

Sparse Representation for Color Image Restoration

- <https://ieeexplore.ieee.org/document/4392496>
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CMSE 491 Project Due 12/14/2022

```
In [3]: 1 # ALL PACKAGES WE NEED FROM PYTHON, add more as we complete notebook:
        2
        3 import numpy as np
        4 import cv2
        5 import scipy.linalg
        6 import matplotlib.pyplot as plt
        7 from skimage.util import random_noise
        8 from skimage import data, img_as_float
        9 from time import time
       10 import argparse
       11 import matplotlib.pyplot as plt
       12 import pdb
       13 from PIL import Image
       14 from numpy import*
       15
       16 # For K-SVD:
       17 from sklearn.feature_extraction.image import extract_patches_2d
       18 from sklearn.feature_extraction.image import reconstruct_from_patches_2d
       19
       20 # For algorithm/Sparse Coding:
       21 from sklearn.linear_model import OrthogonalMatchingPursuit
       22 from sklearn.datasets import make_sparse_coded_signal
       23 from sklearn.decomposition import MiniBatchDictionaryLearning
       24 from skimage.exposure import rescale_intensity
       25 from sklearn.feature_extraction.image import extract_patches_2d
       26
```

General Summary of Sparse Representation for Color Image Restoration:

The authors put forward algorithms addressing several different tasks, such as denoising of color images, denoising with nonuniform noise, inpainting small holes of color images, and demosaicing.

What algorithms/methods will be used? (Project Description)

- Briefly Defines Sparse Coding, OMP, KSVD, Denoising, any remaining methods.
- **Dictionary learning** is the method of learning a matrix, such that we can write a signal as a linear combo of as few columns from the dictionary matrix as possible. This is why it is considered, **Sparse Coding**.
- We intend to use this method, along with **Orthogonal Matching Pursuit**, a sparse approximation algorithm which finds the "best matching" projections of multidimensional data onto the span of an over-complete dictionary, to denoise grayscale images, as well as denoising using SVD methods from class for RGB, like **low rank approximations**. This is where we denoise the data by estimating the underlying structured low-rank matrix by it's singular values, using SVD.
- **INSERT PART C METHOD HERE**

Part A: Denoising Grayscale Images

METHOD EXPLANATION:

For this portion, we first chose a grayscale image, then applied distortion to this image. We then extracted patches from this distorted version, and used these to train/fit the dictionary. We then extracted noisy patches from the image again, and used the trained dictionary to reconstruct the image. This was done using the Orthogonal Matching Pursuit algorithm, which minimizes differences an image and optimizes the process by recursively, such that the resulting image would be denoised by rebuilding the sprase representation, through the OMP method.

How it relates to the research paper, **Sparse Representation for Color Image Restoration**:

In the journal, they use this dictionary method with OMP, but they include an additonal step: K-SVD. This step was difficult for us to implement, due to scikit learn having clustering and k-means methods, but not specifically a K-SVD method. It was above the level of difficulty in this course to recreate their algorithm form scratch to include this step, so we produced results of denoising without it.

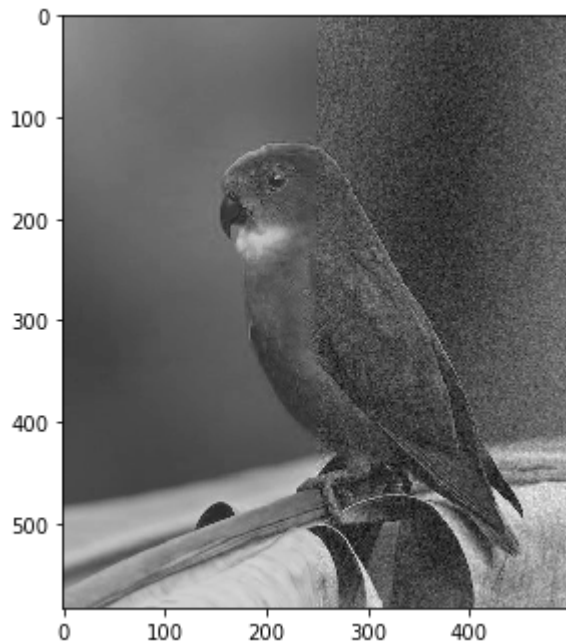
Journal explanation for their use of K-SVD: First, the OMP stops when the approximation reaches a sphere of radius in the patches' space in order not to reconstruct the noise. Then, the SVD selects an "average" new direction for each atom, which rejects noise from the dictionary.

