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## Boolean Algebra

**a.**

$$\begin{aligned}x &= ABC + \overline{A}C \\&= C(AB + \overline{A}) \\&= C[(\overline{A} + B)(A + \overline{A})] \\&= C(\overline{A} + B) \\&= \overline{A}C + BC\end{aligned}$$

**b.**

$$\begin{aligned}y &= (Q + R)(\overline{Q} + \overline{R}) \\&= Q\overline{Q} + Q\overline{R} + R\overline{Q} + R\overline{R} \\&= Q\overline{R} + \overline{Q}R\end{aligned}$$

**c.**

$$\begin{aligned}w &= ABC + A\overline{B}C + \overline{A} \\&= AC(B + \overline{B}) + \overline{A} \\&= \overline{A}C + \overline{A} \\&= \overline{A}(\overline{C} + C) + \overline{A} \\&= (\overline{A} + \overline{C})\overline{A} \\&= \overline{A}\overline{A} + \overline{C}\overline{A} \\&= C + \overline{A}\end{aligned}$$

**d.**

$$q = \overline{RST}(\overline{R + S + T}) = (\overline{R} + \overline{S} + \overline{T})\overline{R.S.T} = \sum \overline{S.T}(\overline{R} + \overline{R}) = \sum \overline{S.T}.\overline{R} = \overline{S.T}.\overline{R}$$

**e.**

$$\begin{aligned}x &= \overline{A}\overline{B}\overline{C} + \overline{A}BC + ABC + A\overline{B}\overline{C} + A\overline{B}C \\&= (A + \overline{A})\overline{B}\overline{C} + (A + \overline{A})BC + A\overline{B}C \\&= \overline{B}\overline{C} + BC + A\overline{B}C \\&= \overline{B}\overline{C} + C(B + A\overline{B})\end{aligned}$$

$$= \overline{B} \overline{C} + CB + CA$$

**f.**

$$\begin{aligned} z &= (B + \overline{C})(\overline{B} + C) + \overline{\overline{A} + B + \overline{C}} \\ &= B\overline{B} + BC + \overline{B} \overline{C} + \overline{C}C + \overline{A}BC \\ &= BC + \overline{B} \overline{C} + \overline{A}BC \\ &= BC + (\overline{B} + \overline{A}B) \overline{C} + \overline{A}BC \\ &= BC + \overline{B} \overline{C} + \overline{A}B \overline{C} + \overline{A}BC \\ &= BC + \overline{B} \overline{C} + \overline{A}B \end{aligned}$$

**g.**

$$\begin{aligned} y &= \overline{C + D} + \overline{A}.C.\overline{D} + A.\overline{B}.\overline{C} + \overline{A}.\overline{B}.C.D + A.C.\overline{D} \\ y &= \overline{C}.\overline{D} + \overline{A}.C(\overline{D} + \overline{B}.D) + A.\overline{B}.\overline{C} + A.C.\overline{D} \\ y &= \overline{C}.\overline{D} + \overline{A}.C(\overline{D} + \overline{D}.\overline{B} + \overline{B}.D) + A.\overline{B}.\overline{C} + A.C.\overline{D} \\ y &= \overline{C}.\overline{D} + \overline{A}.C(\overline{D} + \overline{B}) + A.\overline{B}.\overline{C} + A.C.\overline{D} \\ y &= \overline{C}.\overline{D} + (\overline{A}.C.\overline{D} + A.C.\overline{D}) + \overline{A}.C.\overline{B} + A.\overline{B}.\overline{C} \\ y &= \overline{C}.\overline{D} + C.\overline{D} + \overline{A}.C.\overline{B} + A.\overline{B}.\overline{C} \\ y &= \overline{D} + \overline{A}.C.\overline{B} + A.\overline{B}.\overline{C} \end{aligned}$$

**h.**

$$\begin{aligned} x &= AB(\overline{C}D) + \overline{A}BD + \overline{B}\overline{C}\overline{D} \\ x &= AB(C + \overline{D}) + \overline{A}BD + \overline{B}\overline{C}\overline{D} \\ x &= ABC + AB\overline{D} + \overline{A}BD + \overline{B}\overline{C}\overline{D} \\ x &= ABC + \overline{A}BD + AB\overline{D} + \overline{B}\overline{C}\overline{D} \\ x &= B(AC + \overline{A}D) + \overline{D}(AB + \overline{B}\overline{C}) \end{aligned}$$

## K-map

1.  $F(A,B,C) = \sum(1, 2, 3, 4, 6, 7)$ :

	$\bar{C}$	$C$
$\bar{A}.\bar{B}$	0	1
$\bar{A}.B$	1	1
$A.B$	1	1
$A.\bar{B}$	1	0

1.png

Therefore :  $X = B + A\bar{C} + \bar{A}C$ .

