Calculus 2 172 – Endterm Rview.

Endterm Examination: 90 – 100 minutes, 10 questions. 4 Topics

Topic 01: Triple Integral. Direct Integral or Application: Mass or Total Charge.

Question 1: Oxyz.

Question 2: Cylindrical.

Question 3: Spherical.

Topic 02: Line Integral. Direct Integral or Application: Mass or Work. Independent of path. Green Theorem.

Question 4: Line Integral, 1st type.

Question 5: Line Integral, 2nd type.

Question 6: Independent of path (Line Integral or Conservative Vector Field).

Question 7: Green.

Topic 03: Surface Integral. Direct Integral or Application: Mass or Flux.

Question 8: Surface Integral, 1st type.

Question 9: Surface Integral, 2nd type.

Topic 10: Series.

Question 10: a/ Numerical Series.

b/ Power Series.

Review Exercises

1.1/ Find I = $\iiint\limits_E \left(\frac{x}{2}+\frac{y}{3}\right)dV$ where E = $\{(x, y, z) \mid 0 \le x \le 1; 0 \le y \le 3x; 0 \le z \le 2(x+y)\}$

1.2/ Find the mass of solid E bounded by the plane x + y + z = 1 & coordinates planes with linear density ρ(x, y, z) = $x + 2y + z$

2/ Solid E within $x^2 + y^2 = 4$, above $z = 0$, below $z^2 \le 9x^2 + 9y^2$, ρ(x, y, z) = x^2. Find the mass of E.

3/ Consider solid E enclosed by the sphere $x^2 + y^2 + z^2 = 16$ in the first octant with charge density σ(x, y, z) = $\frac{1}{(x^2+y^2+z^2)^3+1}$. Find total charge.

4/ A wire takes the shape of the curve (C): $y = \frac{x^2}{2}$, $0 \le x \le \sqrt{3}$. Find wire mass if its density at point M(x, y) belong to (C) is $\rho = \sqrt{2y}$.

5.1/ Evaluate $\int\limits_{(C)} y^2dx + xdy$ where (C) is the line segment from $(-5; -3)$ to (0; 2)

5.2/ Find work $\int\limits_{(C)} F \cdot dr$ done by vector field $\vec{F} = \frac{1}{\sqrt{x^2+y^2}}\left(2x\vec{i} + 3y\vec{j}\right)$ along the curve (C): $y = 4\sin x$ from A$\left(\frac{\pi}{6}; 2\right)$ to origin.

6.1/ a/ Show $\int\limits_{(C)} e^{x-y}\left[(1+x+y)dx + (1-x-y)dy\right]$ is independent of path b/ Evaluate $\int\limits_{(C)} e^{x-y}\left[(1+x+y)dx+(1-x-y)dy\right]$ by any (C) from A(1; 2) to B(4; 3)

6.2/ Show that the vector field $\vec{F}=\left(2x^2y^2+y\right)\vec{i}+\left(x^3y-x\right)\vec{j}$ IS NOT conservative. Find $h(x,y)=x^{\alpha}y^{\beta}$ so that the vector field $\vec{F_1}=h(x,y)\cdot\vec{F}$ is conservative. Find the work done by F_1 from A$(1;2)$ to B$(3;5)$.

7.1/ Evaluate the line integral: $\oint\limits_{(C)} 5ydx+3xdy$ where (C) is boundary of region bounded by $y=x^2$, $y=2x$ and oriented counterclockwise.

7.2/ Evaluate $\oint\limits_{(C)} 2\left(x^2+y^2\right)dx+(x+y)^2dy$ where (C) is the contour of a triangle in the positive direction with vertices A(1; 1), B(2; 2) and C(1; 3).

8.1/ Evaluate the integral: $\iint\limits_S \left(x^2+2y^2\right)dS$, (S): $z=1+3x+4y$, lies above $x^2+y^2\le 9$.

8.2/ Consider surface (S) which is the part of the cone $z^2=x^2+y^2$ that lies between the planes $z=1$ and $z=3$. If the linear density at M(x, y, z) belong to the surface is $\rho(x, y, z) = x^2z^2$, find the mass of (S).

9/ Find the flux of $\vec{F}=\left(xze^y;-xze^y;z\right)$ through the surface (S) which is the part of the plane $x+y+z=1$ in the first octant and has downward orientation.

10a/ Find $\lambda > 0$ so $\sum\limits_{n=2}^{\infty}\left[\dfrac{2\lambda+4^n\lambda^2}{3\cdot 5^n}\right]=2$.

10b/ Find the greatest intergen number inside the interval of convergence of

$$\sum_{n=0}^{\infty}(-1)^{n+1}\left(\frac{n-1}{n+1}\right)^{n^2}\cdot\frac{x^n}{4^{n+1}}.$$

Short Answer

1.1/ I = $\int\limits_0^1\int\limits_0^{3x}\int\limits_0^{2(x+y)}\left(\dfrac{x}{2}+\dfrac{y}{3}\right)dzdydx$ = 7.12

1.2/ I = $\int\limits_0^1\int\limits_0^{1-x}\int\limits_0^{1-x-y}(x+2y+z)dzdydx$ = $\dfrac{1}{6}$.