Generate and Display Distorted Image

```
In [10]: 1 # Upload image:
2 img = cv2.imread("bird.jpeg")
3
4 # Turn into 2d array:
5 bird = img[:, :, 0]
6
7 # convert to floats for accurate computations
8 bird = img_as_float(bird)
9 height, width = bird.shape
10
11 # Distort the right half of the image with noise:
12 distorted = bird.copy()
13 distorted[:, width // 2 :] += 0.075 * np.random.randn(height, width // 2)
```

```
In [11]: 1 plt.figure(figsize=(10, 10))
2 plt.subplot(1, 2, 1)
3 plt.imshow(distorted, cmap=plt.cm.gray, interpolation="nearest")
4
5 # Difference from the normal image:
6 difference = distorted - bird
7 diff = np.sqrt(np.sum(difference**2))
8 print("Difference from the normal image: ", diff)
9
10 # Only right side should be distorted!
```

Difference from the normal image: 28.67627521090818



Extracting Patches for Training Dictionary

```
In [14]: 1 from sklearn.feature_extraction.image import extract_patches_2d
2 # Reshape a 2D image into a collection of patches.
3
4 # Extract all reference patches from the left half of the image:
5 t0 = time()
6 patch_size = (7, 7)
7 data = extract_patches_2d(distorted[:, : width // 2], (7, 7))
8 data = data.reshape(data.shape[0], -1)
9
10 # Regularization:
11 data -= np.mean(data, axis=0)
12 data /= np.std(data, axis=0)
13
14 # Patches info:
15 print("Number of patches extracted:", data.shape[0], "Overall Time:", time
16
```

Number of patches extracted: 140788 Overall Time: 0.09081029891967773

Train the Dictionary Using the Reference Patches Extracted Above

```
In [15]: 1 from sklearn.decomposition import MiniBatchDictionaryLearning
2 # Finds a dictionary (a set of atoms) that performs well at sparsely encoding
3
4
5
6 t0 = time()
7 dico = MiniBatchDictionaryLearning(
8     n_components=50, # Number of dictionary elements to extract.
9     batch_size=200, # Number of samples in each mini-batch.
10    alpha=1.0, #Sparsity controlling parameter.
11    max_iter=10, # Maximum number of iterations over the complete dataset
12 )
13 V = dico.fit(data).components_ # Fitting the reference patches to dictionary
14 dt = time() - t0 # Getting the time to run this model.
15
16
17 # Providing the iterations and steps taking by the model forming the dictionary
18 print("Number of iterations: ", dico.n_iter_)
19 print("Number of steps: ", dico.n_steps_, "Over time period: ", dt )
20
```

