad \frac{\partial L}{\partial \dot{\varphi}} = \frac{1}{3} m l^2 \dot{\varphi}$

$\frac{d}{dt}\left(\frac{1}{3} m l^2 \dot{\varphi}\right) = \frac{1}{3} m l^2 \ddot{\varphi}$

$\frac{\partial L}{\partial \varphi} = -\frac{1}{2} m g l \cos(\varphi)$

$\frac{1}{3} m l^2 \ddot{\varphi} = -\frac{1}{2} m g l \cos(\varphi)$

$\frac{1}{3} l \ddot{\varphi} = -\frac{1}{2} g \cos(\varphi)$

$\ddot{\varphi} = -\frac{3}{2} \frac{g}{l} \cos(\varphi)$ $\qquad \int \ddot{\varphi}\, dt = \int -\frac{3}{2} \frac{g}{l} \cos(\varphi)\, dt$

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lahendame teise astme differentsiaalvõrrandi

$$\ddot{\varphi} = -\frac{3}{2}\frac{g}{l}\cos(\varphi)$$

$$\ddot{\varphi} = -\frac{3}{2}\frac{g}{l}\cos(\varphi)$$

aga sellist teise astme ODE-d mulle koolis ei õpetatud lahendama

ehk korrutame tunnis näidatud soovitust.

$$\ddot{\varphi}\dot{\varphi} = \frac{d}{dt}\left(\frac{\dot{\varphi}^2}{2}\right) = -\frac{w_0^2}{2}\frac{d}{dt}(\sin(\varphi))$$

vaatame $\frac{d}{dt}\left(\frac{\dot{\varphi}}{2}\right) = -\frac{w_0^2}{2}\frac{d}{dt}(\sin(\varphi))$

korrutame mõlemad pooled $2\frac{dt}{d}$-ga ja saame

$$\dot{\varphi}^2 = -w_0^2\sin(\varphi)$$

$$\dot{\varphi} = -\sqrt{w_0^2\sin(\varphi)}$$

$\dot{\varphi} = w_0\sqrt{\sin(\varphi)}$ on kiiruse võrrand

alternatiivselt $\dot{\varphi} = \int -\frac{3}{2}\frac{g}{l}\cos(\varphi)\,dt$

lisaülesanneteni mina ei jõudnud vaevu sain põhiülesannete jaoks midagi kirja

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