Projet d'Initiation à la Recherche de 4ème année

Development of a ball balancing system

ABREU DE ANDRADE Heitor
BELAFKI Ayoub
COELHO ROBL Ana Carolina
EYCHENNE Vincent
GARNICA AZA Jose Eduardo
SUAREZ SEPULVEDA Johan Sebastian

4AE-SE

Tutors: CHANTHERY Elodie
LE CORRE Gwendoline

Abstract

The ball and plate system is a widely studied topic in control engineering. This literature review focuses on the comparison of different aspects of this system. The aim is to provide a comprehensive overview of the current state of the art in the field of ball and plate systems and to provide insights into the design and control of these systems.

The aspects studied are the main methods of modeling, control, and motion detection of the system ball.

A detailed analysis of the operation of the cameras and the touch screens for motion detection is presented. In parallel to the study of system development without modeling and the modeling for systems with 4 DOF.

The review also compares the performance of different regulators, including PID, LQR, and sliding mode controllers, in controlling the ball and plate system. This part of the review provides a summary of the design and implementation of each regulator and compares their performance in different scenarios. It also highlights the strengths and weaknesses of each regulator and provides insights into the design of optimal controllers for the ball and plate system.

Keywords: Ball and plate system, Nonlinearities, Regulators, PID, LQR, Sliding mode controller, Control theory, Tracking

1. Introduction

The ball and plate system is a classic example of a non-linear multi-input multi-output (MIMO) control system, which has been widely studied and implemented in various fields such as robotics, mechatronics, and control engineering. The system consists of a plate and a ball, where the plate is used to control the position of the ball. The objective of the ball and plate system is to maintain the ball position at a desired setpoint or to make it follow a trajectory.

The development of the ball and plate system has been the subject of numerous research studies, with a focus on various aspects of the system, such as the methods for detecting the ball position, the mechanical structure and modeling of the system, and the different types of controllers used to regulate the system's behavior. The first part of this literature review will focus on the various methods used for detecting the ball position, including resistive touch screen and camera-based systems.

The second part of the review will focus on the mechanical structure and modeling of the ball and plate system, including the analysis of the system's dynamics and the modeling of the system's behavior.

The final part of the review will cover the different types of controllers used in the ball and plate system, including Proportional-Integral-Derivative (PID) controllers, sliding mode controllers, and Linear-Quadratic Regulator (LQR) controllers. The performance of each type of controller will be compared and analyzed to determine their effectiveness in regulating the ball and plate system.

2. Method of detecting the ball

This section seeks to expose the possible applicable technologies for the detection of the ball. The most common technologies in this case are divided into two mainly. The first one can be summarized as a touch panel that detects the X and Y position of the sphere on the flat surface by means of a reference voltage the corresponding value is found, this will be elaborated later, the second technology is about tracking through cameras to detect the position of the ball, each technology shows advantages and disadvantages one with respect to the other, as well as having the possibility of being used together.

2.1. Touch screen

The goal of touch panels is to find the position in which they are touched, for this there are several concepts but the most used is the resistive touch panel.

These are composed of two layers that contain an electrical circuit. The top layer is pliable, so when pressure is applied, it causes a bend in the surface, altering the resistance in the circuit. This change in resistance can be interpreted as a fluctuation in voltage.[1]

A resistive touch screen operates by sending a voltage across a resistor array. The resistance at the point where the screen is touched by a stylus, pen, or finger is then measured. The 5-wire touch screen panel operates differently, by applying voltage to the corners of the bottom resistive layer and measuring the resistance of the vertical or horizontal network with the 5th wire, which is incorporated into the top sheet. As shown in the following image. [2]

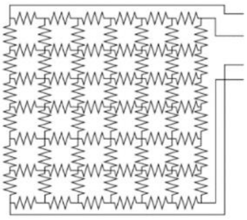

Fig. 1 The five-wire touchscreen resistor network.
In [2, Fig. 5]

A process of calibration and filtering of the signal must be done to have a correct response of precision that allows to have the least possible errors, this is done by means of sampling and a Butterworth low-pass filtering, in [3] there is a detailed explanation step by step on the process for the calibration of the panels for this use and how to do the filtering of the signal, as well as responses to signal treatments.

2.2. Camera

Cameras are used for ball detection, there are many methods for processing the images that are taken in real time to determine the trajectory and how the ball will move over the system, one widely used is the following algorithm.

