Economic AI

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What do economists do?

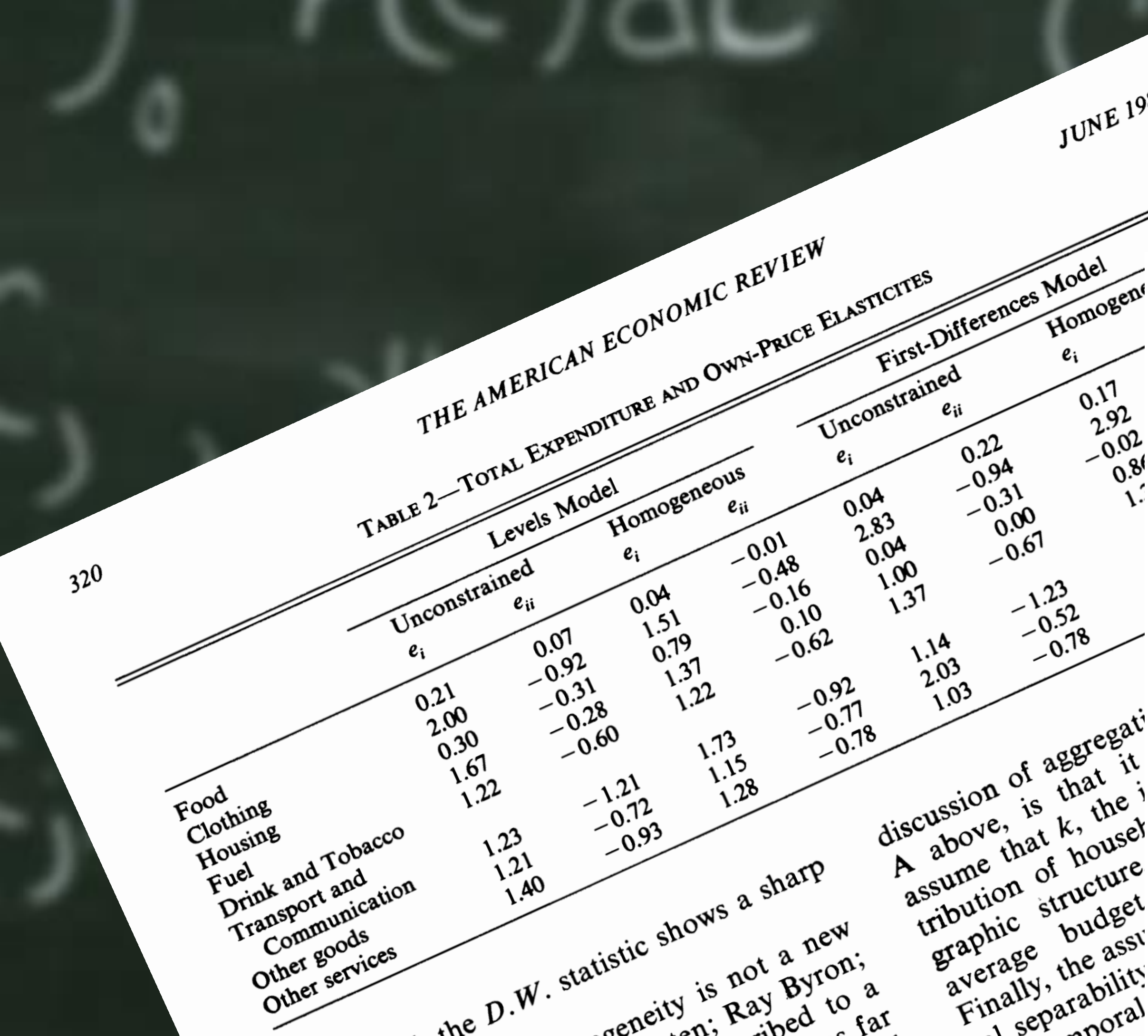

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Table 2—Total Expenditure and Own-Price Elasticites

	Levels Model				First-Differences Model		
	Unconstrained		Homogeneous		Unconstrained		Homogene
	e_i	e_{ii}	e_i	e_{ii}	e_i	e_{ii}	e_i
Food	0.21	0.07	0.04	−0.01	0.04	0.22	0.17
Clothing	2.00	−0.92	1.51	−0.48	2.83	−0.94	2.92
Housing	0.30	−0.31	0.79	−0.16	0.04	−0.31	−0.02
Fuel	1.67	−0.28	1.37	0.10	1.00	0.00	0.8
Drink and Tobacco	1.22	−0.60	1.22	−0.62	1.37	−0.67	1.
Transport and Communication	1.23	−1.21	1.73	−0.92	1.14	−1.23	
Other goods	1.21	−0.72	1.15	−0.77	2.03	−0.52	
Other services	1.40	−0.93	1.28	−0.78	1.03	−0.78	

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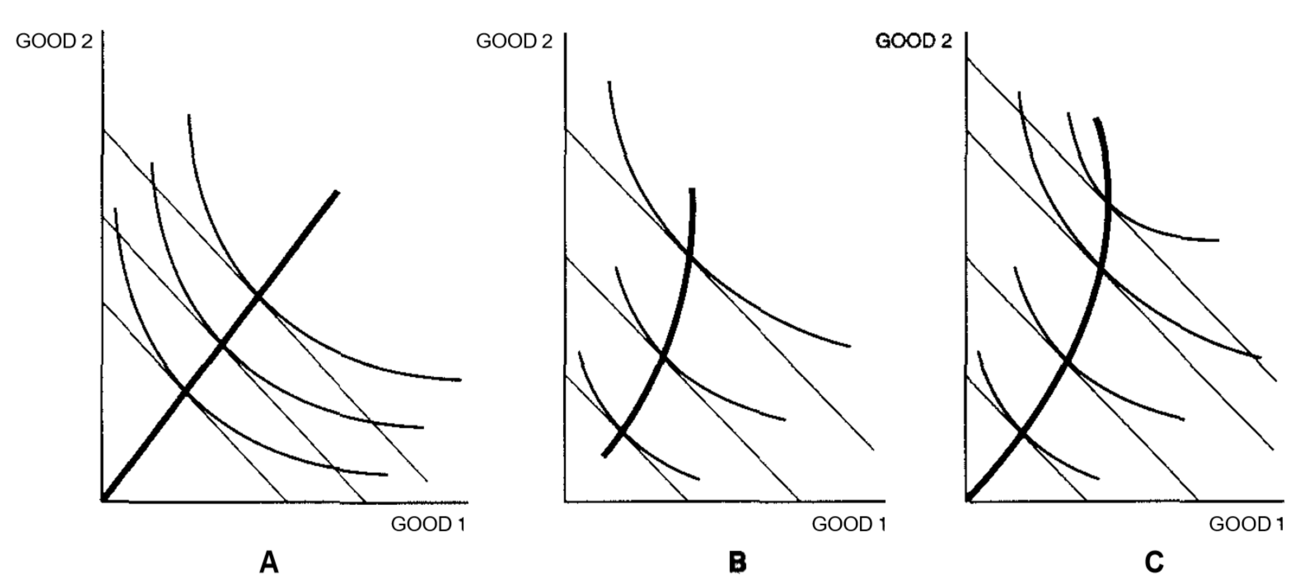
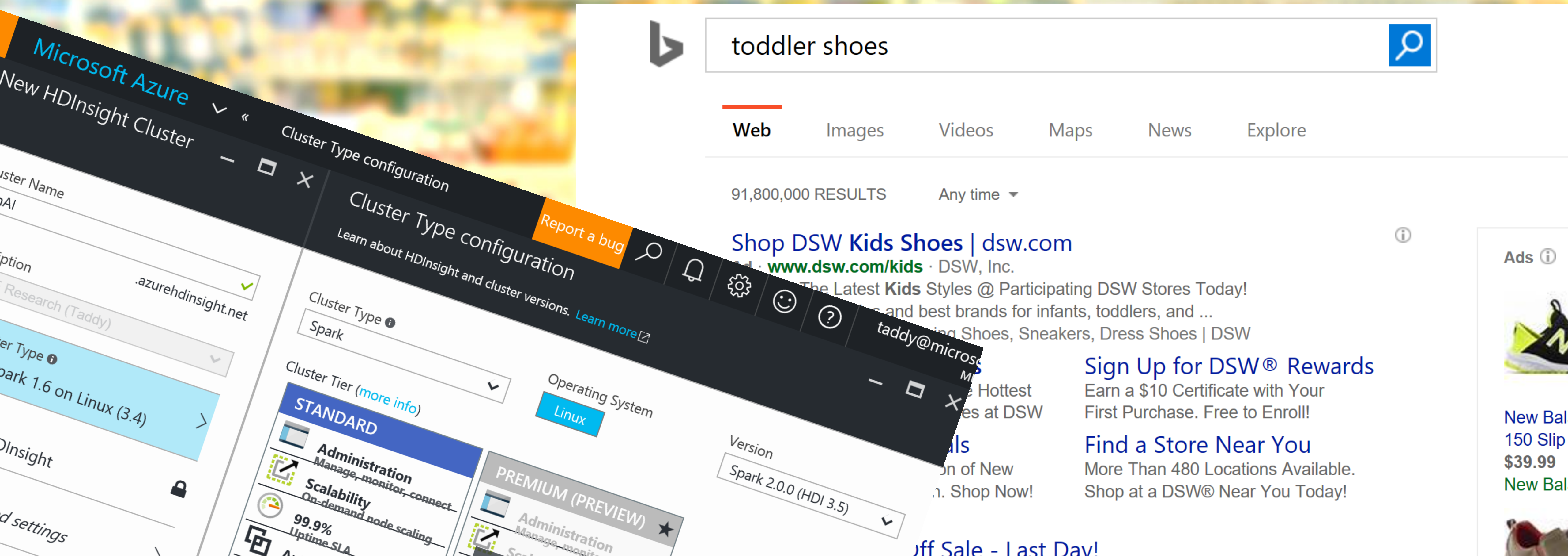


Income expansion paths. Panel **A** depicts unit elastic demands, in panel B good 2 is a luxury good, and in panel *C*, good 1 is an inferior good.

