Département de Génie Électrique & Informatique

Tutored research report

Development of a ball balancing system

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Abstract :

This study addresses the regulation of ball position on a 3-degree-of-freedom (3-DOF) Stewart platform using a Proportional Integral Derivative (PID) control system. Previous research has primarily focused on either 2-DOF or 6-DOF platforms, leaving a significant knowledge gap concerning the application of control techniques to 3-DOF Stewart platforms. This research aims to bridge this gap by investigating the feasibility of PID control for regulating ball position on the platform.

To achieve this, an Arduino-based system incorporating three servos is employed for platform control, with the PID control system implemented in MATLAB Simulink. The ball's position is acquired through a webcam and computer vision, and the data are transmitted serially for processing.

While the results in terms of ball position regulation were unsatisfactory within the scope of this preliminary study due to time constraints and model play, the detection system demonstrated exceptional performance, and the Arduino-based servo control proved highly effective.

As a next step, further research will focus on enhancing the accuracy and robustness of the platform by implementing advanced control techniques such as Linear Quadratic Regulator (LQR), Fuzzy Logic, or Sliding Mode controllers. These methods are expected to address the limitations encountered in this study and improve the regulation of the ball's position on the 3-DOF Stewart platform.

In conclusion, this study presents a preliminary investigation into the regulation of ball position on a 3-DOF Stewart platform using PID control. Although the results were limited by time constraints and model play, the findings indicate the potential for achieving accurate ball position detection. Future research efforts will explore advanced control techniques to overcome the limitations encountered and enhance the platform's regulation capabilities.

Contents

1 Introduction

Our project consists in developing a ball stabilization model using a 3 degree of freedom (3dof) Steward platform. In previous years, other student groups have developed the physical model. This year, we have two main objectives: to stabilize the ball on the model and to make our model usable for the automatic laboratory at the Institut National des Sciences Appliquées (INSA) of Toulouse. In this report, we focus on exploring the regulation of ball position on a 3dof Stewart platform using a Proportional Integral Derivative (PID) control system.

Previous research has primarily concentrated on either 2dof [1, 2] or 6dof [3, 4] Stewart platforms, with limited investigations conducted on the 3dof variant. Addressing this knowledge gap, our research aims to assess the feasibility of employing a PID controller to achieve accurate ball position regulation on a 3dof Stewart platform.

Firstly, we will present the hardware employed and explore the integration of the Arduino microcontroller in the system, explaining its role and functionality in facilitating real-time control and communication with the platform.
Secondly, we will discuss the development and implementation of the ball detection algorithm, highlighting its effectiveness and reliability in accurately determining the position of the ball.
Furthermore, the mathematical model of the 3dof Stewart platform will be presented, elucidating the relationships and equations utilized for control and manipulation.
The report will finally examine the theoretical foundations and principles underlying the PID control system, exploring its potential capabilities and limitations in regulating the ball position. Although we couldn't perform practical experiments in this regard, we will present a comprehensive theoretical framework for understanding the PID control system's functioning and its potential application to the 3dof Stewart platform.

While practical experimentation with the PID control system was limited, this report aims to provide a comprehensive understanding of the various components involved in the regulation of ball position on the 3dof Stewart platform. By focusing on the theoretical calculations, algorithm development, platform modeling, and the integration of the Arduino, this report lays the foundation for future practical implementation and testing.

2 Materials and Methods

2.1 Hardware

2.1.1 Arduino

The Arduino Uno served as a programmed interface between the computer-based control system and the mechanical components of our 3dof Stewart platform, which was at the center of our experimental setup. An Arduino Uno, a board built on the ATmega328P microprocessor, was employed in this study. It includes a 16 MHz quartz crystal, 6 analog inputs, 14 digital input/output pins (of which 6 can be used as PWM outputs), a USB port, a power jack, an In-Circuit Serial Programming (ICSP) header, and a reset button.[5]

The Arduino's ability to translate MATLAB Simulink signals into servo-actionable commands is what makes the experimental setup successful. The Uno was configured to execute these instructions on the 3dof Stewart platform in real-time using the Arduino Integrated Development Environment (IDE). This allowed the PID controller to manage the ball's position.

The maximum current output of the Arduino Uno is one of its drawbacks because it cannot power numerous servos. The Uno's digital or analog pins have a cap on the amount of current they can deliver at roughly 20 mA per pin, with a total output of 200 mA across all pins[5]. This is insufficient for the current-hungry servos utilized in this configuration.

2.1.2 External Power Supply

The servos were powered by an external power source in order to get around this restriction. This configuration prevented the Arduino, which would have most certainly led to system failure due to the Uno's power limitations, from supplying the servos with the necessary current.

To guarantee constant voltage levels and avoid any ground loop issues, the external power supply and the Arduino shared a common ground. This allowed the Arduino to control the servos without also having to supply power for them, ensuring that the Arduino Uno's capabilities were fully utilized within its operational parameters.

2.1.3 Servos

Servo motors, a class of electric motor devices sometimes referred to as "servos," are used in robotics and other sectors due to their accuracy and control skills. A motor, a control circuit, a potentiometer (for position feedback), and gears are the components of a servo. They may be controlled by providing them a particular electrical pulse signal, and they are made to rotate and retain the output shaft in a given position. They are perfect for applications needing precision positioning since they can hold their position even when outside forces try to shift them.

We used Tower Pro MG995R servo motors for this build. These are metal-geared, high-torque servos that provide greater stability and longevity than their plastic-geared equivalents. They are a great option for demanding applications like ours because of their voltage range of 4.8 - 7.2V and their stall torque of roughly 10 kg-cm (at 6V)[6].

RC servo pulses with a range of roughly 500 to 2500 microseconds and a refresh rate of about 50 Hz are accepted by the Tower Pro MG995R. The servo can rotate its output shaft to a precise angle within a range of 180 degrees in response to these input pulses.

The Arduino in our setup converted the control directives from the MATLAB Simulink model into the pulse-width modulation (PWM) signals that the servo motors could interpret. The Stewart platform's position was modified by the servos in response to these signals, allowing for fine control of the ball's placement on the platform.

The Tower Pro MG995R servos' sturdy design and good torque performance were crucial in preserving accurate and stable control of the 3dof Stewart platform, highlighting their efficiency in high-precision control systems.

2.1.4 Serial communication

Serial communication is a form of data transfer where information is transmitted sequentially, one bit at a time, over a single communication channel. It is a fundamental method employed in digital communication systems for the exchange of data between two devices. Serial communication offers several advantages, including simplicity, reliability, and compatibility with a wide range of devices. It is extensively utilized in various applications, such as computer peripherals, embedded systems, and long-distance communication between devices.
Serial communication was employed in our setup to ensure the reliable and real-time transfer of data between the computer and the Arduino. We used the Arduino's onboard hardware serial ports to communicate with the computer. Serial communication must be configured on the transmitter(MATLAB Simulink) and on the receiver(Arduino).

MATLAB Simulink side. Serial communication requires perfect synchronization between the transmitter and the receiver. The processing time in our algorithm being variable, the use of a Zero-Order Hold (ZOH)[1] is necessary to guarantee a perfect synchronization. The Simulink program concatenates the desired angle for each of the servos and transmits them at fixed intervals to the Arduino.

Arduino Uno side. The Arduino board is configured to read at fixed intervals its serial reception buffer. The reading frequency is the same as the transmission frequency configured in the ZOH. The received angles are decomposed and are finally transmitted to their respective servo.

2.2 Ball detection

2.2.1 Image acquisition

The camera used is a "Microsoft LifeCam HD-3000", presented in Figure 1. It is a generic webcam supporting a resolution of up to 1280x720 pixels[7]. Even though faster technologies exist such as touchscreens[8], we decided to keep the camera since it is a cheaper alternative and has frames per second (FPS) and latency properties that are enough for a simple ball detection system.

[1] a Zero-Order Hold (ZOH) approximates a continuous-time signal by maintaining a constant value during each sampling period.

