1 Vectors

1. **Assertion (A) :** The vectors

   $\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$

   $\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$

   $\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$

   represent the sides of a right angled triangle.

   Reason (R) : Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

2. Find the vector equation of the line passing through the point $(2, 3, -5)$ and making equal angles with the co-ordinate axes.

3. Find a vector of magnitude $4\,units$ perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ and hence verify your answer.

4. (a) Find the co-ordinates of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.
   Also, find the perpendicular distance of the given point from the line.

   (b) Find the shortest distance between the lines L_1 & L_2 given below:
   L_1 : The line passing through $(2, -1, 1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$
   $L_2 : \vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

2 Matrices

1. (a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$ then find A^{-1} and hence solve the following system of equations :

$$\begin{aligned} x + 2y - 3z &= 1 \\ 2x - 3z &= 2 \\ x + 2y &= 3 \end{aligned}$$

   (b) Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence solve the system of linear equations :

$$\begin{aligned} x + 2y - 3z &= -4 \\ 2x + 3y + &= 2z = 2 \\ 3x - 3y - 4z &= 11 \end{aligned}$$

3 Differentiation

1. (a) Verify whether the function f defined by

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is continuous at $x = 0$ or not.

   (b) Check for differentiability of the function f defined by $f(x) = |x-5|$, at the point $x = 5$.

2. (a) Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.

   (b) If$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

3. If $x = a\sin^3\theta$, $y = b\cos^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

4. (a) Find the particular solution of the differential equation $\frac{dy}{dx} - 2xy = 3x^2e^{x^2}$; $y(0) = 5$.

   (b) Solve the following differential equation : $x^2dy + y(x+y)dx = 0$

4 Integration

1. (a) Find : $\int \cos^3 x\, e^{log \sin x} dx$

   (b) Find: $\int \frac{1}{5+4x-x^2} dx$

2. (a) Evaluate: $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

   (b) Find: $\int \frac{2x+1}{(x+1)^2\,(x-1)} dx$

3. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration

5 Probability

1. The random variable X has the following probability distribution where a and b area some constants:

X	1	2	3	4	5
$P(X)$	0.2	a	a	0.2	b

   If the mean $E(X) = 3$, then find values of a and b and hence determine $P(X \geq 3)$.

2. A departmental store sends bills to charge its customers once a month bill in time. The store also found that 70% of its customers pay their first month bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.
   Based on the above information, answer the following questions:

   (i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time.
   Find $P(E_1)$, $P(E_2)$.

   (ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E)$.

   (iii) Find the probability of customer paying second month's bill in time.

   (iv) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

6 Trignometry

1. Find the value k if

$$\sin^{-1}\left[k \tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}.$$

7 Circles

1. The area of the circle is increasing at a uniform rate of $2\,cm^2/sec$. How fast is the circumference of the circle increasing when the radius $r = 5\,cm$?

8 Relations

1. (a) Students of a school are taken to a railway museum to learn about railway heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$$R = \{(l_1, l_2) : l_1 \parallel l_2\}$$

On the basis of the above information, answer the following questions:

(i) Find whether the relation R is symmetric or not.

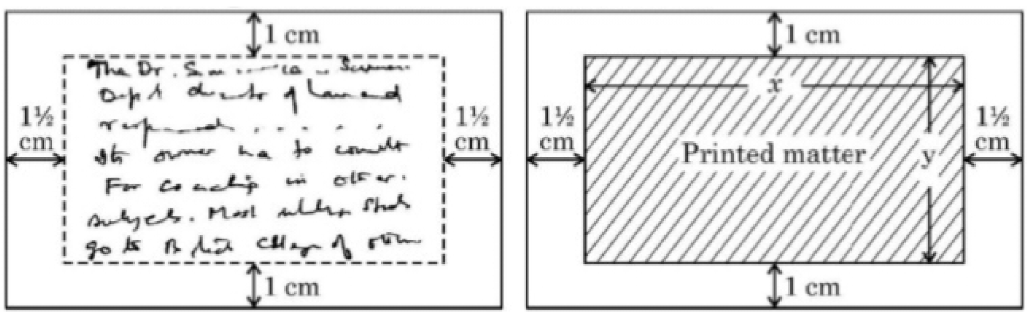
(ii) Find whether the relation R is transitive or not.

(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

(b) Let S be the relation defined by S=$\{(l_1, l_2) : l_1 \perp l_2\}$ check whether the relation S is symmetric and transitive.

9 Calculus

1. A rectangular visiting card is to contain $24\,sq.cm$ of printed matter. The margins at the top and bottom of the card are to be $1\,cm$ and the margins on the left and right are to be $1\frac{1}{2}cm$ as shown below :



On the basis of the above information, answer the following questons:

(i) Write the expression for the area of the visiting card in terms of x.

(ii) Obtain the dimensions of the card of minimum area.

10 Optimization

1. Solve the following L.P.P. graphically :
Maximize $Z = 60x + 40y$
Subject to

$$\begin{aligned} x + 2y &\leq 12 \\ 2x + y &\leq 12 \\ 4x + 5y &\geq 20 \\ x, y &\geq 0 \end{aligned}$$