What do they need to do today?



Microsoft Azure

New HDInsight Cluster

Cluster Type configuration

...uster Name

...AI .azurehdinsight.net

...iption

...Research (Taddy)

...er Type

...park 1.6 on Linux (3.4)

...DInsight

...d settings

Cluster Type configuration

Report a bug

taddy@micros...

Learn about HDInsight and cluster versions. Learn more

Cluster Type: Spark

Operating System: Linux

Version: Spark 2.0.0 (HDI 3.5)

Cluster Tier (more info)

STANDARD

Administration
Manage, monitor, connect

Scalability
On-demand node scaling

99.9%
Uptime SLA

PREMIUM (PREVIEW)

Administration
Manage, monit...

toddler shoes

Web Images Videos Maps News Explore

91,800,000 RESULTS Any time

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More Than 480 Locations Available. Shop at a DSW® Near You Today!

...ff Sale - Last Day!

Ads

New Bal... 150 Slip... $39.99 New Bal...

What can we ML do to help?

The dimension and complexity of the problem space has exploded

We can develop ML to navigate this space: stay safe and automate

Example: Demand System

Suppose that you have transactions ‘t’ on products ‘j’.

Write the quantity bought ‘q’ as

$$q_{tj} = \alpha_{tj} + \gamma_j p_{tj} + e_{tj}$$

a function of utility we can (α_{tj}) and can’t (e_{tj}) see, plus price p_{tj}.

You need to have a model like this to target customers or set prices.

But it’s a system!

For example: There many different products

Demand for j depends on substitutes and complements

Or: where does price come from?

$$\log p_{tj} = \varphi_{tj} + \psi_j q^\star_{tj} + \nu_{tj}$$

and the *demand system* is in equilibrium when $q^\star_{tj} = q_{tj}$

Product Co-occurrence

Ignoring price and observable demand shifters…

$$q_{tj} \;=\; \alpha_{tj} \;+\; \gamma_j \log p_{tj} + \varepsilon_{tj}$$

Classic "data mining" seeks *association rules* in *market baskets*:

Find $j \neq k$ pairs so that $\mathbb{E}\left[q_{tj} q_{tk}\right] \gg \mathbb{E}q_{tj}\mathbb{E}q_{tk}$

We can do better

From word to product embedding

Words are like products and sentences are like baskets

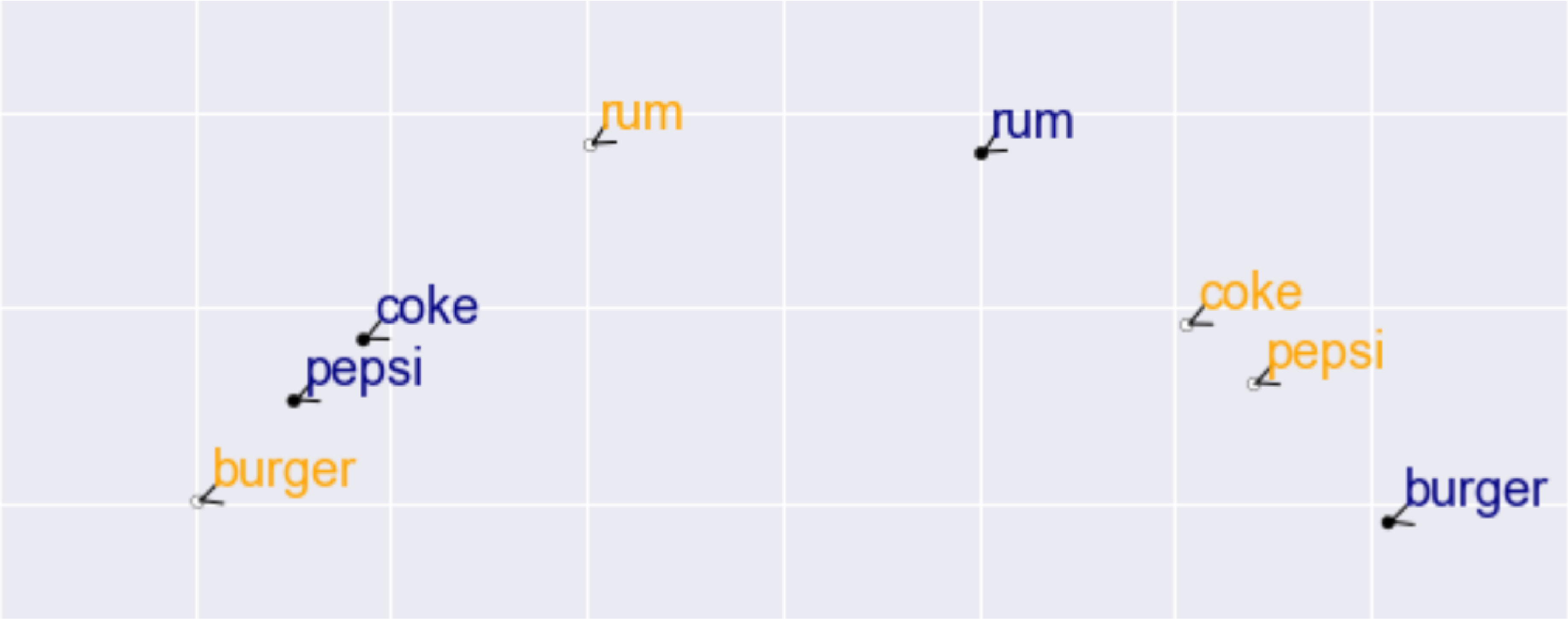
Tools like word2vec and glove map from the J-dimensional discrete space to [two] vector *embeddings* in $\mathbb{R}^S$, and $S \ll J$

$$\max\left\{ \sum_t \sum_{j,k} q_{tj} q_{tk} \boldsymbol{u}_j' \boldsymbol{v}_k \; - \; A(\boldsymbol{U}, \boldsymbol{V}, \boldsymbol{C}) \right\}$$

where $c_{jk} = \sum_t q_{tj} q_{tk}$ and $A(\cdot)$ is a normalizing constant

W2V uses logit model, Glove minimizes $\sum_{j,k} w_{jk} \left(c_{jk} - \boldsymbol{u}_j' \boldsymbol{v}_k\right)^2, w_{jk} = \mathbb{1}[c_{jk} > 0]$

Product Embeddings



substitutes (synonyms) are close in the same vector space
complements (topical words) are close across vector spaces

Moving to a demand system (AID)

It's *almost* ideal:

$$\boldsymbol{s}_t = \boldsymbol{\alpha} + \boldsymbol{\Gamma}\log(\boldsymbol{p}_t) + \boldsymbol{\beta}\log\frac{e_t}{\phi_t} + \boldsymbol{\varepsilon}_t$$

s_{tj} is the budget share for product j in basket t and e_t is the budget

$(e_t = \sum_j \$_{tj}$ and $s_{tj} = \$_{tj}/e_t)$

ϕ_t is the translog price index $\sum_j \log p_{tj}\left[\alpha_j + \sum_k \gamma_{jk} \log p_{tk}\right]$

This is meaningful after aggregation, and its *easier* to estimate…

But not so easy…

In grocery stores, we have 100,000s of products

Each week, at each store, for each product,
we will have different demand and elasticity and income effects.

$$\mathbb{E}s_{tj} = \alpha_{tj} + \sum_k \gamma_{tjk} \log p_{tj} + \beta_{tj} \frac{e_t}{\phi_t}$$

Featurize and Embed

Grab massive $\boldsymbol{x}_{tj}$ that *hierarchically* encode products and transactions

$$\mathbb{E}s_{tj} = \boldsymbol{x}'_{tj}\boldsymbol{\alpha} \ + \boldsymbol{x}'_{tj}\boldsymbol{\delta}\log p_{tj} + \boldsymbol{x}'_{tj}\boldsymbol{\beta}\log\left(\frac{e_t}{\phi_t(\boldsymbol{x}_{tj})}\right) + \boldsymbol{\Gamma}\log\boldsymbol{p}_t$$

and use a symmetric square matrix factorization $\quad \boldsymbol{\Gamma} = \boldsymbol{UV}' + \boldsymbol{VU}'$

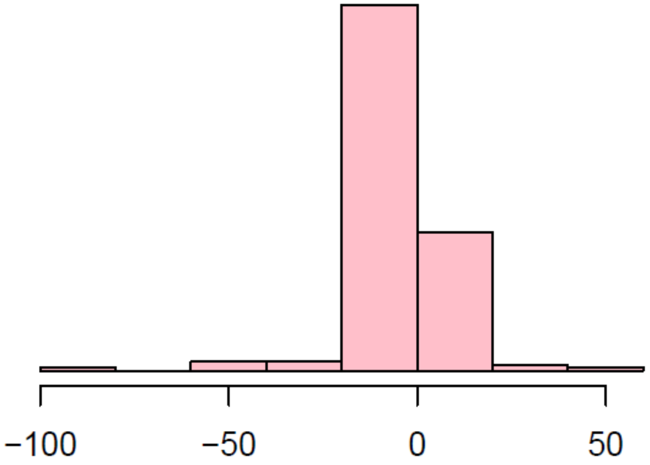
Note that we are using the actual translog and solving a bilinear system

