Handbook of Statistical Methods for Design and Analysis in Sports

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Chapter 1

Hockey Player Performance via Regularized Logistic Regression

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A hockey player's plus-minus measures the difference between goals scored by and against that player's team while the player was on the ice. This measures only a marginal effect, failing to account for the influence of the others he is playing with and against. A better approach would be to jointly model the effects of all players, and any other confounding information, in order to infer a partial effect for this individual: his influence on the box score regardless of who else is on the ice.

This chapter describes and illustrates a simple algorithm for recovering such partial effects. There are two main ingredients. First, we provide a logistic regression model that can predict which team has scored a given goal as a function of who was on the ice, what teams were playing, and details of the game situation (e.g. full-strength or power-play). Since the resulting model is so high dimensional that standard maximum likelihood estimation techniques fail, our second ingredient is a scheme for regularized estimation. This adds a penalty to the objective that favors parsimonious models and stabilizes estimation. Such techniques have proven useful in fields from genetics to finance over the past two decades, and have demonstrated an impressive ability to gracefully handle large and highly imbalanced data sets. The latest software packages accompanying this new methodology – which exploit parallel computing environments, sparse matrices, and other features of modern data structures – are widely available and make it straightforward for interested analysts to explore their own models of player contribution.

This framework allows us to quickly obtain high-quality estimates for the full system of competing player contributions. After introducing the measurement problem in Section 1.1, we detail our regression model in Section 1.2 and the regularization scheme in Section 1.3. The remainder of the chapter analyzes more than a decade of data from the NHL. We fit and interpret our main model, based on prediction of goal scoring, in Section 1.4. This is compared to shot-based analysis, and metrics analogous to Corsi or Fenwick scores, in Section 1.5. Finally, Section 1.6 considers the relationship between our estimated performance scores and player salaries. Overall, we are able to estimate a partial plus-minus metric that occurs on the same scale as plus-minus but controls for other players and confounding variables. This metric is shown to be more highly correlated with salary than the standard (marginal) plus minus. Moreover, we find that the goals-based metric is more correlated with salary than those based upon shots and blocked shots. We conclude in Section 1.7 with thoughts on further extensions, in particular by breaking out of the linear framework to use classification models popular in the Machine Learning literature.

The code for all empirical work in this chapter is provided to the public via a GitHub repository (`https://github.com/TaddyLab/hockey`) and utilizes open source libraries for R [?], particularly the gamlr [?] package from [?].

1.1 Introduction: marginal and partial effects

Hockey is played on ice, but that's not all that sets it apart from seemingly related sports like soccer, basketball, or even field hockey. At least not from an analytics perspective. The unique thing about hockey is the rapid substitutions transpiring continuously during play, as well as at stoppages in play. In the data sets we have compiled, which we discuss in more detail shortly, the median amount of time observed for a particular on-ice player configuration (determined by unique players on the ice for both teams) is a mere eight seconds. Although many "shifts" are much longer than that, a trickle of piecemeal substitutions on both sides, transpiring as play develops, makes it difficult to attribute credit or blame to players for significant events, such as goals or shots.

Plus-minus (PM) is a traditional metric for evaluating player contributions in hockey. It is calculated as the difference, for a given player, between the number of goals scored against the player's team and those scored by the player's team while that player was on the ice. For example, during the 2012-2013 season Stanley Cup Finals, between Boston and Chicago, Duncan Keith of the Chicago Blackhawks was on the ice for 8 goals by Chicago and 4 by Boston, giving him a +4 PM for the series.

The PM score represents what statisticians call a *marginal effect:* the aggregate change in some response (goals for-vs-against) with change in some covariate (a player being on the ice) *without accounting for whatever else changes at the same time*. It is an aggregate measure that averages over the contributions of other factors, such as teammates and opponents. For example, suppose that the three authors of this chapter are added to the Blackhawks roster and that Joel Quenville (the coach of the Blackhawks) makes sure that Duncan Keith is with us on the ice whenever we are playing. Since none of us are anything close to as good at hockey as Keith is, and surely our poor play would allow the other team to score, this will cause Duncan Keith's PM to drop. At the same time, our PMs will be much higher than they would be if we didn't get to play next to Duncan Keith.

Due to its simplicity and minimal data requirements, plus-minus has been a preferred metric for the last fifty-odd years. But since it measures a marginal effect, the plus-minus is impacted by many factors beyond player ability, which is the actual quantity of interest. The ability of a player's teammates, or the quality of opponents, are not taken into account. The amount of playing time is also not factored in, meaning plus-minus stats are much noisier for some players than others. Finally, goalies, teams, coaches, salaries, situations, special teams, etc. – which all clearly contribute to the nature of play, and thus to goals –

are neither accounted for when determining player ability by plus-minus, and nor are they used to explain variation in the goals scored against a particular player or team.

Instead of marginal effects, statisticians are more often interested in *partial effects*: change in the expected response that can be accounted for by change in your variable of interest *after removing the change due to other influential variables.* In the example above, a partial effect for Duncan Keith would be unchanged if he plays with the authors of this article or with the current members of the Blackhawks. In each case, the partial effect will attempt to measure how Duncan Keith can influence the box-score regardless of with whom he skates. Because such partial effects help us predict how Keith would perform on a different team or with a different combination of line-mates, this information is more useful than knowing a marginal effect.

One way that statisticians can isolate partial effects is by running *experiments*. Suppose that now, instead of playing for the Blackhawks, we are coaching them. In order to figure out the value of Keith, we could randomly select different players to join him whenever he is on the ice and send completely random sets of players onto the ice whenever he is not playing. Then, due to the setup of this randomized experiment, Keith's resulting PM score will represent a partial effect – his influence regardless of who he plays with. Of course, no real hockey coach would ever manage their team in this way. Instead, we hockey analysts must make sense of *observational data* that is collected as the games are naturally played, with consistent line mates and offensive-defensive pairings and where Duncan Keith tends to play both with and against the best players available.

Partial effects are measured from observational data through *regression*: you model the response (e.g., goals) as a function of many influential variables (*covariates*; e.g., all of the players on the ice). With rich enough data, we can simultaneously estimate the full set of competing partial effects corresponding to all of our influential variables. This is straightforward when there are only a small number of covariates. However, the standard regression algorithms will fail when the number of covariates is large. This 'high dimensional regression' setting occurs in hockey analysis, where we would like to regress 'goals' onto the set of variables corresponding to whether each NHL player is on the ice (a set of 2500 players in our dataset) while also including effects of team, season, playoffs, and special teams scenarios (e.g. power plays). Moreover, the covariate design is highly *imbalanced*: over the span of several seasons there may be tens of thousands of goals, but players play with and against only a small fraction of other players and the number of unique player configurations is relatively small. Due to the use of player lines, and consistent line match-ups with opponents, where groups of two or three players are consistently on ice together at the same time, the data contain many clusters of individuals who are seldom observed apart. Standard regression algorithms, such as maximum likelihood inference via Fisher scoring, will either massively *over-fit* (e.g. assign large effects to players who rarely play) or simply fail to converge.

However, there has been a tremendous improvement over the past two decades in the techniques available for high dimensional regression analysis. These advancements are driven by the demands of researchers in genetics and finance, for example, for whom resolving partial effects amongst large sets of variables is the key to their science. The most successful approaches introduce some amount of *regularization* to the estimation problem – an additional penalty term that rewards simplicity (e.g., [**?**]). In our context, regularization shrinks towards a model where individual players don't make a huge difference while still allowing for large estimated player effects when the data warrant it. This conforms to what most analysts already believe: many players have a neutral, or "zero", effect (relative to the NHL average), whereas some are stars and others are liabilities. The amount of regularization is chosen to make the model perform as well as possible in out-of-sample prediction and, again, contemporary statistical learning tools are designed to do exactly this – reliably predict the future. To take advantage of these tools, we need only to phrase partial player effect estimation as a regression problem.

