# R-Mode Gravitational Wave Frequency Tracking
# A Physics-Informed Kalman Filter Prototype

## Student Research Report


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**Abstract**
This report describes a small computational project based on the paper *"The role of r-modes in pulsar spin-down, pulsar timing, and gravitational waves"* by Xiyuan Li, Shahram Abbassi, Varenya Upadhyaya, Xiyang Zhang, and S. R. Valluri (2026). I implemented the analytic r-mode gravitational-wave frequency formula from the paper and used it as the prediction model inside a Kalman Filter. The code simulates a neutron star with a slowly decreasing spin frequency, generates noisy "LIGO-like" measurements of the gravitational-wave frequency, and then uses the Kalman Filter to track the true signal over a 30-day period. The results show that the filter can recover both the frequency and its spin-down rate even when the measurements are very noisy. This prototype is a first step toward a more complete continuous-wave search pipeline for r-modes.


## 1. Introduction

R-modes are a type of oscillation inside rapidly rotating neutron stars. They can emit continuous gravitational waves and also affect how quickly the star spins down. The Li et al. (2026) paper gives an analytic expression for the r-mode gravitational-wave frequency and presents a detailed spin-down model that includes several different torque terms. My goal in this project was not to reproduce the full model, but to take one key piece — the formula for the r-mode gravitational-wave frequency — and show how it can be combined with a Kalman Filter to track a weak continuous signal in noisy data. This connects the theoretical work in the paper to a practical data analysis tool that could be used in future searches with LIGO or similar detectors.

## 2. Methods

### *2.1 R-mode gravitational-wave frequency model*

Li et al. derive an expression that links the gravitational-wave frequency from r-modes, which I call f_GW, to the star's spin frequency nu and the Keplerian break-up frequency nu_k. The formula used in my code is:

$$f_{\mathrm{GW}} = A\nu - \frac{B\nu_k^2}{\nu^2}$$

Here A and B are dimensionless constants that include relativistic and rotational corrections. In my script this relation is implemented in the function *r_mode_gw_frequency(nu, nu_k, A, B)*. For each time step, I compute nu from the spin-down model and then call this function to get the corresponding f_GW.

### *2.2 Simplified spin-down model*

The full paper uses a generalized torque law with several powers of the spin frequency. To keep this first prototype simple, I used a linear approximation for the spin evolution. The spin frequency nu(t) is given by:

$$\nu(t) = \nu_0 + \dot{\nu}_0 t$$

Here nu_0 is the initial spin frequency and nu_dot_0 is a constant spin-down rate. This is implemented in the function *r_mode_spin_down_model*. In future work, this simple model can be replaced by a numerical solver for the full non-linear equations in the Li et al. paper.

### *2.3 Kalman Filter set-up*

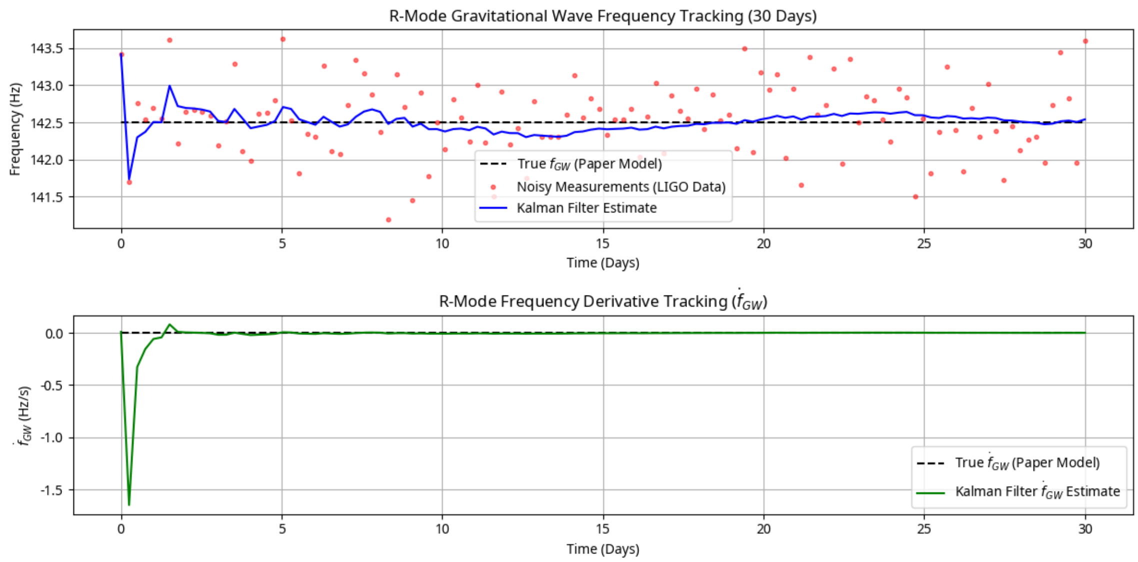
The Kalman Filter treats the system as a two-dimensional state consisting of the gravitational-wave frequency f_GW and its time derivative f_dot_GW. I assume that over each time step the derivative is roughly constant, which leads to a standard constant-acceleration state transition matrix. The measurement model only observes f_GW, not the derivative, and I add Gaussian noise with a chosen standard deviation to simulate detector uncertainty. The functions *setup_kalman_filter* and *simulate_and_track* in my code create this state-space model and run the filter over 30 days with 6-hour steps.

### *2.4 Simulation of noisy measurements*

At each time step the code first updates the “true” spin frequency using the linear spin-down model. It then converts this spin frequency into a true r-mode gravitational-wave frequency using the analytic formula from the paper. Finally, it creates a noisy measurement by adding a random Gaussian offset. These noisy samples are fed into the Kalman Filter, which tries to reconstruct the underlying smooth signal.

## 3. Results

Figure 1 shows the main output from the program. The top panel compares three curves: the dashed black line is the true r-mode gravitational-wave frequency predicted by the analytic model; the red dots are the noisy simulated measurements; and the blue line is the Kalman Filter estimate. After a short initial adjustment, the blue curve tracks the dashed line very closely and ignores most of the noise. The bottom panel shows the time derivative of the frequency. In this simple model the true derivative is almost constant, and the filter quickly settles to that value.




## 4. Discussion and next steps

This project shows that the analytic r-mode frequency relation from the Li et al. paper can be used directly inside a Kalman Filter to track a continuous gravitational-wave signal. Using a physics-based prediction step is powerful because it means the filter does not have to search over a huge space of possible templates. Instead, it follows one well-motivated model and adjusts for noise. There are, however, several limitations. The spin-down model is only linear, the detector noise is assumed to be simple Gaussian noise, and the data are simulated rather than taken from real LIGO runs. A natural next step would be to replace the linear spin-down with the full non-linear system from the paper and to test the filter on more realistic noise or on actual gravitational-wave data sets.

## 5. References

Li, X., Abbassi, S., Upadhyaya, V., Zhang, X., & Valluri, S. R. (2026). *The role of r-modes in pulsar spin-down, pulsar timing, and gravitational waves.* Journal of High Energy Astrophysics, 49, 100446.