

1 Differentiation

1. (a) Verify whether the function f defined by

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is continuous at $x = 0$ or not.

- (b) Check for differentiability of the function f defined by $f(x) = |x - 5|$, at the point $x = 5$.
2. (a) Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.
- (b) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.
3. If $x = a \sin^3 \theta$, $y = b \cos^3 \theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.
4. (a) Find the particular solution of the differential equation $\frac{dy}{dx} - 2xy = 3x^2 e^{x^2}$; $y(0) = 5$.
- (b) Solve the following differential equation :
 $x^2 dy + y(x+y)dx = 0$