Beer Store own-price (compensated) elasticities

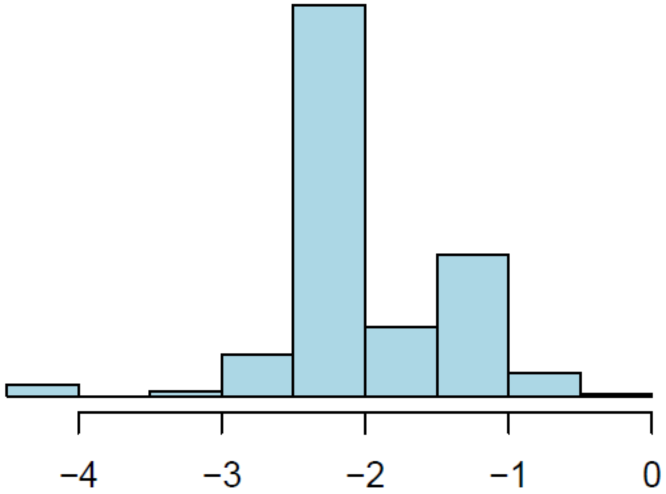
one shared: $x_{tj} = 1$

$$\frac{dq}{dp}\frac{p}{q} = -0.23$$

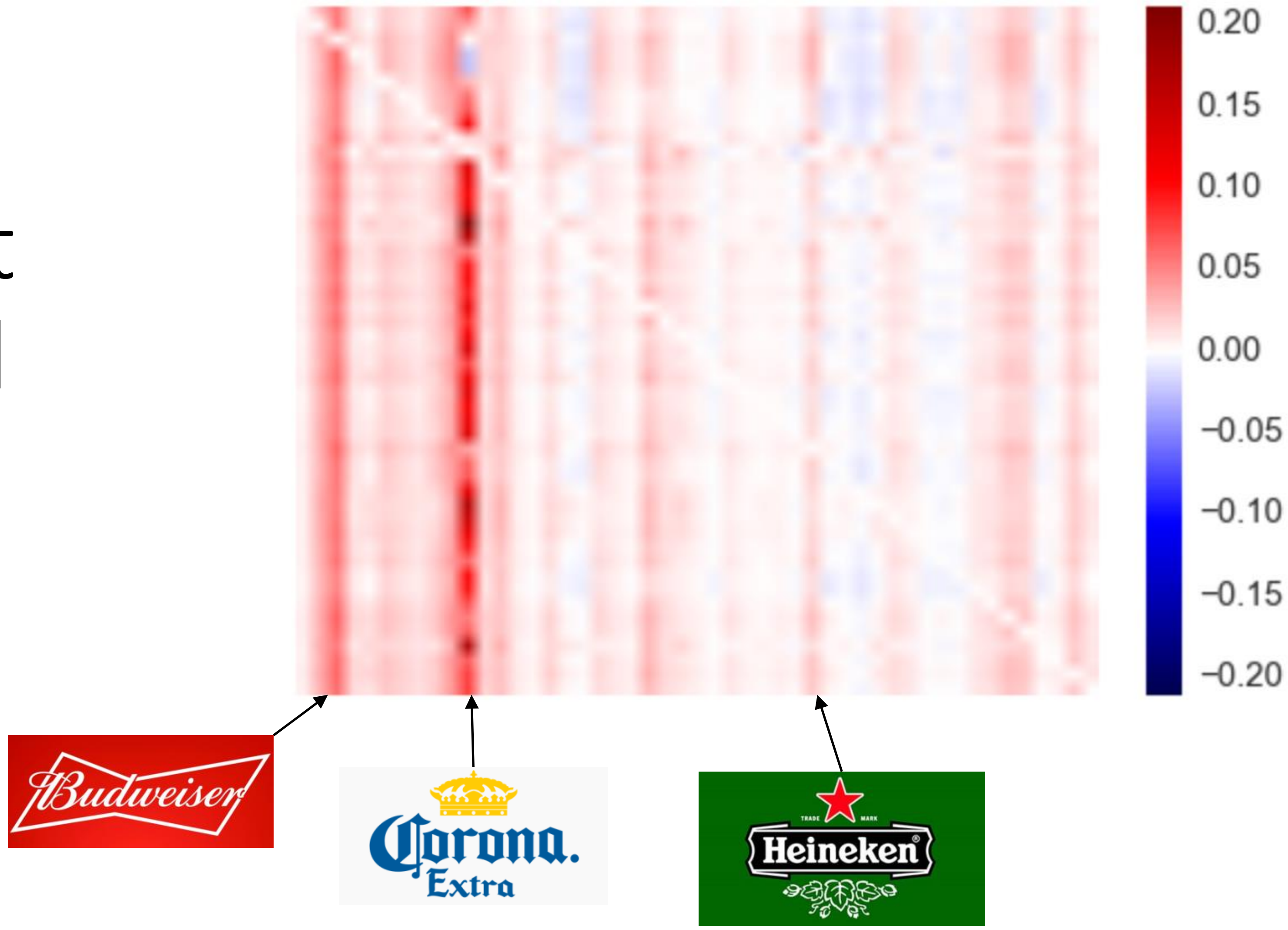
brand-specific: $x_{tjk} = \mathbb{1}_{[k=j]}$



$\boldsymbol{x}_{tj}$= *featurized description*



Beer Store cross-product compensated elasticities



Elasticity matrix (omitting diagonal)

But wait… it’s still a system

$$\boldsymbol{s}_t = \boldsymbol{\alpha} + \boldsymbol{\Gamma}\log(\boldsymbol{p}_t) + \boldsymbol{\beta}\log\frac{e_t}{\phi_t} + \boldsymbol{e}_t$$

Recall: where does price come from?

$$\log p_{tj} = \varphi_{tj} + \psi_j q^{\star}_{tj} + \nu_{tj}$$

and the *demand system* is in equilibrium when $q^{\star}_{tj} = q_{tj}$

This equilibrium introduces `price endogeneity’: $\mathbb{E}[p_{tj}e_{tj}] \neq 0$

Endogenous Errors

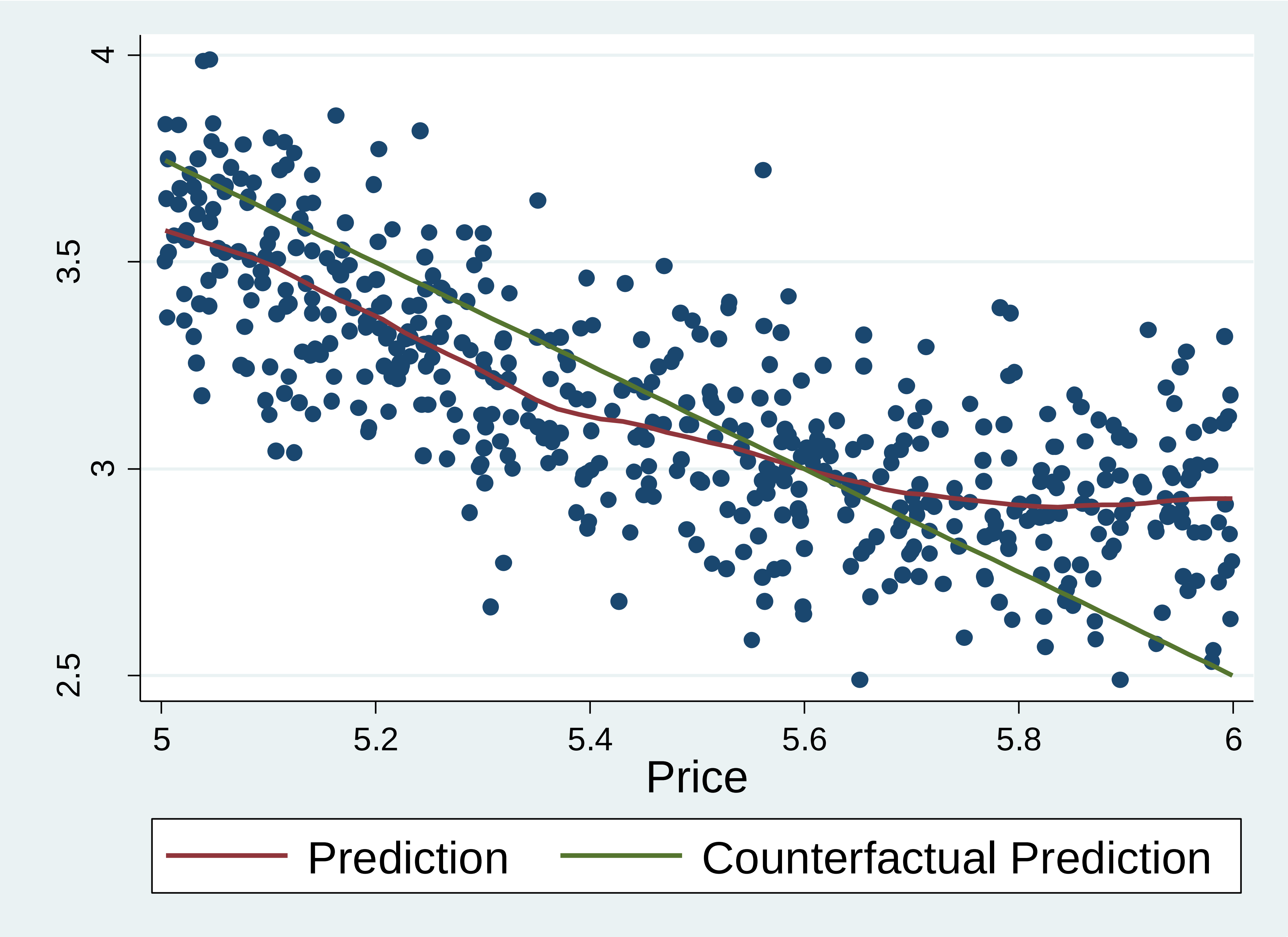
$$y = g(p, \boldsymbol{x}) + e \ \text{ and } \ \mathbb{E}[\, p\, e\,] \neq 0$$

If you estimate this using naïve ML, you'll get

$$E[y|p, \boldsymbol{x}] = E_{e|p}[g(p, \boldsymbol{x}) + e] = g(p, \boldsymbol{x}) + E[e|t, \boldsymbol{x}]$$

This works for prediction. It doesn't work for counterfactual inference:

What happens if I change p independent of e ?