Figure 1: Microsoft LifeCam HD-3000 Webcam [7]

2.2.2 Description of our image recognition system

As our model will be used for practical work, we decided to use exclusively MATLAB Simulink for the whole project. Therefore we have to use it also for the ball detection.
The camera provides images of the ball on the platform to the MATLAB algorithm and the images are processed using the "Blob Analysis" function[9]. In order to keep a simple algorithm and to increase the processing speed, no AI was involved.
For this reason, we had to pick an orange ball whose color is very different from the white plate. This allowed the algorithm to detect the outline of the ball, and from this outline, to compute the x and y coordinates of the center of the ball.

2.2.3 The Blob Analysis function

In MATLAB, the blob analysis function is used to identify and analyze connected regions or "blobs" in an image. Blobs typically represent objects or regions of interest that share similar characteristics, such as color, intensity, or texture.
The blob analysis function in MATLAB offers several operations to extract useful information from blobs, such as their centroid, area, perimeter, bounding box, and more.
The following parameters must be chosen to make the algorithm work properly :

1. Allowable color range: In order to recognize only the orange ball, an appropriate color interval must be used. This range must be wide enough to take into account the different brightness conditions. We have chosen an interval from hsv(5,40,50) to hsv(15,100,100), shown in Figure 2.

Figure 2: Allowable color range : from hsv(5,40,50) to hsv(15,100,100)

2. Image preprocessing: A x5 numeric zoom is applied in order to only analyze the platform
3. Bloc Detection: The allowable area range for the object is set [100, 10000] square pixels. The defined interval prevents the system from confusing the ball with very small orange objects (wire, digital noise)
4. Post-processing: Of all the orange objects that can be identified, only the largest is considered. The function then returns the center of this object.

2.2.4 User interface

In order to allow an easy debugging to the user, an interface allowing to visualize in live the images taken by the camera as well as the position of the ball has been added.
A first function named concatenate is in charge of transforming the position (x,y) and the surface of the ball into a shape
The "Draw circle" block allows to overlay the original image with the shape.
The whole set is then displayed to the user thanks to the "To video display" block.
The Simulink blocks used are shown in Figure 3.

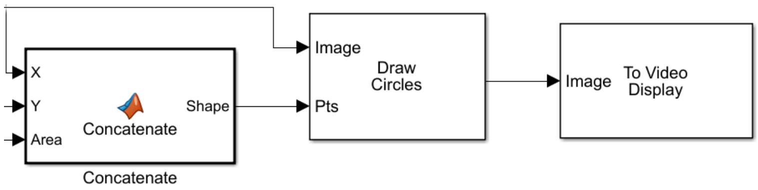

Figure 3: Simulink diagram of the interface

2.3 System modeling

2.3.1 Mathematics modeling

Illustrative graph

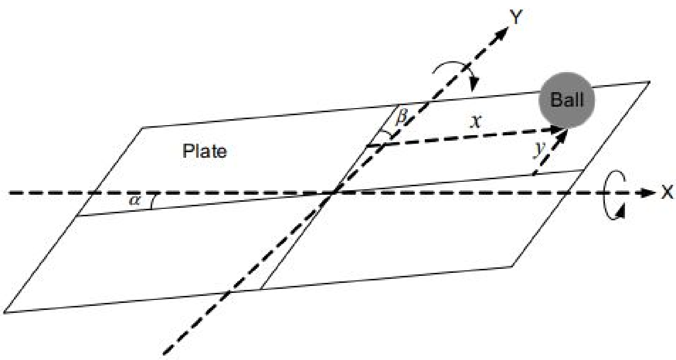

Figure 4: Free body diagram

Mathematical modeling In this case, we use Newton's second law to the free body diagram presented in Figure 4, which states that the sum of forces is equal to mass times acceleration. Applying it to angular acceleration, we obtain the following two differential equations that provide the modeling of the system

$$(m + \frac{I}{r^2})\ddot{x} - m(x\dot{\alpha}^2 + y\dot{\alpha}\dot{\beta}) + mg\sin(\alpha) = 0 \qquad \text{(eqt. 1)}$$

$$(m + \frac{I}{r^2})\ddot{y} - m(y\dot{\beta}^2 + x\dot{\alpha}\dot{\beta}) + mg\sin(\beta) = 0 \qquad \text{(eqt. 2)}$$

Linearization In general, the dynamics of the ball and plate system can be linearized under the following assumptions:

- The ball is considered as a point object with concentrated mass.
- The plate is considered rigid and its deformation is negligible.
- Contact forces between the ball and plate are modeled as normal and tangential forces.
- Angular accelerations of the ball and plate are small.
- The angle of rotation of the plate is small so that we can suppose $\sin\alpha \approx \alpha$ and $\sin\beta \approx \beta$
- The centrifugal force on the ball can be ignored as it is very small compared to gravity.

From these assumptions, the linearized model can be expressed as following:

$$\ddot{x} + \frac{5}{7}g\alpha = 0 \qquad \text{(eqt. 3)}$$

$$\ddot{y} + \frac{5}{7}g\beta = 0 \qquad \text{(eqt. 4)}$$

Equations (3) and (4) are decoupled so that we can consider as 2 different single-input and single-output systems. We can also design a controller by considering a single system and then apply it to both systems because of their similarity. The input and output for each system isα, x and β, y.

From this, Python code[10] uses alpha and beta to generate the angle that each servo motor has to have in order to get the ball in the center of the plate.

2.3.2 Digital twin modeling

For the successful development of the project, it was crucial to create a digital twin of the physical system. This modeling enables precise comprehension of the mechanics of the movement, as well as the system's response to desired inputs. Moreover, it provides the advantage of manipulating the system without the need of direct contact with the physical model. By implementing a digital twin, a deeper understanding of the system's behavior is achieved, ensuring more accurate analysis and facilitating the optimization process.

3D Assembly. The first step to develop the digital twin was to make an assembly of the previously modeled parts [2], for this it was necessary to convert these armature parts into solid parts, then using the Autodesk Inventor software, we were able to modify joint conditions such as extreme conditions, insertion angles and motion geometry between the solid bodies.

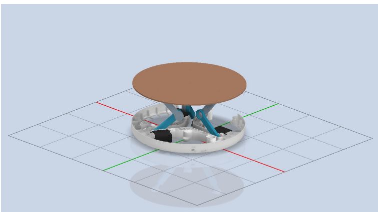

Figure 5: Assembly 3D made by Inventor Software with CAD pieces.

[2]CAD parts modeled in 3D by the last project team

This software allowed us to precisely manipulate the position and joint conditions between components and then export them as a block diagram to the MATLAB Simulink environment using the Simscape[11] library.

Simulink model. Due to the nature of the project its export and development in the MATLAB Simulink environment was imperative, thanks to the use of the Simscape export library it was possible to transform the CAD model into a block diagram that allowed its numerical and precise manipulation to later concatenate the mechanical subsystem obtained with the ball detection system and processing of the input angles.

In the Figure 6,it is evident that the input to the joints [0 to 2] corresponds to the angles to be taken by the servomotor shaft, in this case they are constant but it is there where the connection of this subsystem to the outputs of the ball detection subsystem will be made, The rotational joints [3 ... 5] correspond to the connection between the base arm and the arm connected to the plate, likewise there are 3 spherical joints that join the plate to the second arm allowing the movement of 3 degrees of freedom, shown in the Figure 7 an image of the execution of this subsystem.

