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(Linear Algebra)

1 Matrix Algebra

1.1 Definition

- A $m \times n$ matrix is a table of numbers that contains in m rows and n columns

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{2n} \\ a_{n1} & a_{n2} & a_{n3} & a_{nn} \end{pmatrix} = (a_{ij})_{m \times n}$$

1.2 Definition

- If $a_{ij} = 0$, $\forall i, j$, then A is called a zero matrix and is denoted by 0

1.3 Definition

Let A = $(a_{ij})_{m \times n}$:

- A matrix A that is formed by columns is called a transpose of A

Ex: $\begin{pmatrix} 2 & -1 & 3 \\ 4 & 7 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ -1 & 7 \\ 3 & 8 \end{pmatrix}$

1.4 Definition

- If the number of rows and the number of columns equal, then the matrix is called a square matric of order

- Let A = $(a_{ij})_{m \times n}$ be a square matrix of order n

- All the elements $a_{11}, a_{22}, \ldots, a_{nn}$ is diagonal of A

- A sum of $a_{11}, a_{22}, \ldots, a_{nn}$ is called a trace of A

  A $= \begin{pmatrix} \mathbf{1} & 1 & 3 \\ 4 & \mathbf{2} & 7 \\ 2 & 5 & \mathbf{6} \end{pmatrix}$

  Trace(A) = 1 + 2 + 6 = 9.

- If $a_{ij} = 0, \forall i \neq j$, then A is called a diagonal matrix

  D $= \begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ is a diagonal matrix

- $$\text{If } a_{ij} = \begin{cases} 0, if \text{ i} \neq j \\ 1, if \text{ i} = j \end{cases}$$

  then A is called an identity matrix

2 Matrix Operation

2.1 Equality

A = B $\leftrightarrow$

$$\begin{cases} \text{Same size} \\ \text{Corresponding elements are equal} \end{cases}$$

Ex: A $= \begin{pmatrix} 2 & -1 & 3 \\ 4 & 7 & 8 \end{pmatrix}$

B $= \begin{pmatrix} 2 & x & 3 \\ 4 & 7 & 8 \end{pmatrix}$

A = B when

$$\begin{cases} x = -1 \\ y = 4 \end{cases}$$

2.2 Sum:

$A = (a_{ij})_{m\times n}$
$B = (b_{ij})_{m\times n}$
$\rightarrow A + B = (a_{ij} + b_{ij})_{m\times n}$
Ex:
A = $\begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$

B = $\begin{pmatrix} 1 & -3 \\ 0 & 7 \end{pmatrix}$

$\rightarrow A + B = \begin{pmatrix} 3 & -4 \\ 3 & 11 \end{pmatrix}$

2.3 Multiplication by a number

$A = (a_{ij})_{m\times n}$
$\rightarrow \alpha.A = (\alpha a_{ij})_{m\times n}$
Ex:
A = $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 5 & 4 \end{pmatrix}$

$\rightarrow 3A = \begin{pmatrix} 3 & 6 & 3 \\ 8 & 5 & 12 \end{pmatrix}$

2.4 Matrix Multiplication

$A = (a_{ij})_{m\times n}$; $B = (b_{ij})_{n\times p}$
$A \times B = C = (c_{ij})_{m\times p}$
$c_{ij} = (i^{th}$ rows of A) $\times$ (j^{th} columns of B)
$c_{ij} = (a_{i1} \times a_{i2} \times a_{i3} \ldots \times a_{in}) \begin{pmatrix} b_{1j} \\ b_{2j} \\ \ldots \\ b_{nj} \end{pmatrix}$

$c_{ij} = a_{i1}b_{1j} + a_2b_{2j} + \ldots + a_{in}b_{nj}$

$c_{ij} = \sum_{n}^{k=1} a_{ik}b_{kj}$

Ex:

A $= \begin{pmatrix} 1 & -1 & 3 \\ 2 & 4 & -2 \end{pmatrix}_{2\times 3}$; B $= \begin{pmatrix} 2 & 1 & -1 \\ 5 & 2 & 6 \\ 0 & 1 & -1 \end{pmatrix}_{3\times 3}$

$\rightarrow A \times B = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{pmatrix}_{2\times 3}$