1.2 Regression Model

The goal of our regression analysis will be to estimate a model that relates individual presence on the ice to observable outcomes of interest. We describe the model here for a goals-based analysis, but extend it to shots and other metrics in the analysis sections.

Previous attempts at partial player effect estimation range from standard linear regression (usually on aggregate data) – the adjusted plus-minus scores of [?], [?], and [?] – to the complex hazard model of [?], which proposes a proportional hazards process for game events, allowing partial player effects to be backed out from high resolution game data. Adjusted plus-minus is built from similar ideas for Basketball analysis (see [?] and [?]). Its linear model analysis implies an underlying normality assumption for the error structure; this may be a good approximation for basketball, where scoring is frequent and variability in player configurations is small, but it is inappropriate for disaggregated data with a binary response (e.g., whether an individual goal is for-vs-against the home team). Such misspecification becomes especially problematic when combined with the modern regularization techniques necessary for reliable estimation of high dimensional models. On the other hand, more complex stochastic process modeling requires many additional assumptions on the data generating process and can be difficult to validate in practice; moreover, models such as that of [?] take far longer to run than we wish for our analysis. Some other important contributions to estimating player ability and attributing that to team success include [?, ?, ?].

The goal of our modeling is to provide a correct treatment of the binary 'goals' data without introducing significant additional modeling complexity. In particular, we advocate the simple *logistic regression* framework suggested by [?]. In logistic regression, the average log odds of a goal being scored "for" a particular team is modeled as a linear function of predictor variables which may be comprised of an indicator of player configuration and other quantities, which is otherwise identical to the familiar ordinary (least squares) regression setup. We provide a detailed description of the model here, but refer the reader to [?] or similar texts covering *Generalized Linear Models (GLMs)*, of which logistic regression is a special case. The setup is rather straightforward, easy to extend, and highly interpretable. Estimated coefficients describe contributions to the log odds of goals, and we show that these can be converted back onto the scale of goals, resulting in a adjusted plus-minus statistic, but this time one which is a true partial effect.

Given n goals throughout the National Hockey League (NHL) over some specified time period, define y_i is $+1$ for a goal by the *home* team and -1 for a goal by the *away* team.[1] Say that $\boldsymbol{q_i = \mathrm{p}(y_i = 1) = \mathrm{p}(\textbf{home team scored goal } i)}$. The logistic regression model of player contribution is, for goal i in season s with away team a and home team h,

$$\log\left[\frac{q_i}{1-q_i}\right] = \alpha + \mathbf{u}_i'\boldsymbol{\gamma} + \mathbf{v}_i'\boldsymbol{\varphi} + \mathbf{x}_i'\boldsymbol{\beta}_0 + (\mathbf{x}_i \circ \mathbf{s}_i)'(\boldsymbol{\beta}_s + p_i\boldsymbol{\beta}_p), \tag{1.1}$$

where

- Vector $\mathbf{u}_i$ holds indicators for each team-season (e.g., the Blackhawks in 2012-2013 would correspond to a coordinate of $\mathbf{u}_i$), set $u_{it} = +1$ if team-season t was the home team for goal i, $u_{it} = -1$ for the away team, and $u_{it} = 0$ if team-season t was not on the ice for goal i. This information is included to control for factors beyond the player's control, such as quality of coaching and fan support.

- Vector $\mathbf{v}_i$ holds indicators for various special-teams scenarios (e.g., being short-handed on a penalty kill), again set $v_{ik} = +1$ if the home team is in special-teams scenario k when goal i was scored, $v_{ik} = -1$ if the away team is in scenario k, and $v_{ik} = 0$ if neither team was in scenario k when goal i was scored. We consider 6 non-six-on-six settings (6v5, 6v4, 6v3, 5v4, 5v3, 4v3) and an additional 'pulled goalie' indicator; note that more than 35% of the goals occur on some type of special teams scenario.

- Vector $\mathbf{x}_i$ contains player-presence indicators, set $x_{ij} = 1$ if player j was on the home team and on ice for goal i, $x_{ij} = -1$ for away player j on ice for goal i, and $x_{ij} = 0$ for everyone not on the ice. With $\circ$ denoting the Hadamard (element-wise) product, this player vector is also interacted with

  - season (e.g., 2012-2013) vector $\mathbf{s}_i$, with $s_{ti} = 1$ if goal i was scored in season t, and
  - the post-season indicator p_i for whether or not the goal was scored in the playoffs, with $p_i = 1$ for the playoffs and zero for the regular season.

  By interacting players with seasons and with playoffs in this way, we have the potential to differentiate player ability over time both within (regular v. post-) season(s) and

[1] *home* and *away* are merely organizational devices, creating a consistent binary bifurcation for goals that can be applied across games, seasons, etc. Due to the symmetry in the logit transformation, player effects are unchanged when framing away team probabilities as q_i rather than $1 - q_i$, so we loose no generality by privileging home team goals in this way.

across seasons. There is potential for confounding with team–season effects $\mathbf{u}_i$ however, as very few players change teams during a season.

In this full specification the number of parameters, i.e., $K = 1 + |\boldsymbol{\gamma}| + |\boldsymbol{\varphi}| + |\boldsymbol{\beta}_0| + |\boldsymbol{\beta}_s| + |\boldsymbol{\beta}_p|$ is on the order of the number of players, p, in the league spanning the seasons/games of interest. The exact number depends on modeling aspects, like the number of special teams scenarios (i.e., constant in p), and on quantities like the number of team-seasons which grow more slowly than p.

To explain the coefficients and their interpretation, $\beta_{0j} + \beta_{sj}$ is the regular-season-s effect of player j on the log odds that, given a goal has been scored, the goal was scored by their team. Coefficient $\beta_{0j} + \beta_{sj} + \beta_{pj}$ is the corresponding effect for post-season-s (note that, under the regularization scheme in the next section, β_{pj} will be fixed at zero unless player j reaches the playoffs). These effects are 'partial' in that they control for who else was on the ice, special teams scenarios, and team-season effects – a player's β_{0j} or β_{sj} only need be nonzero if that player effects play above or below the team average for a given season. A test of understanding: what does the intercept α represent in (1.2)?[2]

For intuition, consider a simple "player-only" version of our model that has only who-is-on-the-ice as a time-invariant influence on goal scoring. This is the version of the model that was applied in [**?**]. Then there are no team-season-specific intercepts ($\alpha_{sh} = \alpha_{sa} = 0$), no special teams effects ($\boldsymbol{\varphi} = \mathbf{0}$), and no season-specific player-effect changes ($\boldsymbol{\beta}_s = \mathbf{0}$ and $\boldsymbol{\beta}_p = \mathbf{0}$) so that $\beta_j = \beta_{0j}$ is the constant effect of player j. The log odds that the home team has scored a given goal become

$$\log\left[\frac{q_i}{1-q_i}\right] = \alpha + \beta_{h_{i_1}} + \cdots + \beta_{h_{i_6}} - \beta_{a_{i_1}} - \cdots - \beta_{a_{i_6}}, \tag{1.2}$$

where the subscripts on the coefficients β are as follows: $h_{i_1}, \ldots, h_{i_6}$ are the six players on the ice for the home team and $a_{i_1}, \ldots, a_{i_6}$ indicate the players for the away time.[3] This is the just a re-writing of $\mathbf{x}_i'\boldsymbol{\beta}$ from (1.1), where the vector x_i (of length equal to the number of players) contains the "+1" and "−1" indicators depending on whether that player was on the

[2] It is the home ice advantage: if you do not know anything about who is playing or on the ice, the odds are e^{α} higher that the home team has scored any goal.