There are different steps to follow that can make image processing more efficient, in general a structure is followed that follows the steps shown below: "1) Image Mask, for dimension reduction, 2) Measure, for corner detection, 3) Color Threshold to take into account only orange (ball) and black (platform), 4) Binary Image Inversion, to take into account the dynamics of the ball, not the platform, 5) Advance Morphology, to remove sections of the image that could be mistaken for the sphere and 6) Particle Analysis where the ball center coordinates are taken. "[4, p. 12-13]

The purpose of this design is to capture an image using a camera, and then apply a color block recognition algorithm to determine the target of the ball. It also marks the center of the ball and continuously calculates its relative position on the tablet by creating a plane coordinate system in the captured image field of view and sending the coordinate information. [5] Many of these algorithms are detailed and developed in Python because of its varied and efficient image processing libraries, as well as its high performance in the desired field.

A flowchart from the research work referred to below is shown, in this you can see how the different stages mentioned above are fulfilled and how it is necessary to have a filtering of the image depending on the conditions in which it works. [6]

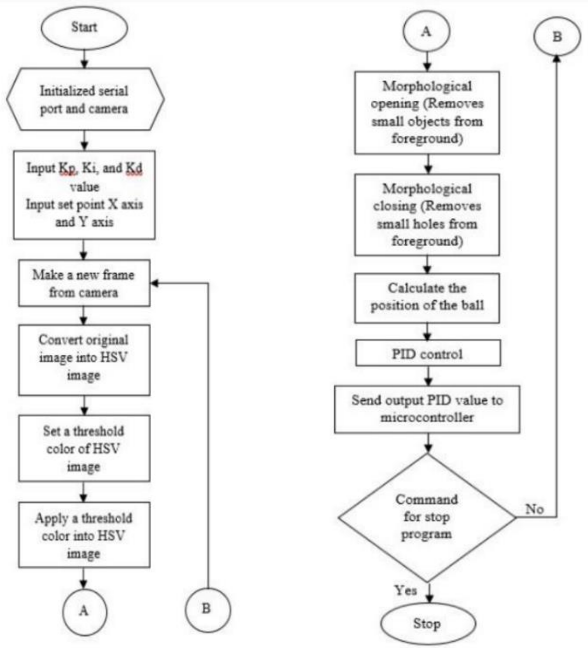

Fig. 2 Flowchart for image processing
In [6, Fig.6]

3. Modeling of the system

3.1. Techniques without modeling

The plate and ball system can be composed of different components that control the plate and the ball, among them an example of easy construction is the use of: computer (working as a pre-filter, controller and receiver/transmitter of date), Arduino(to control the movement of the servo motor), servo motor(controlling the movement of the plate) and webcam (to capture the movements of the ball) [6].

After the analysis of the components that compose the system the next logical step to ensure a good operation would be to model it and then search for the best controller, but not always the modeling of the system is necessary. If the developer chooses to use a PID type control, it is possible that the parameters (Kp, Ki, Kd) can be found by trial and error and without the need for modeling the system [6], [7]. This way it is possible to adjust the Kp parameter (proportional gain) by observing if the ball is managing to move the desired way on the plate (big enough for the ball to move smoothly but not so big as to cause high fluctuation). Then you just adjust the derivative gain (Kd) to adjust the float response and the integral gain (Ki) to adjust the stationary error [6], [7]. The PID type controller is described in more detail in section 4.1 of this paper.

3.2. 4 DOF

“In order to design a good control algorithm for the system, a nominal model needs to be derived. This helps us predict the system behavior”. [8, p. 4]

A 4DOF (degrees of freedom) system refers to a mechanical or physical system that has four independent ways it can move or change. These movements are usually defined by the system's generalized coordinates, which are specific variables that describe the system's state. In a 4DOF system, there are four such variables, and they can be related to various parameters, such as the position, orientation, or velocity of the system's components. [9],[10], [11], [12] proposes an approach using the Lagrangian. Here we have 2 DOF which are related to the motion of the ball and two others to the inclination of the plate. The position of the ball on the plate are given by "x_b" and "y_b" and the rotations of the plate are given by "α" and"β".

The Euler-Lagrange equation is:

$$Q_i = \frac{d}{dt}\frac{\partial T}{\partial \dot{q_i}} - \frac{d}{\partial \dot{q_i}} + \frac{\partial V}{\partial \dot{q_i}} \quad (2)$$

The ball's kinetic energy is:

$$Tb = \frac{1}{2}m_b(\dot{x}_b^2 + \dot{y}_b^2) + \frac{1}{2}I_b(\omega_x^2 + \omega_y^2) \ (3)$$

- q_i represents the coordinate in the "i" direction
- T is the kinetic energy of the system
- V is the potential energy of the system
- Q is the composite force
- m_b is the mass of the ball
- I_b is the moment of inertia of the ball

We assume that the ball rotates without slippage, which allows us to obtain the following equations (3). Once these equations are reintroduced into equation (2), we obtain a new kinetic equation for the ball (4).