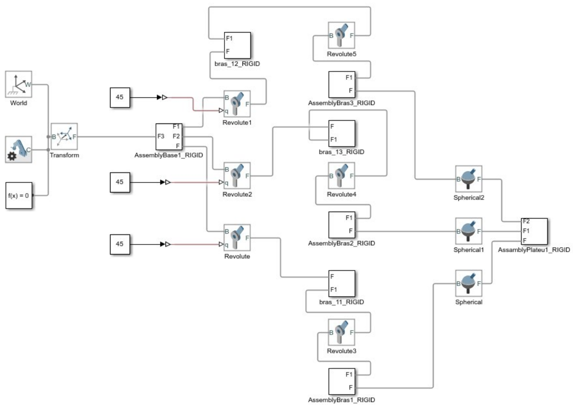

Figure 6: Block diagram obtained by Simscape

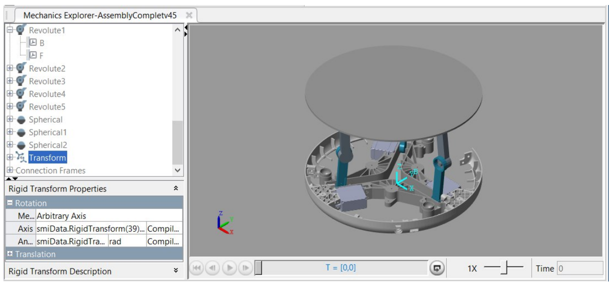

Figure 7: 3D animation generated by Simulink run

2.4 Controller

PID Controller. The PID controller is known for its simplicity and effectiveness in regulating system behavior by continuously adjusting the control input based on error, integral, and derivative terms.The control of dynamic systems plays a crucial role in numerous scientific and industrial processes. Among various control algorithms, the PID controller has gained significant popularity due to its versatility and easy of implementation.

The PID controller comprises three main components: the proportional (P) term, the integral (I) term, and the derivative (D) term. The proportional (P) term in a PID controller contributes to the control action based on the current error between the desired setpoint and the actual system output. It directly multiplies the error by a gain factor, which determines the proportionality between the error and the control output.The integral (I) term addresses steady-state errors by considering the cumulative sum of past errors.The I term acts as a corrective measure to eliminate residual errors that may persist even when the proportional term is applied. The derivative (D) term anticipates the future behavior of the system by measuring the rate of change of the error. It calculates the derivative of the error and multiplies it by a gain factor, the derivative gain.

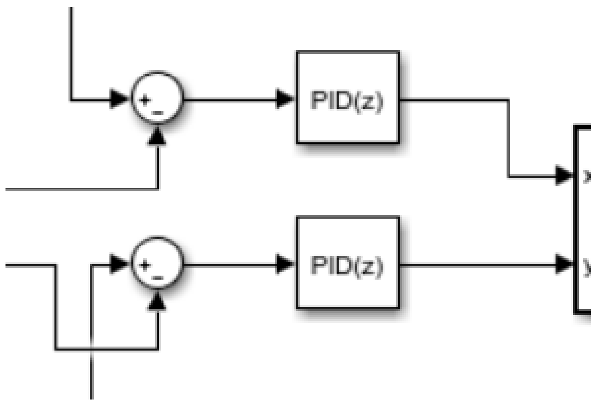

Figure 8: Discrete PID utilized in simulink

Controller implementation. The PID controller for the ball and plate system was implemented using MATLAB Simulink[12] and is presented in Figure 8. The control algorithm was designed to regulate the position of the ball by adjusting the plate's tilt angles. The input of the PID controller is the difference between the position of the ball and the position of the center of the plate, both in pixels, normalized, and its output is used to define the angles α and β of the plate. We use a PID in discrete time[13] since the frequency of the camera and consequently of our system is 30 Hz. The Simulink block used is shown in Figure 8,. And finally, PID gains were estimated with simulink's sisotool[14] tool.

3 Results

3.1 Hardware

The results of our hardware setup demonstrated a successful interplay between the Arduino Uno, the Tower Pro MG995R servos, and the serial communication protocol. However, some operational challenges were observed that affected the overall performance.

3.1.1 Arduino Uno and External Power Supply

The Arduino Uno, coupled with an external power supply, effectively managed the high-power requirements of the servo motors. It successfully converted control commands from the MATLAB Simulink model into signals for the servos, despite the power limitations of its own microcontroller. This successful integration demonstrated the feasibility of using an Arduino as a central control hub even when the power demands of the system exceed its native capabilities.

3.1.2 Servo control

In terms of positional control, the Tower Pro MG995R servos performed commendably within the 3dof Stewart platform setup. They provided a significant level of precision and responsiveness to the Arduino's control signals, consistently rotating and maintaining their output shaft at the required positions.

However, we encountered an issue related to the speed of the servos' response. Despite receiving the correct control signals, the servos showed a delay of about a second before responding, which negatively impacted the real-time control of the platform. This delay was unexpected given the servos' known specifications and introduced additional complexities to the system's performance.

3.1.3 Serial Communication

The serial communication protocol facilitated reliable data transmission between the Arduino Uno and the computer running the MATLAB Simulink model. Despite potential latency risks associated with the protocol, the positional data and control commands were consistently transmitted with minimal delay.

However, in relation to the servo response delay, further investigation into the communication block suggested that the issue might lie in the communication between the Arduino and the servos. Despite receiving correct signal frequency at the communication block input, the delay issue persists, indicating that this could be a communication problem between the Arduino and the servos.

3.2 Ball detection

The ball detection algorithm has given good results under various brightness conditions, thanks to the auto-brightness adjustment in MATLAB Simulink. We verified the algorithm's effectiveness by testing it with a different webcam, which demonstrated the flexibility of our program.
During the utilization of the algorithm, no noticeable lag was detected, ensuring a real-time regulation of the ball's position.

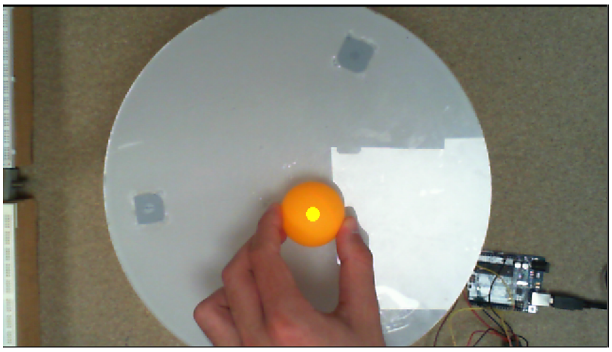

Figure 9: Calculated ball position, marked by a yellow circle on the user interface

However, we encountered a minor drawback where the wood support occasionally got misidentified as the ball. Nevertheless, using a x5 zoom on the camera has mitigated much of the issue. Finally, other solutions are evoked in 4.2.

3.3 Simscape model

After creating a virtual twin of the physical system, a key step was to determine the initial servo angles. It was observed that the angles of the *Revolute Joint* [15] (servos) were reversed compared to the physical system. Therefore, an important adjustment was made by introducing a negative unit gain to establish the correct angle relationship. Following this adjustment, the digital model exhibited perfect response.

As depicted in Figure 10, with all setpoint angles set to 45°, the plate appeared perfectly straight.

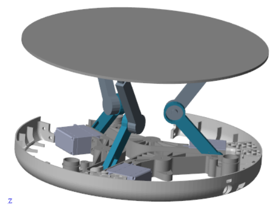

Figure 10: 3D animation of the digital twin model for setpoint angles all equal to 45°

As illustrated in Figure 11, for different setpoint angles (60°, 40°, and 0°), the plate exhibited a tilted position similar to that observed in the real system.

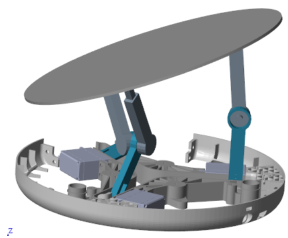

Figure 11: 3D animation of the digital twin model for 60°, 40°, and 0° setpoint angles

3.4 Controller

To design the controller, we utilized Simulink to implement the system model described in section 2.3.1 By leveraging the sisotool, we obtained insightful results pertaining to the PID constants and the system's behavior. Simulink served as a valuable platform for constructing and simulating the model derived from the theoretical framework outlined in 2.3.1. The model incorporated the relevant system dynamics and allowed us to explore various control strategies. By defining the appropriate transfer functions and input-output relationships, we were able to represent the system's behavior accurately within the Simulink environment.

Through the sisotool's graphical interface, we could visualize the system's response to different PID gains and observe the resulting effects on stability, settling time, overshoot, and other relevant performance metrics. By adjusting the PID constants and observing the system's response in real-time, we were able to iteratively fine-tune the controller to achieve the desired behavior.

