INDEFINITE INTEGRAL

SOME BASIC FORMULAS OF INDEFINITE INTEGRALS

- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} ln|\frac{x-a}{x+a}| + C$
- $\int \frac{dx}{\sqrt{x^2 + a}} = ln|x + \sqrt{x^2 + a}| + C$
- $\int \sqrt{a^2 - x^2} dx.$

  Let x = asint, where $\frac{-\pi}{2} \leq t \leq \frac{\pi}{2}$

THE SUBSTITUTION RULE

$$\boxed{\int f(u(x)) du(x) = F(u(x)) + C}$$

INTEGRATE BY PARTS

$$\boxed{\int udv = uv - \int vdu}$$

REDUCTION FORMULA (Công thức truy hồi)

$$\boxed{I_{n+1} = \frac{1}{2na^2} \times \frac{x}{(x^2 + a^2)^n} + \frac{2n-1}{2na^2} \times I_n}$$

Since:

$$I_1 = \int \frac{dx}{x^2 + a^2} = \frac{1}{a} arctan\frac{x}{a} + C$$

When

$$\boxed{I_n = \int \frac{dx}{(x^2 + a^2)^n} (n \in N)}$$

The indefinte integrals of these partial fractions

1. $\int \frac{A}{x-a} dx = Aln|x - a| + C$
2. $\int \frac{A}{(x-a)^n} dx = \frac{A}{(1-n)(x-a)^{n-1}} + C, (n \neq 1)$

3. $\int \frac{Mx+N}{x^2+px+q}dx = \frac{M}{2}\int \frac{(2x+p)dx}{x^2+px+q} + (N - \frac{Mp}{2})\int \frac{dx}{(x+\frac{p}{2})^2+q-\frac{p^2}{4}} = \frac{M}{2}ln|x^2+px+q| + (N - \frac{Mp}{2}) \times \frac{1}{\sqrt{q-\frac{p^2}{4}}} arctan\frac{x+\frac{p}{2}}{\sqrt{q-\frac{p^2}{4}}} + C$

4. $\int \frac{Mx+N}{(x^2+px+q)^m}dx = \frac{M}{2}\int \frac{2x+p}{(x^2+px+q)^m}dx + (N - \frac{Mp}{2})\int \frac{dx}{(x^2+px+q)^m}$
$= \frac{M}{2} \times \frac{(x^2+px+q)^{-m+1}}{-m+1} + (N - \frac{Mp}{2}\int \frac{dx}{(x^2+px+q)^m},$
Where:

$$\int \frac{dx}{(x^2+px+q)^m} = \int \frac{dx}{\left[(x+\frac{p}{2})^2 + (q - \frac{p^2}{4}\right]^m}$$

INTERATION OF RATIONAL FUNCTIONS

$$I = \int \frac{P_n(x)}{Q_m(x)}dx$$

METHOD OF PARTIAL FRACTIONS

1. If $n \geq m$ then we devide $P_n(x)$ into $Q_n(x)$:

$$\frac{P_n(x)}{Q_m(x)} = S(x) + \frac{R(x)}{Q_m(x)}$$

2. If n $<$ m then we factorize the denominator as:

$$\frac{P_n(x)}{Q_m(x)} = \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + ... + \frac{A_k}{(x-a)^k} + ... + \frac{M_1x+N_1}{x^2+px+q} + ... + \frac{M_lx+N_l}{(x^2+px+q)^l}$$

3. $\int \frac{dx}{(x-\alpha)\sqrt{ax^2+bx+c}}$
Solution: Let $x - \alpha = \frac{1}{t}$

4. $\int \frac{dx}{(x^2-a)\sqrt{ax^2+c}}$.
Solution: Let t $= \sqrt{a+\frac{c}{x^2}}$.

