Tutorial Letter 103/0/2024

Formal Logic III

COS3761

Year module

Computer Science Department

This tutorial letter contains the three assignments for COS3761.

BARCODE

ASSIGNMENT 1

Please note: There are 20 questions in this assignment. Choose 1 option out of 4 options. Assignment weighs 30% of the year mark

Propositional logic symbol	**Declarative sentence associated with the symbol**
p	Sam comes to the club
q	Susan comes to the club
r	Titus comes to the club
s	Bob comes to the club

Table 1

QUESTION 1

Using the symbols and their intended meaning given in Table 1, which of the options below is a correct propositional logic translation of the following English sentence?

It is not the case that Sam and Bob come to the club.

Option 1: $\neg (p \wedge s)$

Option 2: $\neg (p \vee s)$

Option 3: $p \wedge s$

Option 4: $\neg p \wedge s$

QUESTION 2

Using the symbols and their intended meaning given in Table 1 which of the options below is a correct propositional logic translation of the following English sentence?

If Susan comes to the club then Bob and Sam come too.

Option 1: $p \wedge s \rightarrow q$

Option 2: $q \rightarrow p \wedge s$

Option 3: $(q \rightarrow p) \wedge s$

Option 4: $q \wedge p \rightarrow s$

.

QUESTION 3

Using the symbols and their intended meaning given in Table 1, which of the options below is a correct propositional logic translation of the following English sentence?

Bob comes to the club if and only if Susan comes and Titus does not come.

Option 1: $(s \rightarrow q \land \neg r)$

Option 2: $(q \land \neg r \rightarrow s)$

Option 3: $(s \leftrightarrow q \land \neg r)$

Option 4: $(q \land \neg r \leftrightarrow s)$

QUESTION 4

Using the symbols and their intended meaning given in Table 1, which of the options below is a correct English translation of the following propositional logic sentence?

$\neg(s \rightarrow \neg q)$

Option 1: It is the case that if Bob comes to the club then Susan comes.

Option 2: It is not the case that if Bob comes to the club then Susan does not come.

Option 3: It is the case that if Bob comes to the club then Susan comes.

Option 4: It is not the case that if Bob comes to the club then Susan comes.

QUESTION 5

Using the symbols and their intended meaning given in Table 1, which of the options below is a correct English translation of the following propositional logic sentence?

A necessary condition for Susan coming to the club, is that, if Bob and Titus aren't coming, Sam comes.

Option 1: $q \rightarrow (\neg s \land \neg r \rightarrow p)$

Option 2: $q \rightarrow (\neg s \land \neg r \land p)$

Option 3: $q \land p \rightarrow (\neg s \land \neg r \land p)$

Option 4: $(\neg s \land \neg r \rightarrow p) \rightarrow q$

QUESTION 6

Using the symbols and their intended meaning given in Table 1, which of the options below is a correct English translation of the following propositional logic sentence?

$(\neg r \land \neg p)$

Option 1: Titus does not come to the club and Sam does not come to the club.

Option 2: Titus does not come to the club and Sam comes to the club.

Option 3: Titus and Sam come to the club.

Option 4: Titus come to the club and Sam does not come to the club.

QUESTION 7

Suppose you have to formally prove the validity of the following sequent using the basic natural deduction rules:

$$p \vee q,\ \neg q \vdash p$$

Write out a proof on some rough paper and then choose the option below that gives a correct strategy.

Option 1:

- Start with the two premises $p \vee q$ and $\neg q$.
- Open an assumption box and assume p.
- Using the copy rule, derive p.
- Assume q.
- Using the ¬ elimination rule, derive ⊥.
- Derive p using the ⊥ elimination rule. Close the assumption box.
- Derive p using the ∨ elimination rule.

Option 2:

- Start with the two premises $p \vee q$ and $\neg q$.
- Open an assumption box and assume p.
- Using the copy rule, derive p. Close the assumption box.
- Open another assumption box and assume q.
- Using the ¬ elimination rule, derive ⊥.
- Derive p using the ⊥ elimination rule. Close the assumption box.
- Derive p using the ∨ elimination rule.

Option 3:

- Start with the two premises $p \vee q$ and $\neg q$.
- Open an assumption box and assume p.
- Using the copy rule, derive p. Close the assumption box.
- Open another assumption box and assume q.
- Using the ¬ elimination rule, derive ⊥.

- Close the assumption box.
- Derive p using the $\vee$ elimination rule.

Option 4:

- Start with the two premises $p \vee q$ and $\neg q$.
- Open an assumption box and assume p.
- Using the copy rule, derive p.
- Assume q.
- Using the copy rule, derive p. Close the assumption box.
- Derive p using the $\vee$ elimination rule.

QUESTION 8

Suppose you have to formally prove the validity of the following sequent using the basic natural deduction rules:

$$p \wedge q \rightarrow r \vdash (p \rightarrow ((p \rightarrow r) \vee (q \rightarrow r))) \vee s$$

First write out a proof on some rough paper and then choose the option below that is a correct proof.