[3] In this setup the goalies *are* included in the calculations, unlike with *plus-minus*.

home or away team, and where all other x_{ij} are zero so that $\sum_j |x_{ij}| = 12$ for full-strength play. See Figure 1.1 for illustration.

$$
\begin{array}{cc}
Y\text{: scoring team} & X\text{: players} \\
y_i \in \{-1, 1\} & x_{ij} \in \{-1, 0, 1\} \\
 & 1 \hspace{20em} n \\
1 & 0\,1\,{-1}\,1\,1\,0\,{-1}\,0\,{-1}\,1\,{-1}\,{-1}\,0\,0 \cdots 0\,1\,0\,1\,1 \\
\vdots & \vdots \\
-1 & 1\,{-1}\,1\,{-1}\,0\,1\,0\,1\,{-1}\,1\,1\,0 \cdots 0\,{-1}\,0\,{-1}\,{-1}
\end{array}
$$

Figure 1.1: Diagram of a simple design matrix for a 'players only model' and two example goals (rows). The two goals are shown in the same season under the same configuration of teams except that the first goal was scored by the home team while the second goal was by the away team. The configurations of players are only differed by the first player since the home team was on a 6v5 power play for the first goal.

The model in 1.1 is simple and transparent; if you wish to control for new variables or situations you just need to add covariates to the logistic regression. In theory, one could fit the model easily in R by typing

```
R> fit <- glm(y~X, family="binomial")
```

But unfortunately, that is problematic on both statistical and computational grounds. Hockey data involving player indicators is too high dimensional (too many covariates) for ordinary logistic regression software, and the design matrices are too highly imbalanced to obtain meaningful (low variance) estimates of player effects. In almost all regressions one is susceptible to the temptations of rich modeling when the data set is large, and our hockey setup is no different. One must be careful not to *over-fit*, wherein parameters are optimized to statistical noise rather than fit to the relationship of interest. And one must be aware of *multicollinearity*, where groups of covariates are correlated with each other making it difficult to identify individual effects, as happens when players are grouped into lines.

A first approach to finding a remedy might be to entertain stepwise regression, e.g., via `step` in R with a stopping rule based upon an information criteria like AIC and BIC.

But that also doesn't work on this data: the calculations take days, and turn up very few non-zero predictors (i.e., players whose presence have any effect on goals). The trouble here is that players can't be judged on their own, since they almost always play with and against eleven others. Therefore the one-at-a-time judgments made by `step` fail to discover many relevant players despite making a combinatorially huge number of such comparisons. Moreover, stepwise regression results are well known to be highly variable: tiny jitter to the data can lead to massive changes in the estimated model. The combined effect is an unstable algorithm that yields overly simple results and takes a very long time to run.

Instead, a crucial contribution of [**?**] is to suggest the use of modern penalized and Bayesian logistic regression models, which biases the estimates of player effects towards zero. In the next section we consider one fast and successful version of these methods: L_1 regularization.

1.3 Regularized Estimation

Our solution is to take a modern regularized approach to regression. If $\eta_i = \log[q_i/(1-q_i)]$ is our linear equation for log odds from (1.1), then the usual maximum likelihood estimation routine (e.g., via glm in R) minimizes the negative log likelihood objective for n goal events

$$l\left(\boldsymbol{\eta};\mathbf{y}\right) = \sum_{i=1}^{n} \log\left(1+\exp[-y_i\eta_i]\right). \tag{1.3}$$

Instead, a *regularized* regression algorithm will minimize a penalized objective, say for example $l\left(\boldsymbol{\eta};\mathbf{y}\right) + n\lambda\sum_{j=1}^{p}\left[c\left(|\beta_{0j}|\right) + c\left(|\beta_{sj}|\right)\right]$, where $\lambda > 0$ controls overall penalty magnitude and $c(\cdot)$ is a coefficient cost function, while n is the total number of goals and p is the number of players. The penalty λ is discussed in some detail shortly. A few common cost functions are shown in Figure 1.2. Those that have a non-differentiable spike at zero (all but ridge) lead to sparse estimators, meaning that many coefficients are set to exactly zero. The curvature of the penalty away from zero dictates the weight of shrinkage imposed on the nonzero coefficients: L_2 costs increase with coefficient size, lasso's L_1 penalty has zero curvature and imposes constant shrinkage, and as curvature goes towards $-\infty$ one approaches

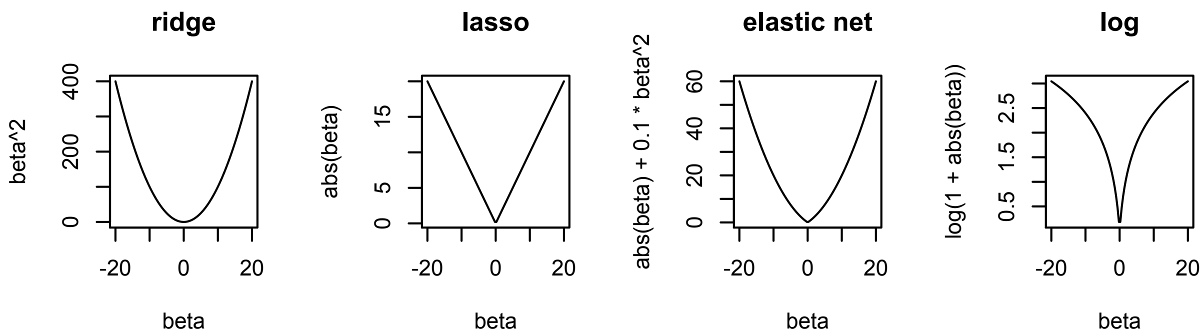


Figure 1.2: From left to right, L_2 'ridge' costs [?], L_1 'lasso' [?], the 'elastic net' mixture of L_1 and L_2 [?], and the log penalty [?].

the L_0 penalty of subset selection.

We will focus on the L_1 penalty for its balance between shrinkage of large signals (players tend not to have huge effects) and a preference for sparsity (we can only measure the nonzero effects of a subset of players). For these and many appealing theoretical reasons (and for computational tractability), the L_1 penalty is by far the most commonly used in contemporary regularized regression; see [?] for a broad-audience overview and [?] for details on the algorithms used in this chapter. Under lasso L_1 penalization, estimation for the unknown parameters in our particular hockey model (1.1) proceeds through optimization of

$$l\left(\boldsymbol{\eta};\mathbf{y}\right) + n\lambda \sum_{j=1}^{p}\left(|\beta_{0j}| + |\beta_{sj}| + |\beta_{pj}|\right). \tag{1.4}$$

It is important to note there that we are *penalizing only the player effects*. The team-season effects ($\boldsymbol{\gamma}$) are unpenalized. This strategy of combining penalized and unpenalized estimation is advocated in, e.g., [?] and [?]. It works nicely whenever you have a subset of covariates for which there is strong data signal (many repeated observations, which we have for team-season and special teams effects) and whose effect you'd like to completely remove from estimation for other coefficients. In this way, we ensure that the player effect estimates are not *polluted* by confounding effects in $\mathbf{u}$ and $\mathbf{v}$.