2/ $\iiint\limits_E x^2dV$. Cylindrical $\Rightarrow \dfrac{96\pi}{5}$.

3/ Total charge = $\iiint\limits_E \dfrac{dV}{\left(x^2+y^2+z^2\right)^3+1}$. Spherical $\Rightarrow \dfrac{\pi}{6}\cdot\arctan 64$.

4/ Mass = I = $\int\limits_{(C)}\rho ds$ = $\int\limits_0^{\sqrt{3}} x\sqrt{1+x^2}dx$ = $\dfrac{7}{3}$

5.1/ Line equation $y = x + 2 \Rightarrow \mathrm{I} = -\frac{5}{6}$

5.2/ $\int\limits_{(C)} \frac{2xdx + 3ydy}{\sqrt{x^2 + y^2 + 1}}$. Substitute: $\begin{cases} y = 4\sin x \\ dy = 4\cos x dx \end{cases} \Rightarrow \int\limits_{\frac{\pi}{6}}^{0} \frac{(2x + 48\sin x \cos x)}{\sqrt{x^2 + 16\sin^2 x + 1}} dx \approx -1.38$

6.1/ $\mathrm{I} = \int\limits_{1}^{4} e^{x-3}(1 + x + 3)dx + \int\limits_{2}^{3} e^{4-y}(-3 - y)dy = 7e - \frac{3}{e}$.

6.2/ $h(x, y) = \frac{1}{xy}$. Work $= 43 + \ln\frac{6}{5}$.

7.1/ Green formula: $\mathrm{I} = \oint\limits_{(C)} 5ydx + 3xdy = -2\iint\limits_{D} dA$. Intercept equation: $x^2 = 2x \Rightarrow$

$\left[\begin{array}{l} x = 0 \\ x = 2 \end{array}\right. \Rightarrow$ D: $\left| \begin{array}{l} 0 \le x \le 2 \\ x^2 \le y \le 2x \end{array}\right. \Rightarrow \mathrm{I} = -2\int\limits_{0}^{2}\int\limits_{x^2}^{2x} dydx = -\frac{8}{3}$.

7.2/ Green formula: $\mathrm{I} = \iint\limits_{D}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA = \iint\limits_{D} 2(x - y)dA = 2\int\limits_{1}^{2}\int\limits_{x}^{3-x}(x - y)dydx = -\frac{4}{3}$.

8.1/ $\mathrm{I} = \iint\limits_{S}(x^2 + 2y^2)dS = \iint\limits_{D}(x^2 + 2y^2)\sqrt{1 + z_x^2 + z_y^2}dA = \sqrt{26}\iint\limits_{D}(x^2 + 2y^2)dA$. Polar

$\Rightarrow \mathrm{I} = \sqrt{26}\int\limits_{0}^{3}\int\limits_{0}^{2\pi} r^3(\cos^2\varphi + 2\sin^2\varphi)d\varphi dr = \frac{243\sqrt{26}\pi}{4}$

8.2/ Mass $= \iint\limits_{(\mathrm{S})} x^2z^2dS$. (S): $\begin{cases} z = \sqrt{x^2 + y^2} \\ 1 \le x^2 + y^2 \le 9 \end{cases} \Rightarrow \frac{364\sqrt{2}}{3}\pi$.

9/ Flux: $\iint\limits_{(S)} \vec{F} \cdot dS = \iint\limits_{D} \vec{F} \cdot \vec{n}dA$. (S): $z = 1 - x - y$ & downward $\Rightarrow \vec{n} = (z_x; z_y; -1) =$

$\vec{n} = (-1; -1; -1) \Rightarrow$ Flux $= -\iint\limits_{D} zdA = \int\limits_{0}^{1}\int\limits_{0}^{1-x}(x + y - 1)dydx = -\frac{1}{6}$.

10a/ Rewrite: $\frac{2\lambda}{3}\sum_{n=2}^{\infty}\left(\frac{1}{5}\right)^n + \frac{\lambda^2}{3}\sum_{n=2}^{\infty}\left(\frac{4}{5}\right)^n = 2$. Find: $\sum_{n=2}^{\infty}\left(\frac{1}{5}\right)^n = \frac{1}{20}$, $\sum_{n=2}^{\infty}\left(\frac{4}{5}\right)^n = \frac{16}{5}$.

Substitute, notice that $\lambda > 0 \Rightarrow \lambda \approx 1.35$.

10b/ $\lim\limits_{n\to\infty}\sqrt[n]{\left|(-1)^{n+1}\left(\frac{n-1}{n+1}\right)^{n^2} \cdot \frac{x^n}{4^{n+1}}\right|} = \frac{|x|}{4}e^{-2} < 1 \Leftrightarrow x \in (-4e^2; 4e^2) \Rightarrow$ Required: 29.