Option 1:

1	$p \wedge q \rightarrow r$	premise
2	p	$\wedge$e 1
3	q	$\wedge$e 1, 2
4	$p \wedge q$	$\wedge$i 2, 3
5	r	$\rightarrow$e 1, 4
6	$q \rightarrow r$	$\rightarrow$i 3 - 5
7	$(p \rightarrow r) \vee (q \rightarrow r)$	$\vee i_2$ 6
8	$p \rightarrow ((p \rightarrow r) \vee (q \rightarrow r))$	$\rightarrow$i 2 – 7
9	$(p \rightarrow ((p \rightarrow r) \vee (q \rightarrow r))) \vee s$	$\vee i_1$ 8

Option 2:

1	$p \wedge q \rightarrow r$	premise
2	p	assumption
3	q	assumption
4	$p \wedge q$	$\wedge$i 2, 3
5	r	$\rightarrow$e 1, 4
6	$q \rightarrow r$	$\rightarrow$i 3 - 5
7	$(p \rightarrow r) \vee (q \rightarrow r)$	$\vee i_2$ 6
8	$p \rightarrow ((p \rightarrow r) \vee (q \rightarrow r))$	$\rightarrow$i 2 – 7
9	$(p \rightarrow ((p \rightarrow r) \vee (q \rightarrow r))) \vee s$	$\vee i_1$ 8

Option 3

1	$p \wedge q \rightarrow r$	premise
2	p	assumption
3	q	assumption
4	$p \wedge q$	$\wedge$i 2, 3
5	r	$\rightarrow$e 1, 4
6	$q \rightarrow r$	$\rightarrow$i 3 - 5
7	$(p \rightarrow r) \vee (q \rightarrow r)$	$\vee i_2$ 6
8	$p \rightarrow ((p \rightarrow r) \vee (q \rightarrow r))$	$\rightarrow$i 2 – 7
9	$(p \rightarrow ((p \rightarrow r) \vee (q \rightarrow r))) \vee s$	$\vee i_1$ 8

Option 4:

1	$p \wedge q \rightarrow r$	premise
2	p	assumption
3	q	assumption
4	$p \wedge q$	$\wedge$i 2, 3
5	r	$\rightarrow$e 1, 4
6	$q \rightarrow r$	$\rightarrow$i 3 - 5
7	$(p \rightarrow r) \vee (q \rightarrow r)$	$\vee i_2$ 6

8	$p \rightarrow ((p \rightarrow r) \vee (q \rightarrow r))$	$\rightarrow$i 2 – 7
9	$(p \rightarrow ((p \rightarrow r) \vee (q \rightarrow r))) \vee s$	$\vee i_1$ 8

QUESTION 9

We have to prove the validity of the following sequent using the rules of natural deduction:

$$p \rightarrow q,\ r \rightarrow \neg t,\ q \rightarrow r \vdash p \rightarrow \neg t$$

First write out a proof yourself on some rough paper. Then look at the proof below where two lines are omitted. Which of the options below gives the correct propositional logic formula and the associated correct rule in both lines?

1	$p \rightarrow q$	premise
2	$r \rightarrow \neg t$	premise
3	$q \rightarrow r$	premise
4		
5	q	$\rightarrow$e 1, 4
6	r	$\rightarrow$e 3, 5
7		
8	$p \rightarrow \neg t$	$\rightarrow$i 4 - 7

Option 1:

4	p	assumption
7	$\neg t$	$\rightarrow$e 2, 6

Option 2:

4	r	assumption
7	$\neg t$	$\rightarrow$e 2 - 6

Option 3:

4	r	assumption
7	p	$\rightarrow$e 2, 6

Option 4:

4	p	assumption
7	$\neg t$	$\neg$i 2

QUESTION 10

We have to prove the validity of the following sequent using the basic natural deduction rules:

$$p \rightarrow q \vdash \neg q \rightarrow \neg p$$

In the proof below the rules that are used in three of the lines are omitted. First write out a proof yourself on some rough paper. Then look at the proof below and choose the option giving the correct rule in each line.

1	$p \rightarrow q$	premise
2	$\neg q$	
3	p	assumption
4	q	$\rightarrow$e 1, 3
5	$\perp$	
6	$\neg p$	$\neg$i 3 - 5
7	$\neg q \rightarrow \neg p$	

Option 1:

2 assumption

5 ¬ e 2

7 → i 2 - 6

Option 2:

2 assumption

5 ¬ e 2

7 → i 1 - 6

Option 3:

2 assumption

5 ¬ e 4, 2

7 → i 2 - 6

Option 4:

2 assumption

5 ¬ e 4, 2

7 → i 2, 6

QUESTION 11

We have to prove the validity of the following sequent using the basic natural deduction rules:

$$p \vee q,\ p \rightarrow (s \wedge t),\ t \rightarrow r,\ q \rightarrow r \vdash r$$

In the proof below the formulas in three of the lines are omitted. First write out a proof yourself on some rough paper. Then look at the proof below and choose the option giving the correct formula in each line.

1	$p \vee q$	premise
2	$p \rightarrow (s \wedge t)$	premise
3	$t \rightarrow r$	premise
4	$q \rightarrow r$	premise
5		assumption
6		$\rightarrow$e 2, 5
7	t	$\wedge_2$e 6
8	r	$\rightarrow$e 3, 7
9	q	assumption
10		$\rightarrow$e 4, 9
11	r	$\vee$e 1, 5 – 8, 9 - 10

Option 1:

5 $p \vee q$

6 $s \wedge t$

10 r

Option 2:

5 p

6 s

10 r

Option 3:

5 p

6 $s \wedge t$

10 $q \vee r$

Option 4:

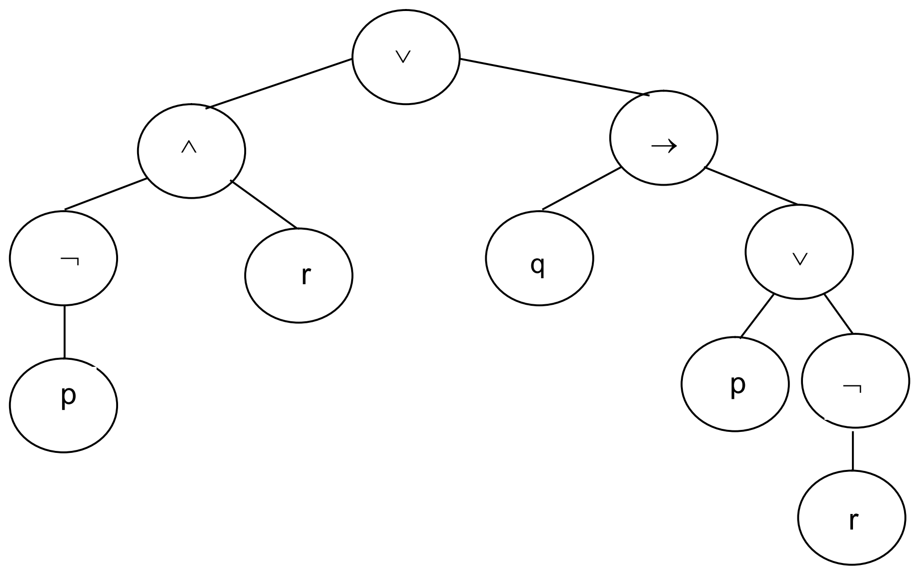
5	p
6	$s \wedge t$
10	r

QUESTION 12

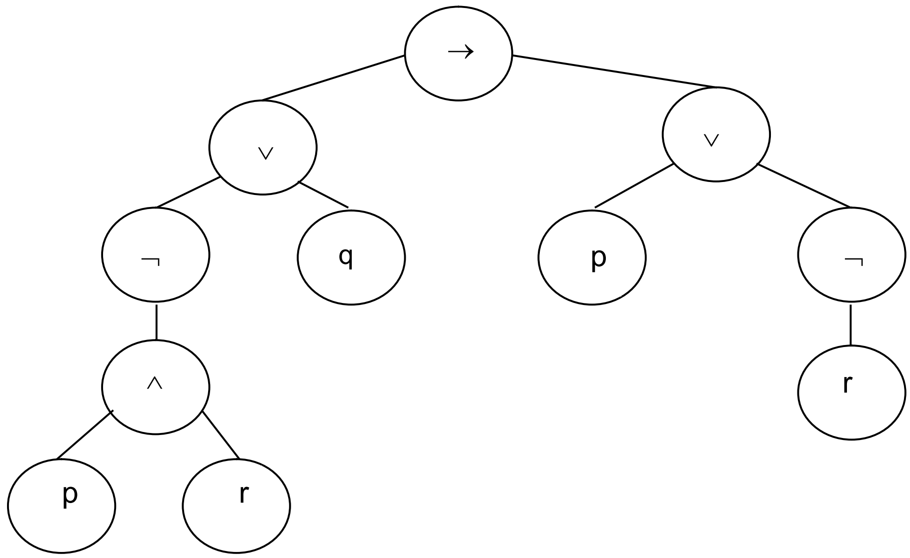
Given the following propositional logic formula, which of the options below is the corresponding parse tree?

$\neg (p \wedge r) \vee (q \rightarrow p \vee \neg r)$

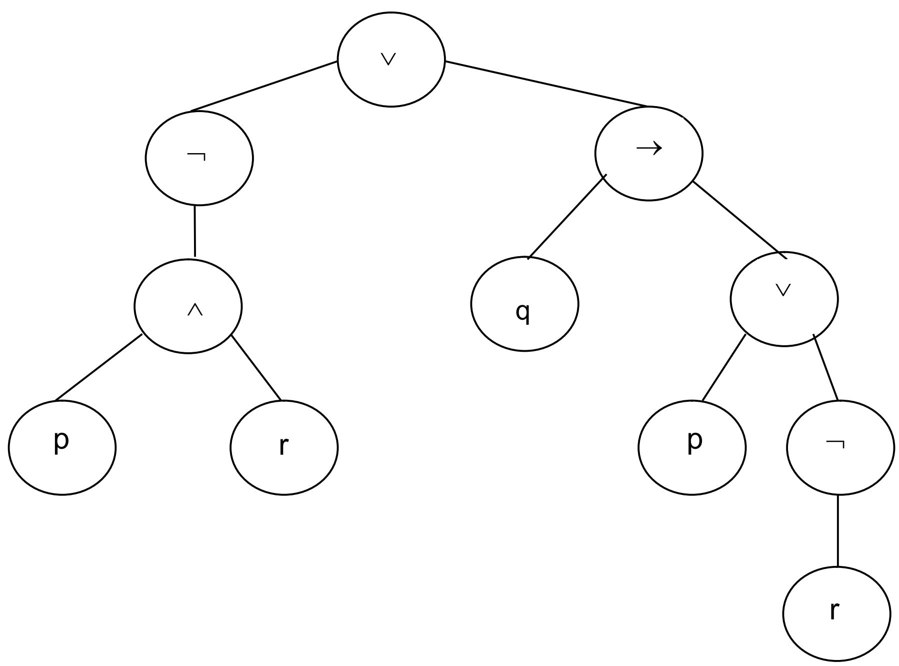
Option 1:



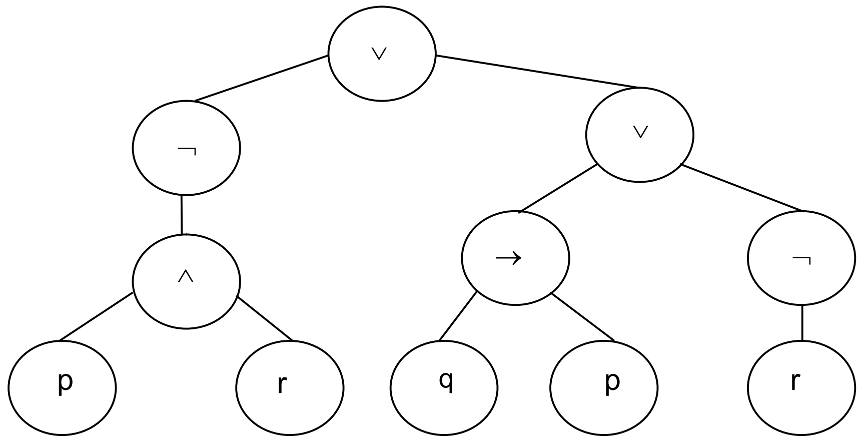
Option 2:



Option 3:

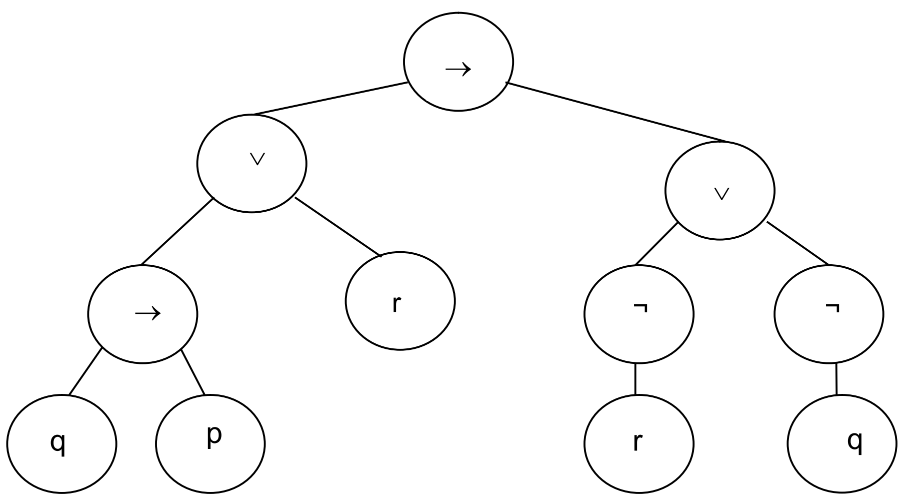


Option 4:



QUESTION 13

Given the following parse tree, which option below gives the associated propositional logic formula?



Option 1: $(q \rightarrow p) \vee (r \rightarrow \neg r \vee \neg q)$

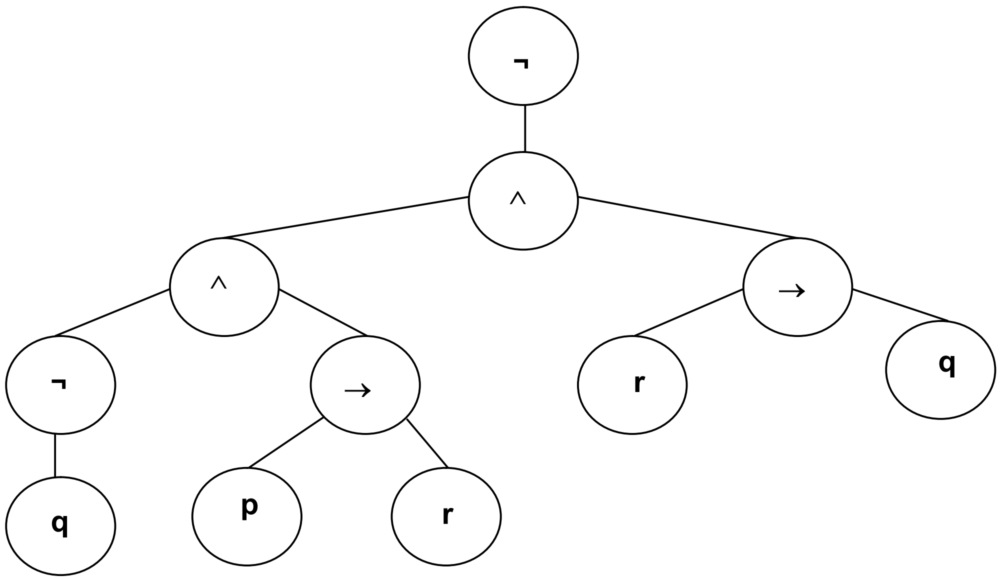
Option 2: $q \rightarrow p \vee r \rightarrow \neg r \vee \neg q$

Option 3: $q \rightarrow (p \vee r \rightarrow \neg r \vee \neg q)$

Option 4: None of the options above gives the correct associated sentence.

QUESTION 14

Given the following parse tree, which option below gives the associated propositional logic sentence?



Option 1: $\neg (\neg (q \wedge (p \rightarrow r)) \wedge (r \rightarrow q))$

Option 2: $\neg (\neg q \wedge (p \rightarrow r)) \wedge (r \rightarrow q)$

Option 3: $\neg ((\neg (q \wedge (p \rightarrow r)) \wedge (r \rightarrow q)))$

Option 4: $\neg ((\neg q \wedge (p \rightarrow r)) \wedge (r \rightarrow q))$

QUESTION 15

Consider the following sequent and then choose the correct option below.

$$(p \wedge q) \rightarrow s, \neg s \vdash p \vee \neg q$$

Option 1: The sequent is valid and can be formally proved using natural deduction rules.

Option 2: The sequent is valid because the relation $(p \wedge q) \rightarrow s, \neg s \vDash p \vee \neg q$ holds as shown by the following valuation: p = F, q = F, s = F.

Option 3: The sequent is not valid because the relation $(p \wedge q) \rightarrow s, \neg s \vDash p \vee \neg q$ does not hold as shown by the following valuation: p = F, q = T, s = F.

Option 4: The sequent is not valid because the relation $(p \wedge q) \rightarrow s, \neg s \vDash p \vee \neg q$ does not hold as shown by the following valuation: p = F, q = T, s = T.

QUESTION 16

Draw the truth tables of the following three propositional logic sentences and then choose the correct option below.

$\neg (p \wedge q) \wedge r$

$(\neg p \vee \neg q) \vee r$

$r \rightarrow (p \vee q)$

Option 1: $\neg (p \wedge q) \wedge r,\ (\neg p \vee \neg q) \vee r \vdash r \rightarrow (p \vee q)$

Option 2: The sentence $r \rightarrow (p \vee q)$ is not semantically entailed by the sentences $\neg(p \wedge q) \wedge r$ and $(\neg p \vee \neg q) \vee r$ because the respective three columns in the truth table are not identical.