4
3.5
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2.5
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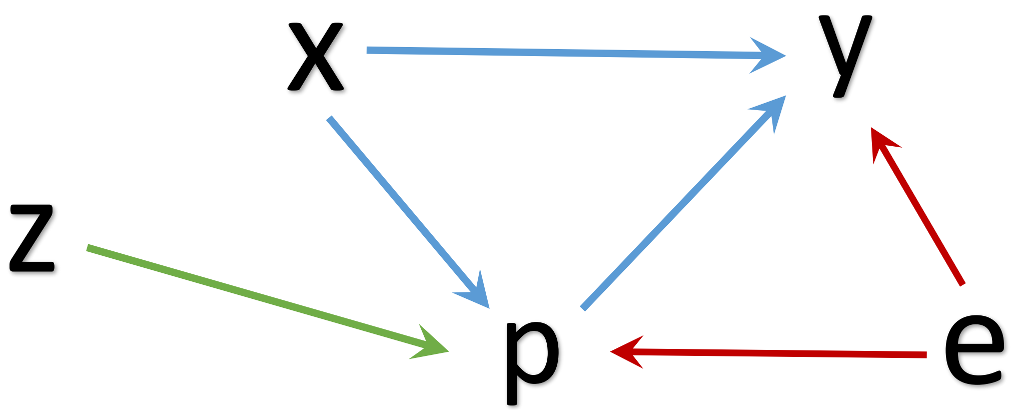
5.6
5.8
6
Price
Prediction
Counterfactual Prediction

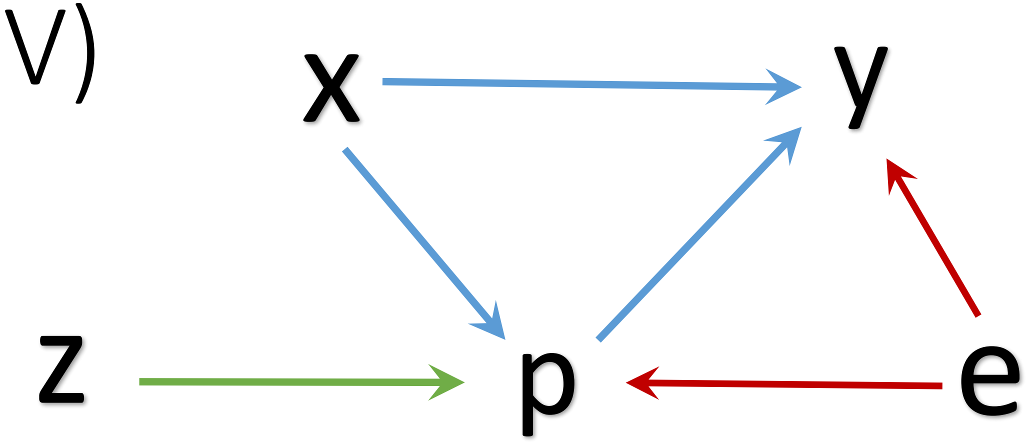
Instrumental Variables (IV)



In IV we have a special $z \perp e$ that influences policy p but not response y.

- Supplier costs that move price independent of demand (e.g., fish, oil)
- Any source of treatment randomization (intent to treat, AB tests, lottery)

Instrumental Variables (IV)



The *exclusion structure* implies

$$E[y|x,z] = E[g(p,x) + e|x,z] = \int g(p,x) d\mathrm{F}(p|x,z)$$

So to solve for *structural* $g(p,x)$ we have a new learning problem

$$\min_{g \in G} \sum \left(y_i - \int g(p,x_i) d\mathrm{F}(p|x_i,z_i) \right)^2$$

$$\min_{g \in G} \sum \left(y_i - \int g(p, x_i) d\mathrm{P}(p | x_i, z_i) \right)^2$$

2SLS:

$p = \beta z + \nu$ and $g(p) = \tau p$ so that $\int g(p) d\mathrm{P}(p|z) = \tau \hat{p} = \tau \hat{\beta} z$

So you first regress p on z then regress y on $\hat{p}$ to recover $\hat{\tau}$.

This requires strict assumptions and homogeneous treatment effects.

$$\min_{g \in G} \sum \left(y_i - \int g(p, x_i) d\mathrm{P}(p | x_i, z_i) \right)^2$$

Or look to nonparametric 2SLS like in Newey and Powell:

$$g(p, x_i) \approx \sum_k \varphi_k(p, x_i) \text{ and } \varphi_k(p, x_i) \approx \sum_j \phi_{kj}(x_i, z_i)$$

But this requires careful crafting and will not scale with $\dim(x)$

$$\min_{g \in G} \sum \left(y_i - \int g(p, x_i) d\mathrm{F}(p | x_i, z_i) \right)^2$$

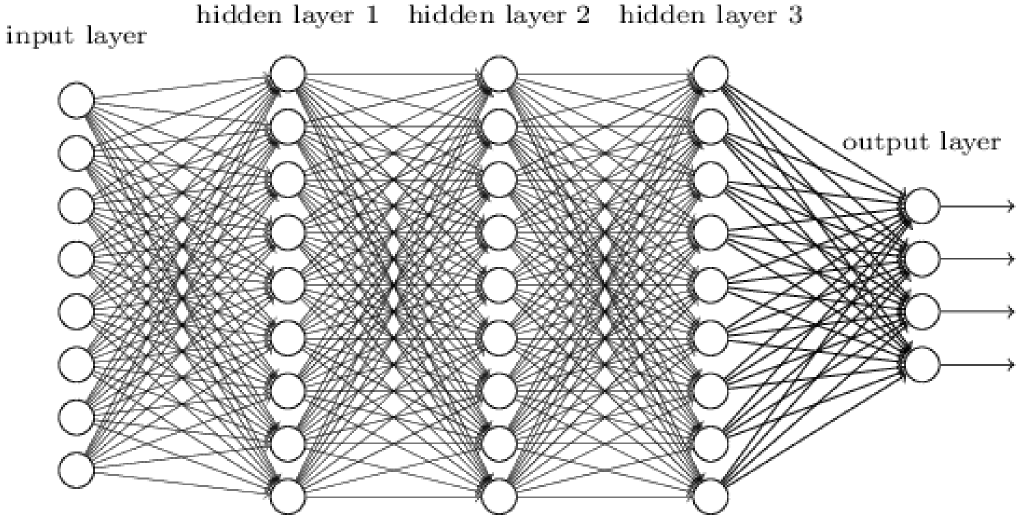
Instead, we propose to target the integral loss function directly

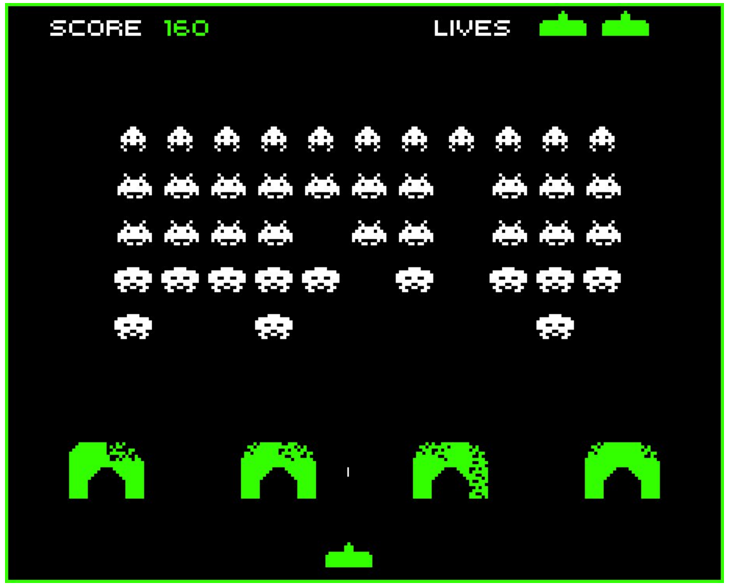
For discrete treatment

- Fit distributions $\hat{\mathrm{F}}(p|x_i, z_i)$ with probability masses $\hat{f}(p_b|x_i, z_i)$
- Train $\hat{g}$ to minimize $\left[\mathrm{y_i} - \sum_b g(\hat{p}_b, x_i) \hat{f}(p_b|x_i, z_i) \right]^2$

And you’ve turned IV into two *generic* machine learning tasks

Learning to love Deep Nets





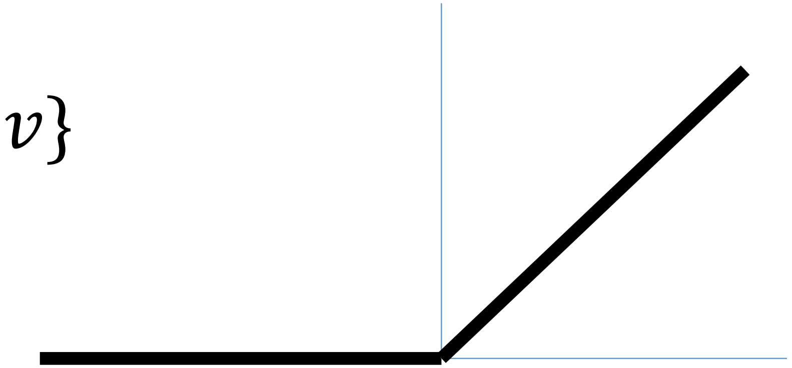
What is a deep net?

$$\hat{y}_i = \sum_k h_k^0(a_{ik}), \qquad a_{ik} = \sum_j w_{kj} z_j\,, \quad z_j = \sum_l h_l^1(b_{il})\,, \ldots$$

