

## INDEFINITE INTEGRAL

### SOME BASIC FORMULAS OF INDEFINITE INTEGRALS

- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$
  - $\int \frac{dx}{\sqrt{x^2 + a}} = \ln|x + \sqrt{x^2 + a}| + C$
  - $\int \sqrt{a^2 - x^2} dx.$
- Let  $x = a \sin t$ , where  $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$

### THE SUBSTITUTION RULE

$$\int f(u(x)) du(x) = F(u(x)) + C$$

### INTEGRATE BY PARTS

$$\int u dv = uv - \int v du$$

### REDUCTION FORMULA (Công thức truy hồi)

$$I_{n+1} = \frac{1}{2na^2} \times \frac{x}{(x^2 + a^2)^n} + \frac{2n-1}{2na^2} \times I_n$$

Since:

$$I_1 = \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + C$$

When

$$I_n = \int \frac{dx}{(x^2 + a^2)^n} (n \in \mathbb{N})$$

### The indefinite integrals of these partial fractions

1.  $\int \frac{A}{x-a} dx = A \ln|x-a| + C$
2.  $\int \frac{A}{(x-a)^n} dx = \frac{A}{(1-n)(x-a)^{n-1}} + C, (n \neq 1)$

$$\begin{aligned}
3. \int \frac{Mx + N}{x^2 + px + q} dx &= \frac{M}{2} \int \frac{(2x + p)dx}{x^2 + px + q} + (N - \frac{Mp}{2}) \int \frac{dx}{(x + \frac{p}{2})^2 + q - \frac{p^2}{4}} = \frac{M}{2} \ln|x^2 + px + q| + (N - \\
&\frac{Mp}{2}) \times \frac{1}{\sqrt{q - \frac{p^2}{4}}} \arctan \frac{x + \frac{p}{2}}{\sqrt{q - \frac{p^2}{4}}} + C \\
4. \int \frac{Mx + N}{(x^2 + px + q)^m} dx &= \frac{M}{2} \int \frac{2x + p}{(x^2 + px + q)^m} dx + (N - \frac{Mp}{2}) \int \frac{dx}{(x^2 + px + q)^m} \\
&= \frac{M}{2} \times \frac{(x^2 + px + q)^{-m+1}}{-m+1} + (N - \frac{Mp}{2}) \int \frac{dx}{(x^2 + px + q)^m},
\end{aligned}$$

Where:

$$\int \frac{dx}{(x^2 + px + q)^m} = \int \frac{dx}{\left[ \left(x + \frac{p}{2}\right)^2 + \left(q - \frac{p^2}{4}\right) \right]^m}$$

## INTERATION OF RATIONAL FUNCTIONS

$$I = \int \frac{P_n(x)}{Q_m(x)} dx$$

## METHOD OF PARTIAL FRACTIONS

1. If  $n \geq m$  then we devide  $P_n(x)$  into  $Q_n(x)$  :

$$\frac{P_n(x)}{Q_m(x)} = S(x) + \frac{R(x)}{Q_m(x)}$$

2. If  $n < m$  then we factorize the denominator as:

$$\frac{P_n(x)}{Q_m(x)} = \frac{A_1}{x - a} + \frac{A_2}{(x - a)^2} + \dots + \frac{A_k}{(x - a)^k} + \dots + \frac{M_1x + N_1}{x^2 + px + q} + \dots + \frac{M_lx + N_l}{(x^2 + px + q)^l}$$

$$3. \int \frac{dx}{(x - \alpha)\sqrt{ax^2 + bx + c}}$$

**Solution:** Let  $x - \alpha = \frac{1}{t}$

$$4. \int \frac{dx}{(x^2 - a)\sqrt{ax^2 + c}}.$$

**Solution:** Let  $t = \sqrt{a + \frac{c}{x^2}}$ .

$$5. \int R(\sin x, \cos x) dx.$$

Let  $t = \tan \frac{x}{2}$ . Then:

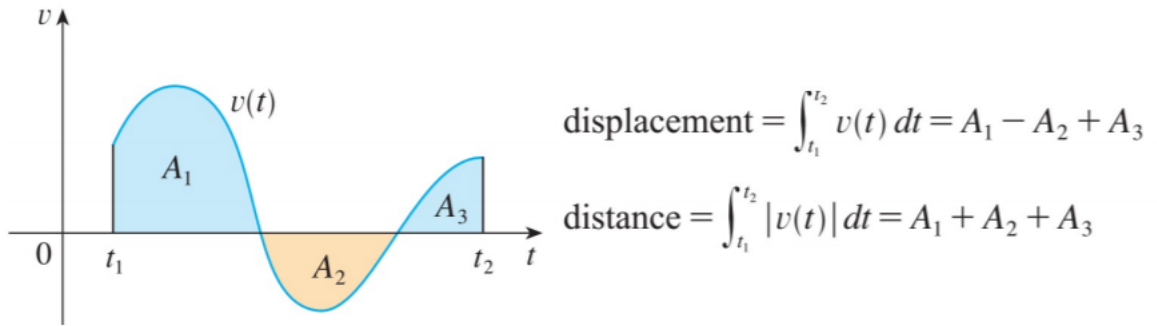
$$\begin{cases} \sin x = \frac{2t}{1+t^2} \\ \cos x = \frac{1-t^2}{1+t^2} \\ \tan x = \frac{2t}{1-t^2} \\ \cot x = \frac{1-t^2}{2t} \end{cases}$$

