

# Regularized Estimation of Hockey Player Performance

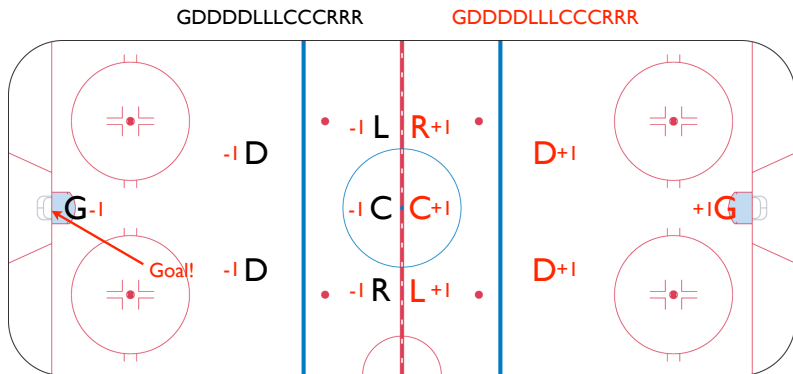
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## Plus-Minus

PM is a running count of, for every goal, a +1 for those on the scoring team and -1 for those on the team scored upon.



It doesn't quite measure player quality, as we haven't controlled for the effects of teammate and opponent quality (or anything else).

## A Regression version of PM

Set up a 'response' variable:

$y_i = +1$  for a *home* team goal,

$y_i = -1$  for an *away* team goal.

We're interested in how individual players affect

$$q_i = p(y_i = 1) = p(\text{home team scored goal } i)$$

The standard model for such problems is logistic regression, say

$$\log \left[ \frac{q_i}{1 - q_i} \right] = \alpha + \beta_{HG} + \beta_{HD} \dots + \beta_{HR} - \beta_{AG} - \dots - \beta_{AR}$$

where  $\beta_{HG}$  is Home-Goalie and  $\beta_{AR}$  is Away-Right-wing, etc.

Then, for player  $j$  and given a goal was scored,  $e^{\beta_j}$  is the multiplier on odds that it was scored by his team if he's on the ice.

We actually use a larger regression model:

$$\log \left[ \frac{q_i}{1 - q_i} \right] = \alpha + \mathbf{u}_i' \boldsymbol{\gamma} + \mathbf{v}_i' \boldsymbol{\varphi} + \mathbf{x}_i' \boldsymbol{\beta}_0 + (\mathbf{x}_i \circ \mathbf{s}_i)' (\boldsymbol{\beta}_s + p_i \boldsymbol{\beta}_p)$$

where

- ▶  $\mathbf{u}_i$  holds indicators for each team-season,
- ▶  $\mathbf{v}_i$  holds indicators for various special-teams scenarios,
- ▶  $\mathbf{x}_i$  contains player-presence indicator,

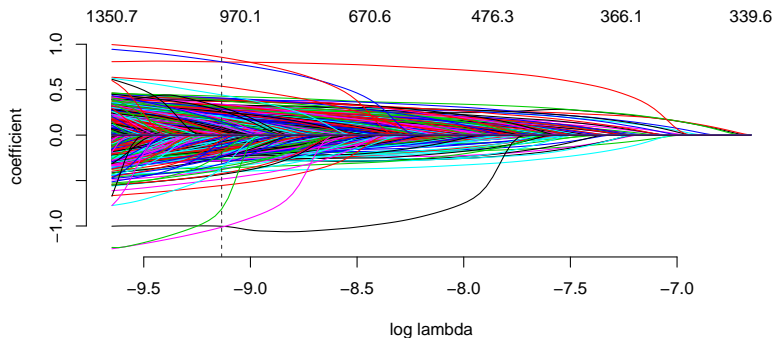
All of these indicators are +1 for home and -1 for away.

Then  $\beta_j$  measures player effect after **controlling** for team strength (e.g., coach or schedule) and on-ice scenarios (e.g., PP or PK).

We also allow deviations in the player effects for specific seasons ( $s_{it}$ ) and in the playoffs ( $p_{it}$ ), but these are seldom 'significant'.

## Regularization

Instead of minimizing deviance, we minimize deviance *plus penalty*  $\lambda|\beta_j|$  on the size of each  $\beta_j$  coefficient. This is called the LASSO.



