

ORDINARY DIFFERENTIAL EQUATIONS

1. Separable Equations

- A **separable equation** is a first-order differential equation, which has a form:

$$P(x)dx + Q(y)dy = 0$$

Solution:

Integrate both sides, we receive:

$$\int P(x)dx + \int Q(y)dy = C$$

2. The differential Equation of the form

$$f_1(x).g_1(y)dx + f_2(x).g_2(y)dy = 0$$

Solution:

If $f_1(x), f_2(x), g_1(y), g_2(y) \neq 0$, then we divide both sides by $f_2(x).g_1(y)$. So: (1)

$$\frac{f_1(x)}{f_2(x)}dx + \frac{g_2(y)}{g_1(y)}dy = 0 \rightarrow \int \frac{f_1(x)}{f_2(x)}dx + \int \frac{g_2(y)}{g_1(y)}dy$$

3. COOL PROBLEM

(Liên quan đến the rate of change cần chú ý)

- The rate of change of the temperature $T(t)$ is the derivative of $\frac{dT}{dt}$
- According to the Newton's law of cooling:

$$\frac{dT}{dt} = k(T - T_s)$$

When k is a constant of proportionality.

Separating the variables we have:

$$\frac{dT}{T - T_s} = kdt \leftrightarrow \frac{dT}{T - T_s} = kdt \quad (2)$$

$$\rightarrow \int \frac{dT}{T - T_s} = k \int dt \rightarrow \ln|T - T_s| = kt + \ln C \quad (3)$$

$$\rightarrow T - T_s = e^{kt + \ln C} = e^{kt} \cdot e^{\ln C} = C e^{kt} \quad (4)$$

4. The first-order linear differential equation

- The first-order **linear** differential equation is one that can be put into the form:

$$\frac{dy}{dx} + P(x).y = Q(x) \quad (5)$$

- To solve the linear differential equation, multiply both side by the **integrate factor** $e^{P(x)dx}$ and integrate both sides:

$$y = e^{-\int P(x)dx} \cdot \int Q(x) \cdot e^{\int P(x)dx} dx + C \quad (6)$$