2.  $F(A,B,C,D) = \sum(1, 3, 4, 5, 6, 7, 12, 13)$ :

	$\bar{C}.\bar{D}$	$\bar{C}.D$	$C.D$	$C.\bar{D}$
$\bar{A}.\bar{B}$	0	1	1	0
$\bar{A}.B$	1	1	1	1
$A.\bar{B}$	1	1	0	0
$A.B$	0	0	0	0

2.png

Therefore :  $X = \bar{A}D + \bar{A}B + B\bar{C}$ .

3.

$$F(A,B,C,D) = \sum(2,5,7,8,10,12,13,15)$$

	$\overline{C}.\overline{D}$	$\overline{C}.D$	$C.D$	$C.\overline{D}$
$\overline{A}.\overline{B}$	0 <sub>0</sub>	0 <sub>1</sub>	0 <sub>3</sub>	1 <sub>2</sub>
$\overline{A}.B$	0 <sub>4</sub>	1 <sub>5</sub>	1 <sub>7</sub>	0 <sub>6</sub>
$A.B$	1 <sub>12</sub>	1 <sub>13</sub>	1 <sub>15</sub>	0 <sub>14</sub>
$A.\overline{B}$	1 <sub>8</sub>	0 <sub>9</sub>	0 <sub>11</sub>	1 <sub>10</sub>

$$\Rightarrow F(A,B,C,D) = BD + A\overline{C}.\overline{D} + \overline{B}C\overline{D}$$

4.

$$F(A,B,C,D) = \sum(0,6,8,9,10,11,13,14,15)$$

Case 1

	$\overline{C}.\overline{D}$	$\overline{C}.D$	$C.D$	$C.\overline{D}$
$\overline{A}.\overline{B}$	1 <sub>0</sub>	0 <sub>1</sub>	0 <sub>3</sub>	0 <sub>2</sub>
$\overline{A}.B$	0 <sub>4</sub>	0 <sub>5</sub>	0 <sub>7</sub>	1 <sub>6</sub>
$A.B$	0 <sub>12</sub>	1 <sub>13</sub>	1 <sub>15</sub>	1 <sub>14</sub>
$A.\overline{B}$	1 <sub>8</sub>	1 <sub>9</sub>	1 <sub>11</sub>	1 <sub>10</sub>

$$\Rightarrow F(A,B,C,D) = AD + AC + BC\overline{D} + \overline{B}.\overline{C}.\overline{D}$$

Case 2

	$\overline{C}.\overline{D}$	$\overline{C}.D$	$C.D$	$C.\overline{D}$
$\overline{A}.\overline{B}$	1 <sub>0</sub>	0 <sub>1</sub>	0 <sub>3</sub>	0 <sub>2</sub>
$\overline{A}.B$	0 <sub>4</sub>	0 <sub>5</sub>	0 <sub>7</sub>	1 <sub>6</sub>
$A.B$	0 <sub>12</sub>	1 <sub>13</sub>	1 <sub>15</sub>	1 <sub>14</sub>
$A.\overline{B}$	1 <sub>8</sub>	1 <sub>9</sub>	1 <sub>11</sub>	1 <sub>10</sub>

$$\Rightarrow F(A,B,C,D) = AD + A\overline{B} + BC\overline{D} + \overline{B}.\overline{C}.\overline{D}$$

5.

AB/CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	0	0
$\bar{A}B$	1	1	1	1
AB	0	1	1	1
$A\bar{B}$	1	1	1	1

$$X = BD + BC + \bar{A}\bar{C}\bar{D} + \bar{A}\bar{B}$$

AB/CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	✓	0	0	0
$\bar{A}B$	1	1	1	1
AB	0	1	1	1
$A\bar{B}$	✓	1	1	1

$$X = AC + AD + \bar{B}\bar{C}\bar{D} + \bar{A}B$$

6.