Number of iterations: 1.0

Number of steps: 118 Over time period: 8.954864740371704

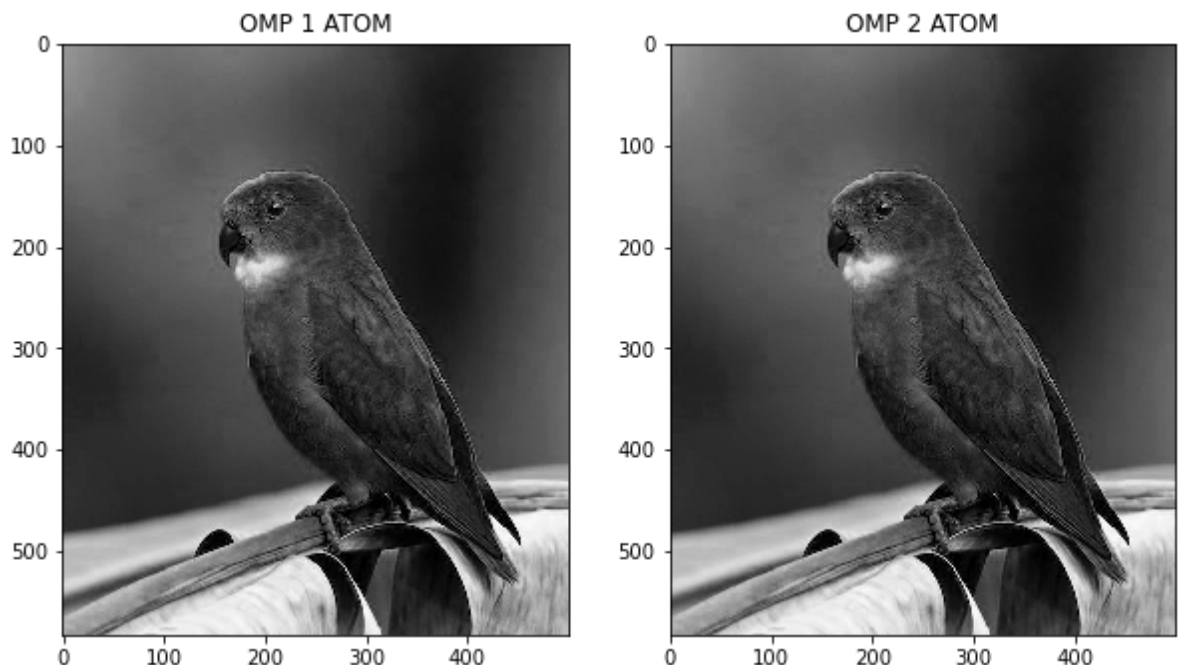
Extract Noisy Patches, Reconstruct them via Dictionary, using OMP method

```
In [16]: 1 # Extracting the patches for transform -> reconstruction again:
2
3 t0 = time() # Tracking time:
4
5 # Reshape a 2D image into a collection of patches.
6 data = extract_patches_2d(distorted[:, width // 2 :], patch_size)
7 # Regularization:
8 data = data.reshape(data.shape[0], -1)
9 intercept = np.mean(data, axis=0)
10 data -= intercept
11
12 # Using the OMP algorithm from the project journal:
13 # 'omp': uses orthogonal matching pursuit to estimate the sparse solution.
14 transform_algorithms = [
15     ("Orthogonal Matching Pursuit\n1 atom", "omp", {"transform_n_nonzero_c
16     ("Orthogonal Matching Pursuit\n2 atoms", "omp", {"transform_n_nonzero_
17 ]
18
19 # Reconstructing the image using the algorithms:
20
21 reconstructions = {}
22
23 # For transform title, omp algorithm, and it's requirements:
24 for title, transform_algorithm, params in transform_algorithms:
25
26     print(title)
27     reconstructions[title] = bird.copy()
28     t0 = time()
29
30     # Set the parameters of this estimator, using OMP:
31     dico.set_params(transform_algorithm=transform_algorithm, **params)
32
33     # Use the reconstructed image patches to Encode the data as a sparse c
34     code = dico.transform(data)
35
36     # We want the the sparse representation and the Components extracted f
37     patches = np.dot(code, V)
38
39     # Portion below taken from scikit example:
40     patches += intercept
41     patches = patches.reshape(len(data), *patch_size)
42
43     # End time of reconstruction:
44     dt = time() - t0
45
46     print("Time: ", dt)
47
48 # Rebuild a sample starting from a sparse dictionary of atoms!
```

