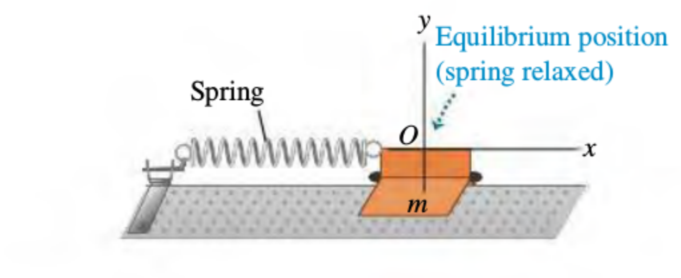
Periodic Motion

Component in Periodic Motion:



1. **Displacement:** x is the x-component of the displacement of the body from the equilibrium and is also the change in the length of the spring.

2. **Restoring Force:** whenever the body is displaced from its equilibrium position, the spring exert the **restoring force** to **restore** it to the equilirium position.

3. **Amplitude:** The amplitude of the motion, denoted by A, is the maximum magnitude of displacement from equilirium, tha maximum value of |x|. It is always positive.

   - The SI unit of A is meter.
   - A complete vibration of **cycle** is one complete round trip: from A to -A and back to A, or from O to A, back through to -A, and back to O.

4. **The Period:** (T) is the time to complete a cycle.

   - It is always positve.
   - The SI unit is second, or "seconds per cycle"

5. **The Frequency:** (f) is the number of cycles in a unit of time.

   - It is always positive.
   - The SI unit is *hertz* (Hz).

$$\boxed{1 \text{ hertz} = 1 \text{ Hz} = 1 \text{ cycle/s} = 1\ s^{-1}}$$

6. **Angular Frequency:** (ω) is 2π times the frequency.

$$\boxed{\omega = 2\pi f}$$

Simple Harmonic Motion

(SHM) "The simplest kind of oscillation occurs when the restoring force F_x is *drectly proportional* to the displacement from equilibrium x"

1. **Restoring Force exerted by an ideal spring:** (there is no friction)

$$\boxed{F_x = -kx}$$

Whether:

- k is constant force (N/m)
- x is displacement. (m)

When the restoring force is directly proportional to the displacement from equilibrium, as given by Eq. (14.3), the oscillation is called **simple harmonic motion (SHM).** The acceleration $a_x = d^2x/dt^2 = F_x/m$ of a body in SHM is

Equation for simple harmonic motion

$$\underbrace{a_x}_{x\text{-component of acceleration}} = \underbrace{\frac{d^2x}{dt^2}}_{\text{Second derivative of displacement}} = -\frac{k}{m}x \tag{14.4}$$

Force constant of restoring force: k
Displacement: x
Mass of object: m

- The acceleration and displacement in SHM always have opposite signs. And this *acceleration* is not constant.
- Not all periodic motions are simple harmonic.
- In simple harmonic the restoring force is proportional to displacement.

- Some structure:

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

where:

$$\begin{cases} \omega \text{ is angular frequency (rad/s)} \\ k \text{ is force constant (N/m)} \\ m \text{ is mass of object (kg)} \end{cases}$$

2. **Relation between frequency and period:**

$$\boxed{f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}}$$

- The period and frequency of simple harmonic are completetely determined by the mass m and the force constant k.
- In simple harmonic motion the period and frequency do not depend on the amplitude A.
- The change of amplitude A will lead to the change of the value of |x|, restoring force and the average speed.

3. **Displacement, Velocity and Acceleration in SHM:**
Displacement in simple harmonic:

$$\boxed{x = Acos(\omega t + \phi)}$$

Where:

$$\begin{cases} x \text{ is displacment (m)} \\ A \text{ is amplitude (m)} \\ \omega \text{ is angular frequency (rad/s)} \\ \phi \text{ is phase angle (rad)} \end{cases}$$

In simple harmornic motion the displacement is a periodic, ***sinusoidal*** *function of time.*