DC/BA	$\overline{B}\overline{A}$	$\overline{B}A$	BA	$B\overline{A}$
$\overline{D}\overline{C}$	1	0	1	1
$\overline{D}C$	0	1	1	0
DC	1	1	1	1
$D\overline{C}$	1	0	1	1

$$X = CA + DC + BA + \overline{C}\overline{A}$$

DC/BA	$\overline{B}\overline{A}$	$\overline{B}A$	BA	$B\overline{A}$
$\overline{D}\overline{C}$	1	0	1	1
$\overline{D}C$	0	1	1	0
DC	1	1	1	1
$D\overline{C}$	1	0	1	1

$$X = CA + DC + \overline{C}\overline{A} + \overline{C}B$$

DC/BA	$\overline{B}\overline{A}$	$\overline{B}A$	BA	$B\overline{A}$
$\overline{D}\overline{C}$	1	0	1	1
$\overline{D}C$	0	1	1	0
DC	1	1	1	1
$D\overline{C}$	1	0	1	1

$$X = CA + BA + \overline{C}\overline{A} + D\overline{A}$$



DC/BA	$\overline{B}\overline{A}$	$\overline{B}A$	BA	$B\overline{A}$
$\overline{D}\overline{C}$	1	0	1	1
$\overline{D}C$	0	1	1	0
DC	1	1	1	1
$D\overline{C}$	1	0	1	1

$$X = CA + \overline{C}\overline{A} + D\overline{A} + \overline{C}B$$

7.

DC/BA	$\bar{B}\bar{A}$	$\bar{B}A$	BA	$B\bar{A}$
$\bar{D}\bar{C}$	1	1	0	0
$\bar{D}C$	1	1	1	0
DC	0	1	1	1
$D\bar{C}$	1	0	0	1

$$X = \bar{D}\bar{B} + CA + DCB + D\bar{C}\bar{A}$$

DC/BA	$\bar{B}\bar{A}$	$\bar{B}A$	BA	$B\bar{A}$
$\bar{D}\bar{C}$	1	1	0	0
$\bar{D}C$	1	1	1	0
DC	0	1	1	1
$D\bar{C}$	1	0	0	1

$$X = \bar{D}\bar{B} + CA + DB\bar{A} + D\bar{C}\bar{A}$$

DC/BA	$\bar{B}\bar{A}$	$\bar{B}A$	BA	$B\bar{A}$
$\bar{D}\bar{C}$	1	1	0	0
$\bar{D}C$	1	1	1	0
DC	0	1	1	1
$D\bar{C}$	1	0	0	1

$$X = \bar{D}\bar{B} + CA + DB\bar{A} + \bar{C}\bar{B}\bar{A}$$

8.

- Case 1:

		BA			
		00	01	11	10
DC	00	X	1	X	1
	01	X	1	0	0
	11	1	X	X	X
	10	X	0	0	1

$$x_1 = \overline{C} \overline{A} + \overline{D} \overline{B} + DC$$

- Case 2:

		BA			
		00	01	11	10
DC	00	X	1	X	1
	01	X	1	0	0
	11	1	X	X	X
	10	X	0	0	1

$$x_2 = \overline{C} \overline{A} + \overline{D} \overline{C} + C\overline{B}$$

- Case 3:

		BA			
		00	01	11	10
DC	00	X	1	X	1
	01	X	1	0	0
	11	1	X	X	X
	10	X	0	0	1

$$x_3 = \overline{C} \overline{A} + \overline{D} \overline{B} + C\overline{B}$$

- Case 4:

		BA			
		00	01	11	10
DC	00	X	1	X	1
	01	X	1	0	0
	11	1	X	X	X
	10	X	0	0	1

$$x_4 = D\overline{A} + \overline{D} \overline{C} + \overline{D} \overline{B}$$

- Case 5:

		BA			
		00	01	11	10
DC	00	X	1	X	1
	01	X	1	0	0
	11	1	X	X	X
	10	X	0	0	1

$$x_5 = D\bar{A} + \bar{D}\bar{C} + C\bar{B}$$

- Case 6:

		BA			
		00	01	11	10
DC	00	X	1	X	1
	01	X	1	0	0
	11	1	X	X	X
	10	X	0	0	1

$$x_6 = \bar{C}\bar{A} + \bar{D}\bar{B} + \bar{B}\bar{A}$$

- Case 7:

		BA			
		00	01	11	10
DC	00	X	1	X	1
	01	X	1	0	0
	11	1	X	X	X
	10	X	0	0	1

$$x_7 = D\bar{A} + \bar{C}\bar{A} + \bar{D}\bar{B}$$

## KMap (cont)

1

$$1. F(A,B,C,D) = \sum m(0, 1, 2, 5, 7, 8, 10, 14, 15) + d(3, 13)$$

	$\overline{C}\overline{D}$	$\overline{C}D$	$C\overline{D}$	$CD$
$\overline{A}\overline{B}$	1	1	x	1
$\overline{A}B$	0	1	1	0
$A\overline{B}$	0	x	1	1
$AB$	1	0	0	1

$$F = B'D' + A'D + ABC$$

2

$$2) F(A, B, C, D) = \prod M(1, 3, 4, 5, 11, 12, 14, 15) \cdot D(0, 6, 7, 8)$$

Solution

	$\overline{C}\overline{D}$	$\overline{C}D$	$CD$	$C\overline{D}$
$\overline{A}\overline{B}$	x	0	0	1
$\overline{A}B$	0	0	x	x
$AB$	0	1	0	0
$A\overline{B}$	x	1	0	1

$$\Rightarrow F(A, B, C, D) = \overline{A}D + B\overline{D} + CD$$

	$\bar{C}\bar{D}$	$\bar{C}D$	$CD$	$C\bar{D}$
$\bar{A}\bar{B}$	x	0	0	1
$\bar{A}B$	0	0	x	x
$AB$	0	1	0	0
$A\bar{B}$	x	1	0	1

$$\Rightarrow F(A, B, C, D) = \bar{A}\bar{C} + B\bar{D} + CD$$

3

$$3. F(A, B, C, D) = \sum m(1, 3, 6, 8, 11, 14) + d(2, 4, 5, 13, 15)$$

	$\bar{C}\bar{D}$	$\bar{C}D$	$CD$	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	5	7	6
$AB$	12	13	15	14
$A\bar{B}$	8	9	11	10

$$F = A'B'D + B'CD + BCD' + AB'C'D'$$

$$F = AB'C'D' + A'B'D + BCD' + ACD$$

$$F = AB'C'D' + A'C'D + B'CD + BCD'$$

4.

- Case 1:



		CD			
		00	01	11	10
AB	00	X	0	X	X
	01	1	0	0	0
	11	1	1	0	X
	10	X	0	0	1

$$x_1 = \overline{C} \overline{D} + A\overline{D} + AB\overline{C}$$

- Case 2:

		CD			
		00	01	11	10
AB	00	X	0	X	X
	01	1	0	0	0
	11	1	1	0	X
	10	X	0	0	1

$$x_2 = \overline{C} \overline{D} + \overline{B} \overline{D} + AB\overline{C}$$

5.

		<i>BA</i>			
		00	01	11	10
<i>DC</i>	00	0	1	0	1
	01	1	<i>X</i>	<i>X</i>	0
	11	<i>X</i>	0	1	0
	10	1	0	0	<i>X</i>

$$\begin{aligned}
 &F(D,C,B,A) \\
 &= \prod M(0,3,6,9,11,13,14) \cdot D(5,7,10,12) \\
 &= (A + B + C + D)(\bar{B} + \bar{A} + C)(\bar{D} + \bar{A} + B)(\bar{C} + \bar{B} + A)
 \end{aligned}$$

6.

Case 1:

		<i>BA</i>			
		00	01	11	10
<i>DC</i>	00	1	1	0	0
	01	1	<i>X</i>	<i>X</i>	1
	11	<i>X</i>	0	<i>X</i>	1
	10	<i>X</i>	<i>X</i>	<i>X</i>	1

$$\begin{aligned}
 &F(D,C,B,A) \\
 &= \Sigma m(0,1,4,6,10,14) + d(5,7,8,9,11,12,15) \\
 &= \bar{D}\bar{B} + BC + BD
 \end{aligned}$$

Case 2:

		$BA$			
		00	01	11	10
$DC$	00	1	1	0	0
	01	1	X	X	1
	11	X	0	X	1
	10	X	X	X	1

$$\begin{aligned}
 &F(D,C,B,A) \\
 &= \Sigma m(0,1,4,6,10,14) + d(5,7,8,9,11,12,15) \\
 &= \bar{B}\bar{D} + BD + \bar{A}C
 \end{aligned}$$

Case 3:

		$BA$			
		00	01	11	10
$DC$	00	1	1	0	0
	01	1	X	X	1
	11	X	0	X	1
	10	X	X	X	1

$$\begin{aligned}
 &F(D,C,B,A) \\
 &= \Sigma m(0,1,4,6,10,14) + d(5,7,8,9,11,12,15) \\
 &= \bar{B}\bar{D} + BD + C\bar{D}
 \end{aligned}$$

Case 4:

		<i>BA</i>			
		00	01	11	10
<i>DC</i>	00	1	1	0	0
	01	1	X	X	1
	11	X	0	X	1
	10	X	X	X	1

$$\begin{aligned}
 &F(D,C,B,A) \\
 &= \Sigma m(0,1,4,6,10,14) + d(5,7,8,9,11,12,15) \\
 &= \bar{B}\bar{D} + BC + \bar{A}D
 \end{aligned}$$