Orthogonal Matching Pursuit
1 atom
Time: 3.615778684616089
Orthogonal Matching Pursuit
2 atoms
Time: 7.581718683242798

Display Results

```
In [19]: 1 # Pull results:
2 omp_1 = reconstructions['Orthogonal Matching Pursuit\n1 atom']
3 omp_2 = reconstructions['Orthogonal Matching Pursuit\n2 atoms']
4
5 # Plot:
6 plt.figure(figsize=(10, 10))
7
8 plt.subplot(1, 2, 1)
9 plt.title("OMP 1 ATOM")
10 #difference = omp_1-face
11 #diff = np.sqrt(np.sum(difference**2))
12 #print("Difference from the normal image: ",diff)
13 plt.imshow(omp_1, cmap=plt.cm.gray, interpolation="nearest")
14
15 plt.subplot(1, 2, 2)
16 #difference = omp_2-face
17 #diff = np.sqrt(np.sum(difference**2))
18 #print("Difference from the normal image: ",diff)
19 plt.title('OMP 2 ATOM')
20 plt.imshow(omp_2, cmap=plt.cm.gray, interpolation="nearest")
21 plt.show()
22
23 # Difference is minimal as image was easily reconstructed!
```



B. Denoising of Color Images

```
In [31]: 1 # Read in the image:
          2 image = cv2.imread("bird.jpeg")
          3
          4 # Extract the red, green, and blue channels:
          5 (B, G, R) = cv2.split(image)
          6
          7 # Get the image dimensions:
          8 h,w,d = image.shape
          9 print("width = {}, height = {}, depth = {}".format(w, h, d))
```

width = 500, height = 583, depth = 3

```
In [32]: 1 #display color image
          2 image_rgb = cv2.cvtColor(image, cv2.COLOR_BGR2RGB)
          3 image_rgb = np.array(image_rgb)
          4 plt.imshow(image_rgb)
          5 plt.axis('off')
```

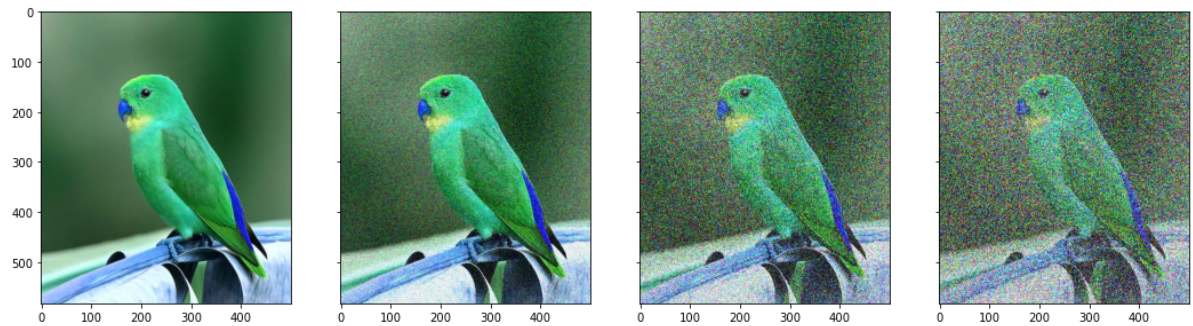
Out[32]: (-0.5, 499.5, 582.5, -0.5)



Adding Noise:

```
In [33]: 1 sigma = 0.155
2 original = img_as_float(image) #convert to floats for accurate computation
3
4 noisy1 = random_noise(original, var=sigma**2)
5 noisy2 = random_noise(original, var=sigma**1)
6 noisy3 = random_noise(original, var=sigma**0.5)
7
8 fig, ax = plt.subplots(nrows=1, ncols=4, figsize=(18, 15), sharex=True, sh
9 ax[0].imshow(original)
10 ax[1].imshow(noisy1)
11 ax[2].imshow(noisy2)
12 ax[3].imshow(noisy3)
```