Velocity in Harmonic Motion:

$$\boxed{v_x = \frac{dx}{dt} = -\omega Asin(\omega t + \phi)}$$

Acceleration in Harmonic Motion:

$$\boxed{a_x = \frac{dv}{dt} = -\omega^2 Acos(\omega t + \phi)}$$

The maximum speed in SHM:

$$\boxed{v_{max} = \omega A}$$

The maximum acceleration in SHM:

$$\boxed{a_{max} = \omega^2 A}$$

When a body is passing through the equilibrium position so that $x = 0$, the velocity equals its maximum value v_{max} or $-v_{max}$ (depend on which way the body is moving) and the acceleration is zero.
When a body is at its most positive displacement or most negative position, the velocity is zero and the body is instantaneously at rest. At these point the resotring force $F_x = -kx$ and the accleration have its maximum value.

Find phase angle at intial time:
We have:

$$v_{0x} = -\omega A sin\phi$$
$$x_0 = A cos\phi$$

$\rightarrow \dfrac{v_{0x}}{x_0} = \dfrac{-\omega A sin\phi}{A cos\phi}$
$\rightarrow \phi = arctan(-\dfrac{v_{0x}}{\omega x_0})$

To find the amplitude A in SHM:

$$\boxed{A = \sqrt{x_0^2 + \frac{v_{0x}^2}{\omega^2}}}$$

Or:

$$\boxed{A = \sqrt{\frac{a^2}{\omega^4} + \frac{v_{0x}^2}{\omega^2}}}$$

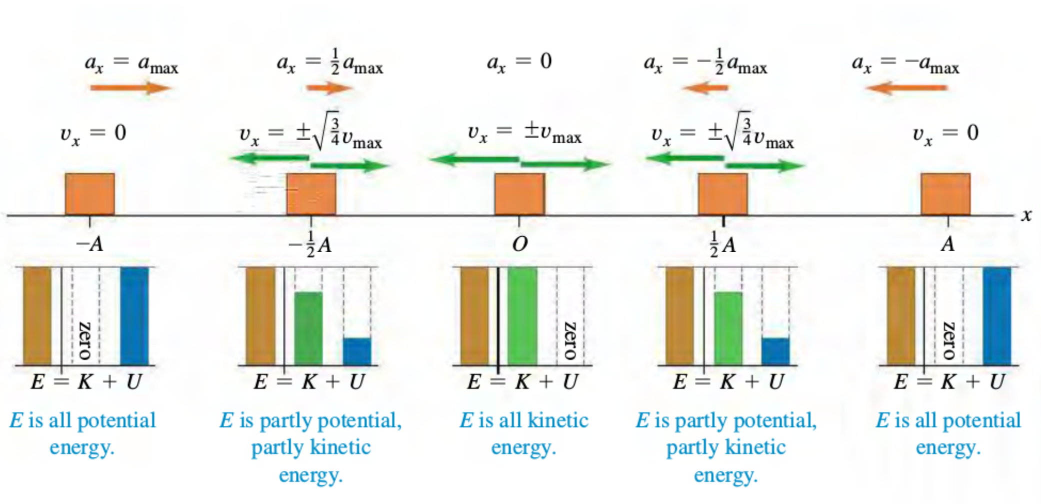
Energy in Simple Harmonic Motion:

Since only horizontal force is the conservative force exerted by an ideal spring and the vertival do no work so total mechanical energy of the system is conserved.

$$\boxed{E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2}$$

For:

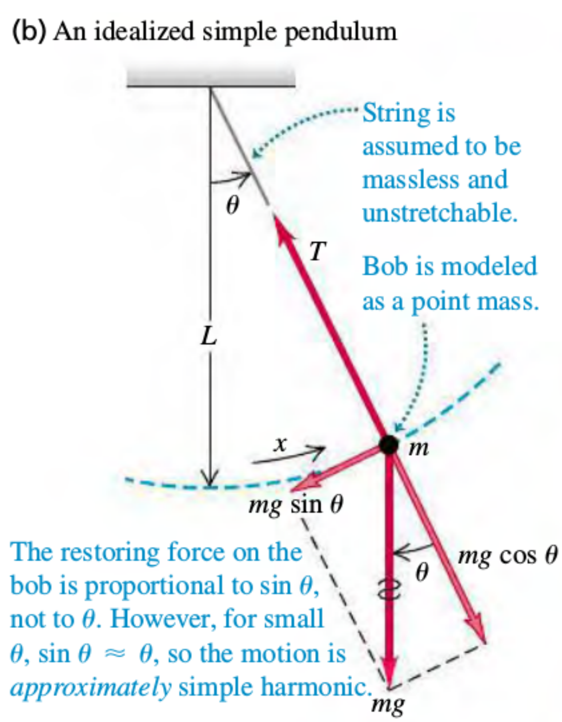
$$\begin{cases} E \text{ is total mechanical energy (J)} \\ m \text{ is velocity (m/s)} \\ k \text{ is force constant (N/m)} \\ x \text{ is displacement (m)} \\ A \text{ is amplitude (m)} \end{cases}$$



$$\boxed{v_x = \omega\sqrt{A^2 - x^2}}$$

Simple Pendulum

A simple pendulum is an idealized model consisting of a point mass suspended by a massless, unstretchable string. When a point mass pulled to one side of its straight-down equilibrium position and released, it oscillates about the equilibrium position.



1. **Restoring Force:** is the tangential component of the net force.

$$F_0 = -mgsin\theta$$

   - Gravity provides the restoring force F_0.
   - The tension T merely acts to make the point mass move in the arc.
   - Since F_0 is proportional to $sin\theta$, so the motion is not simple harmonic, but if the angle θ is small ($< 10°$), the pendulum is now in simple harmonic:

$$F_0 = -mg\theta = -mg\frac{x}{L} = -\frac{mg}{L}x$$

     For:

$$k = \frac{mg}{L}$$

     For: L is the length of the pendulum

2. **Angular Frequency:**

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg/L}{m}} = \sqrt{\frac{g}{L}}$$

3. **Frequency of Simple Pendulum:**

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

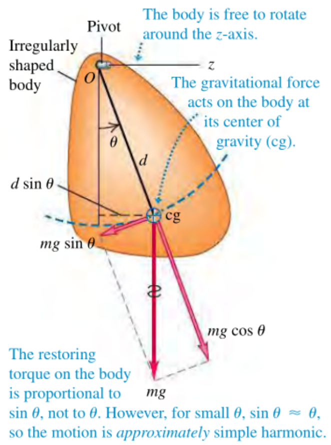
4. **Period of Simple Pendulum:**

$$T = 2\pi\sqrt{\frac{L}{g}}$$

   - The change of g (↑) leads to (↑) of the restoring force, the frequency and (↓) of the period.
   - The motion of pendulum is only **approximately** simple harmonic.

Physical Pendulum

*A physical pendulum is any **real** pendulum. When a body is displaced from equilibrium by and angle θ, the distance from O to the center of gravity is **d**, the moment of inertia of the body is **I**.*



The restoring torque of physical pendulum:

$$\boxed{\tau = -mg(dsin\theta)}$$

- When θ is small ($< 10°$) the motion of physical pendulum can be approximately harmonic motion.

$$\boxed{\tau = -mg(d\theta)}$$

1. **Angular Frequency:**

$$\boxed{\omega = \sqrt{\frac{mgd}{I}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{I}{mgd}}}$$

For:

$$\begin{cases} m \text{ is mass (kg)} \\ g \text{ is accleration due to gravity } (m/s^2) \\ I \text{ is moment of inertia } (kg.m^2) \end{cases}$$

Damped oscillation

The oscillation that its amplitude is decreased gradually form time to time is called damped oscillation.