Option 3: The sentence $r \rightarrow (p \vee q)$ is not semantically entailed by the sentences $\neg(p \wedge q) \wedge r$ and $(\neg p \vee \neg q) \vee r$ because of their respective truth values for the valuation $p = F, q = F, r = T$.

Option 4: The sentence $r \rightarrow (p \vee q)$ is semantically entailed by the sentences $\neg(p \wedge q) \wedge r$ and $(\neg p \vee \neg q) \vee r$ because of their respective truth values for the valuation $p = T, q = F, r = T$.

QUESTION 17

Consider the following:

$A, B \models C$

where A, B and C are propositional logic sentences. Choose the correct option below.

Option 1: $A, B \models C$ means that C is true whenever both A and B are true.

Option 2: $A, B \models C$ means that both A and B are true whenever C is true.

Option 3: $A, B \models C$ means that C will only be true if both A and B are true.

Option 4: $A, B \models C$ means that A, B and C have identical columns in a truth table.

Option 5: None of the options above is correct.

QUESTION 18

Suppose the HORN algorithm is used to determine whether the following propositional logic sentence is satisfiable or not:

$$(p \wedge q \wedge s \to p) \ \wedge\ (q \wedge r \to p) \ \wedge\ (p \wedge s \to s) \ \wedge\ (T \to r)$$

After the first step has been executed, we have the following (underlining is used to indicate marking):

$$(p \wedge q \wedge s \to p) \ \wedge\ (q \wedge r \to p) \ \wedge\ (p \wedge s \to s) \ \wedge\ (\underline{T} \to r)$$

Which of the options below gives the situation after the next step has been completed?

Option 1: $(p \wedge q \wedge s \to p) \ \wedge\ (q \wedge \underline{r} \to p) \ \wedge\ (p \wedge s \to s) \ \wedge\ (\underline{T} \to \underline{r})$

Option 2: $(\underline{p} \wedge q \wedge s \to p) \ \wedge\ (q \wedge r \to p) \ \wedge\ (\underline{p} \wedge s \to s) \ \wedge\ (\underline{T} \to r)$

Option 3: $(\underline{p} \wedge q \wedge s \to \underline{p}) \ \wedge\ (q \wedge \underline{r} \to \underline{p}) \ \wedge\ (\underline{p} \wedge s \to s) \ \wedge\ (\underline{T} \to \underline{r})$

Option 4: $(p \wedge q \wedge s \to p) \ \wedge\ (q \wedge r \to p) \ \wedge\ (p \wedge s \to s) \ \wedge\ (\underline{T} \to \underline{r})$

QUESTION 19

Suppose the HORN algorithm is used to determine whether the following propositional logic sentence is satisfiable or not:

$$(p \wedge q \to \bot) \ \wedge\ (q \wedge s \ \to\ T) \ \wedge\ (T \ \to\ q) \ \wedge\ (q \wedge p \wedge r \to s)$$

After the first step has been executed, we have the following (underlining is used to indicate marking):

$$(p \wedge q \to \bot) \ \wedge\ (q \wedge s \ \to\ \underline{T}) \ \wedge\ (\underline{T} \ \to\ q) \ \wedge\ (q \wedge p \wedge r \to s)$$

Which of the options below gives the situation after the next step has been completed?

Option 1: $(p \wedge \underline{q} \to \bot) \ \wedge\ (\underline{q} \wedge \underline{s} \ \to\ \underline{T}) \ \wedge\ (\underline{T} \ \to\ \underline{q}) \ \wedge\ (\underline{q} \wedge p \wedge r \to \underline{s})$

Option 2: $(\underline{p} \wedge q \to \bot) \ \wedge\ (q \wedge s \ \to\ \underline{T}) \ \wedge\ (\underline{T} \ \to\ q) \ \wedge\ (q \wedge \underline{p} \wedge r \to s)$

Option 3: $(\underline{p} \wedge \underline{q} \to \bot) \ \wedge\ (\underline{q} \wedge s \ \to\ \underline{T}) \ \wedge\ (\underline{T} \ \to\ \underline{q}) \ \wedge\ (\underline{q} \wedge \underline{p} \wedge r \to s)$

Option 4: $(p \wedge \underline{q} \to \bot) \ \wedge\ (\underline{q} \wedge s \ \to\ \underline{T}) \ \wedge\ (\underline{T} \ \to\ \underline{q}) \ \wedge\ (\underline{q} \wedge p \wedge r \to s)$

QUESTION 20

Which one of the options given below is not a well formed formula.

Option 1:. P

Option 2 : $\neg\neg p$

Option 3 : $(p \wedge p)$

Option 4 : $(\neg p \leftrightarrow r \vee s)$

ASSIGNMENT 02

Please note: Weight of contribution to year mark is 40%

Predicate symbols	
S(x)	x is a student
C(x)	x is a Computer subject
G(x)	x is a Geometry subject
T(x,y)	x takes y
L(x, y)	x loves y
k	Kevin
b	Bill

Table 2

QUESTION 1 **[15]**

Use the predicate, function and constant symbols and their intended meanings given in Table 2 to translate the English sentences given below into predicate logic:

Question 1.1

No students love Kevin.

Question 1.2

Every student loves some student.

Question 1.3

Every student who takes a Computer subject also takes a Geometry subject.

Question 1.4

Kevin takes a Computer subject while Bill does not.

Question 1.5

There is at least a student who takes a Geometry subject

QUESTION 2 **[12]**

Use the predicate, function and constant symbols and their intended meanings given in Table 2 and translate the following sentences of predicate logic into English:

Question 2.1

$\exists x(S(x) \land T(x, G) \land \forall y(S(y) \land T(y, G) \rightarrow x = y))$

Question 2.2

$\exists x(C(x) \land \forall y.(S(y) \rightarrow \neg T(y, x)))$

Question 2.3

$\forall x \exists y\ T(x, y)$

Question 2.4

$\forall x[S(x) \rightarrow L(k,x)]$

QUESTION 3 **[7]**

Let

- P and Q be two predicate symbols, each with two arguments,
- f a function symbol with one argument and
- c a constant symbol.
- x, y, z are variables

For each of the following, state whether it is a term or a well-formed formula (wff) or neither. If it is neither a term nor a wff, state the reason.