Case 5:

		<i>BA</i>			
		00	01	11	10
<i>DC</i>	00	1	1	0	0
	01	1	X	X	1
	11	X	0	X	1
	10	X	X	X	1

$$\begin{aligned}
 &F(D,C,B,A) \\
 &= \Sigma m(0,1,4,6,10,14) + d(5,7,8,9,11,12,15) \\
 &= \bar{B}\bar{D} + \bar{A}C + D\bar{A}
 \end{aligned}$$

Case 6:

		<i>BA</i>			
		00	01	11	10
<i>DC</i>	00	1	1	0	0
	01	1	X	X	1
	11	X	0	X	1
	10	X	X	X	1

$$\begin{aligned}
 &F(D,C,B,A) \\
 &= \Sigma m(0,1,4,6,10,14) + d(5,7,8,9,11,12,15) \\
 &= \bar{B}\bar{D} + D\bar{A} + C\bar{D}
 \end{aligned}$$

Case 7:

		<i>BA</i>			
		00	01	11	10
<i>DC</i>	00	1	1	0	0
	01	1	X	X	1
	11	X	0	X	1
	10	X	X	X	1

$$\begin{aligned}
 &F(D,C,B,A) \\
 &= \Sigma m(0,1,4,6,10,14) + d(5,7,8,9,11,12,15) \\
 &= \bar{B}\bar{D} + BC + \bar{C}D
 \end{aligned}$$

Case 8:

		<i>BA</i>			
		00	01	11	10
<i>DC</i>	00	1	1	0	0
	01	1	X	X	1
	11	X	0	X	1
	10	X	X	X	1

$$\begin{aligned}
 &F(D,C,B,A) \\
 &= \Sigma m(0,1,4,6,10,14) + d(5,7,8,9,11,12,15) \\
 &= \bar{B}\bar{D} + \bar{C}D + C\bar{A}
 \end{aligned}$$

Case 9:

		<i>BA</i>			
		00	01	11	10
<i>DC</i>	00	1	1	0	0
	01	1	X	X	1
	11	X	0	X	1
	10	X	X	X	1

$$\begin{aligned}
 &F(D,C,B,A) \\
 &= \Sigma m(0,1,4,6,10,14) + d(5,7,8,9,11,12,15) \\
 &= BD + C\bar{A} + \bar{B}\bar{C}
 \end{aligned}$$

Case 10:

		<i>BA</i>			
		00	01	11	10
<i>DC</i>	00	1	1	0	0
	01	1	X	X	1
	11	X	0	X	1
	10	X	X	X	1

$$\begin{aligned}
 &F(D,C,B,A) \\
 &= \Sigma m(0,1,4,6,10,14) + d(5,7,8,9,11,12,15) \\
 &= BD + \bar{C}\bar{B} + C\bar{D}
 \end{aligned}$$

Case 11:

		<i>BA</i>			
		00	01	11	10
<i>DC</i>	00	1	1	0	0
	01	1	X	X	1
	11	X	0	X	1
	10	X	X	X	1

$$\begin{aligned}
 &F(D,C,B,A) \\
 &= \Sigma m(0,1,4,6,10,14) + d(5,7,8,9,11,12,15) \\
 &= \bar{C}\bar{B} + C\bar{A} + D\bar{A}
 \end{aligned}$$

Case 12:

		<i>BA</i>			
		00	01	11	10
<i>DC</i>	00	1	1	0	0
	01	1	X	X	1
	11	X	0	X	1
	10	X	X	X	1

$$\begin{aligned}
 &F(D,C,B,A) \\
 &= \Sigma m(0,1,4,6,10,14) + d(5,7,8,9,11,12,15) \\
 &= \bar{C}\bar{B} + C\bar{D} + D\bar{A}
 \end{aligned}$$

Case 13:

		<i>BA</i>			
		00	01	11	10
<i>DC</i>	00	1	1	0	0
	01	1	X	X	1
	11	X	0	X	1
	10	X	X	X	1

$$\begin{aligned}
 &= \Sigma m(0,1,4,6,10,14) + d(5,7,8,9,11,12,15) \\
 &= \bar{C}\bar{B} + C\bar{A} + D\bar{C}
 \end{aligned}$$



7.