$$\dot{x}_b = r_b\omega_y \, , \dot{y}_b = r_b\omega_x \quad (4)$$

- r_b is the radius of the ball

The ball's kinetic energy without slippage is:

$$Tb = \frac{1}{2}(m_b + \frac{I_b}{r_b^2})(\dot{x}_b^2 + \dot{y}_b^2) \qquad (5)$$

The potential energy difference of the ball relative to the horizontal plane passing through the center of the inclined plate is given by:

$$V_x = m_b g x_b sin(\alpha)$$
$$V_y = m_b g y_b sin(\beta)$$

Assuming that there is no spinning around the z-axis, the plate's kinetic energy encompasses both its rotational kinetic energy and that of the ball. This expression can be formulated as:

$$T_p = \frac{1}{2}(I_p + I_b)(\dot{\alpha}^2 + \dot{\beta}^2) + \frac{1}{2}m_b(x_b\dot{\alpha} + y_b\dot{\beta})^2 \quad (6)$$

Dynamical equations of the system are obtained using Lagrange equation [9], [10], [11], [12]:

<u>Equation of the ball movement on the plate:</u>

$$(m_b + \frac{I_b}{r_b^2})\ddot{x}_b - m_b(x_b\dot{\alpha}^2 + y_b\dot{\alpha}\dot{\beta}) + m_b g sin(\alpha) = 0 \ (7)$$

$$(m_b + \frac{I_b}{r_b^2})\ddot{y}_b - m_b(y_b\dot{\beta}^2 + x_b\dot{\alpha}\dot{\beta}) + m_b g sin(\beta) = 0 \ (8)$$

Equation (6) and (7) highlight the correlation between the ball's acceleration, the rotational angle, and the angular velocity of the plate.

<u>Equation showing the effect of external torque on the entire system:</u>

$$\tau_x = (I_p + I_b + m_b x_b^2)\ddot{\alpha} + 2m_b x_b \dot{x}_b \dot{\alpha} + m_b x_b y_b \ddot{\beta} + m_b \dot{x}_b y_b \dot{\beta} + m_b x_b \dot{y}_b \dot{\beta} + m_b g x_b cos(\alpha) \ (9)$$

$$\tau_y = (I_p + I_b + m_b y_b^2)\ddot{\beta} + 2m_b y_b \dot{y}_b \dot{\beta} + m_b x_b y_b \ddot{\alpha} + m_b \dot{x}_b y_b \dot{\alpha} + m_b x_b \dot{y}_b \dot{\alpha} + m_b g y_b cos(\beta) \ (10)$$

As the equations (9) and (10) are nonlinear, it will be difficult to use them to control the system. Since we only consider the angles α and β, we can focus solely on equations (7) and (8). Knowing that at steady state, α and β must be equal to 0 and that sin α ≈ α, sin β ≈ β, we obtain using equations (7) and (8):

$$x : (m_b + \frac{I_b}{r_b^2})\ddot{x} + m_b g\alpha = 0 \quad (11)$$

$$y : (m_b + \frac{I_b}{r_b^2})\ddot{y} + m_b g\beta = 0 \quad (12)$$

- $I_b = \frac{2}{5}m_b r_b^2$

4. Applied controls

4.1. PID

Even if it is possible to obtain the PID parameters by trial and error, "the best PID parameters can be determined using the dynamic model of the system" [7, p. 58]. In [6], a method is given to calculate the PID parameters.

In order to simplify the control of the system, we assume linear independence between the x and y axis. This allows us to transform the MIMO system into two Single Input – Single Output (SISO) systems. [6], [8]. Nevertheless, "This assumption is reasonable, if the two axes were perfectly perpendicular to each other. But due to hardware tolerances, plate unevenness and play in the system this is never the case in reality. Neglecting the cross-coupling effects between x and y axis is one of the major causes of periodic disturbances in the BaP system"[8 p. 4].

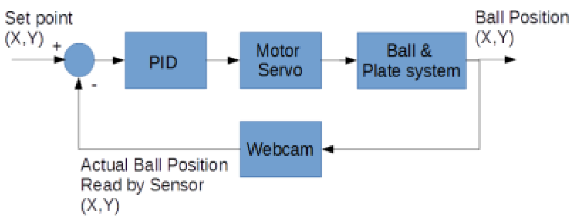

Fig. 3 Block Diagram of Ball and Plate Control System without Pre-filter
In [6, Fig. 2]

The PID control law in continuous time is of the form:

$$u(t) = K_P e(t) + K_I \int e(t)dt + K_D \frac{d}{dt}e(t) \quad (12)$$

And we can derive the recurrence equation to be implemented in the microcontroller:

$$OutputPID[n] = P[n] + I[n] + D[n]$$

$$P[n] = Kp.e[n]$$

$$I[n] = Ki.e[n] + I[n-1]$$

$$D[n] = Kd.(e[n] - e[n-1]) \quad (13)$$

To find the PID parameters, we can use the Ziegler Nichols method for closed loop, which consists of finding the ultimate gain (Ku) and ultimate period (Pu) and deriving the constants from the Zeigler Nichols table.