Out[33]: <matplotlib.image.AxesImage at 0x1db9bbcce80>



Noise to Signal Ratio:

```
In [34]: 1 SNR_noisy1 = (np.linalg.norm(image)/np.linalg.norm(noisy1-image))**2
2 print('SNR of noisy image 1 = ', SNR_noisy1)
3
4 SNR_noisy2 = (np.linalg.norm(image)/np.linalg.norm(noisy2-image))**2
5 print('SNR of noisy image 2 = ', SNR_noisy2)
6
7 SNR_noisy3 = (np.linalg.norm(image)/np.linalg.norm(noisy3-image))**2
8 print('SNR of noisy image 3 = ', SNR_noisy3)
```

```
SNR of noisy image 1 = 1.0077862310288288
SNR of noisy image 2 = 1.0076566133443017
SNR of noisy image 3 = 1.007564722190207
```

Regular SVD:


```

In [35]: 1 # Create matrix values of image:
          2
          3 A = np.matrix(image[:, :, 1])
          4
          5 # Get SVD:
          6 U, e, Vt = np.linalg.svd(A) # E is the vector of singular values!
          7 U = np.matrix(U)
          8 Vt = np.matrix(Vt)
          9
         10 # Generate compressed image using singular values:
         11 e_matrix = np.zeros((583, 500)) # Same size as A
         12 for i in range(len(e_matrix[-1])):
         13     e_matrix[i, i] = e[i]
         14
         15 # Display/Generate New Image:
         16 print(U*e_matrix*Vt)
         17 new_im = U*e_matrix*Vt

[[170. 170. 170. ... 123. 123. 123.]
 [170. 170. 170. ... 123. 123. 123.]
 [170. 170. 170. ... 123. 123. 123.]
 ...
 [196. 195. 194. ... 217. 217. 213.]
 [194. 194. 191. ... 218. 218. 214.]
 [191. 191. 189. ... 217. 217. 212.]]

```

```

In [36]: 1 #This should be true to make sure SVD is correct:
          2 np.allclose(A, new_im)

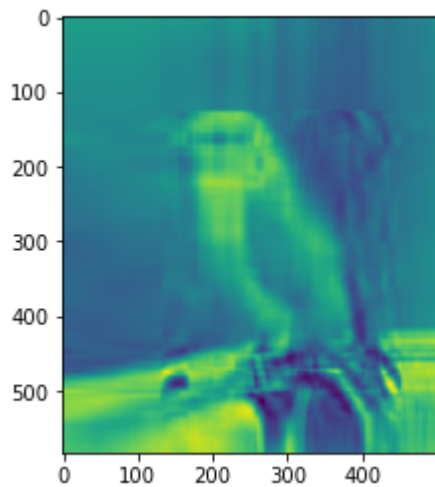
```

Out[36]: True

We are now going to make a new image but only keep the, N, biggest singular values, while setting all of the rest to zero. First we define a new vector, S, consisting of the first N = 10 singular values:

```
In [37]: 1 s = np.zeros((e.shape))
2 s[0:10] = e[0:10]
3
4 # Sigma matrix should be the same size as the original A matrix with mostl
5 S = np.zeros(A.shape)
6
7 # The upper left diagonal of the Sigma matrix should be the singular value
8 S[:len(s), :len(s)] = np.diag(s)
9
10 # Compressed image
11 I = U*S*Vt
12
13 plt.imshow(I)
```