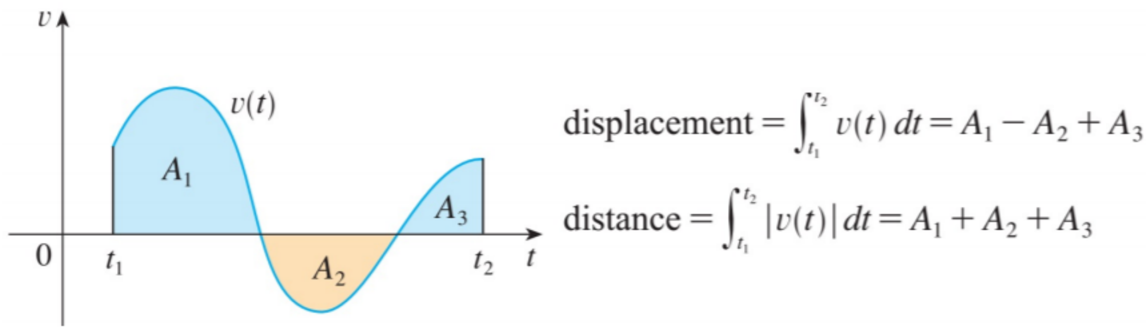
5. $\int R(sinx, cosx)dx$.
Let $t = tan\frac{x}{2}$. Then:

$$
\begin{cases}
sinx = \dfrac{2t}{1+t^2} \\
cosx = \dfrac{1-t^2}{1+t^2} \\
tanx = \dfrac{2t}{1-t^2} \\
cotx = \dfrac{1-t^2}{2t}
\end{cases}
$$

DEFINITE INTEGRAL

1. **Newton-Leibniz Formula**
   If f(x) is continous on $[a, b]$, then:

$$\boxed{\int_a^b f(x)dx = \int_a^b F'(x)dx = F(x)|_a^b = F(a) - F(b)}$$



2. **Integration by parts for Definite Integrals**

$$\boxed{\int_a^b udv = uv|_a^b - \int_a^b vdu}$$

3. **The Substition Rule for Definite Integrals**

$$\boxed{\int_a^b f(u(x)).u'(x)dx = \int_\alpha^\beta f(t)dt}$$

   For $\alpha = u(a); \beta = u(b)$

4. **Integral of Symmertric Functions**
   - If f(x) is ***odd funtion***:

$$\boxed{\int_{-a}^a f(x)dx = 0}$$

   - If f(x) is ***even function***:

$$\boxed{\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx}$$

APLICATION OF DEFINITE INTEGRAL

1. **Area**
   The **area** between the curves $\mathbf{y = f(x)}$ and $\mathbf{y = g(x)}$ and between $\mathbf{x = a}$ **and** $\mathbf{x = b}$ is:

$$\boxed{A = \int_a^b |f(x) - g(x)| dx}$$

2. **The volume of the solid in x-axis**

$$\boxed{V_x = \pi \int_a^b f^2(x) dx}$$

3. **The volume of the solid in y-axis**

$$\boxed{V_y = \pi \int_a^b g^2 y dy}$$

(Áp dụng cho phần hình cần tính nằm ở phía trên)

Beside:

$$\boxed{V_y = 2\pi \int_a^b |xf(x)| dx}$$

(Áp dụng cho phần hình nằm ở phía dưới)

4. **The Arc Length Formula**

$$\boxed{L = \int_a^b \sqrt{1 + f'(x)^2} dx}$$

5. **Surface Area**

$$\boxed{S = 2\pi \int_a^b f(x) \sqrt{1 + f'(x)^2} dx}$$

IMPROPER INTEGRAL

1. **Indefinite Interval**

$$\boxed{I = \lim_{b \to \infty} \int_a^b f(x)dx}$$

- If the limit exists then, the improper integral is **convergent**.
- If the limit doesn't exist, the imroper integral is **divergent**.

NEWTON - LEIBLIZ'S FORMULA

$$\boxed{\int_a^{\infty} f(x)dx = F(\infty) - F(a) = F(x)|_a^{\infty}}$$

Solution for:

$$I = \int_1^{\infty} \frac{dx}{x^{\alpha}}$$

- If $\alpha > 1$ then $I = \displaystyle\int_1^{\infty} \frac{dx}{x^{\alpha}}$ **converges**.
- If $\alpha \leq 1$ then $I = \displaystyle\int_1^{\infty} \frac{dx}{x^{\alpha}}$ **diverges**.

2. **Improper Integral type II**