<u>Instruction:</u> Type your answer to the following quetion provided by Latex and submit a zipped file (included .pdf file and .tex file) to E-learning by group (only 4-5 member in each group). Only team leader will submit it. One page per problem. Please use the solution template provided)

GROUP ... —-MEMBER LIST			
No.	**Name**	ID	Role
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Problem 1 [5pt] Prove the following statements

1. $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$
   1)Suppose that:$x \in \overline{A \cap B \cap C}$
   $\leftrightarrow x \in \overline{A} \cap x \in \overline{B} \cap x \in \overline{C}$
   $\leftrightarrow x \notin A \cap x \notin B \cap x \notin C$
   $\leftrightarrow x \in \neg A \cap B \cap C$
   $\leftrightarrow x \in (\overline{A} \cup \overline{B} \cup \overline{C})$
   $\rightarrow Q.E.D$
   2) Membership Table:

A	**B**	**C**	$\overline{A \cap B \cap C}$	$\overline{A} \cup \overline{B} \cup \overline{C}$
1	1	1	0	0
0	0	0	1	1
0	1	1	0	0
0	1	0	0	0
1	0	0	0	0
1	0	1	0	0
0	0	1	0	0
1	1	0	0	0

2. $P(A) \subseteq P(B)$ if and only if$A \subseteq B$
   Suppose that $A \nsubseteq B$ and$A \in P(A), B \in P(B)$
   $\rightarrow P(A) \nsubseteq P(B)$so if$A \subseteq B$then$P(A) \subseteq P(B)$

3. $(B - A) \cup (C - A) = (B \cup C) - A$
   Suppose that: A= $\{1, 2, 3\}$
   B= $\{3, 4, 5\}$
   C= $\{2, 7, 8\}$
   We have: B - A = $\{4, 5\}$
   C - A = $\{7, 8\}$
   $\rightarrow (B - A) \cup (C - A) = \{4, 5, 7, 8\}$ (1)
   We have: $B \cup C = \{2, 3, 4, 5, 7, 8\}$
   $\rightarrow (B \cup C) - A = \{4, 5, 7, 8\}$ (2)
   From (1), (2) $\rightarrow Q.E.D$

4. Explain why $A \times B \times C$ and $(A \times B) \times C$ are not the same.
   Suppose that:
   A= $\{1\}$
   B= $\{3, 4\}$
   C= $\{5, 6\}$
   We have $A \times B \times C = \{(1, 3, 5), (1, 3, 6), (1, 4, 5), (1, 4, 6)\}$ (1)
   We also have $(A \times B) = \{(1, 3), (1, 4)\}$

$\rightarrow (A \times B) \times C = \{(1,5),(1,6),(3,5),(3,6),(4,5),(4,6)\}$ (2)
$\therefore$ (1) and (2) are different
$\rightarrow Q.E.D$

Problem 2 [5pt] The symmetric differrence of A and B, denoted by $A \oplus B$, is the set containing those elements in either A or B, but not in both A and B.

1. Show that$A \oplus B = (A \cup B)(A \cap B)$
   We define that $A \oplus B = (A - B) \cup (B - A)$
   Suppose that:
   A= $\{1, 2, 3\}$
   B= $\{3, 4, 5\}$
   We have: A - B =$\{1, 2\}$
   B - A =$\{4, 5\}$
   $\rightarrow (A - B) \cup (B - A) = \{1, 2, 4, 5\}$ (1)
   Beside:$(A \cup B)(A \cap B) = \{1, 2, 4, 5\}$ (2)
   (1)&(2) $\therefore A \oplus B = (A \cup B)(A \cap B)$

2. What can you say about the sets A and B if$A \oplus B = A$?
   A is the set and B is the empty set or B is the set and A is the empty set
   Definitely: $A \oplus B = (A - B) \cup (B - A)$
   So if $B = \emptyset$ then $A - B = A - \emptyset = A$
   $B - A = \emptyset - A = \emptyset$
   $\rightarrow (A - B) \cup (B - A) = A \cup \emptyset = A$
   $\therefore Q.E.D$

3. If A, B, C are sets, does it follow that$A \oplus (B \oplus C) = (A \oplus B) \oplus C$?
   Use Membership Table:

A B C	$B \oplus C$	$A \oplus B$	$A \oplus (B \oplus C)$	$(A \oplus B) \oplus C$
1 1 1	0	0	1	1
1 1 0	1	0	0	0
1 0 1	1	1	0	0
1 0 0	0	1	1	1
0 1 1	0	1	0	0
0 1 0	1	1	1	1
0 0 1	1	0	1	1
0 0 0	0	0	0	0

Problem 3 [5pt] What can you say about the sets A and B if we know that

1. $A \cup B = A$?
2. $A \cap B = A$?
3. $A - B = A$?
4. $A \cap B = B \cap A$?
5. $A - B = B - A$?

Problem 4 [5pt] Find the domain and range of these functions. Note that in each case, find the domain, determine the set of set of elements assigned values by the function.

1. The function that assigns to each bit string the number of ones in the string mins the number of zeros in the string
2. The function that assigns to each bit string twice the number of zeros in the string.
3. The function that assigns the number of bits left over when a bit string is split into bytes (which are blocks of 8 bits)
4. The function that assigns tp each possitve integer the largest perfect square not exceeding this integer

Problem 5 [5pt] Determine whether each of these functions is a bijection from **R** to **R**

1. f(x) = -3x +4
2. f(x) = -3x^2 +7
3. f(x) = $\dfrac{x+1}{x+2}$
4. f(x) = $x^5 + 1$
5. f(x) = $\dfrac{x^5+1}{x^2+2}$