		<i>CB</i>				<i>CB</i>			
		00	01	11	10	00	01	11	10
<i>ED</i>	00	0	1	1	0	0	X	0	0
	01	0	X	0	0	0	1	0	0
	11	X	0	0	1	1	X	0	1
	10	0	0	0	1	0	0	0	1
		<i>A</i> = 0				<i>A</i> = 1			

$$\begin{aligned}
 &F(E,D,C,B,A) \\
 &= \Sigma m(1,3,10,14,21,26,28,30) + d(5,12,17,29) \\
 &= \bar{C}\bar{B}\bar{E} + C\bar{B}E + \bar{A}\bar{D}\bar{E}B + DE\bar{B}
 \end{aligned}$$

8.  $F(A, B, C, D) = \Pi M(0, 2, 3, 4, 7, 8)$

- Case 1:

		CD			
		$\overline{C}.\overline{D} \quad \overline{C}D \quad CD \quad C\overline{D}$			
AB	$\overline{A}.\overline{B}$	0	1	0	0
	$\overline{A}B$	0	1	0	1
	$A\overline{B}$	1	1	1	1
	$AB$	0	1	1	1

Groups

(0, 2)	$A + B + D$
(0, 4)	$A + C + D$
(0, 8)	$B + C + D$
(3, 7)	$A + \overline{C} + \overline{D}$

Call x is the result of the simplification and we have:

- $x = (A + B + D).(A + C + D).(B + C + D).(A + \overline{C} + \overline{D})$

- Case 2:

		CD			
		$\overline{C}.\overline{D}$	$\overline{C}D$	$CD$	$C\overline{D}$
AB	$\overline{A}.\overline{B}$	0	1	0	0
	$\overline{A}B$	0	1	0	1
	$A\overline{B}$	1	1	1	1
	$AB$	0	1	1	1

Groups

(3, 2)	$A + B + \overline{C}$
(0, 4)	$A + C + D$
(0, 8)	$B + C + D$
(3, 7)	$A + \overline{C} + \overline{D}$

Call x is the result of the simplification so we have:

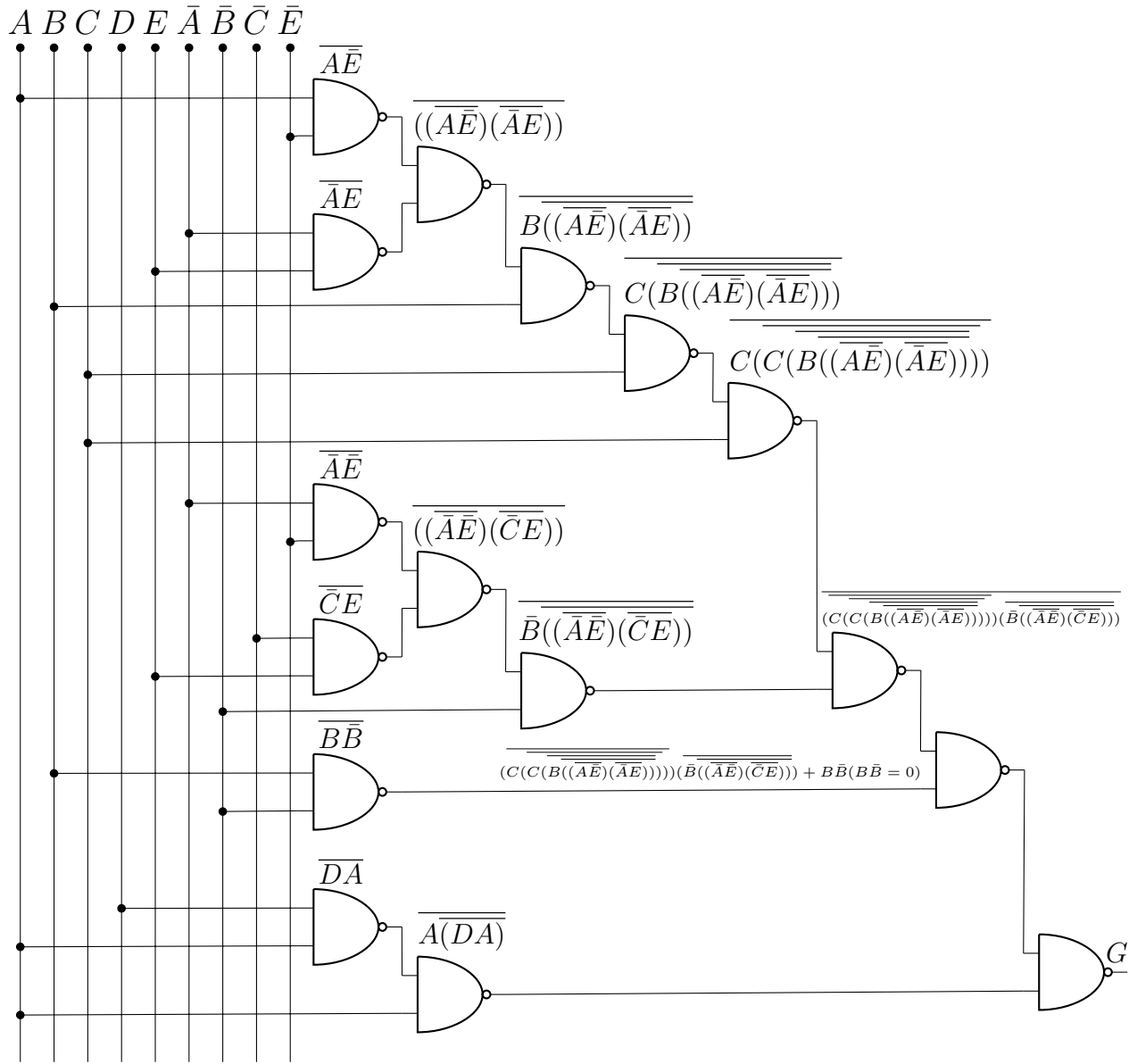
- $x = (A + B + \overline{C}).(A + C + D).(B + C + D).(A + \overline{C} + \overline{D})$