1. **Displacement of Damped oscillation:**

$$x = Ae^{-(b/2m)t} cos(\omega' t + \phi)$$

For:

$$\begin{cases} x \text{ is displacement (m)} \\ A \text{ is initial amplitude (m)} \\ b \text{ is damping constant} \\ m \text{ is mass (kg)} \\ \omega' \text{ is angular frequency of damped oscillation (rad/s)} \end{cases}$$

2. **Angular Frequency:**

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

- When $\omega' = 0$:

$$\frac{k}{m} - \frac{b^2}{4m^2} \text{ or } b = \sqrt{km}$$

- At this time, the condition is called **critical damping**. The system is no longer oscillates but returns to its equilibrium position.
- If $b > 2\sqrt{km}$, the condiiton is now **overdamping**. The system is no longer oscillates, but it returns to equlibrium more slowly than **critical damping**.

$$x = C_1 e^{-a_1 t} + C_2 e^{-a_2 t}$$

where C_1 and C_2 are constants that depend on the initial conditions and a_1 and a_2 are constants determined by m, k, and b.

- When b is less than the critical value, the condition is now **underdamping**. The system oscilates with steadily decreasing amplitude.

Energy in Damped Oscillation

In damped oscillation, the damping force is non-conservative; the mechanical energy of the system is not constant but decrease continously, aproaching zero after a long time.

$$\frac{dE}{dt} = -bv_x^2$$

Forced Oscillations and Resonance

1. **Forced Oscillation:** (Damped Oscillation with a Periodic Driving Force)

- If we apply a **periodic driving force** with angular frequency ω_d to a damped harmornic oscillator, the motion that results is called **a forced oscillation** or **driven oscillation.**

- The value of ω_d is independent with the natural ω.
- When $\omega_d = \omega$, the driven force is working with:

$$F(t) = F_{max} cos\omega_d t$$

- **Amplitude of a forced oscillator:**

$$\boxed{A = \frac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}}$$

2. **Resonance:**
   - When angular frequency at driving force ω_d equals to the natural angular frequency ω_0, that condition is called **resonance**.

Mechanical Wave

A mechanical wave is a disturbance that travels through some material or substance called the ***medium*** *for the wave.*

1. **Tranverse wave:** (Sóng ngang) is a moving wave that consists of oscillations occuring **perpendicular** to the direction of energy transfer (or the propagation of the wave).
2. **Longitudinal wave:** (Sóng dọc) is a moving wave that the oscillation is the same direction with the propagation of the wave.

Periodic Wave:

Periodic Tranverse Waves:

- There is different between wave motion and particle motion: In wave motion, the wave moves with **constant speed** along the string, while the motion of the particle is **simple harmonic** and **perpendicular** to the length of the string.

1. **Wave length:** (λ) of the wave is the distance from the one crest(đỉnh sóng) to the next, or one trough (bụng sóng) to the next, or from any point to the corresponding point of the next wave shape.

$$\boxed{v = \lambda f = \frac{\lambda}{T}}$$

For:

$$\begin{cases} v \text{ is wave speed (m/s)} \\ \lambda \text{ is wave length (m)} \\ f \text{ is frequency (Hz)} \\ T \text{ is period (s)} \end{cases}$$

- The wave speed v is determined entirely by the property of the medium not λ and f

Mathematical Description of a Wave

Wave function of the Sinusoidal wave:

1. **Wave function for the Sinusodial wave in x+ direction:**

$$\boxed{y(x,t) = Acos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]}$$

Or:

$$\boxed{y(x,t) = Acos(kx - \omega t)}$$

2. **Wave function for the Sinusodial wave in x- direction:**

$$\boxed{y(x,t) = Acos\left[2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)\right]}$$

Or:

$$\boxed{y(x,t) = Acos(kx + \omega t)}$$

For: k is a **wave number**. And:

$$\boxed{k = \frac{2\pi}{\lambda}}$$

We also have:

$$\boxed{\omega = vk \textbf{ or } v = \frac{\omega}{k}}$$

3. **Particle Velocity and Acceleration in a Sinusoidal Wave**
For:

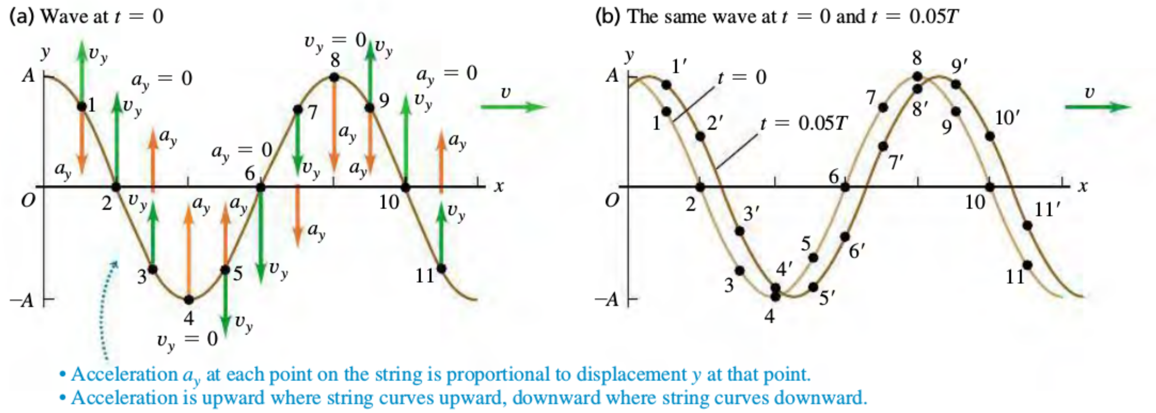
$$y(x,t) = Acos(kx - \omega t)$$

- We have the velocity of the particle in a wave:

$$v_y(x,t) = \frac{\delta y(x,t)}{\delta t} = \omega Asin(kx - \omega t)$$

- For Acceleration of the particle in a wave:

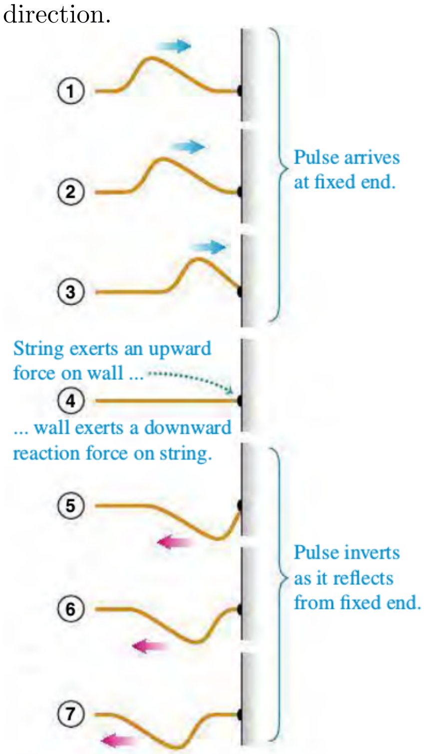
$$a_y(x,t) = \frac{\delta v_y(x,t)}{\delta t} = -\omega^2 A cos(kx - \omega t)$$



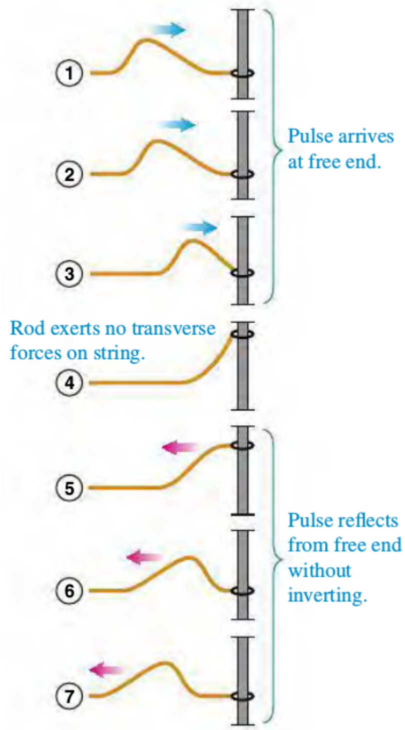
Wave Interference:

When two or more waves pass through the same region at the same time, that condition is called ***wave interference****.*

- When the arriving force exerts a force in a fixed end (wall) will create the reflected wave to the reverse direction.



- When the arriving force exerts a force in a free end (free to move up or down, the reflected force will not inverted.



- **Principle of supperposition:** When two or more waves overlap, the total displacement is the sum of the displacements of the individuals.