Exam Physics of Modern Technology (WBPH027-05)

Friday 27 January 2023, 8:30 – 10:30

- This exam contains 3 questions on 5 pages and is designed for a **2-hour duration**.
- For each of the 3 questions (parts Guimarães, Van der Wal, Giuntoli), **please start your answer on a new separate answer sheet**.
- Write your name and student number on **all answer sheets** that you turn in.
- Clearly write the total number of answer sheets that you turn in on your first answer sheet.
- Each question is 20 points, the points per sub-question are marked.

Useful constants:

Electron mass	$m_e = 9.109 \cdot 10^{-31}$ kg
Electron charge	$-e = -1.602 \cdot 10^{-19}$ C
Speed of light in vacuum	$c = 299792458$ ms^{-1} (exact)
Planck's constant	$h = 6.626 \cdot 10^{-34}$ Js $= 4.136 \cdot 10^{-15}$ eVs
Planck's reduced constant	$\hbar = 1.055 \cdot 10^{-34}$ Js $= 6.582 \cdot 10^{-16}$ eVs
Energy units	1 eV $= 1.602 \cdot 10^{-19}$ J

Question 1 (part Guimarães) [20 points]

A) Consider an electron microscope with an acceleration voltage of 100 kV. Assuming that the electrons are non-relativistic electrons, their wavelength (in nanometers) is given by $\lambda = {}^{1.22}/_{\sqrt{E}}$, with E given in eV.

4pts **a1)** As you know, the resolution of an electron microscope is limited by its spherical aberration, given by $r = \alpha^3 C_s$, where α is the half-opening angle of the lens with spherical aberration constant C_s. Derive an equation for the optimum resolution of this microscope by putting the Rayleigh criterion resolution equal to the spherical aberration resolution. Here assume $NA \approx \alpha$.

3pts **a2)** Calculate the spherical aberration-limited resolution you obtained in the previous question using the non-relativistic wavelength for the electrons and a typical $C_s = 1$ mm.

5pts **B)** A proton radiation therapy machine is tuned to emit protons with an energy of 100 MeV. What is the velocity of this proton beam? Give your answer in units of the speed of light. *Tip*: The total (relativistic) energy of the proton is a sum of its relativistic rest energy with the energy from the acceleration voltage. Use the proton mass $m_p = 1.67 \times 10^{-27}$ kg.

C) In a flash memory device, the data is stored by injecting electrons into a floating gate electrode.

2pts **c1)** How much does a gigabyte of data weigh? Assume the weight of the data is fully given by the weight of the electrons in a floating gate electrode of area 100x100 nm^2, and that you need an electron density of about $5 \times 10^{12}/\text{cm}^2$ to write one bit of data.
Tip: Remember that 1 byte = 8 bits.

3pts **c2)** Using the tunneling current flux current given by $J_e = A_i \exp(-L_i)$, with $A_i = \frac{V_i}{4\pi\hbar t^2}$ and $L_i = \frac{t}{\hbar}\sqrt{8m_e V_i}$, calculate the flux of electrons from the floating gate to the channel. Here $\hbar$ is the reduced Planck's constant (value on page 1), m_e is the effective electron mass (value on page 1), t is the dielectric barrier thickness, and V_i the barrier height (in energy). Assume $V_{FG} = 3.00$ eV, $V_{CH} = 3.50$ eV, and $t = 3.0$ nm.

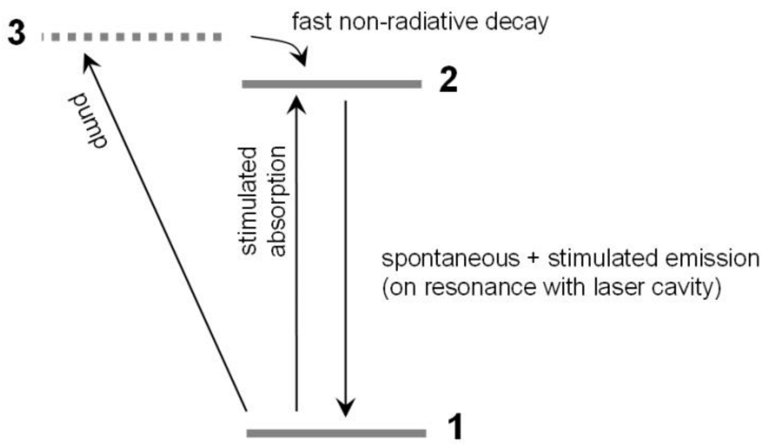
Tip A: If needed use, 1 Joule = 1 kg m^2/s^2.
Tip B: You might need to calculate just one of the fluxes if you give a reason to it.

3pts **c3)** How long would you have before all the electrons stored in the device of question c1) "leak out"?

Question 2 (part Van der Wal)   [20 points]

A) - Knowledge questions about lasers

In the study material the operational mechanism of the laser is explained with help of the following figure. In this figure the labels 1, 2 and 3 refer to energy levels of states that can be occupied by an electron.



3pts **a1)** Explain the following in about 5 lines of text:
Describe where the electrons are in this system if there is a suitable population inversion for the laser to be in operation (that is, for the laser to be lasing).