## 2-input NAND gates.

The following function is in minimum sum of products form. Implement it using only two-input NAND gates. No gate may be used as a NOT gate.

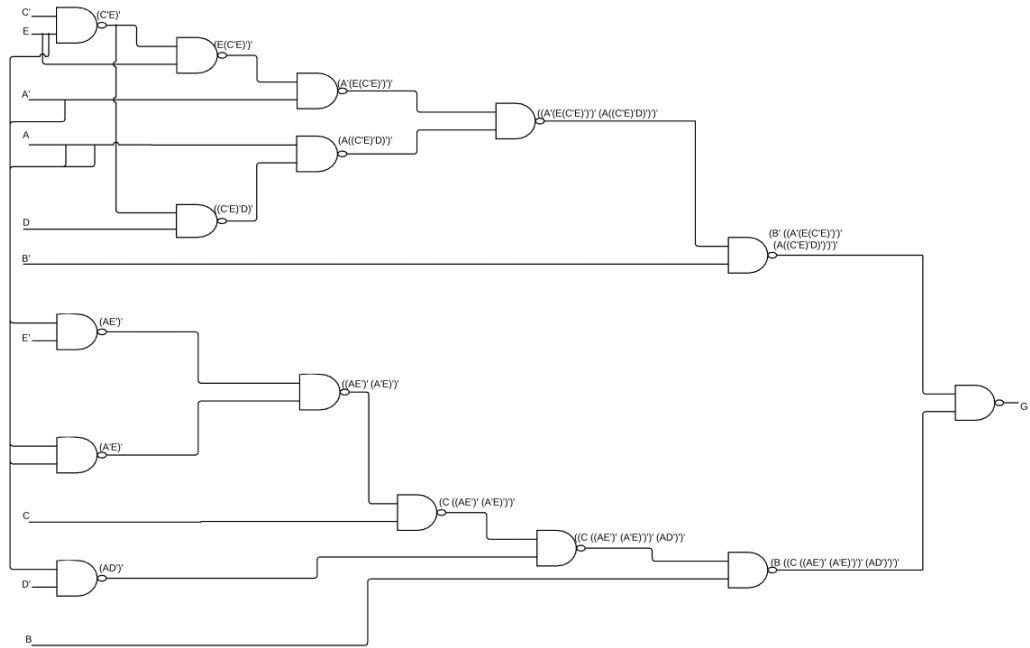
### 1 — First way to simplify and draw G circuit