Enumerate a 'path' of models for different  $\lambda$ , and use the one that predicts best out-of-sample. **This is how modern statistics works.** See the `glmnet` package for R, and `help(hockey)` for this example.

## Partial PM and FP

We use the estimated logistic regression model to produce ‘partial’ (i.e., without effect of confounders) versions of standard statistics.

- ▶ For-Probability:  $PFP_j = \frac{e^{\beta_j}}{(1 + e^{\beta_j})}$
- ▶ Plus-Minus:  $PPM_j = G_j PFP_j - G_j (1 - PFP_j)$   
where  $G_j$  is the total goals with player  $j$  on-ice.

PFP measures the player’s average contribution, and PPM scales this up by # of goals (which is a rough surrogate for time-on-ice).

We also fit models for non-goal response  $y_i$ , like shots or corsi events. This gives partial versions of stats based on those metrics.

# Goal-based performance analysis, ordered by PPM: Studs

| Rank | Player            | Season    | Team | PFP  | FP   | PPM   | PM |
|------|-------------------|-----------|------|------|------|-------|----|
| 1    | PETER FORSBERG    | 2002-2003 | COL  | 0.68 | 0.77 | 55.52 | 85 |
| 2    | SIDNEY CROSBY     | 2009-2010 | PIT  | 0.60 | 0.64 | 43.47 | 60 |
| 3    | DOMINIK HASEK     | 2005-2006 | OTT  | 0.59 | 0.67 | 42.45 | 80 |
| 4    | SIDNEY CROSBY     | 2008-2009 | PIT  | 0.60 | 0.61 | 42.26 | 48 |
| 5    | SIDNEY CROSBY     | 2005-2006 | PIT  | 0.60 | 0.62 | 41.86 | 52 |
| 6    | PETER FORSBERG    | 2005-2006 | PHI  | 0.68 | 0.77 | 40.67 | 61 |
| 7    | PAVEL DATSYUK     | 2007-2008 | DET  | 0.60 | 0.72 | 39.49 | 87 |
| 8    | PAVEL DATSYUK     | 2008-2009 | DET  | 0.60 | 0.67 | 39.49 | 69 |
| 9    | SIDNEY CROSBY     | 2006-2007 | PIT  | 0.60 | 0.72 | 35.62 | 79 |
| 10   | MARK STREIT       | 2008-2009 | NYI  | 0.59 | 0.56 | 35.08 | 24 |
| 11   | MATT MOULSON      | 2011-2012 | NYI  | 0.60 | 0.61 | 34.92 | 37 |
| 12   | LUBOMIR VISNOVSKY | 2010-2011 | ANA  | 0.58 | 0.66 | 34.52 | 70 |
| 13   | ALEX OVECHKIN     | 2008-2009 | WAS  | 0.57 | 0.66 | 34.46 | 80 |
| 14   | JOE THORNTON      | 2009-2010 | SJS  | 0.60 | 0.65 | 33.91 | 52 |
| 15   | JOE THORNTON      | 2010-2011 | SJS  | 0.60 | 0.64 | 33.91 | 48 |
| 16   | ONDREJ PALAT      | 2013-2014 | TAM  | 0.64 | 0.66 | 32.75 | 37 |
| 17   | PAVEL DATSYUK     | 2006-2007 | DET  | 0.60 | 0.71 | 32.61 | 70 |
| 18   | JOE THORNTON      | 2002-2003 | BOS  | 0.60 | 0.64 | 32.17 | 47 |
| 19   | JOE THORNTON      | 2007-2008 | SJS  | 0.60 | 0.71 | 32.17 | 69 |
| 20   | ANDREI MARKOV     | 2007-2008 | MON  | 0.57 | 0.60 | 31.9  | 47 |

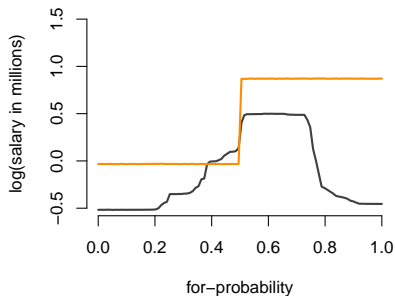
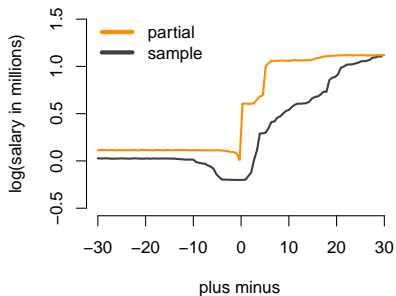
### Goal-based performance analysis, ordered by PPM: Duds