The sisotool allowed us to gain a deep understanding of the interplay between the PID constants and the system's dynamic response. We carefully analyzed the trade-offs between stability and performance, seeking an optimal balance that minimized overshoot, ensured fast response times, and maintained robustness in the face of disturbances or uncertainties.

Overall, the combined use of Simulink and the sisotool provided us with a systematic approach to design and optimize the PID controller. The simulation environment enabled us to accurately represent the system's behavior, while the sisotool empowered us to fine-tune the PID constants and achieve a control strategy that met the desired performance criteria.

The simulation results yielded insightful findings. However, due to an issue encountered with the physical prototype, its physical implementation was not feasible. Additionally, it is crucial for the controlled system's response to exhibit a restrained nature, avoiding excessive aggressiveness. This consideration is essential as an overly aggressive response could lead to extraordinarily high velocities and accelerations, rendering the system uncontrollable.The system response is as follows:

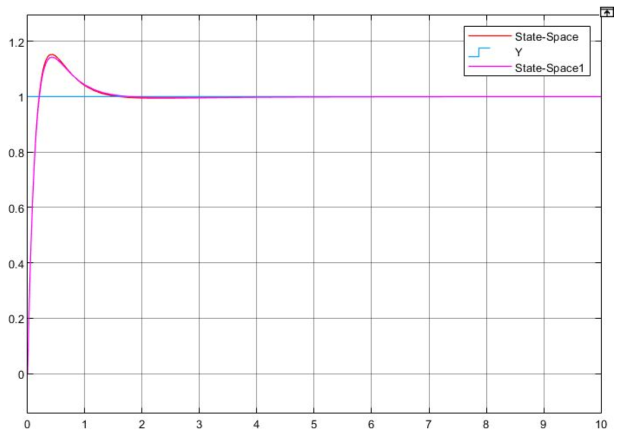

Figure 12: Modeled system response

Considering the requirement for a restrained system response, it is essential to strike a balance between stability and performance. A highly aggressive response may result in rapid and drastic changes in velocity and acceleration, making it difficult to maintain control over the system. By aiming for a more moderate and controlled response, we can ensure that the system remains within manageable limits as we can see in Figure 12, allowing for effective control and ensuring the safety and stability of the overall operation. The optimal gains obtained are as follows: $k_p = 3.467$, $k_i = 1.595$, $k_d = 1.414$.[3]

[3] The gains k_p, k_i, and k_d correspond respectively to the parameters (P), (I), and (D) mentioned in Section 2.4 .

4 Discussion

4.1 Hardware operation

The results obtained from our experimental hardware setup highlight both the capabilities and limitations of using an Arduino Uno, Tower Pro MG995R servos, and a serial communication protocol to control a 3dof Stewart platform.

External power supply. Despite the inherent power limitations of the Arduino Uno, our setup demonstrated that it could serve as a highly functional control hub when coupled with an external power supply. This finding underscores the Arduino's flexibility and adaptability in complex control systems that demand more power than it can natively provide.

Servo. Our servos, while demonstrating their precision and accuracy in positional control, encountered issues related to response speed. A noticeable delay of approximately a second was observed in their reaction time, which had an adverse impact on the real-time control of the Stewart platform. Given the Tower Pro MG995R servos' known specifications and previous performances, this delay was unexpected and, thus, suggests an area requiring further investigation.

Serial communication. Serial communication proved to be a reliable medium for data transmission between the computer running the MATLAB Simulink model and the Arduino Uno. However, we suspect the communication between the Arduino and the servos might be implicated in the observed servo response delay. While the input signal frequency at the communication block was confirmed as correct, the persistent response delay indicates a potential issue in the communication pathway to the servos.

Taken together, these results suggest that while our current hardware configuration is a promising starting point for controlling a 3dof Stewart platform, improvements are necessary for better real-time control and responsiveness. Specifically, resolving the servo response delay is crucial for enhancing the system's overall performance. This challenge provides a direction for future work, where more advanced control strategies and possibly different hardware configurations should be explored to increase the platform's accuracy and robustness. In doing so, the findings from this research will serve as a foundation for more sophisticated Stewart platform control systems.

4.2 Ball detection algorithm

As seen in 3.2, the ball detection algorithm has given good results under various brightness conditions. The user interface implemented for the project is really helpful, providing real-time display of the ball's position. Throughout its utilization, no noticeable lag was detected, ensuring a real-time regulation of the ball's position.
As said in 3.2, we encountered a minor drawback where the wood support occasionally got misidentified as the ball. This issue can be resolved by repainting the wood surface in white or by using a green ball, which would provide a distinct color contrast for accurate detection and prevent false identification.
Overall, the ball detection algorithm proved to be reliable and efficient, exhibiting strong performance in different conditions. With the suggested remedies for the detection issue, the algorithm can further enhance its accuracy and usability.

4.3 Digital twin model

Due to time constraints, only the plate was modeled in the project. As observed in 3.3, the plate demonstrates a perfect response to the input angle. However, in order to make the model more comprehensive, incorporating the ball into the Simulink/Simscape model would be a valuable addition. One possible approach could be to consider the ball as a separate rigid body with its own dynamics and interaction with the plate. This would involve defining the ball's mass, size, and properties such as friction and elasticity. Additionally, the forces and torques exerted on the ball by the plate could be modeled based on their contact and interaction. By integrating the ball into the existing model, a more accurate representation of the entire system could be achieved, allowing for a more comprehensive analysis and control design.

5 Conclusion

In conclusion, our project aimed to develop a 3DOF Stewart platform with ball position regulation. Throughout the course of our work, we made several key findings.

Firstly, the ball detection algorithm using a camera proved to be highly effective. It successfully detected and tracked the ball's position, providing reliable data for our control system.
Secondly, the Simscape digital twin model of the system worked well, accurately representing the dynamics of the platform. However, it is important to note that the model does not include the ball, which limits its comprehensive representation of the entire system.
Lastly, we encountered challenges in setting up the PID controller for the real model. Due to various difficulties, we were unable to fully implement and optimize the controller in practice. This serves as an area for future improvement and further development.

Overall, while we achieved success in certain aspects, such as the ball detection algorithm and the performance of the Simscape digital twin model, our project faced limitations in the practical implementation of the PID controller. Moving forward, addressing these challenges and refining the control system would be crucial to fully realize the potential of the 3DOF Stewart platform with ball position regulation using a PID controller.

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Projet d'Initiation à la Recherche de 4ème année

Development of a ball balancing system

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Abstract

The ball and plate system is a widely studied topic in control engineering. This literature review focuses on the comparison of different aspects of this system. The aim is to provide a comprehensive overview of the current state of the art in the field of ball and plate systems and to provide insights into the design and control of these systems.

The aspects studied are the main methods of modeling, control, and motion detection of the system ball.

A detailed analysis of the operation of the cameras and the touch screens for motion detection is presented. In parallel to the study of system development without modeling and the modeling for systems with 4 DOF.

The review also compares the performance of different regulators, including PID, LQR, and sliding mode controllers, in controlling the ball and plate system. This part of the review provides a summary of the design and implementation of each regulator and compares their performance in different scenarios. It also highlights the strengths and weaknesses of each regulator and provides insights into the design of optimal controllers for the ball and plate system.

Keywords: Ball and plate system, Nonlinearities, Regulators, PID, LQR, Sliding mode controller, Control theory, Tracking

1. Introduction

The ball and plate system is a classic example of a non-linear multi-input multi-output (MIMO) control system, which has been widely studied and implemented in various fields such as robotics, mechatronics, and control engineering. The system consists of a plate and a ball, where the plate is used to control the position of the ball. The objective of the ball and plate system is to maintain the ball position at a desired setpoint or to make it follow a trajectory.

The development of the ball and plate system has been the subject of numerous research studies, with a focus on various aspects of the system, such as the methods for detecting the ball position, the mechanical structure and modeling of the system, and the different types of controllers used to regulate the system's behavior. The first part of this literature review will focus on the various methods used for detecting the ball position, including resistive touch screen and camera-based systems.