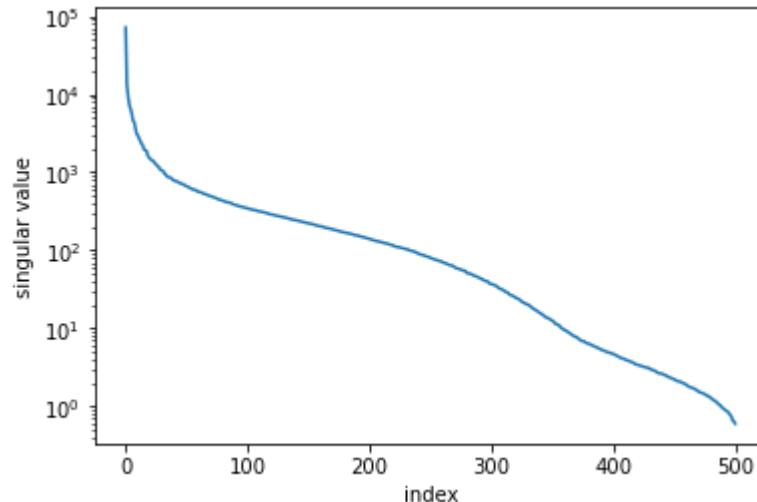
Out[37]: <matplotlib.image.AxesImage at 0x1db9c0b9340>



```
In [38]: 1 rmse = np.sqrt(((np.array(A) - np.array(I)) ** 2).mean())
2 print(rmse)
3
4 plt.plot(e)
5 plt.gca().set_yscale('log')
6 plt.xlabel('index')
7 plt.ylabel('singular value')
8
```

18.285161364307527

Out[38]: Text(0, 0.5, 'singular value')



C. Color Image Inpainting (Also optional, if denoising grayscale/rgb is too short of code)

Image inpainting is the art of modifying an image in an undetectable form, and it often refers to the filling-in of holes of missing information in the image.

```
In [ ]: 1 # PART C: Do example of Color Image Inpainting.
```

HELPFUL LINKS FOR PROJECT: (Sources)

(UPDATE CODE BLOCK WHEN FINISHED)

- Sparse Representation for Color Image Restoration LINK:
<https://ieeexplore.ieee.org/document/4392496>
[.\(https://ieeexplore.ieee.org/document/4392496\)](https://ieeexplore.ieee.org/document/4392496)

Definitions:

For Part A/B

- https://en.wikipedia.org/wiki/Gaussian_noise (https://en.wikipedia.org/wiki/Gaussian_noise)
- https://en.wikipedia.org/wiki/Sparse_dictionary_learning
[.\(https://en.wikipedia.org/wiki/Sparse_dictionary_learning\)](https://en.wikipedia.org/wiki/Sparse_dictionary_learning)

- <https://en.wikipedia.org/wiki/K-SVD> (<https://en.wikipedia.org/wiki/K-SVD>)

For Part C/D

- <https://en.wikipedia.org/wiki/Inpainting> (<https://en.wikipedia.org/wiki/Inpainting>)
- <https://en.wikipedia.org/wiki/Demosaicing#:~:text=A%20demosaicing%20>
(<https://en.wikipedia.org/wiki/Demosaicing#:~:text=A%20demosaicing%20>)
(also%20de%2Dmosaicing,CFA%20interpolation%20or%20color%20reconstruction.

METHODS FOR DENOISING/K-SVD: Part A/B

- <https://github.com/Deepayan137/K-svd> (<https://github.com/Deepayan137/K-svd>)
- <https://stackoverflow.com/questions/48202304/python-implementation-of-k-svd-algorithm>
(<https://stackoverflow.com/questions/48202304/python-implementation-of-k-svd-algorithm>)

Methods for Image Inpainting: Part C

- <https://www.geeksforgeeks.org/image-inpainting-using-opencv/>
(<https://www.geeksforgeeks.org/image-inpainting-using-opencv/>)

Methods for Image Demosaicing: Part D*

- <https://pypi.org/project/colour-demosaicing/> (<https://pypi.org/project/colour-demosaicing/>)
- <https://www.colour-science.org/colour-demosaicing/> (<https://www.colour-science.org/colour-demosaicing/>)

In []:

1	
---	--