And so-on until you get down to a bottom layer $\{f_l(x_i)\}_l$

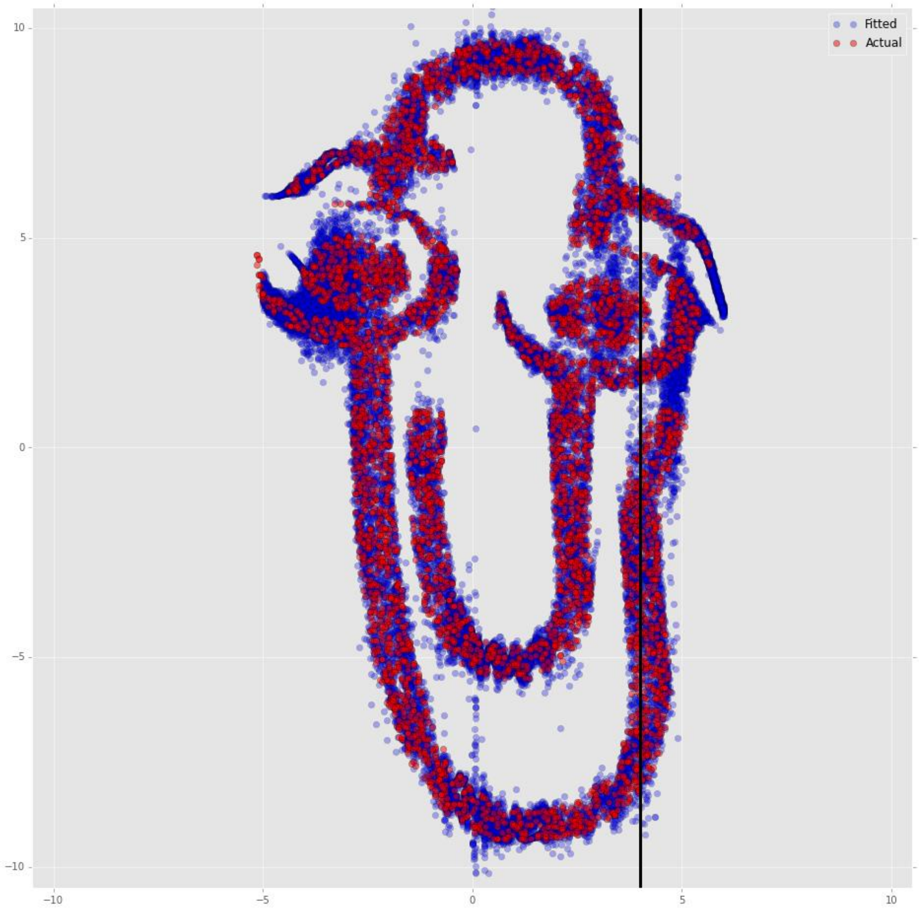
Many different variations here: recursive, convolutional, …

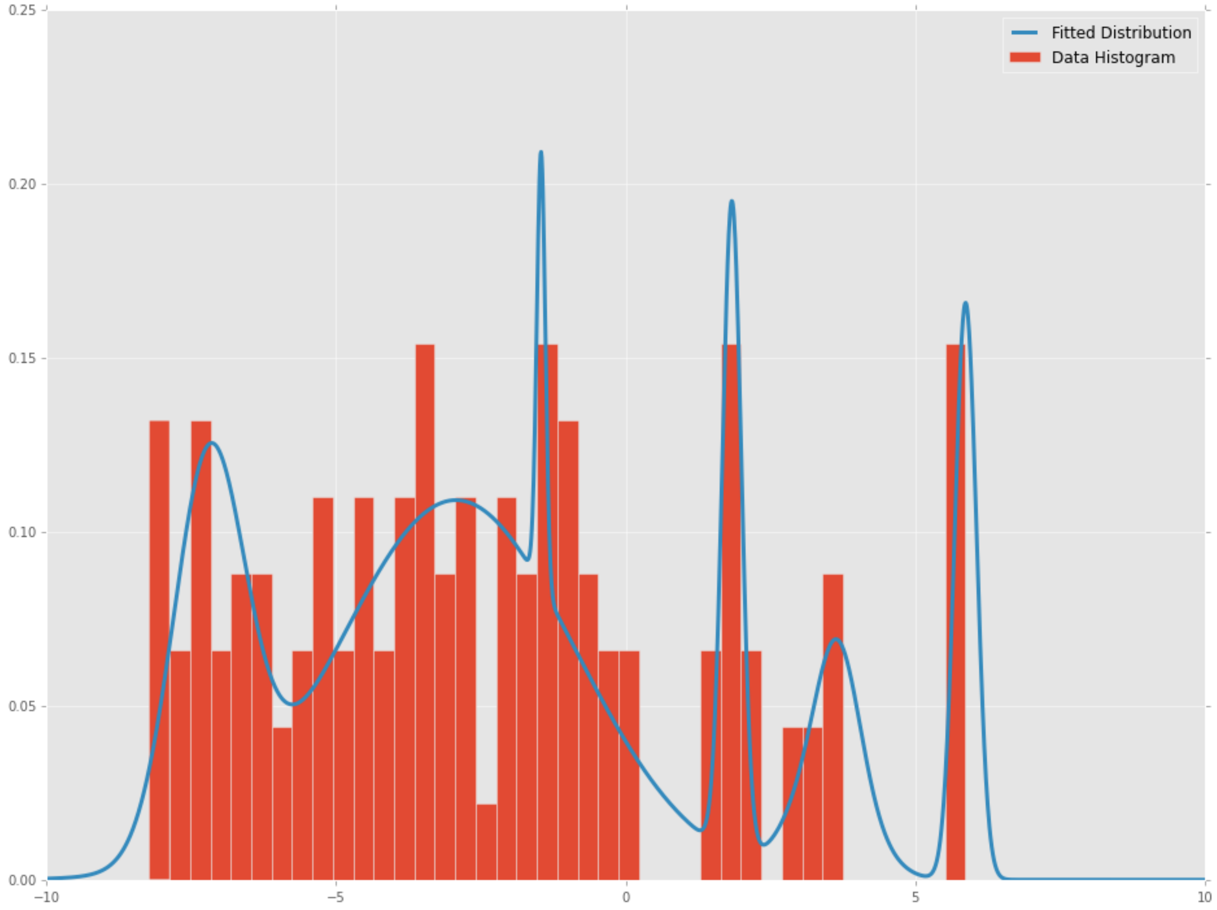
Apart from the bottom, usually $h(v) = \max\{0, v\}$



e.g., first-stage learning for $\mathrm{F}(p|x_i, z_i)$

Bishop 96: Final layer of network parametrizes a mixture of Gaussians





The second stage involves an integral loss function

If p is not discrete or can take many values, we can't just integrate

Brute force just samples from $\widehat{\mathrm{F}}(p|x_i, z_i)$ and you take gradients on

$$\frac{1}{N}\sum_i \left(y_i - \frac{1}{B}\sum_b g(\hat{p}_{ib}, x_i; \theta)\right)^2, \quad \hat{p}_{ib} \sim \widehat{\mathrm{F}}(p|x_i, z_i)$$

But this is inefficient

And more generally, it's inefficient even *without* the integral…

Stochastic Gradient Descent

You have loss $L(\boldsymbol{D},\ \theta)$ where $\boldsymbol{D} = [\boldsymbol{d}_1\ \ldots \boldsymbol{d}_N]$

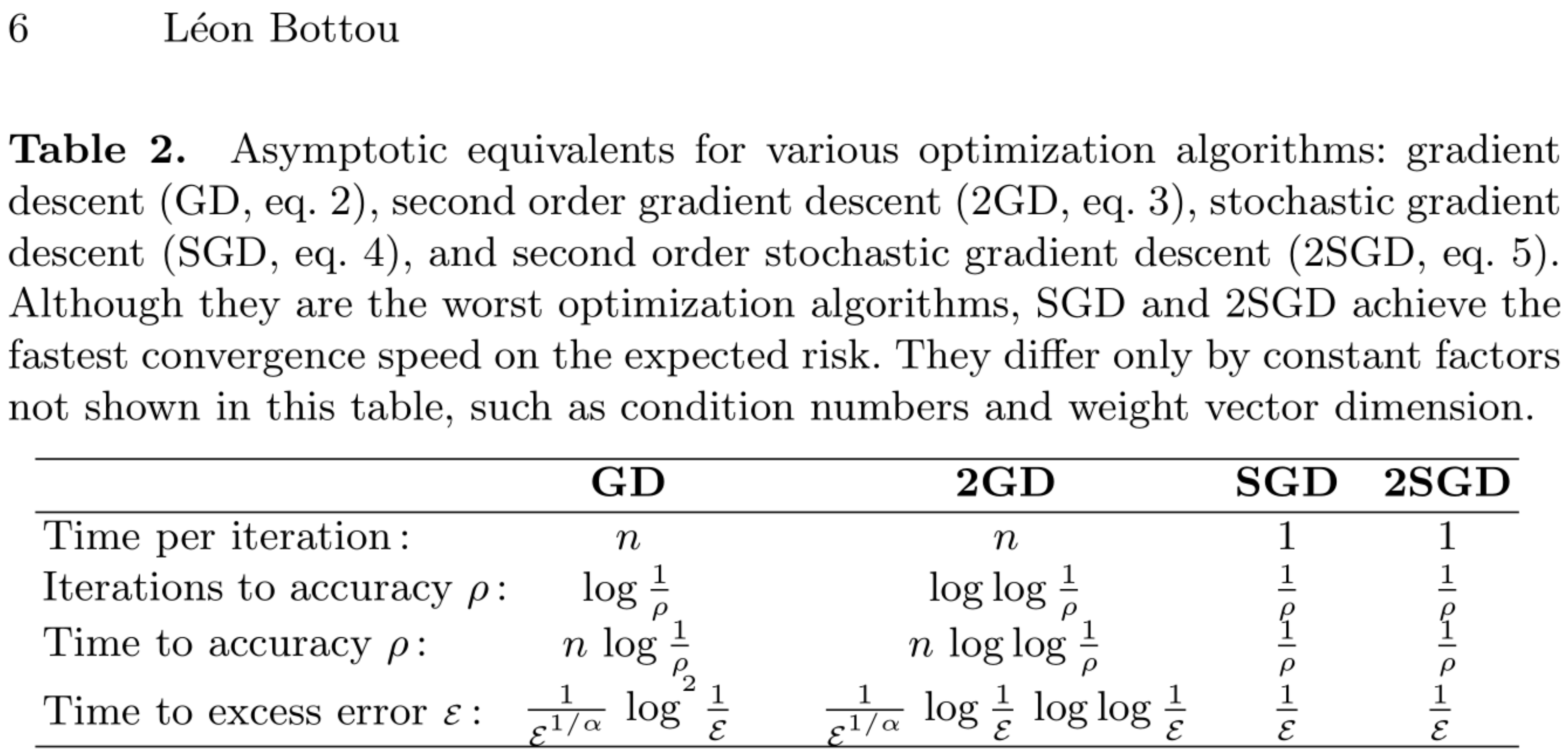
In the usual GD, you iteratively descend

$$\theta_t = \theta_{t-1} - \boldsymbol{C}_t \nabla L(\boldsymbol{D}, \theta_{t-1})$$