The second part of the review will focus on the mechanical structure and modeling of the ball and plate system, including the analysis of the system's dynamics and the modeling of the system's behavior.

The final part of the review will cover the different types of controllers used in the ball and plate system, including Proportional-Integral-Derivative (PID) controllers, sliding mode controllers, and Linear-Quadratic Regulator (LQR) controllers. The performance of each type of controller will be compared and analyzed to determine their effectiveness in regulating the ball and plate system.

2. Method of detecting the ball

This section seeks to expose the possible applicable technologies for the detection of the ball. The most common technologies in this case are divided into two mainly. The first one can be summarized as a touch panel that detects the X and Y position of the sphere on the flat surface by means of a reference voltage the corresponding value is found, this will be elaborated later, the second technology is about tracking through cameras to detect the position of the ball, each technology shows advantages and disadvantages one with respect to the other, as well as having the possibility of being used together.

2.1. Touch screen

The goal of touch panels is to find the position in which they are touched, for this there are several concepts but the most used is the resistive touch panel.

These are composed of two layers that contain an electrical circuit. The top layer is pliable, so when pressure is applied, it causes a bend in the surface, altering the resistance in the circuit. This change in resistance can be interpreted as a fluctuation in voltage.[1]

A resistive touch screen operates by sending a voltage across a resistor array. The resistance at the point where the screen is touched by a stylus, pen, or finger is then measured. The 5-wire touch screen panel operates differently, by applying voltage to the corners of the bottom resistive layer and measuring the resistance of the vertical or horizontal network with the 5th wire, which is incorporated into the top sheet. As shown in the following image. [2]

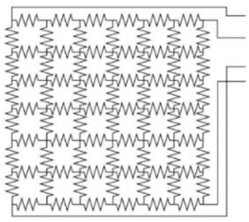

Fig. 1 The five-wire touchscreen resistor network.
In [2, Fig. 5]

A process of calibration and filtering of the signal must be done to have a correct response of precision that allows to have the least possible errors, this is done by means of sampling and a Butterworth low-pass filtering, in [3] there is a detailed explanation step by step on the process for the calibration of the panels for this use and how to do the filtering of the signal, as well as responses to signal treatments.

2.2. Camera

Cameras are used for ball detection, there are many methods for processing the images that are taken in real time to determine the trajectory and how the ball will move over the system, one widely used is the following algorithm.

There are different steps to follow that can make image processing more efficient, in general a structure is followed that follows the steps shown below: "1) Image Mask, for dimension reduction, 2) Measure, for corner detection, 3) Color Threshold to take into account only orange (ball) and black (platform), 4) Binary Image Inversion, to take into account the dynamics of the ball, not the platform, 5) Advance Morphology, to remove sections of the image that could be mistaken for the sphere and 6) Particle Analysis where the ball center coordinates are taken. "[4, p. 12-13]

The purpose of this design is to capture an image using a camera, and then apply a color block recognition algorithm to determine the target of the ball. It also marks the center of the ball and continuously calculates its relative position on the tablet by creating a plane coordinate system in the captured image field of view and sending the coordinate information. [5] Many of these algorithms are detailed and developed in Python because of its varied and efficient image processing libraries, as well as its high performance in the desired field.

A flowchart from the research work referred to below is shown, in this you can see how the different stages mentioned above are fulfilled and how it is necessary to have a filtering of the image depending on the conditions in which it works. [6]

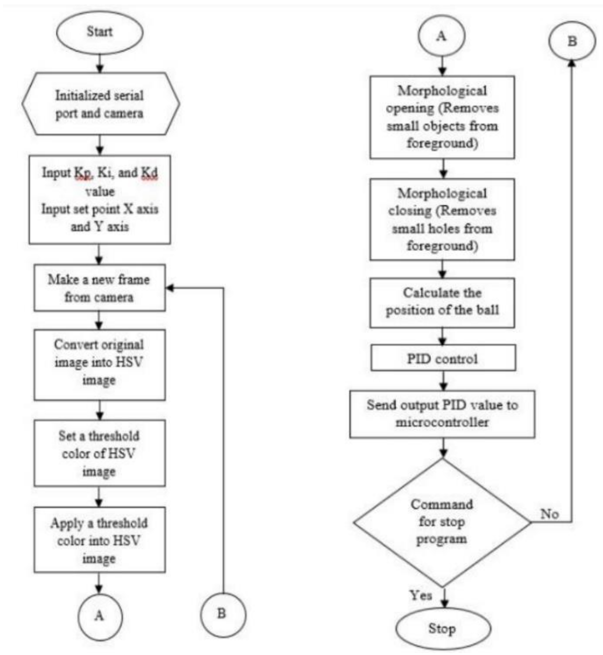

Fig. 2 Flowchart for image processing
In [6, Fig.6]

3. Modeling of the system

3.1. Techniques without modeling

The plate and ball system can be composed of different components that control the plate and the ball, among them an example of easy construction is the use of: computer (working as a pre-filter, controller and receiver/transmitter of date), Arduino(to control the movement of the servo motor), servo motor(controlling the movement of the plate) and webcam (to capture the movements of the ball) [6].

After the analysis of the components that compose the system the next logical step to ensure a good operation would be to model it and then search for the best controller, but not always the modeling of the system is necessary. If the developer chooses to use a PID type control, it is possible that the parameters (Kp, Ki, Kd) can be found by trial and error and without the need for modeling the system [6], [7]. This way it is possible to adjust the Kp parameter (proportional gain) by observing if the ball is managing to move the desired way on the plate (big enough for the ball to move smoothly but not so big as to cause high fluctuation). Then you just adjust the derivative gain (Kd) to adjust the float response and the integral gain (Ki) to adjust the stationary error [6], [7]. The PID type controller is described in more detail in section 4.1 of this paper.

3.2. 4 DOF

“In order to design a good control algorithm for the system, a nominal model needs to be derived. This helps us predict the system behavior”. [8, p. 4]

A 4DOF (degrees of freedom) system refers to a mechanical or physical system that has four independent ways it can move or change. These movements are usually defined by the system's generalized coordinates, which are specific variables that describe the system's state. In a 4DOF system, there are four such variables, and they can be related to various parameters, such as the position, orientation, or velocity of the system's components. [9],[10], [11], [12] proposes an approach using the Lagrangian. Here we have 2 DOF which are related to the motion of the ball and two others to the inclination of the plate. The position of the ball on the plate are given by "x_b" and "y_b" and the rotations of the plate are given by "α" and"β".

<u>The Euler-Lagrange equation is:</u>

$$Q_i = \frac{d}{dt}\frac{\partial T}{\partial \dot{q_\iota}} - \frac{d}{\partial \dot{q_\iota}} + \frac{\partial V}{\partial \dot{q_\iota}} \quad (2)$$

<u>The ball's kinetic energy is:</u>

$$Tb = \frac{1}{2}m_b\,(\dot{x}_b^2 + \dot{y}_b^2) + \frac{1}{2}I_b\,(\omega_x^2 + \omega_y^2) \ (3)$$

- q_i represents the coordinate in the "i" direction
- T is the kinetic energy of the system
- V is the potential energy of the system
- Q is the composite force
- m_b is the mass of the ball
- I_b is the moment of inertia of the ball

We assume that the ball rotates without slippage, which allows us to obtain the following equations (3). Once these equations are reintroduced into equation (2), we obtain a new kinetic equation for the ball (4).

$$\dot{x}_b = r_b\omega_y \,, \dot{y}_b = r_b\omega_x \quad (4)$$

- r_b is the radius of the ball

<u>The ball's kinetic energy without slippage is:</u>

$$Tb = \frac{1}{2}(m_b + \frac{I_b}{r_b^2})(\dot{x}_b^2 + \dot{y}_b^2) \qquad (5)$$