Question 3.1 $P(x, Q(x,y))$

Question 3.2 $\exists x\, \exists c\, P(x, c)$

Question 3.3 $\exists y\, (Q(x, y) \wedge (f(x) \vee f(y)))$

Question 3.4 $P(x, c) \vee \exists x\, Q(x)$

Question 3.5 $\forall x\, P(x, f(c)) \rightarrow \exists y\, Q(x, y)$

Question 3.6 $Q(x, f(x)) \rightarrow P(f(x), y)$

Question 3.7 $f(f(f(y)))$

QUESTION 4 **[10]**

Let φ be the formula

$$\exists x\, (\neg P(x, y) \rightarrow Q(x, y, z)) \rightarrow \forall y\, \exists z\, Q(x, x, x) \wedge P(x, y)$$

where P is a predicate symbol with two arguments and Q is a predicate symbol with three arguments.

Question 4.1 **(4)**

Draw the parse tree of the formula and indicate the free and bound variables.

Question 4.2 **(6)**

Suppose f is a function symbol with one argument. For each of the following substitutions, state whether it will create a problem. If there is no problem, write down the substituted formula. If there will be a problem, state how you would solve it and then write down the substituted formula.

Question 4.2.1 $\varphi[f(z) / x]$

Question 4.2.2 $\varphi[f(z) / y]$

Question 4.2.3 $\varphi[f(x) / y]$

QUESTION 5 **[5]**

Show that the following set of formulas is consistent by constructing either a mathematical or a non mathematical model where both formulas are true. Take *A*, the universe of concrete values, as the set of all integers. Note that the formulas involve two predicate symbols and a function symbol.

$\forall x\,(Q(x, f(x)) \rightarrow \neg Q(f(x), x))$

$\exists x\,(S(x) \wedge \neg \forall y\, Q(x, y))$

.

QUESTION 6 **[6]**

You are given the sentence

$\forall x\, \forall y\, ((R(x, y) \vee R(y, x)) \rightarrow \neg (R(x, y) \wedge R(y, x)))$

where R is a predicate with two arguments.

Question 6.1

Show that the sentence is satisfiable. In other words: Construct either a mathematical or a non mathematical model where the sentence is true.

Question 6.2

Show that the sentence is not valid. In other words: Construct either a mathematical or a non mathematical model where the sentence is false.

QUESTION 7 **[4]**

Given the sentence

$\forall x\, \exists y\, (R(x, y) \wedge R(y, y))$

does the model *M* below satisfy it? Explain your answer.

$A = \{a, b, c, d\}$

$R^M = \{(a, a), (b, a), (c, a), (d, b), (b, b)\}$

QUESTION 8 **[6]**

In this question you have to show that the validity of a sequent cannot be proved by finding a model where all formulas to the left of $\vdash$ evaluate to T but the formula to the right of $\vdash$ evaluates to F.

Question 8.1

Show that the validity of the following sequent

$\forall x\,(R(x) \rightarrow Q(x)) \vdash \forall x\,(R(x) \vee Q(x))$

cannot be proved by finding a *mathematical model* where the formula to the left of $\vdash$ evaluates to T but the formula to the right of $\vdash$ evaluates to F.

Question 8.2

Show that the validity of the following sequent

$\forall x\, \forall y\,(S(x, y) \rightarrow \neg S(y, x)) \vdash \forall x\, S(x, x)$

cannot be proved by finding a *non-mathematical model* where both formulas to the left of $\vdash$ evaluate to T but the formula to the right of $\vdash$ evaluates to F.

QUESTION 9 **[35]**

Using the rules of natural deduction, prove the validity of the following sequents in predicate logic. In all cases, number your steps, indicate which rule you are using and indicate subproof boxes clearly.

Question 9.1 **(8)**

$\exists x\, F(x) \vee \exists x\, G(x) \vdash \exists x\,(F(x) \vee G(x))$

Question 9.2 **(8)**

$\forall x\,(P(x) \vee Q(x)), \forall x(\neg P(x)) \vdash \forall x\, Q(x)$

Question 9.3 **(8)**

$\forall x\, \forall y\, \forall z\,((S(x, y) \wedge S(y, z)) \rightarrow S(x, z)), \forall x\, \neg S(x, x) \vdash \forall x\, \forall y\,(S(x, y) \rightarrow \neg S(y, x))$

Question 9.4 **(4)**

$\forall x\, \forall y\,(Q(y) \rightarrow F(x)) \vdash \exists y\, Q(y) \rightarrow \forall x\, F(x)$

Question 9.5 **(7)**

$\forall x\,(P(x) \wedge Q(x))\;(x) \vdash \forall x\,(P \rightarrow Q(x))$

ASSIGNMENT 03

Please note: There are 20 questions in this assignment. Choose 1 option out of 4 options. Assignment weighs 30% of the year mark