Ku is the gain capable of generating a sustained sinusoidal response and Pu is the period of that response.[13]

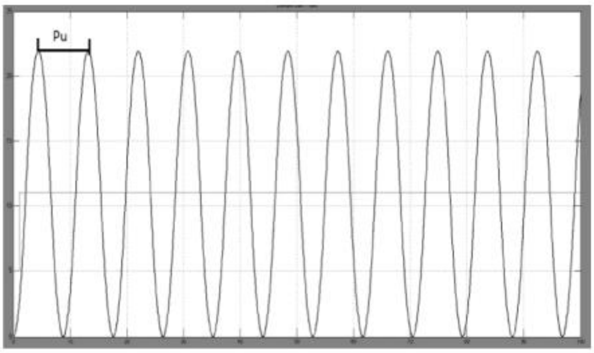

Fig. 4 Sustained oscillation for Ku = 3.4 (X Axis)
In [13, Fig. 9]

In this method, Ku = 3.4 and Pu = 3.75 seconds was obtained, but the results were not satisfactory, with a large settling time of approximately 34 seconds.

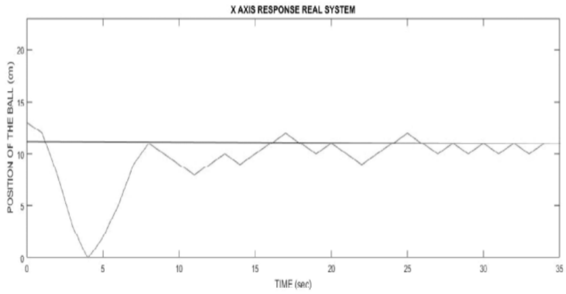

Fig. 5 Closed loop response of real system in X
In [13, Fig.11]

With manual adjustments, it was possible to achieve a more satisfactory settling time of 4.6 seconds [13].

On the other hand, the modeling of the system allows us to know that it is type 1, and that the dominant constants of the controller will be P and D. Seeing that it is possible to find the constants with trial and error done by focusing first on the proportional gain until we have a response that reaches the reference, then adjust the derivative gain to reduce oscillations, and finally add the integral gain to reduce the error steady state.

Another strategy seen in PID control is the use of a low-pass filter on the reference signal to reduce the oscillation of the PID controller.

controller	K_c	τ_I	τ_D
P	0.5 K_{CU}	–	–
PI	0.45 K_{CU}	$P_U/1.2$	–
PID	0.6 K_{CU}	$P_U/2$	$P_U/8$

Table 1: Ziegler Nichols tuning parameters
In [13, Tab. 1]

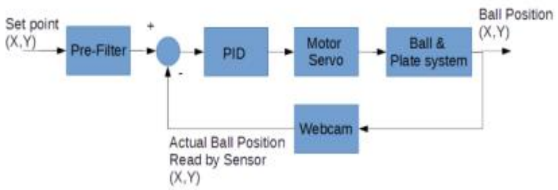

Fig. 6 Block Diagram of Ball and Plate Control System with Pre-filter

The pre-filter used was a first-order low-pass modeled by the following equations:

$$P(s) = \frac{a}{s + a}$$

$$P(z) = \frac{(1-a)^z}{z - a}$$

$$OP[n] - a.OP[n-1] = (1-a).SP[n] \quad (14)$$

Comparing Figure 7 and 8, we see that the filter reduces overshoot, and although it has a slower response, it makes the tracking more robust. [6]

Fig. 7 Response of the system without pre-filter (4 different setpoint)
In [6 Fig.10]

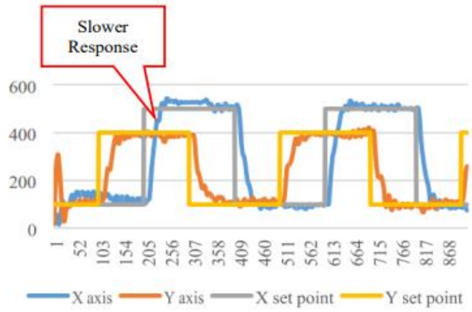

Fig. 8 Responses of the system with pre-filter (4 Different Setpoint)
In [6 Fig. 11]