In SGD, you instead follow *noisy* but *unbiased* sample gradients

$$\theta_t = \theta_{t-1} - \boldsymbol{C}_t \nabla L(\{\boldsymbol{d}_{t_b}\}_{b=1}^{B}, \theta_{t-1})$$

Why SGD? You get what you need faster



6 Léon Bottou

Table 2. Asymptotic equivalents for various optimization algorithms: gradient descent (GD, eq. 2), second order gradient descent (2GD, eq. 3), stochastic gradient descent (SGD, eq. 4), and second order stochastic gradient descent (2SGD, eq. 5). Although they are the worst optimization algorithms, SGD and 2SGD achieve the fastest convergence speed on the expected risk. They differ only by constant factors not shown in this table, such as condition numbers and weight vector dimension.

	GD	**2GD**	**SGD**	**2SGD**
Time per iteration :	n	n	1	1
Iterations to accuracy ρ :	$\log \frac{1}{\rho}$	$\log \log \frac{1}{\rho}$	$\frac{1}{\rho}$	$\frac{1}{\rho}$
Time to accuracy ρ :	$n \log \frac{1}{\rho}$	$n \log \log \frac{1}{\rho}$	$\frac{1}{\rho}$	$\frac{1}{\rho}$
Time to excess error $\mathcal{E}$:	$\frac{1}{\mathcal{E}^{1/\alpha}} \log^2 \frac{1}{\mathcal{E}}$	$\frac{1}{\mathcal{E}^{1/\alpha}} \log \frac{1}{\mathcal{E}} \log \log \frac{1}{\mathcal{E}}$	$\frac{1}{\mathcal{E}}$	$\frac{1}{\mathcal{E}}$

$$\mathcal{E} \;=\; \mathcal{E}_{\text{app}} + \mathcal{E}_{\text{est}} + \mathcal{E}_{\text{opt}} \;\sim\; \mathcal{E}_{\text{app}} + \left(\frac{\log n}{n}\right)^{\alpha} + \rho$$

Learning rates

$$\theta_t = \theta_{t-1} - \boldsymbol{C}_t \nabla L(\{\boldsymbol{d}_{t_b}\}_{b=1}^{B}, \theta_{t-1}) = \theta_{t-1} - \boldsymbol{C_t} \nabla L_t$$

What is $\boldsymbol{C}_t$? Hopefully not $\mathrm{c_t}\mathbf{I}$ …

Ideally, $\boldsymbol{C}_t \to t^{-1}\boldsymbol{C}$ and $\boldsymbol{C}^{-1} = -\nabla\nabla L(\boldsymbol{D}, \theta^{\star})$, but that's infeasible

ADAGRAD (for convex loss)

$$\boldsymbol{C}_t = \mathrm{diag}\left(\sqrt{\sum_t \nabla L_{tj}^2}\right) \Rightarrow \sum_t\big(err(\theta_t) - err(\theta^{\star})\big) = O(\|\theta^{\star}\|\mathrm{tr}(\mathbf{C}_t))$$

We use heuristic innovations on this (ADAM, momentum, etc)

SGD for integral loss functions

Our one-observation stochastic gradient is

$$\nabla L(d_i, \theta) \;= -2\left(y_i \;-\; \int g_\theta(p, x_i) d\hat{F}(p|x_i, z_i)\right)\; \int g_\theta{}'(p, x_i) d\hat{F}(p|x_i, z_i)$$

Do SGD by pairing each observation with *two independent* treatment draws

$$\nabla\hat{L}(d_i, \theta) = -2(y_i - g_\theta(\hat{p}_{i1}, x_i))\, g_\theta'(\hat{p}_{i2}, x_i), \qquad \hat{p}_{ib} \sim \hat{\mathrm{F}}(p|x_i, z_i)$$

So long as the draws are independent, $\mathbb{E}\nabla\hat{L}(d_i, \theta)$ = $\mathbb{E}\nabla L(d_i, \theta)$ = $L(\boldsymbol{D}, \theta)$

Lower variance grad ⇒ faster/better convergence

e.g., in most architectures you want to fit F and g in two-stages

And with MC integration, consider:

$$\nabla\hat{L}\left(\{\boldsymbol{d}_b\}_{b=1}^{B},\theta\right) = \sum_{b=1}^{B} \left(y_i - g_\theta(\hat{p}_{b1},x_i)\right) g'_\theta(\hat{p}_{b2},x_i)$$

$$\nabla\tilde{L}^B(\boldsymbol{d},\theta) = \left(y_i - \sum_{b=1}^{B} g_\theta(\hat{p}_{b1},x_i)\right) \sum_{b=1}^{B} g'_\theta(\hat{p}_{b2},x_i)$$

Both involve the same number of operations, but

$\mathrm{var}\nabla\hat{L}(\{\boldsymbol{d}_b\}_{b=1}^{B},\theta) = \frac{\mathrm{var}\nabla\hat{L}(\boldsymbol{d},\theta)}{B}$ while $\mathrm{var}\nabla\tilde{L}^B(\boldsymbol{d},\theta) \approx \frac{E\left[\mathrm{var}\nabla\tilde{L}^1(\boldsymbol{d},\theta)\right]}{B} + \mathrm{var}(\nabla L(\boldsymbol{d},\boldsymbol{\theta}))$

Aside: we can use SGD more in econ …

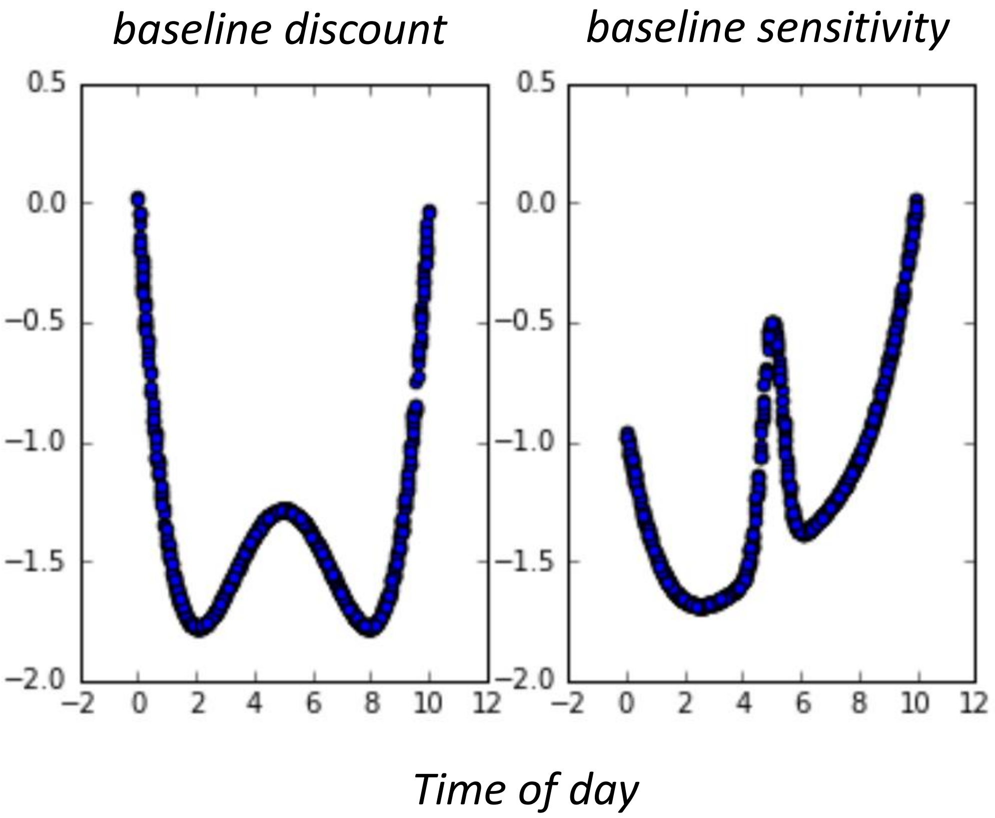
There are a ton of setups where we use simulation to solve

$$\min_{\beta} \sum \left(y_i - \int g(x_i; \theta) d\mathrm{P}(\theta | \beta) \right)^2$$