Moreover, consider the covariates on $\boldsymbol{\beta}_0$, $\boldsymbol{\beta}_s$, and $\boldsymbol{\beta}_p$ in (1.1): for the latter two, $\mathbf{x}_i$ interacts with additional binary indicators, such that β_{0j} acts on more nonzero terms than

β_{sj}, which itself acts on more nonzero terms than β_{pj}. Thus there is less signal associated with the season and playoff player effect innovations than with the player baseline effects, so that these will tend to be estimated at zero unless there is significant evidence that a player has become better or worse across seasons or in a given post-season.

Penalty size, λ, acts as a *squelch*: canceling noise to focus on the true input signal. Large λ lead to very simple model estimates, while as $\lambda \to 0$ we approach maximum likelihood estimation. Since you don't know optimal λ, practical application of penalized estimation requires a *regularization path*: a $p \times T$ field of $\hat{\boldsymbol{\beta}}$ estimates obtained while moving from high to low penalization along $\lambda^1 > \lambda^2 \ldots > \lambda^T$. These paths begin at λ^1 set to infimum λ such that (1.4) is minimized at $\hat{\boldsymbol{\beta}} = \mathbf{0}$, and proceed down to some pre-set λ^T (e.g., $\lambda^T = 0.01\lambda^1$).

A common tool for choosing the optimal λ – that for which we report estimated player effects – is cross validation (CV). In CV, the path of coefficients is repeatedly fit to data subsamples and used to predict the response on the left-out data. The λ leading to minimum error is then selected as optimal. In this chapter, we instead use an analytic alternative to CV that yields models that perform as well or better out-of-sample. The corrected Akaike Information Criterion (AICc), proposed in [**?**], is defined as

$$AICc = 2\sum_{i=1}^{n} l(\hat{\boldsymbol{\eta}}_\lambda; \mathbf{y}) + \frac{2kn}{n-k-1},$$

where $\hat{\boldsymbol{\eta}}_\lambda$ are the estimated log odds under penalty λ, via estimates of parameters $(\hat{\alpha}, \hat{\boldsymbol{\varphi}}, \hat{\boldsymbol{\beta}}_{0,s,p})$ in equation (1.1), and $k \leq K$ is the number of non-zero estimated coefficients (likely far fewer than the total number of parameters K, as many players cannot be distinguished from the league average) at this penalty. See [**?**] and [**?**] for details on AICc selection in this context; we find AICc preferable to CV because it is computationally efficient (you only need to optimize once) and because there is no random Monte Carlo variation – it always gives the same answer on the same data. However, all of our ideas here apply if you wish to use CV selection instead.

The `gamlr` package [**?**] can be called via `R` to implement this procedure:

```
R> fit <- gamlr(X, Y, standardize=FALSE, family="binomial")
```

For CV, just replace `gamlr` with `cv.gamlr`. The `standardize=FALSE` flag tells `gamlr` to *not* weight the coefficient penalties by the standard deviation of the corresponding covariate (i.e., to use penalty $\lambda|\beta_j|$ instead of $\lambda\,\mathrm{sd}(\mathbf{x}_j)|\beta_j|$); this is appropriate here because such standardization would *up-weight* the influence of players who rarely play (and have low $\mathrm{sd}(\mathbf{x}_j)$) relative to those who have a lot of ice time (and thus high $\mathrm{sd}(\mathbf{x}_j)$). The software exploits sparsity in our player effects (`X`) via the `Matrix` library for R, and is extremely fast to run: no examples in this article require more than a few seconds of computation. Estimated coefficients at optimal λ are available as `coef(fit)`.

One natural way to understand regularized regression is through the lens of Bayesian posterior inference. Judiciously chosen prior distributions lend stability to the fitted model, which is crucial in contexts where the number of quantities being estimated is large. In our setting where larger β-values indicate large positive or negative contributions to player ability, it makes sense to choose a prior that encourages coefficients to center around zero, a so-called *shrinkage* prior. Our *a priori* belief is that most players are members of the rank-and-file: their contribution to goals is *neutral* (e.g., zero on the log-odds scale), and that only a handful of stars (and liabilities) have a strong contribution to the chances of scoring (or letting in) goals. From the perspective of point estimation, adding a prior on β_j centered at zero is equivalent to adding a penalty term for $\beta_k \neq 0$ in our objective function. Different choices of priors correspond to different penalty functions on $\beta_j \neq 0$; a Laplace prior distribution on each β_j corresponds to our L_1 penalty in (1.4). The posterior density is the product of likelihood and prior, and on the log scale that product becomes a a sum. So maximizing the posterior to obtain posterior modes is equivalent to maximizing the log likelihood plus a penalty term which is the log of the prior. Conversely, minimizing (1.4) may be interpreted as Bayesian posterior maximization.

1.4 Analysis: goal-based effects

This section attempts to quantify the performance of hockey players using data from the NHL. It extends the analysis in [**?**], which used a smaller dataset, assumed constant player effects, and did not control for team-season effects. The data, downloaded from `http://www.nhl.com`, comprise of play-by-play NHL game data for regular and playoff games during 11 seasons of 2002-2003 through 2013-2014[4]. The data capture all signifigant events in every single game, such as change, goal, shot, blocked shot, miss shot, penalty and etc. There were $p = 2439$ players involved in $n = 69449$ goals.

The analysis proceeds through estimation of the model from (1.1),

$$\log\left[\frac{q_i}{1-q_i}\right] = \alpha + \mathbf{u}_i'\boldsymbol{\gamma} + \mathbf{v}_i'\boldsymbol{\varphi} + \mathbf{x}_i'\boldsymbol{\beta}_0 + (\mathbf{x}_i \circ \mathbf{s}_i)'(\boldsymbol{\beta}_s + p_i'\boldsymbol{\beta}_p),$$

by minimization of the implied penalized deviance in (1.4). The estimated player coefficients for each season s are then available as $\beta_{0j} + \beta_{sj}$, the combination of a baseline effect plus a season-specific innovation. These effects represent the estimated change in the log odds that, given a goal was scored, the goal was scored by player j's team.

The estimated effects – our β_{0j} and β_{sj} – might be tough for non-statisticians to interpret. One option is to translate from the scale of log-odds to that of probabilities. In particular, we define the 'partial for-%' functional as

$$\mathrm{PFP}_{sj} = \left(1 + \exp[-\beta_{0j} - \beta_{sj}]\right)^{-1}. \tag{1.5}$$

This feeds the player effects through a logit link to obtain the probability that, given a goal was scored in season s, it was scored by player j's team, *if we know nothing else other than that player j was on the ice*. It lives on the same scale as the commonly used *for-%* (FP) statistic: the total number of events by a given player's team *divided* by the total number of events by either team, while that player was on the ice. Like PM, FP is a marginal effect that does not account for who else was on the ice and other confounding factors. Hence, PFP is the partial effect version of FP.