| Rank  | Player          | Season    | Team | PFP  | FP   | PPM    | PM  |
|-------|-----------------|-----------|------|------|------|--------|-----|
| 10184 | PATRICK LALIME  | 2008-2009 | BUF  | 0.43 | 0.44 | -15.79 | -15 |
| 10185 | JACK JOHNSON    | 2007-2008 | LOS  | 0.45 | 0.39 | -15.82 | -34 |
| 10186 | BRETT CLARK     | 2011-2012 | TAM  | 0.44 | 0.35 | -16.93 | -47 |
| 10187 | NICLAS HAVELID  | 2008-2009 | ATL  | 0.45 | 0.39 | -16.97 | -40 |
| 10188 | JACK JOHNSON    | 2010-2011 | LOS  | 0.45 | 0.53 | -17.21 | 9   |
| 10189 | JACK JOHNSON    | 2011-2012 | LOS  | 0.45 | 0.5  | -17.21 | -1  |
| 10190 | P. J. AXELSSON  | 2008-2009 | BOS  | 0.41 | 0.49 | -17.35 | -1  |
| 10191 | BRYAN ALLEN     | 2006-2007 | FLA  | 0.45 | 0.45 | -17.9  | -17 |
| 10192 | JACK JOHNSON    | 2009-2010 | LOS  | 0.45 | 0.49 | -19.46 | -4  |
| 10193 | PATRICK LALIME  | 2005-2006 | STL  | 0.43 | 0.40 | -19.77 | -29 |
| 10194 | ALEXANDER EDLER | 2013-2014 | VAN  | 0.37 | 0.27 | -20.49 | -35 |
| 10195 | PATRICK LALIME  | 2007-2008 | CHI  | 0.43 | 0.49 | -22.29 | -4  |
| 10196 | TIM THOMAS      | 2009-2010 | BOS  | 0.43 | 0.46 | -24.22 | -16 |
| 10197 | ANDREJ MESZAROS | 2006-2007 | OTT  | 0.42 | 0.48 | -27.32 | -6  |
| 10198 | BRYCE SALVADOR  | 2008-2009 | NJD  | 0.35 | 0.37 | -34.4  | -31 |
| 10199 | PATRICK LALIME  | 2002-2003 | OTT  | 0.43 | 0.58 | -37.81 | 47  |
| 10200 | PATRICK LALIME  | 2003-2004 | OTT  | 0.43 | 0.56 | -37.81 | 37  |
| 10201 | NICLAS HAVELID  | 2006-2007 | ATL  | 0.34 | 0.44 | -62.64 | -22 |
| 10202 | NICLAS HAVELID  | 2005-2006 | ATL  | 0.33 | 0.40 | -65.94 | -41 |
| 10203 | JAY BOUWMEESTER | 2005-2006 | FLA  | 0.33 | 0.42 | -69.62 | -32 |



For abundant detail and analysis, see our handbook chapter [Hockey Performance via Regression](#) by Gramacy, Taddy, and Tian.

e.g., look at average player salary by standard and partial statistics



## Bargains!

### Top-15 undervalued players in 2013-2014

| Rank | Player              | Team | Goals per million |
|------|---------------------|------|-------------------|
| 1    | ONDREJ PALAT        | TAM  | 58.27             |
| 2    | RYAN NUGENT-HOPKINS | EDM  | 19.81             |
| 3    | GABRIEL LANDESKOG   | COL  | 16.74             |
| 4    | TYLER TOFFOLI       | LOS  | 16.72             |
| 5    | GUSTAV NYQUIST      | DET  | 9.08              |
| 6    | JADEN SCHWARTZ      | STL  | 8.43              |
| 7    | ERIC FEHR           | WAS  | 7.51              |
| 8    | ANDREW MACDONALD    | NYI  | 7.48              |
| 9    | BENOIT POULIOT      | NYR  | 6.43              |
| 10   | BRAD BOYES          | FLA  | 6.01              |
| 11   | TOMAS TATAR         | DET  | 5.83              |
| 12   | AL MONTOYA          | WPG  | 5.79              |
| 13   | BRANDON SAAD        | CHI  | 5.5               |
| 14   | FRANS NIELSEN       | NYI  | 5.5               |
| 15   | JAROMIR JAGR        | NJD  | 4.73              |