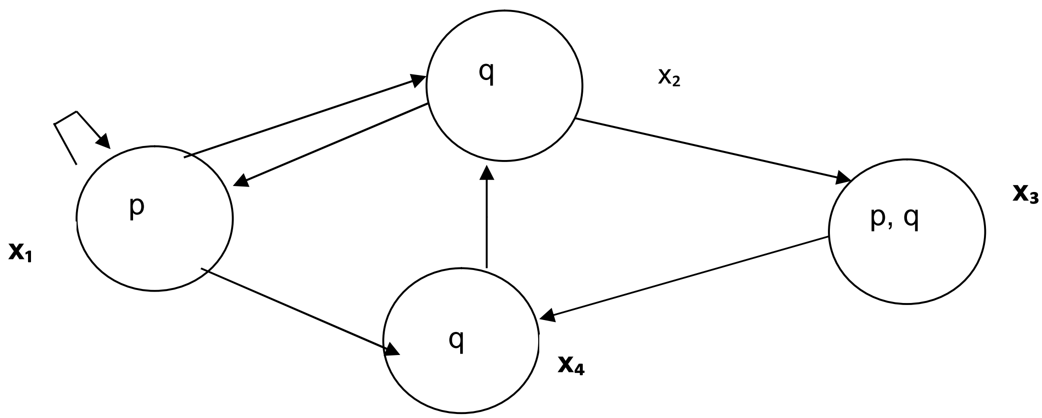


Figure 1: Kripke model used in Questions 1, 2, 3, 4 and 5

QUESTION 1

In which world of the Kripke model in Figure 1 is the formula $\Diamond p \wedge \Box q$ true?

Option 1: world x_1

Option 2: world x_2

Option 3: world x_3,

Option 4: Options 1 and 3 are true.

QUESTION 2

Which of the following does not hold in the Kripke model in Figure 1?

Option 1: $\mathbf{x_1} \Vdash \Diamond \Diamond p$

.*Option 2*: $x_2 \Vdash \Box p$

.*Option 3*: $x_3 \Vdash \Box p \wedge \Box q$

.*Option 4*: $x_4 \Vdash \Box\Box p$

QUESTION 3

Which of the following holds in the Kripke model given in Figure 1?

Option 1: $x_1 \Vdash \Box p$

.*Option 2*: $x_2 \Vdash \Diamond(p \vee q)$

.*Option 3*: $x_3 \Vdash \Diamond p \wedge \Box \neg q$

Option 4: $x_4 \Vdash \Box (p \wedge q)$

QUESTION 4

Which of the following formulas is true in the Kripke model given in Figure 1?

Option 1: $\Diamond p$

Option 2: $\Box q$

.*Option 3*: $\Box \Diamond q$

.*Option 4*: $\Box p$

QUESTION 5

Which of the following formulas is false in the Kripke model given in Figure 1?

Option 1: $p \vee q$

Option 2: $\Box \Diamond p$

Option 3: $\Box (p \vee q)$

Option 4: $p \vee \Diamond q$

QUESTION 6

If we interpret $\Box\ \phi$ as "It ought to be that ϕ", which of the following formulas correctly expresses the English sentence

It ought to be the case that if it rains outside then it is permitted to take leave from work.

where p stands for the declarative sentence "It rains outside " and q stands for “take leave from work”?

Option 1: $\Box\ (p \rightarrow \neg \Box \neg q)$

Option 2: $\Box(p \rightarrow \neg \Diamond q)$

Option 3: $\Box\ p \rightarrow \Diamond \neg q$

Option 4: $\Box\ p \rightarrow \Diamond q$

QUESTION 7

If we interpret $\Box\ \phi$ as "It is necessarily true that ϕ", why should the formula scheme $\Box\ \phi \rightarrow \Box \Box\ \phi$ hold in this modality?

Option 1: Because for all formulas ϕ, it is necessarily true that if ϕ then ϕ.

Option 2: Because for all formulas ϕ, if ϕ is necessarily true, then it is necessary that it is necessarily true.

Option 3: Because for all formulas ϕ, if ϕ is not possibly true, then it is true.

Option 4: Because for all formulas ϕ, ϕ is necessarily true if it is true.

QUESTION 8

If we interpret $\Box\ \phi$ as "the agent knows ϕ", why should the formula scheme $\Box\ \phi \rightarrow \Box \Box\ \phi$ hold in this modality?

Option 1: If the agent knows something he knows that he knows it.

Option 2: If the agent knows something it doesn’t mean that he knows.

Option 3: if the agent does not know something, he again knows that he knows it.

Option 4: if the agent knows something, he knows that he does not know it.

QUESTION 9

If we interpret $\Box\,\phi$ as "˝it is necessarily true", which of the following formula is not valid?

Option 1: $\Box\, p \rightarrow p$

Option 2: $\Box\, p \vee \Box\neg\, p$

Option 3: $\Box\, p \rightarrow \Diamond\, p$

Option 4: $\Diamond\, p \rightarrow \Box\,\Diamond\, p$

QUESTION 10

If we interpret $\Box\,\phi$ as "agent A believes ϕ", what is the modal translation of the English sentence

If agent A believes p then he believes that agent B does not believe q.

Option 1: $\Box\, p \rightarrow \Box\, q$

Option 2: $\Box\, p \rightarrow \neg\Box\, q$

Option 3: $\Box\, p \rightarrow \neg\Box\neg\, q$

Option 4: $\Box\, p \rightarrow \Box\neg\, q$

QUESTION 11

If we interpret $\Box\,\phi$ as "Agent A believes ϕ", English translation of the formula $\Box\, p \rightarrow \Box\,\neg\, q$?

Option 1: If Agent A believes ϕ then Agent B believes not ϕ.

Option 2: If Agent A believes ϕ then Agent B does not believe ϕ.

Option 3: If Agent A believes ϕ then Agent B believes ϕ.

Option 4: If Agent A believes ϕ then Agent B does not believe not ϕ.

QUESTION 12

If we interpret K_i as "agent i knows ϕ", the formula scheme $\neg\phi \rightarrow K_1\ K_2\ \phi$ *means*

Option 1: If ϕ is true then agent 1 knows that agent 2 knows ϕ.

Option 2: If ϕ is false then agent 1 knows that agent 2 knows ϕ.

Option 3: If ϕ is true then agent 1 knows that he knows ϕ.

Option 4: If ϕ is false then agent 1 knows that he knows ϕ.