$$\begin{aligned}
 G &= ABC\bar{E} + \bar{A}\bar{B}\bar{E} + \bar{B}\bar{C}E + \bar{A}BCE + A\bar{D} \\
 &= ABC\bar{E} + \bar{A}BCE + \bar{A}\bar{B}\bar{E} + \bar{B}\bar{C}E + A\bar{D} \\
 &= BC(A\bar{E} + \bar{A}E) + \bar{B}(\bar{A}\bar{E} + \bar{C}E) + A\bar{D} \\
 &= BC(A\bar{E} + \bar{A}E) + C\bar{C} + \bar{B}(\bar{A}\bar{E} + \bar{C}E) + B\bar{B} + A\bar{D} + A\bar{A} \\
 &= C(\bar{C} + B(A\bar{E} + \bar{A}E)) + \bar{B}(\bar{A}\bar{E} + \bar{C}E) + B\bar{B} + A(\bar{D} + \bar{A}) \\
 &= (C(C.(B((A\bar{E}).(\bar{A}E))))).(\bar{B}((\bar{A}\bar{E}).(\bar{C}E))).((B\bar{B})).(A(\bar{D}\bar{A})) \\
 &= [(C(C.(B((A\bar{E}).(\bar{A}E))))).(\bar{B}((\bar{A}\bar{E}).(\bar{C}E))) + (B\bar{B})].(A(\bar{D}\bar{A})) \\
 &= [(C(C.(B((A\bar{E}).(\bar{A}E))))).(\bar{B}((\bar{A}\bar{E}).(\bar{C}E)))].(A(\bar{D}\bar{A}))
 \end{aligned} \tag{1}$$



**2 — Second way to simplify and draw G circuit**

$$\begin{aligned}
G &= ABC\bar{E} + \bar{A}\bar{B}\bar{E} + \bar{B}\bar{C}E + \bar{A}BCE + A\bar{D} \\
&= ABC\bar{E} + \bar{A}\bar{B}\bar{E} + \bar{B}\bar{C}E + \bar{A}BCE + A\bar{D}(B + \bar{B}) \\
&= ABC\bar{E} + \bar{A}\bar{B}\bar{E} + \bar{B}\bar{C}E + \bar{A}BCE + A\bar{D}B + A\bar{D}\bar{B} \\
&= A\bar{D}\bar{B} + \bar{A}\bar{B}\bar{E} + \bar{B}\bar{C}E + \bar{A}BCE + A\bar{D}B + ABC\bar{E} \\
&= \bar{B}(\bar{A}\bar{E} + \bar{C}E + A\bar{D}) + B(AC\bar{E} + \bar{A}CE + A\bar{D}) \\
&= \bar{B}(\bar{A}\bar{E} + \bar{C}E(A + \bar{A}) + A\bar{D}) + B(AC\bar{E} + \bar{A}CE + A\bar{D}) \\
&= \bar{B}(\bar{A}\bar{E} + \bar{C}EA + \bar{C}E\bar{A} + A\bar{D}) + B(C(\bar{A}\bar{E} + \bar{A}E) + A\bar{D}) \\
&= \bar{B}(\bar{A}\bar{E} + \bar{C}E\bar{A} + \bar{C}EA + A\bar{D}) + B(C(\bar{A}\bar{E} + \bar{A}E) + A\bar{D}) \\
&= \bar{B}(\bar{A}(\bar{E} + \bar{C}E) + A(\bar{C}E + \bar{D})) + B(C(\bar{A}\bar{E} + \bar{A}E) + A\bar{D}) \\
&= \bar{B}(\bar{A}(\overline{\overline{E.CE}}) + A(\overline{\overline{CE.D}})) + B(C(\overline{\overline{AE.AE}}) + A\bar{D}) \\
&= \bar{B}(\bar{A}(\overline{\overline{E.CE}}).\overline{\overline{A(\overline{\overline{CE.D}})}}) + B(C(\overline{\overline{AE.AE}}).\overline{\overline{A\bar{D}}}) \\
&= \bar{B}(\bar{A}(\overline{\overline{E.CE}}).\overline{\overline{A(\overline{\overline{CE.D}})}}).\overline{\overline{B(C(\overline{\overline{AE.AE}}).\overline{\overline{A\bar{D}}})}}
\end{aligned} \tag{2}$$



$$\begin{aligned}
 K &= \bar{x}\bar{y}b + \bar{x}y\bar{b} + x\bar{y}\bar{b} + xyb \\
 &= \bar{x}\bar{y}b + x\bar{y}\bar{b} + \bar{x}y\bar{b} + xyb \\
 &= \bar{y}(\bar{x}b + x\bar{b}) + y(\bar{x}\bar{b} + x b) \\
 &= \overline{\overline{\bar{y}(\bar{x}b + x\bar{b})}} \cdot \overline{\overline{y(\bar{x}\bar{b} + x b)}} \\
 &= \overline{\overline{\bar{y}((\bar{x}b) \cdot (\bar{x}\bar{b}))}}) \cdot \overline{\overline{y((\bar{x}\bar{b}) \cdot (x b))}}
 \end{aligned} \tag{3}$$