[4]Season 2004-2005 was a lockout that resulted in a cancellation

An important feature of the standard PM statistic that differs from both *for-%* and β or PFP is that it – in a limited sense – accounts not only for player ability but also the amount that they play. For example, a player with a very high PM must both perform well *and* maintain this level of performance over an extended period of time (assuming that you need to be on the ice a long time to be on the ice for many goals). Conversely, similarly estimated β values for two players might hide the fact that one of these two logs much more ice time and is thus more valuable to the team.[5] It is therefore also important to be able to translate our partial player effects back to same scale as PM, and we do this in the *partial plus-minus* (PPM). Suppose player j was on the ice for g_{sj} total goals (for or against) during season s; then the PPM is defined

$$\mathrm{PPM}_{sj} = g_{sj}\mathrm{PFP}_{sj} - g_{sj}(1 - \mathrm{PFP}_{sj}) = g_{sj}(1 - 2\mathrm{PFP}_{sj}). \tag{1.6}$$

Just as PFP is is the partial effect version of FP, PPM is the partial effect analogue to PM's marginal effect.

Table 1.1 provides a list of top and bottom players listed by their PPM, along with the corresponding β effects and their standard PM. Since these PPMs and effects are calculated for each season, players will occur repeatedly in the table; for example, Sidney Crosby has 4 of the top 10 best player-seasons since 2002; he has been consistently the best, or near best, player in the league. The number one player-season since 2002 by PPM is Peter Forsberg in 2002-2003, with a PPM of 55.5. This is around 25% better than the 2nd best PPM: Crosby's 43.5 in 2009-2010. These tabulated effects are all calculated based upon regular season performance alone. However, for all 10,000 player seasons, using goals data, we never see enough signal to conclude that a given player was significantly better or worse in the post-season than in the regular season. That is, the $\hat{\beta}_{pj}$ are all zero and we have no evidence of 'clutch' players who improve their play in the post-season. At the same time, many of the β_{sj} *are* estimated at nonzero values: there is measurable signal indicating that player performance changes across seasons.

[5] Note that, due to the role of the penalty in our regularized estimation scheme, players with little ice time tend to have their effect estimated at zero; thus, the difference between β or PFP and the PPM statistics should be less dramatic than the difference between FP and PM statistics. In a fully Bayesian analysis, such as the `reglogit` approach discussed in our conclusion, one would be able to separate posterior uncertainty about β from the issue of the number of goals for which a player is on ice; for example, the `reglogit` approach yields posterior uncertainty over a player's PPM.

Goal-based performance analysis

Rank	Player	Season	Team	PFP	FP	PPM	PM
1	PETER FORSBERG	2002-2003	COL	0.68	0.77	55.52	85
2	SIDNEY CROSBY	2009-2010	PIT	0.60	0.64	43.47	60
3	DOMINIK HASEK	2005-2006	OTT	0.59	0.67	42.45	80
4	SIDNEY CROSBY	2008-2009	PIT	0.60	0.61	42.26	48
5	SIDNEY CROSBY	2005-2006	PIT	0.60	0.62	41.86	52
6	PETER FORSBERG	2005-2006	PHI	0.68	0.77	40.67	61
7	PAVEL DATSYUK	2007-2008	DET	0.60	0.72	39.49	87
8	PAVEL DATSYUK	2008-2009	DET	0.60	0.67	39.49	69
9	SIDNEY CROSBY	2006-2007	PIT	0.60	0.72	35.62	79
10	MARK STREIT	2008-2009	NYI	0.59	0.56	35.08	24
11	MATT MOULSON	2011-2012	NYI	0.60	0.61	34.92	37
12	LUBOMIR VISNOVSKY	2010-2011	ANA	0.58	0.66	34.52	70
13	ALEX OVECHKIN	2008-2009	WAS	0.57	0.66	34.46	80
14	JOE THORNTON	2009-2010	SJS	0.60	0.65	33.91	52
15	JOE THORNTON	2010-2011	SJS	0.60	0.64	33.91	48
16	ONDREJ PALAT	2013-2014	TAM	0.64	0.66	32.75	37
17	PAVEL DATSYUK	2006-2007	DET	0.60	0.71	32.61	70
18	JOE THORNTON	2002-2003	BOS	0.60	0.64	32.17	47
19	JOE THORNTON	2007-2008	SJS	0.60	0.71	32.17	69
20	ANDREI MARKOV	2007-2008	MON	0.57	0.60	31.9	47
21	PETER FORSBERG	2003-2004	COL	0.68	0.72	31.47	39
22	JOE THORNTON	2008-2009	SJS	0.60	0.67	31.21	56
23	PETER FORSBERG	2006-2007	PHI	0.68	0.68	31.12	32
24	PAVEL DATSYUK	2005-2006	DET	0.60	0.74	30.85	75
25	ROBERT LANG	2003-2004	WAS	0.60	0.66	30.8	50
10184	PATRICK LALIME	2008-2009	BUF	0.43	0.44	-15.79	-15
10185	JACK JOHNSON	2007-2008	LOS	0.45	0.39	-15.82	-34
10186	BRETT CLARK	2011-2012	TAM	0.44	0.35	-16.93	-47
10187	NICLAS HAVELID	2008-2009	ATL	0.45	0.39	-16.97	-40
10188	JACK JOHNSON	2010-2011	LOS	0.45	0.53	-17.21	9
10189	JACK JOHNSON	2011-2012	LOS	0.45	0.5	-17.21	-1
10190	P. J. AXELSSON	2008-2009	BOS	0.41	0.49	-17.35	-1
10191	BRYAN ALLEN	2006-2007	FLA	0.45	0.45	-17.9	-17
10192	JACK JOHNSON	2009-2010	LOS	0.45	0.49	-19.46	-4
10193	PATRICK LALIME	2005-2006	STL	0.43	0.40	-19.77	-29
10194	ALEXANDER EDLER	2013-2014	VAN	0.37	0.27	-20.49	-35
10195	PATRICK LALIME	2007-2008	CHI	0.43	0.49	-22.29	-4
10196	TIM THOMAS	2009-2010	BOS	0.43	0.46	-24.22	-16
10197	ANDREJ MESZAROS	2006-2007	OTT	0.42	0.48	-27.32	-6
10198	BRYCE SALVADOR	2008-2009	NJD	0.35	0.37	-34.4	-31
10199	PATRICK LALIME	2002-2003	OTT	0.43	0.58	-37.81	47
10200	PATRICK LALIME	2003-2004	OTT	0.43	0.56	-37.81	37
10201	NICLAS HAVELID	2006-2007	ATL	0.34	0.44	-62.64	-22
10202	NICLAS HAVELID	2005-2006	ATL	0.33	0.40	-65.94	-41
10203	JAY BOUWMEESTER	2005-2006	FLA	0.33	0.42	-69.62	-32

Table 1.1: Top-25 and bottom-20 player-seasons when ranked by their regular-season PPM.

The ranking in Table (1.1) differs dramatically from those in [**?**]. This occurs because we're now *controlling* for additional non-player confounding factors (e.g., coaching through team-season effects) and we allow the player performance to change over time rather than be fixed at a single 'career' value. To help intuition on why such control and model flexibility is useful, we note that Sidney Crosby's β effects drop significantly (he falls out of the top 5 for any season) if you do not control for the special teams effects. This occurs because he spends a lot of time on the penalty kill (short-handed), which makes it easier for him to get scored upon through no fault of his own. As another example, many of the goalies have large PPM if you do not control for team-season effects; since the goalie is almost always on the ice, they act as a surrogate for aggregate team performance unless you explicitly control for it (unfortunately for Patrick Lalime, there is still enough variation at goal to measure the effect and PPM for some goalies).