The following natural deduction proof (with some reasons missing) is referred to in Questions 13, 14 and 15:

1	$\neg \Box \neg (p \to q)$	premise
2	$\Box p$	
3	$\Box \neg q$	
4	$p \to q$	
5	p	$\Box$e 2
6	q	$\to$e 4, 5
7	$\neg q$	$\Box$e 3
8	$\bot$	$\neg$e 6, 7
9	$\neg (p \to q)$	$\neg$i 4 - 8
10	$\Box \neg (p \to q)$	$\Box$i 4 - 9
11	$\bot$	$\neg$e 10, 1
12	$\neg \Box \neg q$	$\neg$i 3 - 11

QUESTION 13

How many times are $\neg$ elimination and $\to$ introduction rules used in the above proof?

Option 1: None

Option 2: $\neg$ elimination and $\to$ introduction are both used only once.

Option 3: $\neg$ elimination is used only once but $\to$ introduction twice.

Option 4: $\neg$ elimination is used twice but $\to$ introduction only once.

QUESTION 14

What is the correct reason for steps 1, 2 of the above proof?

Option 1:	2	premise
	3	assumption
Option 2:	2	premise
	3	¬e 1
Option 3:	2	assumption
	3	¬e 1
Option 4:	2	assumption
	3	□i 2

QUESTION 15

What sequent is proved by the above proof?

Option 1: $\vdash \Box\, p \rightarrow \Diamond\, p$

Option 2: $\vdash \Box\, p \;\rightarrow\; \neg \Box \neg\, q$

Option 3: $\vdash \neg \Box \neg\, q$

Option 4: No proof

The following incomplete natural deduction proof is referred to in Questions 16 and 17:

1	$\Box\,(p \wedge q)$	assumption
2	$p \wedge q$	
3	p	$\wedge$ e2
4	q	$\wedge$e2
5	$\Box p$	$\Box$ i3
6	$\Box q$	$\Box$ i4
7	$\Box p \wedge \Box\, q$	
8	$\Box\,(p \wedge q) \rightarrow \Box\, p \wedge \Box\, q$	

QUESTION 16

What formulas and their reasons are missing in steps 3 and 4 of the above proof?

Option 1: 2 p∧ q □ e1
7 □p∧□ q ∧ i5,6

Option 2: 2 p∧ q assumption
7 □p∧□ q →i 5,6

Option 3: 2 p∧ q □ e1
7 □p∧□ q ∧i 2

Option 4: 2 p∧ q assumption
7 □p∧□ q ∧i 5,6

QUESTION 17

What rule is used in line 8?

Option 1: → e

Option 2: ¬e

Option 3: → i

Option 4: ∧ i

QUESTION 18

What proof strategy would you use to prove the following sequent:

$\Box (p \wedge q) \vdash_{KT4} \Box \Box p \wedge \Box \Box q$

Option 1:

- Open a solid box and start with □ (p ∧ q) as an assumption.
- Use axiom T to remove the □ to get p ∧ q.
- Use ∧ elimination twice to obtain the separate atomic formulas.
- Use axiom 4 twice, i.e. once on each atomic formula, to add a □ to each.
- Use axiom 4 twice, i.e. once on □ p and once on □ q, to get □ □ p and □ □ q.
- Combine □ □ p and □ □ q using ∧ introduction.
- Close the solid box to get the result.

Option 2:

- Start with $\Box(p \land q)$ as a premise.
- Use axiom T to remove the $\Box$ to get $p \land q$.
- Open a dashed box and use $\land$ elimination twice to obtain the separate atomic formulas.
- Use axiom 4 twice, i.e. once on each atomic formula, to add a $\Box$ to each.
- Close the dashed box and use $\Box$ introduction twice, i.e. once on $\Box p$ and once on $\Box q$, to get $\Box\Box p$ and $\Box\Box q$.
- Combine $\Box\Box p$ and $\Box\Box q$ using $\land$ introduction.

Option 3:

- Start with $\Box(p \land q)$ as a premise.
- Open a dashed box and use $\Box$ elimination to get $p \land q$.
- Use $\land$ elimination twice to obtain the separate atomic formulas.
- Close the dashed box and use $\Box$ introduction twice, i.e. once on each atomic formula.
- Use axiom 4 twice, once on $\Box p$ and once on $\Box q$, to get $\Box\Box p$ and $\Box\Box q$.
- Combine $\Box\Box p$ and $\Box\Box q$ using $\land$ introduction.

Option 4:

- Open a solid box and start with $\Box(p \land q)$ as an assumption.
- Open a dashed box and use $\Box$ elimination to get $p \land q$.
- Use $\land$ elimination twice to obtain the separate atomic formulas.
- Use axiom 4 twice, i.e. once on each atomic formula, to add a $\Box$ to each.
- Close the dashed box and use $\Box$ introduction twice, i.e. once on $\Box p$ and once on $\Box q$, to get $\Box\Box p$ and $\Box\Box q$.
- Close the solid box to get the result.

QUESTION 19

If we interpret $K_i\,\phi$ as "Agent i knows ϕ", what is the English translation of the formula $\neg K_1 K_2 (p \land q)$

Option 1: Agent 1 knows that agent 2 doesn't know that p and q.

Option 2: Agent 1 doesn't know that agent 2 knows p and q.

Option 3: If agent 1 knows that agent 2 doesn't know p and q.

Option 4: If agent 1 doesn't know that agent 2 knows p and q.

QUESTION 20

If we interpret $K_i \phi$ as "Agent i knows ϕ", what formula of modal logic is correctly translated to English as

If agent 1 knows p then agent 2 doesn't know q.

Option 1: $K_1 p \rightarrow K_2 \neg q$

Option 2: $\neg (K_1 p \wedge K_2 q)$

Option 3: $K_1 (p \rightarrow \neg K_2 q)$

Option 4: $K_1 \neg K_2 (p \rightarrow q)$

END OF QUESTIONS FOR ASSIGNMENTS 1, 2 & 3