Random coefficient models, or wider class in Pakes and Pollard

Sampling SGD is a perfect fit here

heterogeneous price effects

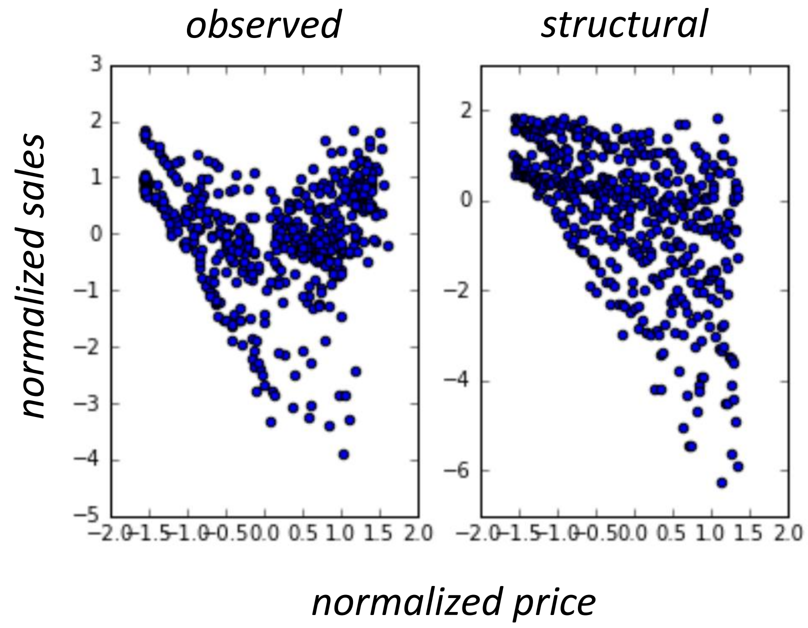


baseline discount

baseline sensitivity

Time of day

- ‘time’ dependent prices and elasticity
- 7 different customer types multiply discounts, sensitivities, and demand