4.2. Sliding mode control

The Sliding Mode Control law is in the form:

$$u_S(t) = b^{-1}\left(\frac{d^2}{dt}x_d(t) - \lambda\dot{e}(t) - \alpha.s(t) - \beta.sgn(s(t))\right) \quad (15)$$

And we can divide it into two parts, one to keep the system on the sliding surface, and one to force the system to go to the sliding surface. [2]

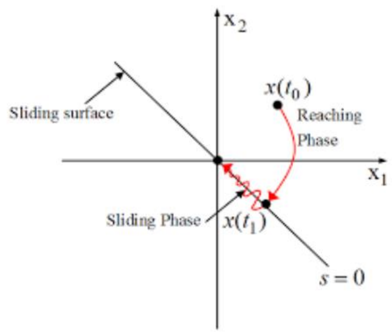

Fig. 9 Phase plane response of an SMC system.
In [2, Fig. 3]

This second part has the form -beta*sgn(s(t)), and this discontinuity caused by sgn(s(t)) in the control law can be harmful to the motors. This effect is called chattering and must be reduced by smoothing the control law with a saturation function. [9]

Also in the control law, s is the sliding function, b = gain of the transfer function of the modeled system, xd = reference. The parameters to be determined are lambda (slope of the desired sliding surface), alpha and beta. The 3 are derived from the input reference, time-varying uncertainty and dynamic mathematical model of the ball and plate system. [2], [14]

The sliding mode controller results are shown below. To stabilize the ball in a required position, the results are satisfactory with no oscillations or unwanted overshoot. The circular trajectory result is also satisfying with an error of less than 2mm on a 5cm radius desired circle.

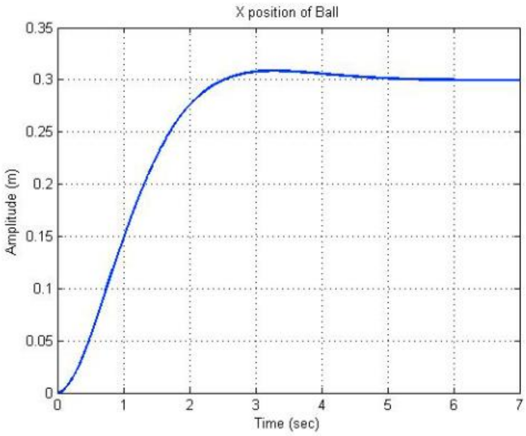

Fig. 10: The position of ball on the x-axis with sliding mode controller on simulation
In: [9, Fig. 5]

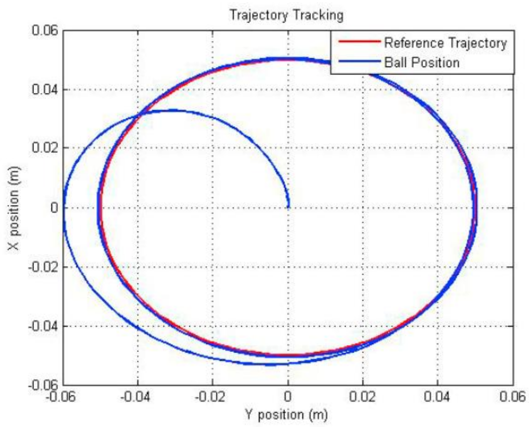

Fig. 11 : Circular trajectory tracking with sliding mode controller on real system
In [9, Fig. 6]

On the experimental system, the sliding mode controller results are shown below, achieving ess = 0.5mm and ts = 0.52second to stabilize the ball in a required position. And on a circular trajectory, the error was less than 2mm. [9]

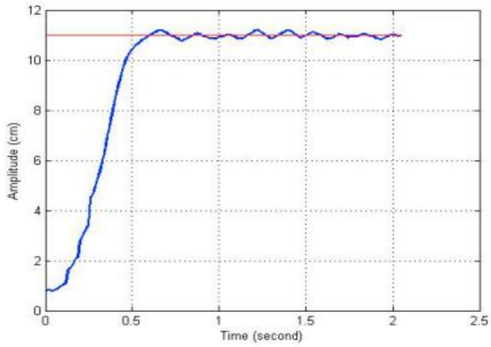

Fig. 12 The position of ball on the x-axis with Sliding Mode Controller on real system.
In [9, Fig. 14]

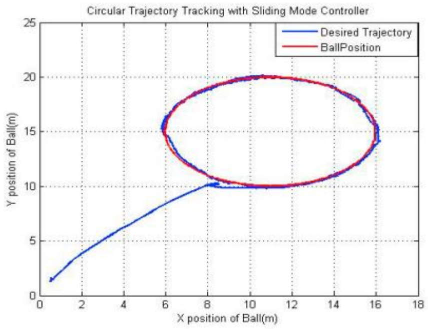

Fig. 13 Circular Trajectory Tracking with Sliding Mode Controller on real system.
In [9, Fig. 15]

For the tracking of the circular trajectory, the error obtained are relatively similar between the real system and the simulation. This shows us the robustness characteristic of the sliding mode controller.