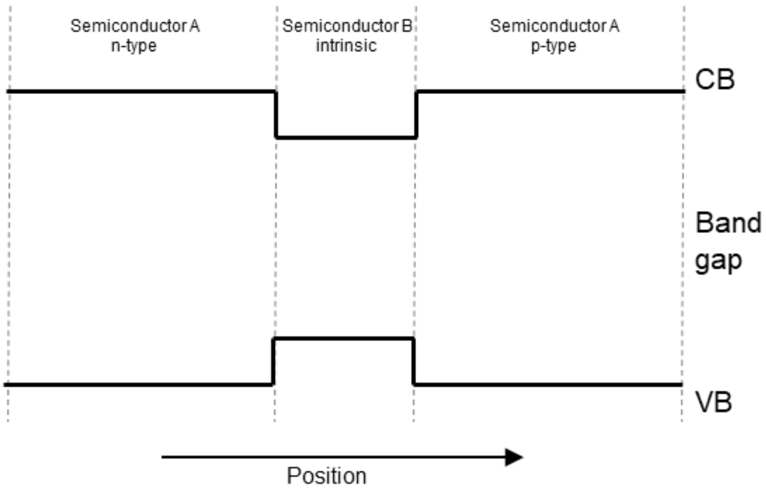
3pts **a2)** Explain the following in about 5 lines of text:
Why is a population inversion needed for the operational mechanism of a laser?

3pts **a3)** The pumping that takes care of maintaining a population inversion is in the figure (above) a process that excites electrons from level 1 to level 3. Is it also possible that the laser works properly if such a pump drives transitions of electrons from level 1 to level 2? (Assume that the laser still uses this 3-level system.) Explain your answer in about 5 lines of text.

B) - Question about LED system

A company produces LEDs where the semiconductor material consists of 3 layers: a layer of n-type of semiconductor A, a layer of intrinsic semiconductor B, and a layer of p-type of semiconductor A.

The following figure presents a sketch of the band diagram of this system as a function of position (CB is conduction band, VB is valance band). The electrical contacts are on the left and right of this diagram. Because of optical coatings, all photons that come out travel to the right.



In material A electron-hole recombination does almost not occur. In material B electron-hole recombination occurs fast, and is described by an optical spontaneous emission time of τ = 20 ns. You can assume that the system is 100% efficient.

2pts **b1)** To get light out of this system you need to apply an electrical current between the contacts. Explain whether this system will function best when the electrical current runs from left to right, or from right to left.

3pts **b2)** If you apply a very short pulse of electrical current at time t = 0, the number of electrons in material B, *n(t)*, shows the following behavior:

$$n(t) = n(0)e^{-\frac{t}{\tau}}.$$

Show that the expression for $n(t)$ is a solution to the rate equation (differential equation)

$$\frac{dn(t)}{dt} = -\frac{1}{\tau}\,n(t).$$

3pts **b3)** For the case of question b2), how long does it take until *n(t)* drops to 5% of the initial value right after the pulse?

3pts **b4)** Now assume that you operate the LED by applying a constant electrical current to it. This brings electrons in material B at a rate Q (which has units 1/s). For this situation, the relevant rate equation becomes

$$\frac{dn(t)}{dt} = -\frac{1}{\tau}\ n(t) + Q.$$

To find out what the steady-state value is for number of photons that come out per second (that is, the average value when applying a constant current) you need to solve:

$$\frac{dn(t)}{dt} = 0.$$

Calculate the steady-state number of photons that comes out per second for the case that the current is 1 mA.

Question 3 (part Giuntoli) [20 points]

Fused filament 3D-printers are typically plugged to the electric system with a power cord, but would it be efficient to build a portable printer that is powered by batteries? With simplifying assumptions, you can estimate it in a few steps.
Even without/with a wrong answer from previous steps, you can solve the following questions by keeping a literal expression or using whatever value you found.

A) At equilibrium, the viscosity $\eta_{eq}(T)$ of the thermoplastic polymer used in the filament varies with temperature following a Vogel-Fulcher-Tamman equation with parameters $\eta_0 = 6.738$ Poise, $B = 1000$ K, and $T_{VF} = 300$ K. Here, η_0 is the high-temperature limit of the viscosity *at equilibrium*. Under shear, the viscosity decreases following a cross model with a $\lambda = 1$ s, shear thinning exponent $n = 0.5$, and viscosity in the limit of zero shear rate equal to η_{eq}. The printer applies a shear rate of $\dot{\gamma} = 81$ Hz.

3pts **a1)** What should be the equilibrium viscosity η_{eq} to have a viscosity $\eta_{shear} = 100$ Poise during extrusion?

4pts **a2)** At which temperature $\bar{T}$ should the polymer filament be heated to achieve η_{eq}?

B) For a simple printer, we assume that all electrical power is needed to heat the polymer filament. The filament is heated by a cartridge heater, a large resistor which dissipates thermal heat Q per unit time equal to its electrical power P (Joule heating). Assume no losses, so that all the power of the cartridge is converted into heat for the filament. For an infinitesimal input heat dQ, the temperature increases following the law $dQ = C(T)dT$, where $C(T) = 7.444 \times 10^{-4} \times T$ is the polymer heat capacity, measured in Joule/Kelvin. The filament remains in touch with the cartridge for a time $t = 2$ s.

4pts **b1)** How much heat $\bar{Q}$ must be provided to the polymer filament to go from room temperature (298 K) to the target temperature $\bar{T}$?

2pts **b2)** What is the required power $\bar{P}$ of the cartridge heater?

Tip 1: note that unlike the ideal gas case, the heat capacity $C(T)$ is not constant but varies linearly with temperature. You thus need to perform an integral between room temperature and $\bar{T}$.
Tip 2: you can use dimensional analysis to check how Q and P are related.

C) Consider the cartridge heater as a resistance $R = 7.5\ \Omega$ connected in a circuit to multiple AA batteries in series. Each battery has internal resistance $r = 0.5\ \Omega$, $Emf = 1.5$ V, and total charge $C = 2000$ mAh.

1pts c1) Draw the equivalent circuit.

2pts c2) For a given power $\bar{P}$, what are the values for the current I and the voltage drop V_{cart} through the cartridge?

2pts c3) How many AA batteries are needed to supply power $\bar{P}$ to it?

2pts c4) For how long can the batteries supply power to the printer until they are depleted?