<u>The potential energy difference of the ball</u> relative to the horizontal plane passing through the center of the inclined plate is given by:

$$V_x = m_b g x_b sin(\alpha)$$
$$V_y = m_b g y_b sin(\beta)$$

Assuming that there is no spinning around the z-axis, <u>the plate's kinetic energy</u> encompasses both its rotational kinetic energy and that of the ball. This expression can be formulated as:

$$T_p = \frac{1}{2}(I_p + I_b)(\dot{\alpha}^2 + \dot{\beta}^2) + \frac{1}{2}m_b(x_b\dot{\alpha} + y_b\dot{\beta})^2 \quad (6)$$

Dynamical equations of the system are obtained using Lagrange equation [9], [10], [11], [12]:

<u>Equation of the ball movement on the plate:</u>

$$(m_b + \frac{I_b}{r_b^2})\ddot{x}_b - m_b(x_b\dot{\alpha}^2 + y_b\dot{\alpha}\dot{\beta}) + m_b g sin(\alpha) = 0 \ (7)$$

$$(m_b + \frac{I_b}{r_b^2})\ddot{y}_b - m_b(y_b\dot{\beta}^2 + x_b\dot{\alpha}\dot{\beta}) + m_b g sin(\beta) = 0 \ (8)$$

Equation (6) and (7) highlight the correlation between the ball's acceleration, the rotational angle, and the angular velocity of the plate.

<u>Equation showing the effect of external torque on the entire system:</u>

$$\tau_x = (I_p + I_b + m_b x_b^2)\ddot{\alpha} + 2m_b x_b \dot{x}_b \dot{\alpha} + m_b x_b y_b \ddot{\beta} + m_b \dot{x}_b y_b \dot{\beta} + m_b x_b \dot{y}_b \dot{\beta} + m_b g x_b cos(\alpha) \ (9)$$

$$\tau_y = (I_p + I_b + m_b y_b^2)\ddot{\beta} + 2m_b y_b \dot{y}_b \dot{\beta} + m_b x_b y_b \ddot{\alpha} + m_b \dot{x}_b y_b \dot{\alpha} + m_b x_b \dot{y}_b \dot{\alpha} + m_b g y_b cos(\beta) \ (10)$$

As the equations (9) and (10) are nonlinear, it will be difficult to use them to control the system. Since we only consider the angles α and β, we can focus solely on equations (7) and (8). Knowing that at steady state, α and β must be equal to 0 and that sin α ≈ α, sin β ≈ β, we obtain using equations (7) and (8):

$$x : (m_b + \frac{I_b}{r_b^2})\ddot{x} + m_b g\alpha = 0 \quad (11)$$

$$y : (m_b + \frac{I_b}{r_b^2})\ddot{y} + m_b g\beta = 0 \quad (12)$$

- $I_b = \frac{2}{5}m_b r_b^2$

4. Applied controls

4.1. PID

Even if it is possible to obtain the PID parameters by trial and error, "the best PID parameters can be determined using the dynamic model of the system" [7, p. 58]. In [6], a method is given to calculate the PID parameters.

In order to simplify the control of the system, we assume linear independence between the x and y axis. This allows us to transform the MIMO system into two Single Input – Single Output (SISO) systems. [6], [8]. Nevertheless, "This assumption is reasonable, if the two axes were perfectly perpendicular to each other. But due to hardware tolerances, plate unevenness and play in the system this is never the case in reality. Neglecting the cross-coupling effects between x and y axis is one of the major causes of periodic disturbances in the BaP system"[8 p. 4].

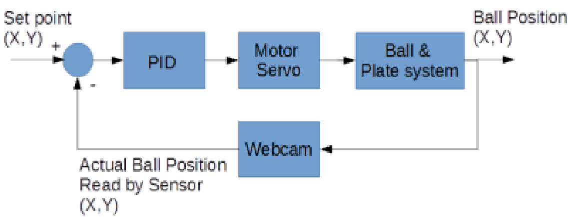

Fig. 3 Block Diagram of Ball and Plate Control System without Pre-filter
In [6, Fig. 2]

The PID control law in continuous time is of the form:

$$u(t) = K_P e(t) + K_I \int e(t)dt + K_D \frac{d}{dt}e(t) \quad (12)$$

And we can derive the recurrence equation to be implemented in the microcontroller:

$$OutputPID[n] = P[n] + I[n] + D[n]$$

$$P[n] = Kp.e[n]$$

$$I[n] = Ki.e[n] + I[n-1]$$

$$D[n] = Kd.(e[n] - e[n-1]) \qquad (13)$$

To find the PID parameters, we can use the Ziegler Nichols method for closed loop, which consists of finding the ultimate gain (Ku) and ultimate period (Pu) and deriving the constants from the Zeigler Nichols table.

Ku is the gain capable of generating a sustained sinusoidal response and Pu is the period of that response.[13]

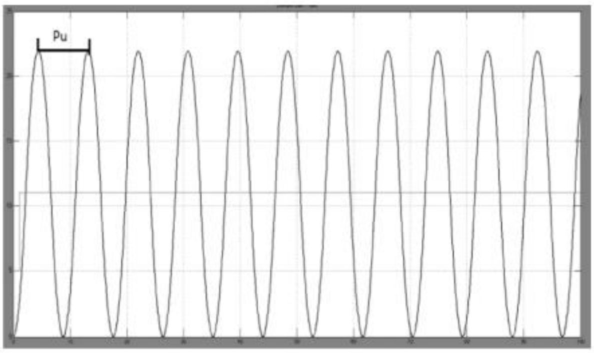

Fig. 4 Sustained oscillation for Ku = 3.4 (X Axis)
In [13, Fig. 9]

In this method, Ku = 3.4 and Pu = 3.75 seconds was obtained, but the results were not satisfactory, with a large settling time of approximately 34 seconds.

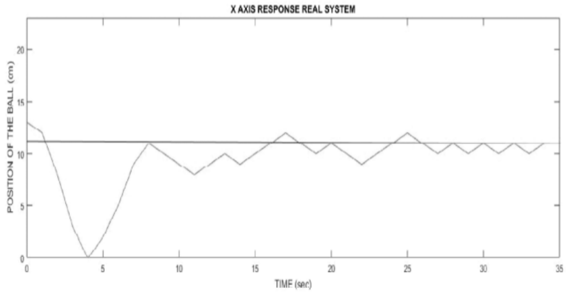

Fig. 5 Closed loop response of real system in X
In [13, Fig.11]

With manual adjustments, it was possible to achieve a more satisfactory settling time of 4.6 seconds [13].

On the other hand, the modeling of the system allows us to know that it is type 1, and that the dominant constants of the controller will be P and D. Seeing that it is possible to find the constants with trial and error done by focusing first on the proportional gain until we have a response that reaches the reference, then adjust the derivative gain to reduce oscillations, and finally add the integral gain to reduce the error steady state.

Another strategy seen in PID control is the use of a low-pass filter on the reference signal to reduce the oscillation of the PID controller.

controller	K_c	τ_I	τ_D
P	0.5 K_{CU}	–	–
PI	0.45 K_{CU}	$P_U/1.2$	–
PID	0.6 K_{CU}	$P_U/2$	$P_U/8$

Table 1: Ziegler Nichols tuning parameters
In [13, Tab. 1]

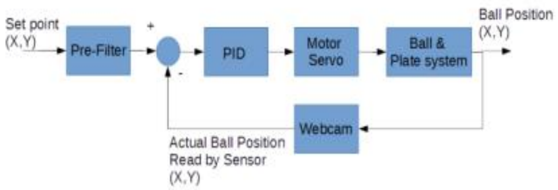

Fig. 6 Block Diagram of Ball and Plate Control System with Pre-filter

The pre-filter used was a first-order low-pass modeled by the following equations:

$$P(s) = \frac{a}{s + a}$$

$$P(z) = \frac{(1-a)^z}{z - a}$$

$$OP[n] - a.OP[n-1] = (1-a).SP[n] \quad (14)$$

Comparing Figure 7 and 8, we see that the filter reduces overshoot, and although it has a slower response, it makes the tracking more robust. [6]