We also include comparison of shot and goal-based metrics:

| PFP player rankings |                    |           |      |      | Corsi-based         |           |      |      |
|---------------------|--------------------|-----------|------|------|---------------------|-----------|------|------|
| Rank                | Player             | Season    | Team | PFP  | Player              | Season    | Team | PFP  |
| 1                   | PETER FORSBERG     | 2002-2003 | COL  | 0.68 | DAVID VAN DER GULIK | 2010-2011 | COL  | 0.64 |
| 2                   | PETER FORSBERG     | 2005-2006 | PHI  | 0.68 | DAVID BOOTH         | 2012-2013 | VAN  | 0.63 |
| 3                   | PETER FORSBERG     | 2003-2004 | COL  | 0.68 | DANIEL SEDIN        | 2012-2013 | VAN  | 0.62 |
| 4                   | PETER FORSBERG     | 2006-2007 | PHI  | 0.68 | ALEXANDER SEMIN     | 2003-2004 | WAS  | 0.61 |
| 5                   | PETER FORSBERG     | 2007-2008 | COL  | 0.68 | DANIEL SEDIN        | 2010-2011 | VAN  | 0.60 |
| 6                   | PETER FORSBERG     | 2010-2011 | COL  | 0.68 | MIKHAIL GRABOVSKI   | 2010-2011 | TOR  | 0.60 |
| 7                   | ONDREJ PALAT       | 2013-2014 | TAM  | 0.64 | DANIEL SEDIN        | 2007-2008 | VAN  | 0.60 |
| 8                   | ONDREJ PALAT       | 2012-2013 | TAM  | 0.64 | DANIEL SEDIN        | 2008-2009 | VAN  | 0.60 |
| 9                   | TYLER TOFFOLI      | 2013-2014 | LOS  | 0.63 | DANIEL SEDIN        | 2011-2012 | VAN  | 0.60 |
| 10                  | TYLER TOFFOLI      | 2012-2013 | LOS  | 0.63 | PATRIK ELIAS        | 2010-2011 | NJD  | 0.60 |
| 11                  | VINCENT LECAVALIER | 2006-2007 | TAM  | 0.61 | SIDNEY CROSBY       | 2013-2014 | PIT  | 0.60 |
| 12                  | VINCENT LECAVALIER | 2003-2004 | TAM  | 0.61 | DANIEL SEDIN        | 2009-2010 | VAN  | 0.60 |
| 13                  | SIDNEY CROSBY      | 2009-2010 | PIT  | 0.60 | JUSTIN WILLIAMS     | 2010-2011 | LOS  | 0.60 |
| 14                  | SIDNEY CROSBY      | 2008-2009 | PIT  | 0.60 | DANIEL SEDIN        | 2013-2014 | VAN  | 0.60 |
| 15                  | SIDNEY CROSBY      | 2005-2006 | PIT  | 0.60 | PATRIC HORNQVIST    | 2013-2014 | NSH  | 0.60 |
| 16                  | PAVEL DATSYUK      | 2007-2008 | DET  | 0.60 | PAVEL DATSYUK       | 2012-2013 | DET  | 0.60 |
| 17                  | PAVEL DATSYUK      | 2008-2009 | DET  | 0.60 | ALEX STEEN          | 2011-2012 | STL  | 0.60 |
| 18                  | SIDNEY CROSBY      | 2006-2007 | PIT  | 0.60 | BRAD RICHARDSON     | 2011-2012 | LOS  | 0.60 |
| 19                  | MATT MOULSON       | 2011-2012 | NYI  | 0.60 | ERIC FEHR           | 2008-2009 | WAS  | 0.60 |
| 20                  | JOE THORNTON       | 2009-2010 | SJS  | 0.60 | TYLER TOFFOLI       | 2013-2014 | LOS  | 0.60 |

and much else.

## Wrap-up

The model is very transparent and easy to estimate.

Check out `gamlr` and the hockey example:

```
data(hockey)
x <- cBind(config,team,player)
y <- goal$homegoal
fit <- gamlr(x, y, free=1:(ncol(config)+ncol(team)),
             standardize=FALSE, family="binomial")
```

We take no stand about what stats lead to wins,  
and don't attempt to model full game action.

But this simple regression tells you a lot about who is contributing.

**Thanks!**