4.3. LQR regulator

LQR regulators are very frequently used for ball-on-plate processes. Like the PID regulator, the LQR regulator is linear. Nevertheless, the LQR controller is multivariable, i.e., “the manipulated variables are calculated taking into account all interactions of process variables” [15, p.17]. It is therefore a good choice for MIMO systems. Reference [15] gives us an overview of the performance of the LQR regulator. A first approach consists in choosing experimentally its settings using the pole placement method. A second method involves optimizing in MATLAB using the Dynamic Local Grid Refinement (DLGR) function. Using the DLRG method allows to eliminate the overshoot at the cost of a longer setting time. The obtained trajectories are the following:

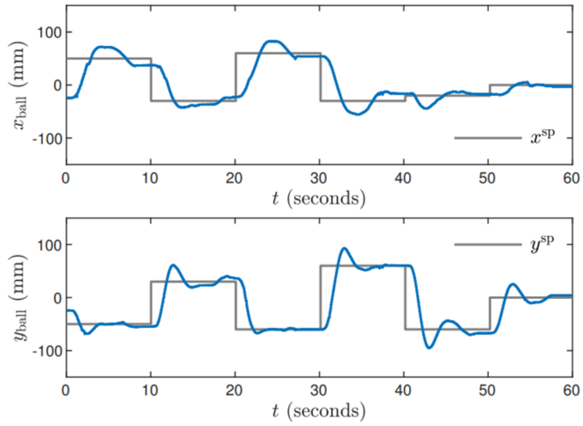

Fig. 14: Experimental results for the LQR controller using the pole placement method. Adapted from [15 Fig.10]

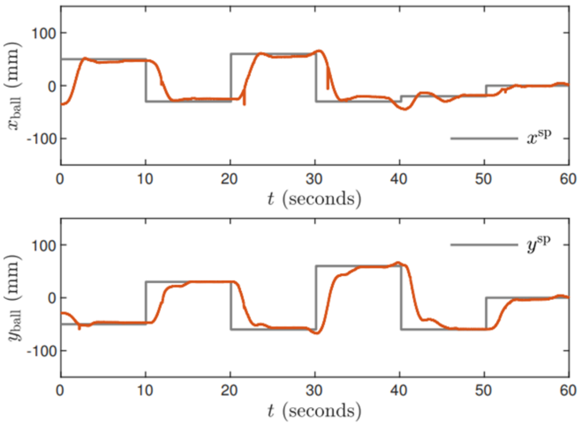

Fig. 15: Experimental results for the LQR controller using the DLGR method. Adapted from [15 Fig. 11]

In [9], another method is used to determine the parameters of the LQR controller. The feedback gain is based upon the minimization of a quadratic cost function:

$$J = \frac{1}{2} * \int_{0}^{+\infty} (x^T\, Qx + u^T Ru)dt$$

Solving Riccati equation allows the state-feedback gain. It should be noted that this method requires the use of a system model. Using Matlab Simulink, the following simulation results are obtained:

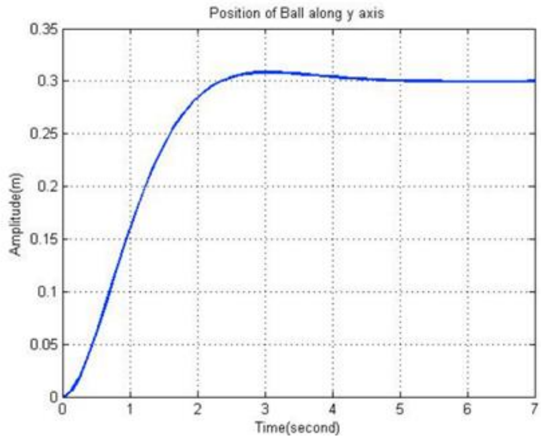

Fig. 16: Simulation result for stabilizing the ball in a desired position. In [9 Fig. 3]

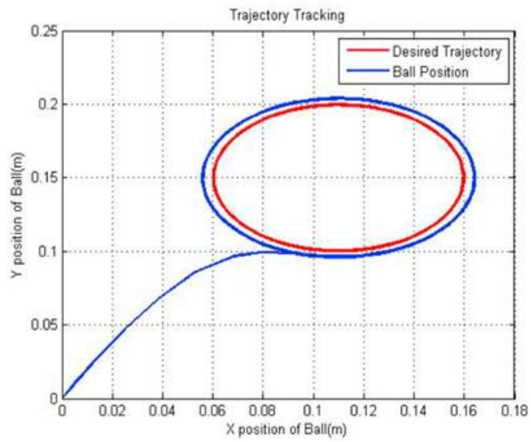

Fig. 17: Simulation result for tracking of a circular trajectory. In [9 Fig. 4]