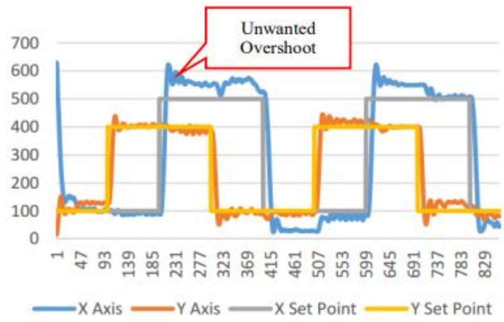

Fig. 7 Response of the system without pre-filter (4 different setpoint)
In [6 Fig.10]

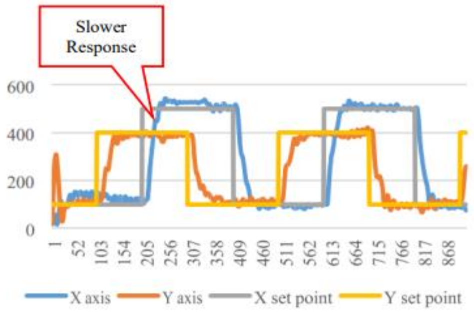

Fig. 8 Responses of the system with pre-filter (4 Different Setpoint)
In [6 Fig. 11]

4.2. Sliding mode control

The Sliding Mode Control law is in the form:

$$u_S(t) = b^{-1}(\frac{d^2}{dt}x_d(t) - \lambda\dot{e}(t) - \alpha\,.s(t) - \beta.sgn(s(t))) \quad (15)$$

And we can divide it into two parts, one to keep the system on the sliding surface, and one to force the system to go to the sliding surface. [2]

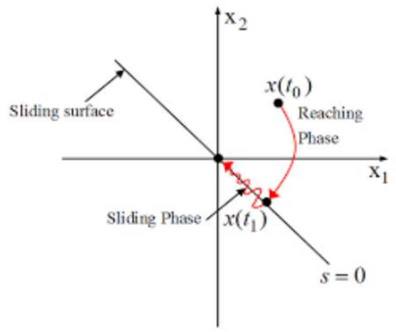

Fig. 9 Phase plane response of an SMC system.
In [2, Fig. 3]

This second part has the form -beta*sgn(s(t)), and this discontinuity caused by sgn(s(t)) in the control law can be harmful to the motors. This effect is called chattering and must be reduced by smoothing the control law with a saturation function. [9]

Also in the control law, s is the sliding function, b = gain of the transfer function of the modeled system, xd = reference. The parameters to be determined are lambda (slope of the desired sliding surface), alpha and beta. The 3 are derived from the input reference, time-varying uncertainty and dynamic mathematical model of the ball and plate system. [2], [14]

The sliding mode controller results are shown below. To stabilize the ball in a required position, the results are satisfactory with no oscillations or unwanted overshoot. The circular trajectory result is also satisfying with an error of less than 2mm on a 5cm radius desired circle.

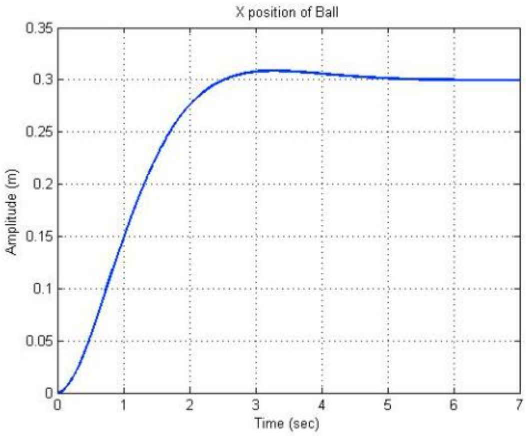

Fig. 10: The position of ball on the x-axis with sliding mode controller on simulation In: [9, Fig. 5]

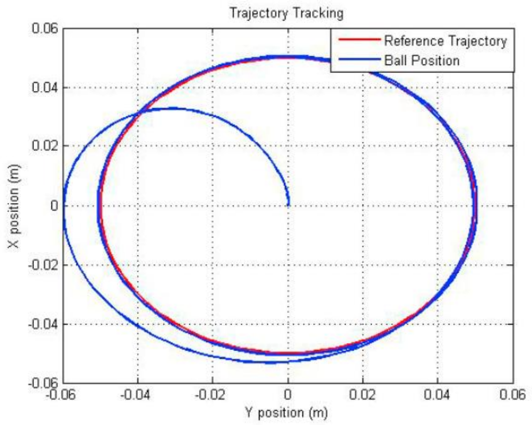

Fig. 11 : Circular trajectory tracking with sliding mode controller on real system In [9, Fig. 6]

On the experimental system, the sliding mode controller results are shown below, achieving ess = 0.5mm and ts = 0.52second to stabilize the ball in a required position. And on a circular trajectory, the error was less than 2mm. [9]

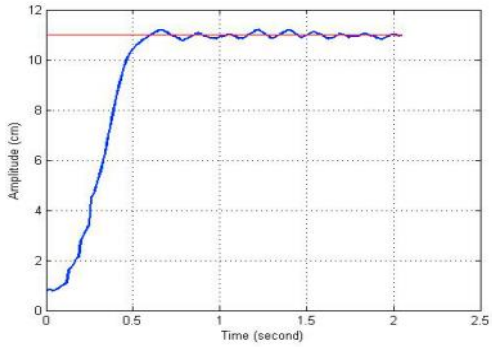

Fig. 12 The position of ball on the x-axis with Sliding Mode Controller on real system. In [9, Fig. 14]

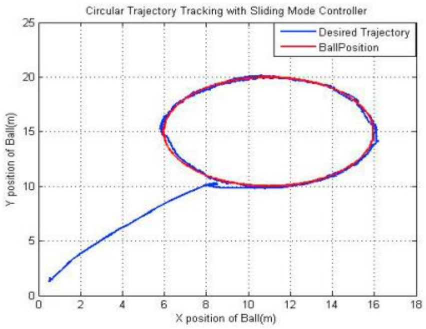

Fig. 13 Circular Trajectory Tracking with Sliding Mode Controller on real system. In [9, Fig. 15]

For the tracking of the circular trajectory, the error obtained are relatively similar between the real system and the simulation. This shows us the robustness characteristic of the sliding mode controller.

4.3. LQR regulator

LQR regulators are very frequently used for ball-on-plate processes. Like the PID regulator, the LQR regulator is linear. Nevertheless, the LQR controller is multivariable, i.e., "the manipulated variables are calculated taking into account all interactions of process variables" [15, p.17]. It is therefore a good choice for MIMO systems. Reference [15] gives us an overview of the performance of the LQR regulator. A first approach consists in choosing experimentally its settings using the pole placement method. A second method involves optimizing in MATLAB using the Dynamic Local Grid Refinement (DLGR) function. Using the DLRG method allows to eliminate the overshoot at the cost of a longer setting time. The obtained trajectories are the following:

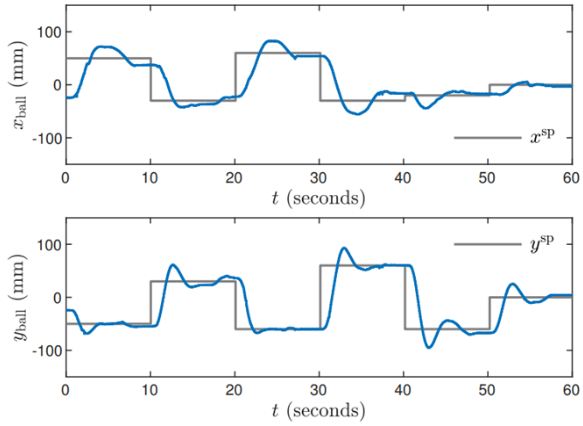

Fig. 14: Experimental results for the LQR controller using the pole placement method. Adapted from [15 Fig.10]

