(2)

a)
```
def fibonacci_1(n):
    if n<0 :                                  ——— 1
        print("Invalid input")
    elif n<=1 :                               ——— 1
        return n
    else
        return fibonacci_1(n-1)+fibonacci_1(n-2)
                                              ——— T(n-1)+T(n-2)
n = int(input("Enter a number:"))
n_th_fib = fibonacci_1(n)
print("The {} th fibonacci number is {}.".
      format(n, nth_fib))
```

$$T(n) = 1 + 1 + T(n-1) + T(n-2)$$
$$= 2 + T(n-1) + T(n-2)$$

Assume,

$$T(n-2) \approx T(n-1) \quad \therefore T(n) = 2T(n-1) + c$$

$$\therefore T\,n = \begin{cases} 1 & n <= 2 \\ 2T(n-1)+c & n > 2 \end{cases}$$

Now,

$$T(n) = 2T(n-1) + c$$
$$= 2\left[2T(n-2) + c\right] + c$$
$$= 2^2 T(n-2) + 2c + c$$

$\because T(n) = 2T(n-1) + c$

$\therefore T(n-1) = 2T(n-2) + c$

$\therefore T(n-2) = 2T(n-3) + c$

$\therefore T(n-3) = 2T(n-4) + c$

$= 2^2 [2T(n-3)+c] + 2c + c$

$= 2^3 T(n-3) + 2c^2 + 2c + c$

$\vdots$

$= 2^k T(n-k) + 2^{k-1} c + 2^{k-2} c + \ldots + 2c + c$

Assume;

$n - K = 0$

$\therefore n = K$

$\therefore T(n) = 2^n T(n-n) + 2^{n-1} c + 2^{n-2} c$

$= 2^n \{T(0) + 2^{n-1} \cdot c + 2^{n-2} c$

$= 2^n * 1 + 1 + 2 + 2^2 + \ldots + 2^{k-1}$

$= 2^n + 2^k - 1$

$= 2^n + 2^n - 1$

$= 2^{n+1}$

$= O(2^n)$

$\therefore$ Time complexcity $= O(2^n)$

```
def fibonacci_2(n):
    if n<0:                                    ——— 1
        return "Invalid input"
    if n<=1:                                   ——— 1
        return n
    fib = [0]*(n+1)
    fib[0] = 0
    fib[1] = 1
    for i in range(2, n+1):                    ——— n
        fib[i] = fib[i-1] + fib[i-2]
    return fib[n]
n = int(input("Enter a number:"))
nth_fib = fibonacci_2(n)
print("The {}th fibonacci number is {}.".format
                    (n, nth_fib))
```

$\therefore T(n) = 1+1+n$

$= 2+n$

$\div n+c$

$\therefore$ Time complexcity $O(n)$

Now,
implementation 1, time complexcity is $O(2^n)$ and in implentation 2 time complexcity is $O(n)$

Here,

$$O(n) < O(2^n)$$

∴ Implementation 2 is better than 1 because it will take less time.