Another change from [**?**] is that we are ranking players here by PPM rather than by β; as described above, this rewards those with more ice-time. For comparison, Table 1.2 ranks players by their PFP (which is equivalent to ranking by β); the table includes both the goal-based metrics from this section and the shot-based metrics from our next section. While PFP and PPM are clearly related quantities, we do see some major differences. For example, Tyler Toffoli (ranks 9 and 10 by goal-based PFP) was a breakout star in 2013-2014 with the Los Angeles Kings; this was his first full season, after playing only a portion of 2012-2013 in the NHL. As a rookie, his ice time was relatively limited; however he clearly has talent and this is reflected in his β and PFP but less in his PPM. On the other hand, players ranked at the bottom by PPM in Table 1.1 are those who have a negative β *and* get a large amount of ice-time. There are many players who have lower PFPs than Jack Johnson's 0.45 (e.g., John McCarthy at 0.38 and Thomas Pock at 0.40), but they do not get to play as much and thus don't show up in our bottom 20.

PFP player rankings

	goal-based				Corsi-based			
Rank	Player	Season	Team	PFP	Player	Season	Team	PFP
1	PETER FORSBERG	2002-2003	COL	0.68	DAVID VAN DER GULIK	2010-2011	COL	0.64
2	PETER FORSBERG	2005-2006	PHI	0.68	DAVID BOOTH	2012-2013	VAN	0.63
3	PETER FORSBERG	2003-2004	COL	0.68	DANIEL SEDIN	2012-2013	VAN	0.62
4	PETER FORSBERG	2006-2007	PHI	0.68	ALEXANDER SEMIN	2003-2004	WAS	0.61
5	PETER FORSBERG	2007-2008	COL	0.68	DANIEL SEDIN	2010-2011	VAN	0.60
6	PETER FORSBERG	2010-2011	COL	0.68	MIKHAIL GRABOVSKI	2010-2011	TOR	0.60
7	ONDREJ PALAT	2013-2014	TAM	0.64	DANIEL SEDIN	2007-2008	VAN	0.60
8	ONDREJ PALAT	2012-2013	TAM	0.64	DANIEL SEDIN	2008-2009	VAN	0.60
9	TYLER TOFFOLI	2013-2014	LOS	0.63	DANIEL SEDIN	2011-2012	VAN	0.60
10	TYLER TOFFOLI	2012-2013	LOS	0.63	PATRIK ELIAS	2010-2011	NJD	0.60
11	VINCENT LECAVALIER	2006-2007	TAM	0.61	SIDNEY CROSBY	2013-2014	PIT	0.60
12	VINCENT LECAVALIER	2003-2004	TAM	0.61	DANIEL SEDIN	2009-2010	VAN	0.60
13	SIDNEY CROSBY	2009-2010	PIT	0.60	JUSTIN WILLIAMS	2010-2011	LOS	0.60
14	SIDNEY CROSBY	2008-2009	PIT	0.60	DANIEL SEDIN	2013-2014	VAN	0.60
15	SIDNEY CROSBY	2005-2006	PIT	0.60	PATRIC HORNQVIST	2013-2014	NSH	0.60
16	PAVEL DATSYUK	2007-2008	DET	0.60	PAVEL DATSYUK	2012-2013	DET	0.60
17	PAVEL DATSYUK	2008-2009	DET	0.60	ALEX STEEN	2011-2012	STL	0.60
18	SIDNEY CROSBY	2006-2007	PIT	0.60	BRAD RICHARDSON	2011-2012	LOS	0.60
19	MATT MOULSON	2011-2012	NYI	0.60	ERIC FEHR	2008-2009	WAS	0.60
20	JOE THORNTON	2009-2010	SJS	0.60	TYLER TOFFOLI	2013-2014	LOS	0.60

Table 1.2: Top 20 player-seasons by goal and Corsi-based PFP.

1.5 Analysis: comparison to shot-based metrics

The analysis above is built around the event of a 'goal'; this is the most reasonable baseline analysis, as it removes any subjectivity about whether or not the statistics are related to team performance – you score more you win. However, it has recently become popular in hockey analysis to consider alternative metrics that are built from shots and other events; see [**?**] for a review. The most popular of such statistics is Corsi, which counts the number of events that are goals, shots on goal, missed shots, or blocked shots. Fenwick is another statistic; it is Corsi but without counting blocked shots. Although we have seen no evidence that Corsi or Fenwick events are more useful in predicting team performance than goal-based metrics, they do offer a big advantage to the statistician: they lead to a larger sample size, so that you can hopefully better identify the competing influences of different players and confounding factors. Our data contain $n_c = 1,329,679$ Corsi events and $n_f = 1,034,154$ Fenwick events; this is an order of magnitude more events than the $n = 69449$ goals.

The standard way to report Corsi and Fenwick for a given player is as the *for-%* (FP)

Corsi-based performance analysis

Rank	Player	Season	Team	PFP	FP	PPM	PM
1	DANIEL SEDIN	2010-2011	VAN	0.60	0.65	615.14	876
2	ERIC STAAL	2008-2009	CAR	0.58	0.59	605.41	619
3	MIKHAIL GRABOVSKI	2010-2011	TOR	0.60	0.57	597.05	465
4	JOE THORNTON	2011-2012	SJS	0.59	0.61	596.37	742
5	ALEX OVECHKIN	2009-2010	WAS	0.59	0.66	575.72	1047
6	DANIEL SEDIN	2007-2008	VAN	0.60	0.63	562.11	685
7	DANIEL SEDIN	2008-2009	VAN	0.60	0.62	547.83	680
8	RYAN KESLER	2010-2011	VAN	0.58	0.59	530.05	649
9	SIDNEY CROSBY	2009-2010	PIT	0.57	0.62	517.86	815
10	DANIEL SEDIN	2011-2012	VAN	0.60	0.67	510.16	880
11	HENRIK ZETTERBERG	2011-2012	DET	0.58	0.60	497.04	596
12	CLAUDE GIROUX	2010-2011	PHI	0.58	0.56	487.53	347
13	ZACH PARISE	2008-2009	NJD	0.58	0.64	486.45	843
14	JOE THORNTON	2010-2011	SJS	0.58	0.60	482.72	647
15	ALEX STEEN	2010-2011	STL	0.59	0.61	475.5	561
16	LUBOMIR VISNOVSKY	2010-2011	ANA	0.56	0.56	474.91	446
17	ERIC STAAL	2010-2011	CAR	0.56	0.56	473.92	415
18	JUSTIN WILLIAMS	2011-2012	LOS	0.59	0.63	471.53	717
19	ALEX OVECHKIN	2007-2008	WAS	0.56	0.65	463.14	1094
20	PATRIK ELIAS	2010-2011	NJD	0.60	0.60	461.75	461
21	SIDNEY CROSBY	2013-2014	PIT	0.60	0.61	459.92	480
22	DUSTIN BYFUGLIEN	2010-2011	ATL	0.56	0.60	456.04	705
23	JAROMIR JAGR	2007-2008	NYR	0.58	0.65	455.78	911
24	ALEX OVECHKIN	2008-2009	WAS	0.56	0.64	455.46	1065
25	JASON BLAKE	2008-2009	TOR	0.58	0.55	454.7	278
10605	MIKE COMMODORE	2008-2009	CBS	0.43	0.42	-447.91	-537
10606	SCOTT HANNAN	2011-2012	CGY	0.42	0.40	-451.04	-591
10607	CHRIS PHILLIPS	2007-2008	OTT	0.43	0.40	-454.09	-644
10608	JAY BOUWMEESTER	2005-2006	FLA	0.44	0.46	-457.01	-305
10609	KARLIS SKRASTINS	2008-2009	COL	0.43	0.38	-457.54	-754
10610	KARLIS SKRASTINS	2009-2010	DAL	0.42	0.39	-464.49	-655
10611	MATTIAS OHLUND	2008-2009	VAN	0.42	0.47	-465.36	-212
10612	MATTIAS OHLUND	2006-2007	VAN	0.43	0.48	-470.03	-147
10613	SCOTT HANNAN	2008-2009	COL	0.43	0.38	-478.83	-788
10614	DOUGLAS MURRAY	2009-2010	SJS	0.42	0.47	-486.16	-184
10615	SCOTT HANNAN	2007-2008	COL	0.42	0.42	-507.7	-504
10616	FILIP KUBA	2011-2012	OTT	0.42	0.49	-509.74	-77
10617	NICLAS HAVELID	2007-2008	ATL	0.41	0.35	-516.86	-883
10618	JOHNNY ODUYA	2008-2009	NJD	0.42	0.51	-522.4	51
10619	DOUGLAS MURRAY	2010-2011	SJS	0.40	0.48	-540.83	-117
10620	DION PHANEUF	2006-2007	CGY	0.42	0.49	-552.69	-42
10621	NICLAS HAVELID	2008-2009	ATL	0.40	0.40	-562.65	-604
10622	SERGEI GONCHAR	2006-2007	PIT	0.42	0.52	-586.55	174
10623	PAUL MARTIN	2008-2009	NJD	0.39	0.55	-695.83	283
10624	BRYCE SALVADOR	2008-2009	NJD	0.32	0.42	-912.17	-407