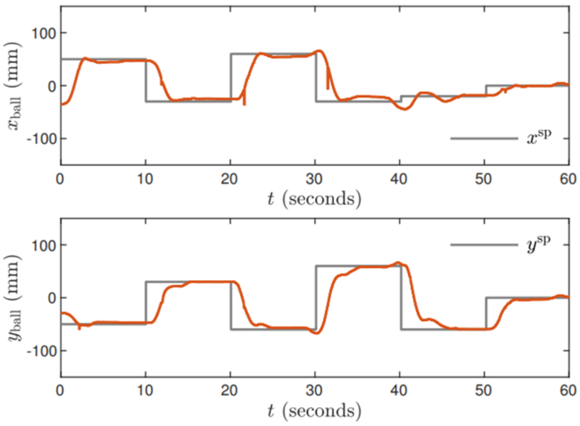

Fig. 15: Experimental results for the LQR controller using the DLGR method. Adapted from [15 Fig. 11]

In [9], another method is used to determine the parameters of the LQR controller. The feedback gain is based upon the minimization of a quadratic cost function:

$$J = \frac{1}{2} * \int_{0}^{+\infty} (x^T Qx + u^T Ru)dt$$

Solving Riccati equation allows the state-feedback gain. It should be noted that this method requires the use of a system model. Using Matlab Simulink, the following simulation results are obtained:

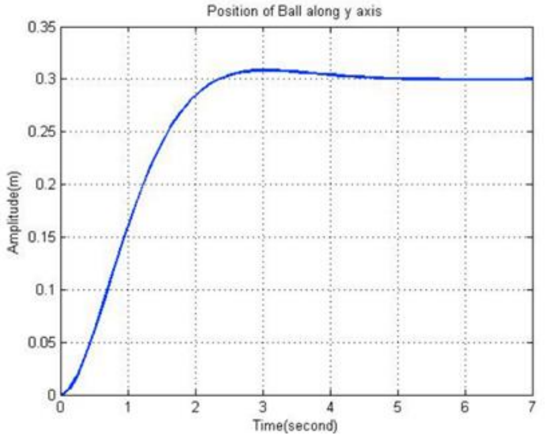

Fig. 16: Simulation result for stabilizing the ball in a desired position. In [9 Fig. 3]

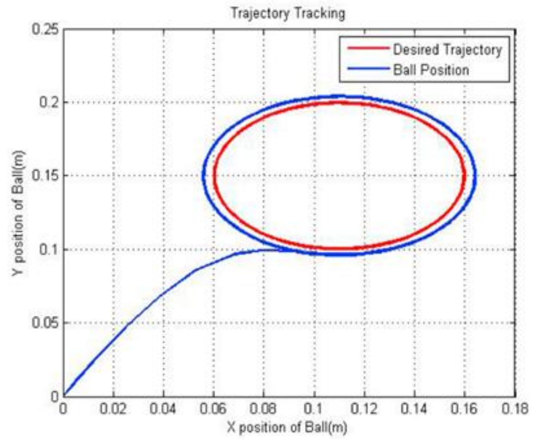

Fig. 17: Simulation result for tracking of a circular trajectory. In [9 Fig. 4]

The obtained error during the tracking of a circular trajectory is less than 5mm. Experimental results are given in [9 Fig. 12] and [9 Fig. 13]. The tracking of the circular trajectory gives an error smaller than 6mm. The trajectory is nevertheless slightly deformed. This illustrates that although the LQR is a linear corrector, it remains effective on systems with nonlinearities such as the ball-and-plate system.

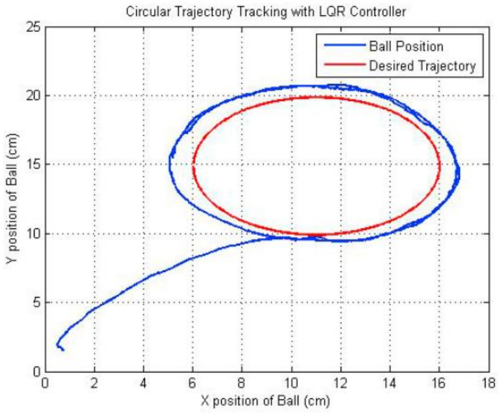

Fig. 18: Experimental result for tracking of a circular trajectory.
In [9 Fig. 13]

4.4. Comparison of controller performance

Reference [15] offers us a first comparison between PID and LQR controllers. One of the major problems for the PID is the derivative component (D), “If the calculations are performed using the ball position measurement, the derivative part causes the whole system to vibrate strongly, even when the ball reaches the set-points.” [15, p.14]. Another problem with PID is that it is very sensitive to measurement errors. The LQR controller is not affected by this type of error. “For the set-point tracking task, the LQR controllers perform much better than P, PD, PI, and PID ones” [15, p.15].

Reference [4] compares PD and sliding mode regulators. Using a Kalman filter permits to reduce oscillations of the PD regulator [4], [15]. When following a circular path, a large amount of error appears for the PD controller. On the other hand, the sliding mode regulator can make the ball follow the circular trajectory with low error. The control action of the PD controller is smoother than that of SM because SM suffers from the chattering effect.[4]

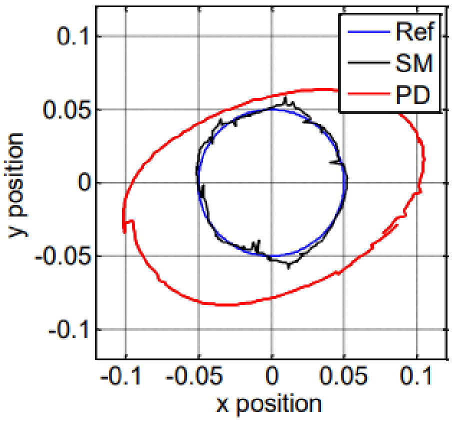

Fig. 19: Experimental result for tracking of a circular trajectory
In [4 Fig. 14]

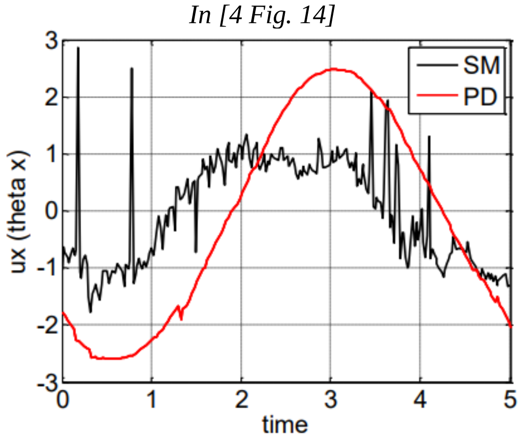

Fig. 20: Experimental result for the control action for X axis
In [4 Fig. 15]

The results obtained in [2] are identical to those obtained in [9]: “comparison between the proposed strategies illuminates the capability of the Sliding Mode control to solve the problem of static and dynamic tracking for nonlinear systems with high complexity as it overcomes modeling imprecision and parameter variations.” [9, p. 52]. The sliding mode controller is the most robust and will therefore fit our modeling inaccuracies.
On the other hand, the PID does not provide accurate results but allows an easy implementation and does not require preliminary modeling.

The following table shows the different characteristics of the studied controllers in the case of a ball and plate system:

	PID	SM	LQR
Ease of implementation	++	--	--
Position control	-	++	+
Trajectory tracking	--	++	+

Table 2: Comparison of the controllers studied.

5. Conclusion

In conclusion, this literature review has discussed the development of a ball and plate system. The first part of the review examined two methods of detecting the ball, touch screen and camera, and highlighted their advantages and limitations. The second part of the review explored the modeling of the system, with the Lagrangian approach identified as the best method for modeling the ball and plate system. Finally, different controllers such as PID, LQR, and sliding mode control were discussed in the third part of the review. Based on our analysis, we conclude that while the PID controller can be used for the ball and plate system, the sliding mode controller would be a better choice due to its robustness and ability to handle disturbances.

Overall, this literature review highlights the importance of choosing the appropriate detection method, modeling approach, and controller for the ball and plate system to achieve optimal performance. Future studies can further investigate the use of different controllers and their impact on the ball and plate system's performance.

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