The obtained error during the tracking of a circular trajectory is less than 5mm. Experimental results are given in [9 Fig. 12] and [9 Fig. 13]. The tracking of the circular trajectory gives an error smaller than 6mm. The trajectory is nevertheless slightly deformed. This illustrates that although the LQR is a linear corrector, it remains effective on systems with nonlinearities such as the ball-and-plate system.

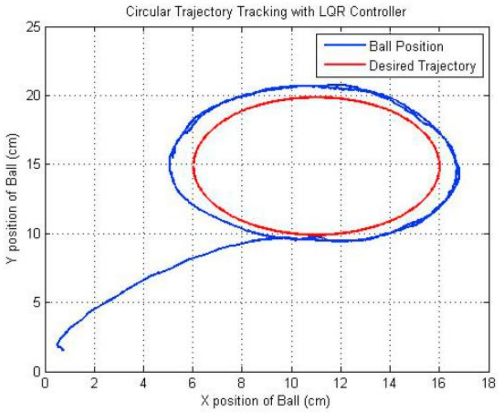

Fig. 18: Experimental result for tracking of a circular trajectory.
In [9 Fig. 13]

4.4. Comparison of controller performance

Reference [15] offers us a first comparison between PID and LQR controllers. One of the major problems for the PID is the derivative component (D), "If the calculations are performed using the ball position measurement, the derivative part causes the whole system to vibrate strongly, even when the ball reaches the set-points." [15, p.14]. Another problem with PID is that it is very sensitive to measurement errors. The LQR controller is not affected by this type of error. "For the set-point tracking task, the LQR controllers perform much better than P, PD, PI, and PID ones" [15, p.15].

Reference [4] compares PD and sliding mode regulators. Using a Kalman filter permits to reduce oscillations of the PD regulator [4], [15]. When following a circular path, a large amount of error appears for the PD controller. On the other hand, the sliding mode regulator can make the ball follow the circular trajectory with low error. The control action of the PD controller is smoother than that of SM because SM suffers from the chattering effect.[4]

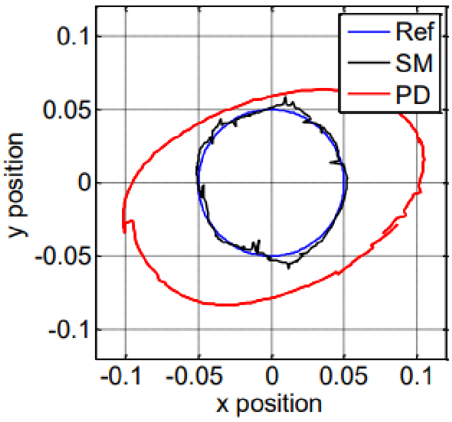

Fig. 19: Experimental result for tracking of a circular trajectory
In [4 Fig. 14]

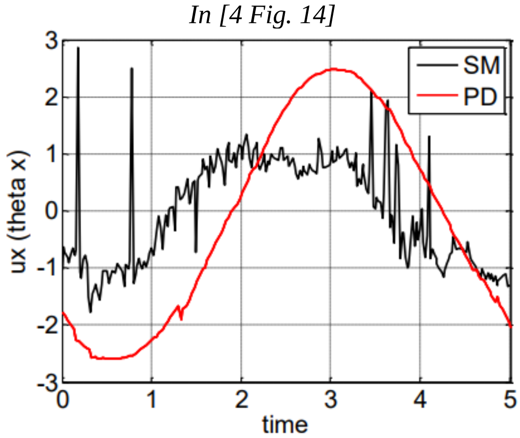

Fig. 20: Experimental result for the control action for X axis
In [4 Fig. 15]

The results obtained in [2] are identical to those obtained in [9]: "comparison between the proposed strategies illuminates the capability of the Sliding Mode control to solve the problem of static and dynamic tracking for nonlinear systems with high complexity as it overcomes modeling imprecision and parameter variations." [9, p. 52]. The sliding mode controller is the most robust and will therefore fit our modeling inaccuracies.
On the other hand, the PID does not provide accurate results but allows an easy implementation and does not require preliminary modeling.

The following table shows the different characteristics of the studied controllers in the case of a ball and plate system:

	PID	SM	LQR
Ease of implementation	++	--	--
Position control	-	++	+
Trajectory tracking	--	++	+

Table 2: Comparison of the controllers studied.