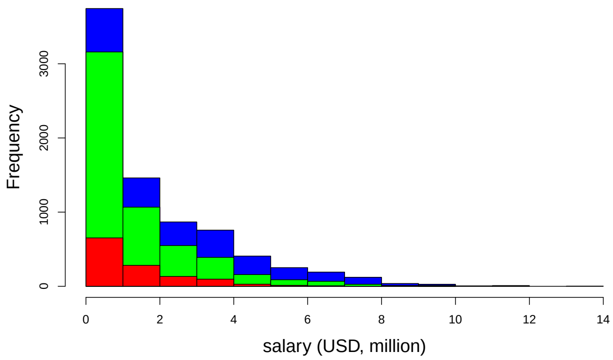
Table 1.3: Top 25 and bottom 20 players by Corsi-based PPM.

described above. Again, since the FP score does not reward players for the amount of time that they spend on the ice, we also consider both Corsi and Fenwick versions of the plus-minus statistic. Of course, all of these statistics – Corsi-FP, Corsi-PM, etc. – measure marginal effects. They are thus subject to the same criticisms as the original PM: they fail to control for the influence of other players and confounding factors, and are thus less useful than a partial effect for predicting and measuring player performance. However, we can apply the exact same regression analysis that we've used above for goal events to derive *partial* versions of the Corsi and Fenwick statistics: simply replace y_i with a response calculated from Corsi or Fenwick events. For example, a Corsi regression applies the model as in (1.1) but for response $y_i = +1$ if the event was a Corsi event (shot, goal, blocked shot) by the home team and $y_i = -1$ if it was a Corsi event by the away team. The partial *for-%* and plus-minus formulas of (1.5-1.6) can similarly be applied to obtain Corsi-PPF and Corsi-PPM values.

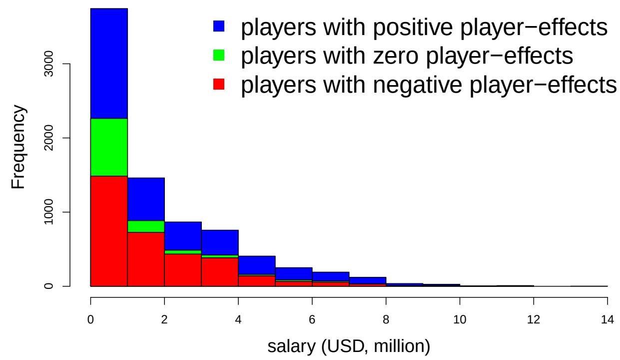
The results for regular season Corsi-based performance analysis are in Table 1.3 and on the right side of Table 1.2. Comparison to the goal-based rankings shows a distinctly different set of players are at both the top and bottom. For example, Daniel Sedin is a prominent player who ranks highly in multiple seasons under Corsi-PPM but does not appear in the top-20 for goals-PPM (his best goals-PPM is a still respectable 19.45 in 2010-2011, which ranks 152^{nd} across all player-seasons). At this point we are not looking to argue for either the goal or Corsi based metrics as 'best'; however, the fact that they do differ dramatically should be a bit troubling for those who wish to focus exclusively on Corsi statistics (since only goal differentials dictate who wins the game). In the next section, we consider a comparison between all of our metrics and an outside measure of player quality: salary.

1.6 Analysis: the relationship between salary and performance

In our final analysis section, we consider how our partial effect statistics relate to the *market* value of a player's worth as represented by their annual salary. The salary numbers are obtained through a combination of the databases maintained at blackhawkzone.com and hockeyzoneplus.com; we are able to obtain annual salaries for 80% of the player-seasons in



(a) Goals-based



(b) Corsi-based

Figure 1.3: Distribution of estimated player effects and salaries over all 11 seasons.

our dataset, including almost all player-seasons where the player was on ice for more than a couple of goals. The histograms in Figure 1.3 show, for the salary distribution over all 11 seasons, how the estimated player effects are distributed between negative, neutral (zeros), and positive. Results are shown for both Goal and Corsi based regressions. In both cases, the ratio of positive to negative effects increases with salary. The main difference between the two plots is that fewer players have zero estimated effects under Corsi – this is a result of its much larger event sample.

Figure 1.4 takes a deeper look at the relationship between salary and performance for two of our goal-based metrics: the goal plus-minuses (PM and PPM) and for-percentages (FP and PFP). For each metric, we use nonparametric Bayesian regression to fit expected log salary as a function of that metric. In particular, we apply the `tgp` package [?, ?] to obtain posterior means for the Bayesian regression trees of [?]. The trees are fit to salary and performance data that has first been aggregated by player, such that these surfaces represent the expected log average salary (per player) conditional upon their average performance across the years that they played. This aggregation is done to minimize dependence between observations, and because we assume that a player's salary is not determined by a single season.

The surfaces in 1.4 expose some interesting differences between the partial and marginal performance statistics. In the plus-minus case (PM and PPM), the two curves are similar – they both show little relationship between salary and performance for negative plus-minus, while salaries rise with positive performance. However, there is a larger jump up from zero for PPM than for PM. This occurs because the regression estimation only assigns a positive non-zero effect for statistically significant performances, so that players with small but positive PM values have their PPM shrunk to zero. The difference between FP and PFP is more dramatic. In the case of marginal FP, the salaries are highest for players with with FP in the middle of the range (near and above 0.5). This occurs because FPs outside of that range occur only for players with little ice-time. In contrast, the PFP relationship with salary is very simple: salaries are low for PFP below 0.5, and high above that number. If you make it more likely than not that a goal-scored is a goal-for, you can expect to make more money.