## DEFINITE INTEGRAL

### 1. Newton-Leibniz Formula

If  $f(x)$  is continuous on  $[a, b]$ , then:

$$\int_a^b f(x)dx = \int_a^b F'(x)dx = F(x)|_a^b = F(a) - F(b)$$



### 2. Integration by parts for Definite Integrals

$$\int_a^b u dv = uv|_a^b - \int_a^b v du$$

### 3. The Substitution Rule for Definite Integrals

$$\int_a^b f(u(x)) \cdot u'(x) dx = \int_{\alpha}^{\beta} f(t) dt$$

For  $\alpha = u(a); \beta = u(b)$

### 4. Integral of Symmetric Functions

- If  $f(x)$  is *odd function*:

$$\int_{-a}^a f(x) dx = 0$$

- If  $f(x)$  is *even function*:

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

## Application of Definite Integral

### 1. Area

The **area** between the curves  $y = f(x)$  and  $y = g(x)$  and between  $x = a$  and  $x = b$  is:

$$A = \int_a^b |f(x) - g(x)| dx$$

### 2. The volume of the solid in x-axis

$$V_x = \pi \int_a^b f^2(x) dx$$

### 3. The volume of the solid in y-axis

$$V_y = \pi \int_a^b g^2(y) dy$$

(Áp dụng cho phần hình cần tính nằm ở phía trên)

Beside:

$$V_y = 2\pi \int_a^b |xf(x)| dx$$

(Áp dụng cho phần hình nằm ở phía dưới)

### 4. The Arc Length Formula

$$L = \int_a^b \sqrt{1 + f'(x)^2} dx$$

### 5. Surface Area

$$S = 2\pi \int_a^b f(x) \sqrt{1 + f'(x)^2} dx$$

## IMPROPER INTEGRAL

### 1. Indefinite Interval

$$I = \lim_{b \rightarrow \infty} \int_a^b f(x) dx$$

- If the limit exists then, the improper integral is **convergent**.
- If the limit doesn't exist, the improper integral is **divergent**.

#### NEWTON - LEIBLIZ'S FORMULA

$$\int_a^\infty f(x) dx = F(\infty) - F(a) = F(x)|_a^\infty$$

Solution for:

$$I = \int_1^\infty \frac{dx}{x^\alpha}$$

- If  $\alpha > 1$  then  $I = \int_1^\infty \frac{dx}{x^\alpha}$  **converges**.
- If  $\alpha \leq 1$  then  $I = \int_1^\infty \frac{dx}{x^\alpha}$  **diverges**.

### 2. IMPROPER TYPE II

## PHYSICAL APPLICATION OF THE INTEGRAL

### 1. Work

- The work done by the object and is given by by Newton's second law.

$$F = ma = m \frac{d^2 s}{dt^2}$$

- The work done is defined to be the product of the force  $F$  and the distance  $D$ :

$$W = F \times D$$

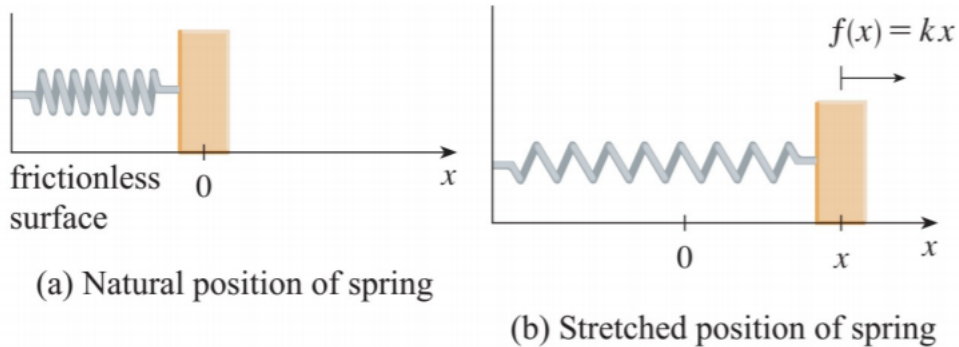
- The work done by the variable force  $f(x)$  in moving the object along the  $x$ -axis from  $\mathbf{a}$  to  $\mathbf{b}$

$$W = \int_a^b f(x) dx$$

### 2. Hook's Law

$$f(x) = kx$$

$k$  is the spring constant.



### 3. The average value of the function

$$F_{aver} = \frac{1}{b-a} \int_a^b f(x) dx$$

- If  $f$  is continuous on  $[a, b]$ , then there exist a number  $c \in [a, b]$  such that:

$$f(c) = f_{aver} = \frac{1}{b-a} \int_a^b f(x) dx$$