5. Conclusion

In conclusion, this literature review has discussed the development of a ball and plate system. The first part of the review examined two methods of detecting the ball, touch screen and camera, and highlighted their advantages and limitations. The second part of the review explored the modeling of the system, with the Lagrangian approach identified as the best method for modeling the ball and plate system. Finally, different controllers such as PID, LQR, and sliding mode control were discussed in the third part of the review. Based on our analysis, we conclude that while the PID controller can be used for the ball and plate system, the sliding mode controller would be a better choice due to its robustness and ability to handle disturbances.

Overall, this literature review highlights the importance of choosing the appropriate detection method, modeling approach, and controller for the ball and plate system to achieve optimal performance. Future studies can further investigate the use of different controllers and their impact on the ball and plate system's performance.

6. Bibliographical references

[1] C. Ham et M. M. Taufiq, « Development of a Ball and Plate System », présenté à 2015 ASEE Annual Conference & Exposition, juin 2015, p. 26.518.1-26.518.13, doi: 10.18260/p.23857

[2] D. Núñez, G. Acosta Amaya, et J. A. Jiménez Builes, « Control of a ball-and-plate system using a State-feedback controller », Ingeniare: Revista Chilena de Ingeniería, vol. 28, no 1, p. 6-15, 2020.

[3] A. Koszewnik, K. Troc, et M. Słowik, « PID controllers design applied to positioning of ball on the Stewart platform », Acta Mechanica et Automatica, vol. 8, no 4, p. 214-218, 2014.

[4] L. J. Saud et M. M. Alwan, « Design and Implementation of Classical Sliding Mode Controller for Ball and Plate System », Journal of Engineering, vol. 23, no 6, p. 74-92, mai 2017.

[5] S. Kizir, Z. Bingül, C. Oysu, et S. Küçük, « DEVELOPMENT AND CONTROL OF A HIGH PRECISION STEWART PLATFORM », Uluslararası Teknolojik Bilimler Dergisi, vol. 3, no 1, p. 51-59, mars 2011.

[6] A. Adiprasetya et A. Wibowo, « Implementation of PID controller and pre-filter to control non-linear ball and plate system », sept. 2016, p. 174-178, doi: 10.1109/ICCEREC.2016.7814965.

[7] Y. Jian et C. Qin, « Design of Rolling Ball Control System Based on Image Recognition », J. Phys.: Conf. Ser., vol. 1865, no 4, p. 042037, avr. 2021, doi: 10.1088/1742-6596/1865/4/042037.

[8] G. Quan, « Transfer Learning in Frequency-Domain ILC. Application to a Ball and Plate System », Master Thesis, ETH Zurich, 2020 [Online]. Disponible sur : https://www.research-collection.ethz.ch/handle/20.500.11850/486556. [Accessed: Feb. 09, 2023]

[9] A. Kassem, H. Haddad, et C. Albitar, « Commparison Between Different Methods of Control of Ball and Plate System with 6DOF Stewart Platform », IFAC-PapersOnLine, vol. 48, no 11, p. 47-52, janv. 2015, doi:10.1016/j.ifacol.2015.09.158.

[10] A. Knuplez, A. Chowdhury, et R. Svecko, « Modeling and control design for the ball and plate system », in IEEE International Conference on Industrial Technology, 2003, déc. 2003, vol. 2, p. 1064-1067 Vol.2, doi: 10.1109/ICIT.2003.1290810.

[11] W. Hongrui, T. Yantao, F. Siyan, et S. Zhen, « Nonlinear control for output regulation of ball and plate system », in 2008 27th Chinese Control Conference, juill. 2008, p. 382-387, doi: 10.1109/CHICC.2008.4605473.

[12] F. Zheng, X. Li, S. Wang, et D. Ding, « Position control of ball and plate system based on switching mechanism », in 2011 IEEE International Conference on Automation and Logistics (ICAL), août 2011, p. 233-237, doi: 10.1109/ICAL.2011.6024719.

[13] J. Kumar, N. Showme, M. Aravind, and R. Akshay, “Design and control of ball on plate system,” vol. 9, pp. 765–778, Jan. 2016.

[14] H. Liu et Y. Liang, « Trajectory tracking sliding mode control of ball and plate system », in 2010 2nd International Asia Conference on Informatics in Control, Automation and Robotics (CAR 2010), mars 2010, vol. 3, p. 142-145, doi: 10.1109/CAR.2010.5456649.

[15] K. Zarzycki et M. Ławryńczuk, « Fast Real-Time Model Predictive Control for a Ball-on-Plate Process », Sensors, vol. 21, no 12, p. 3959, 2021, doi: 10.3390/s21123959.