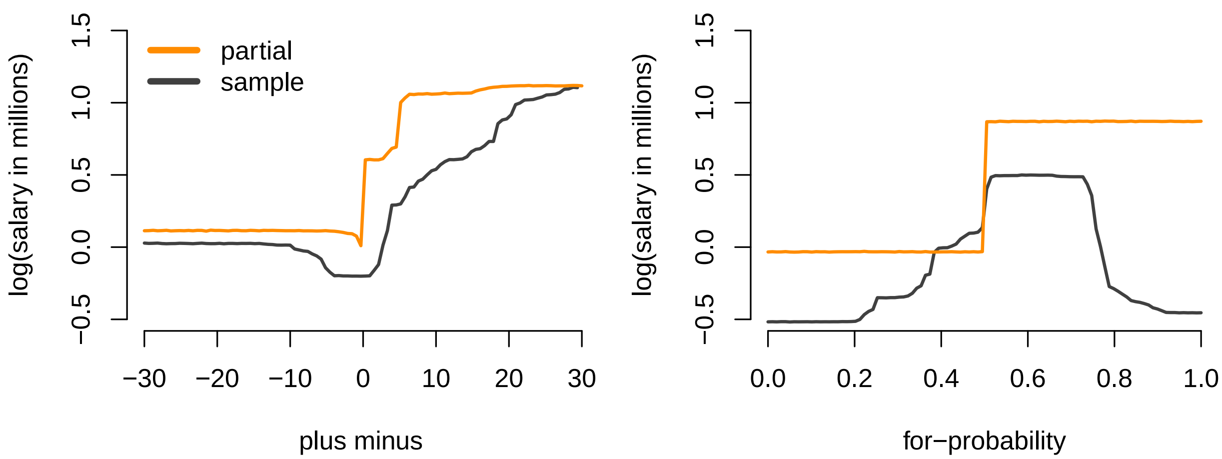


Figure 1.4: Nonparametric regression for log average salary (per player) onto their average performance metric: partial (PPM, PFP) and sample/marginal (PFP, FP) analogues.

Finally, we close with a look at the 'highest value' players in the 2013-2014 season: those for whom their salary was low relative to their goal-based PPM. Table 1.4 lists the top 20 players by goal-PPM/salary – that is, the top players measured by their goals-added-per-dollar. The cheapest player contribution, by a massive margin, is from Ondrej Palat of Tampa Bay. Palat was an inexpensive 7th round draft pick in 2011, making \$500 thousand per year. After spending two seasons in the minors, he moved up to the NHL in 2013-2014 and had a season good enough to be nominated for rookie-of-the-year. The Lightning re-signed Palat to a new contract in 2014; he now averages around \$3.5 million per year.

Rank	Player	Team	Goals per million
1	ONDREJ PALAT	TAM	58.27
2	RYAN NUGENT-HOPKINS	EDM	19.81
3	GABRIEL LANDESKOG	COL	16.74
4	TYLER TOFFOLI	LOS	16.72
5	GUSTAV NYQUIST	DET	9.08
6	JADEN SCHWARTZ	STL	8.43
7	ERIC FEHR	WAS	7.51
8	ANDREW MACDONALD	NYI	7.48
9	BENOIT POULIOT	NYR	6.43
10	BRAD BOYES	FLA	6.01
11	TOMAS TATAR	DET	5.83
12	AL MONTOYA	WPG	5.79
13	BRANDON SAAD	CHI	5.5
14	FRANS NIELSEN	NYI	5.5
15	JAROMIR JAGR	NJD	4.73
16	LOGAN COUTURE	SJS	4.7
17	RADIM VRBATA	PHO	4.4
18	DAVID PERRON	EDM	4.1
19	HENRIK LUNDQVIST	NYR	3.76
20	ANDREI MARKOV	MON	3.5

Table 1.4: Top 20 value players as ranked by PPM/salary.

1.7 Conclusion

We have provided a sketch for how modern techniques in regularized logistic regression, developed originally to address challenging large-scale problems in genetics, finance, and text mining, can be used to calculate partial player effects in hockey. We have argued that such partial effects are a better measure of player ability compared to the classic plus-minus statistic, and have the benefit of being interpretable on the same scale as plus-minus. We have shown how the framework is flexible, allowing one to control for many aspects of situational play (special teams, overtime, playoffs), and personnel/season (coaches, salaries, season-years). A comparison was provided to the popular Corsi and Fenwick alternatives to plus-minus, and we argued that a recent emphasis in the literature on shots (and blocked shots), does not in general compare favorably to the traditional goals focus in this framework.

Our development has focused on point-estimation via the `gamlr` package, which infers parameters under L_1 penalization. Another software package offering similar features is `glmnet` [?]. The difference between `gamlr` and `glmnet` is in what options they provide on top of the standard L_1 penalty. As detailed in [?], including an example analysis of this same hockey data, gamlr provides a 'gamma-lasso' algorithm for *diminishing bias* penalization: the penalty on coefficients automatically diminishes for strong signals. If you believe that the current analysis has over-shrunk the influence of, say, total stars like Sidney Crosby or Pavel Datsyuk, then gamlr and [?] will offer a preferable analysis framework. On the other hand, if you think that all players should be shrunk *closer* to zero – perhaps you believe that the current results over-state the effect of a few stars – then the elastic net penalization scheme of [?] and glmnet will be preferable. In either case, simple L_1 penalization provides a useful reference baseline analysis.

Beyond changes to the penalty specification, we think that there can be considerable value in moving from point-estimation to a fully Bayesian analysis. [?] included exploration of the player effect posterior in their earlier analysis of a related model. They apply the reglogit package [?] for R, which implements the Gibbs sampling strategy from [?]. The software takes advantage of sparse matrix libraries (slam [?]), and is multi-threaded via OpenMP to engage multiple processors simultaneously. It combines two scale-mixture of normals data-augmentation schemes, one for the logit [?] and one for the Laplace prior [?]. Obtaining T samples from the full posterior, is straightforward using the following R code

```
bfit <- reglogit(T=T, y=Y, X=X, normalize=FALSE)
```

The full posterior sample for β, residing in bfit, is available for calculation of posterior means and covariances of player effects and other posterior functionals relevant to player performance. For example, [?] use the posterior *probability* that one player is better than another as a basis for ranking players, and can even provide posterior credible intervals around these rankings.[6] This information can be used to construct teams of players under budget constraints and subsequently describe the probability that those teams will score more goals than their opponents.

Another option is to depart from the restrictions of linear modeling. Anecdotally, some in the sports analytics community (not just in hockey) have embraced a framework built around random forests [?]. An advantage of decision trees, on which random forests are based, is that they naturally explore interactions between predictors – e.g., between players

[6] see., e.g, https://github.com/TaddyLab/hockey/blob/master/results/blog/logistic_pranks_betas.csv

and other effects in the hockey analysis. The bagging procedure – averaging across many trees – provides a mechanism for avoiding over-fit: structure that is not persistent across trees is eliminated by the averaging. Such work also fits with our above advocacy of fully Bayesian analysis: [?] describes random forests as approximating a Bayesian nonparametric posterior over trees, while Bayesian additive regression trees (BART [?]) provide an alternative tree-based scheme that can be extended to logistic regression via the latent variable techniques in [?]. Finally, the proportional hazards model [?], mentioned above in Section 1.2, attempts to reproduce more completely the stochastic processes behind scoring in a hockey game. Their fully Bayesian analysis accounts for a wide set of game information, including the time-on-ice information that we are only roughly accounting for in our PPM statistic.

Regardless of these and other possible complex extensions, we argue strongly that our simple L_1 penalized logistic regression has much to recommend it. The model is very simple to interpret and relies upon minimal restrictive assumptions on the process of a hockey game. Our measures are also much faster to compute than any of the alternatives. These qualities make sophisticated real-time analysis of player effects possible as games and seasons progress.