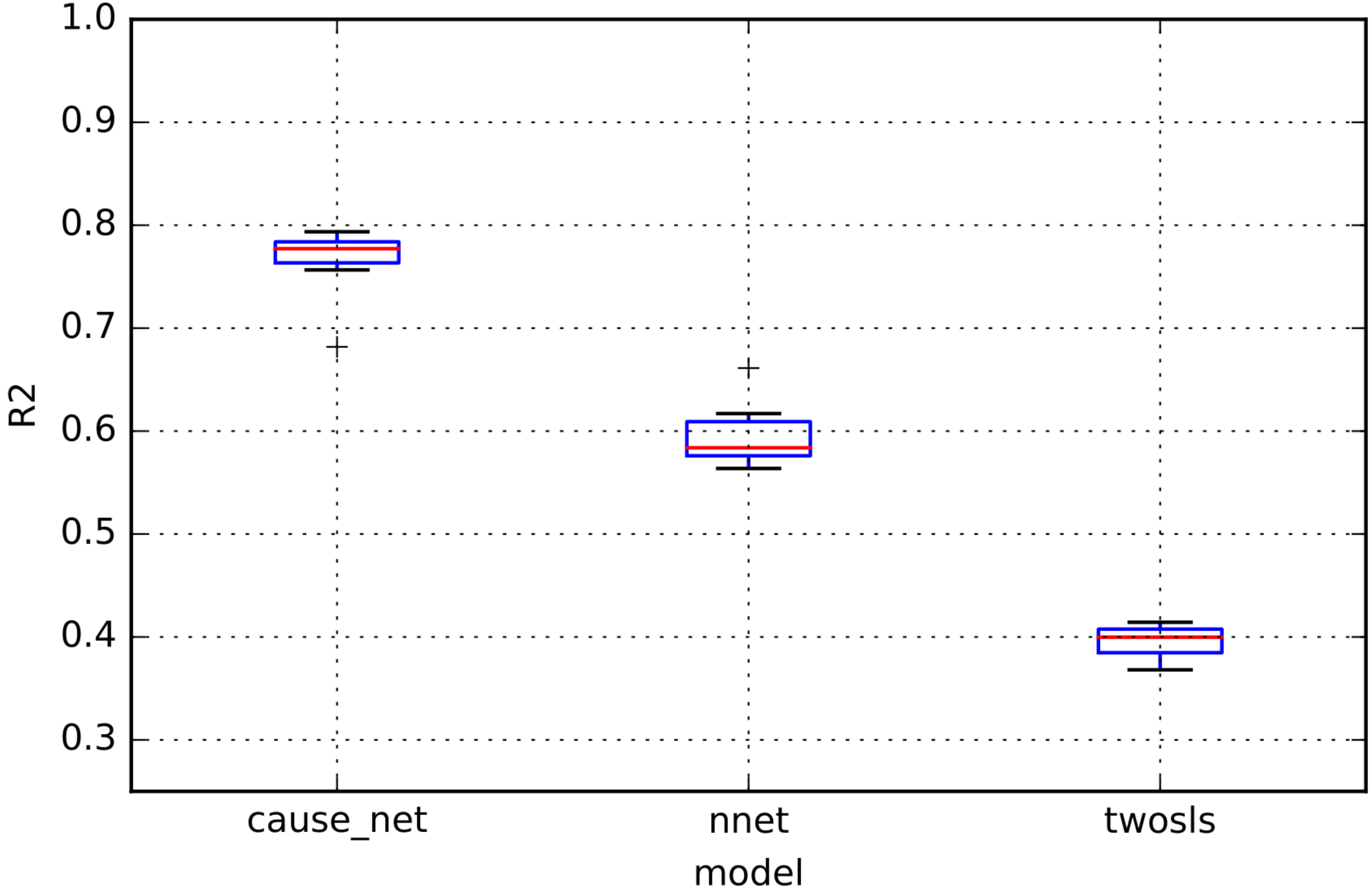
observed

structural

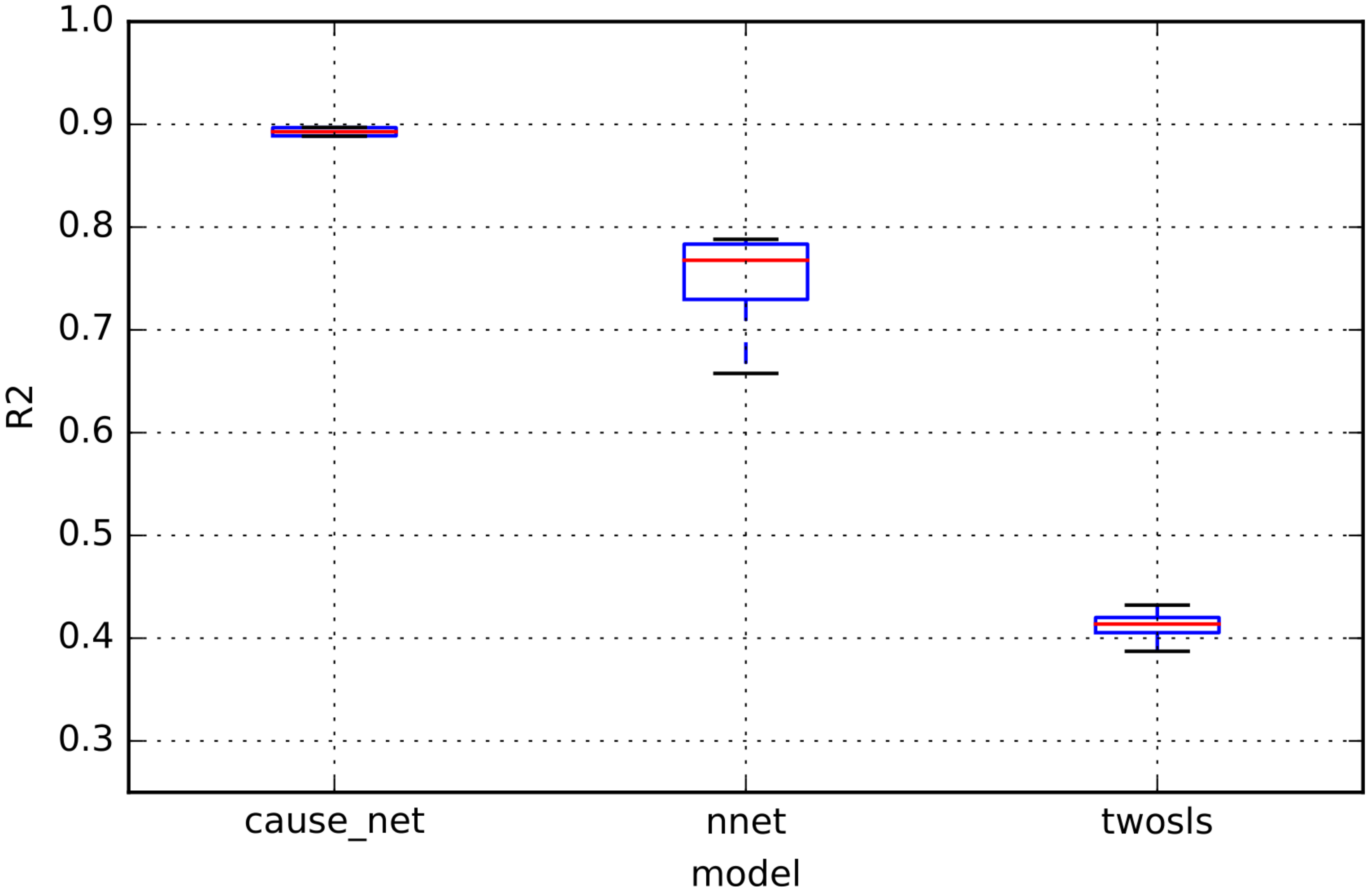
normalized sales

normalized price

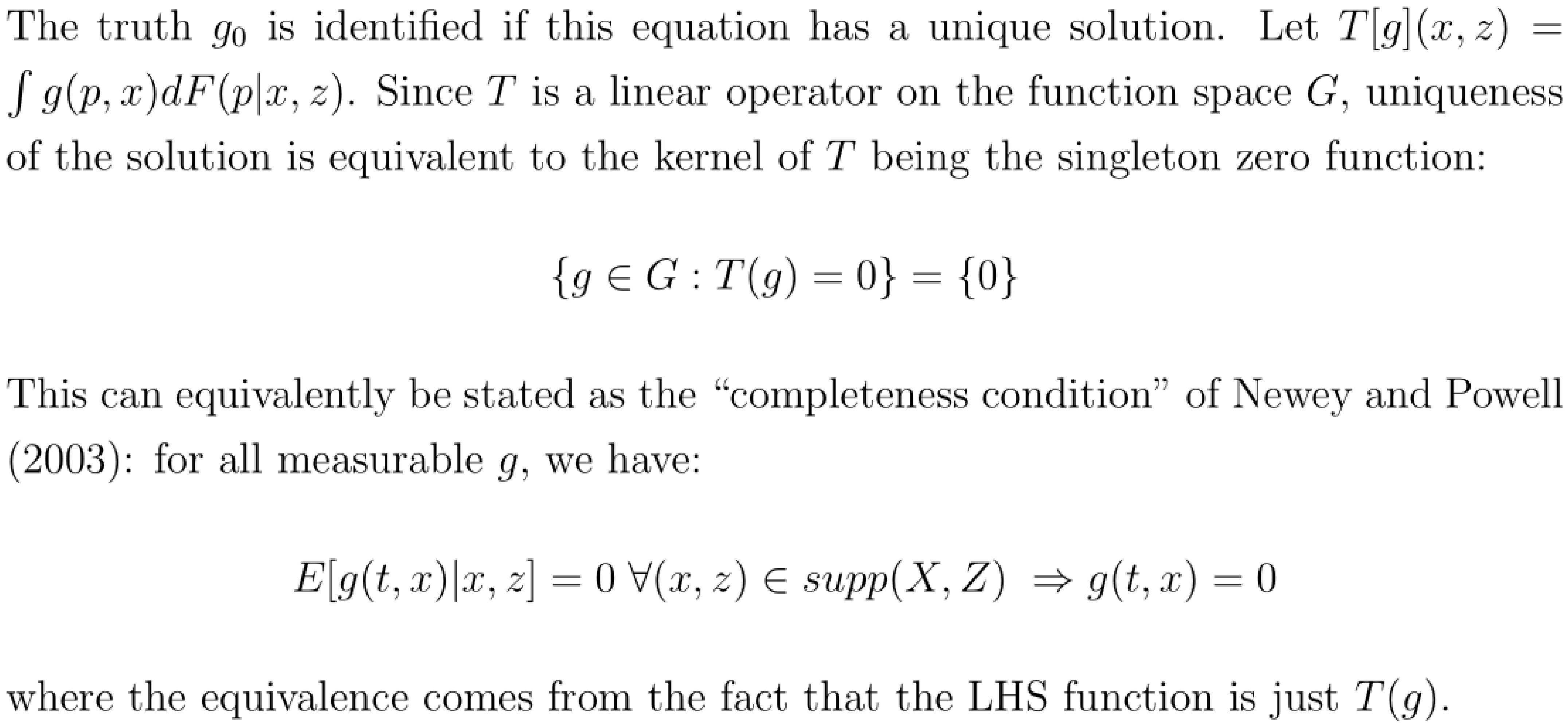
N=1000



N=5000



Identification



The truth g_0 is identified if this equation has a unique solution. Let $T[g](x,z) = \int g(p,x)dF(p|x,z)$. Since T is a linear operator on the function space G, uniqueness of the solution is equivalent to the kernel of T being the singleton zero function:

$$\{g \in G : T(g) = 0\} = \{0\}$$

This can equivalently be stated as the "completeness condition" of Newey and Powell (2003): for all measurable g, we have:

$$E[g(t,x)|x,z] = 0 \ \forall (x,z) \in supp(X,Z) \ \Rightarrow g(t,x) = 0$$

where the equivalence comes from the fact that the LHS function is just $T(g)$.

Consistency

Our is just a minimum distance estimator conditional upon draws from the 1st stage

Consistency has $\|\bar{g} - \bar{g}^{\star}\| \rightarrow 0$ after averaging over the instrument distribution

The relevant theory is in Newey and Powell (03) and Chen and Pouzo (12)

Promise to grow both nets at rates $k(n) \longrightarrow \infty$, $k/n \longrightarrow 0$, then approximation theorems for neural nets (e.g. Hornik 91) imply density around the truth

For treatment (and 1st stage) continuous, also need # mixture components to grow

Ads Application

Taken from Goldman and Rao (2014)

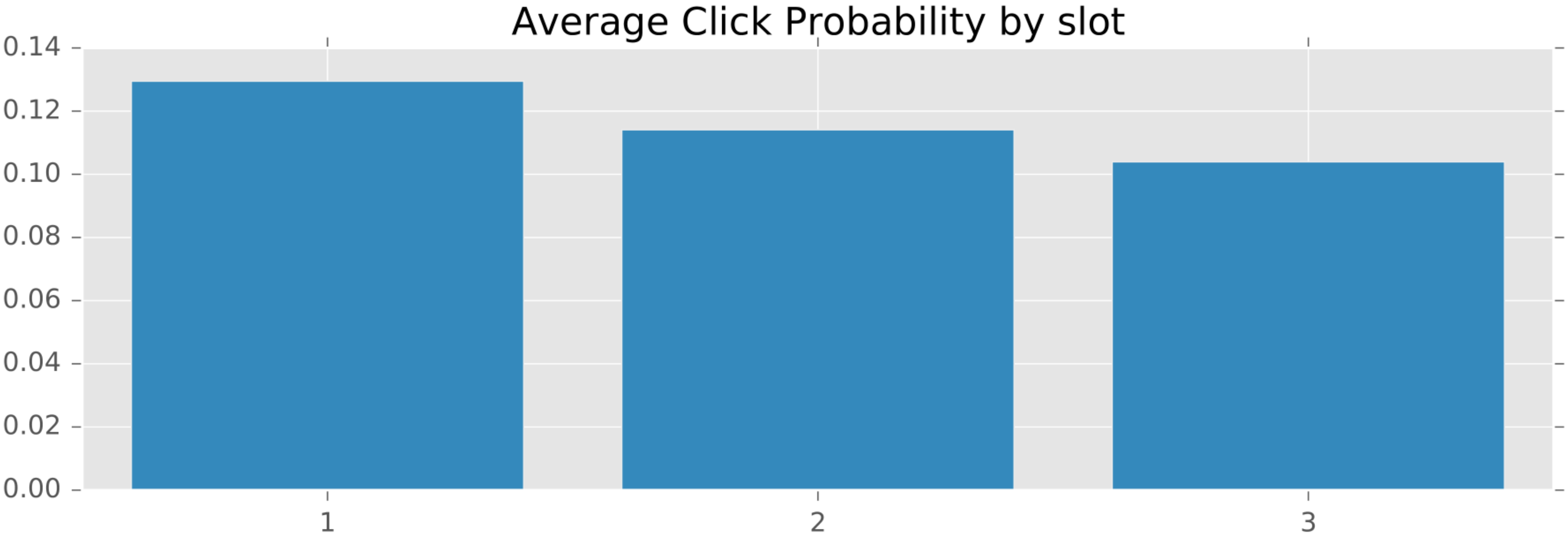
We have 74 mil click-rates over 4 hour increments for 10k search terms

Treatment: ad position 1-3

Instrument: background AB testing (bench of ~ 100 tests)

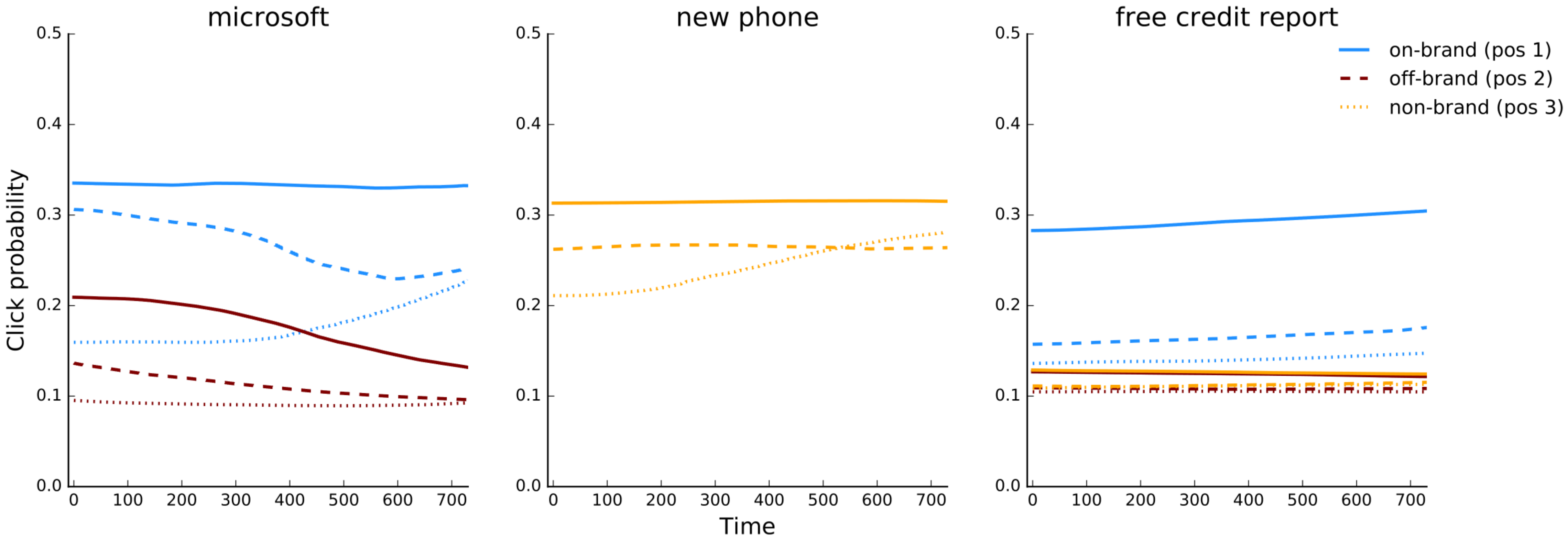
Covariates: advertiser id and ad properties, search text, time period

Average Treatment Effects



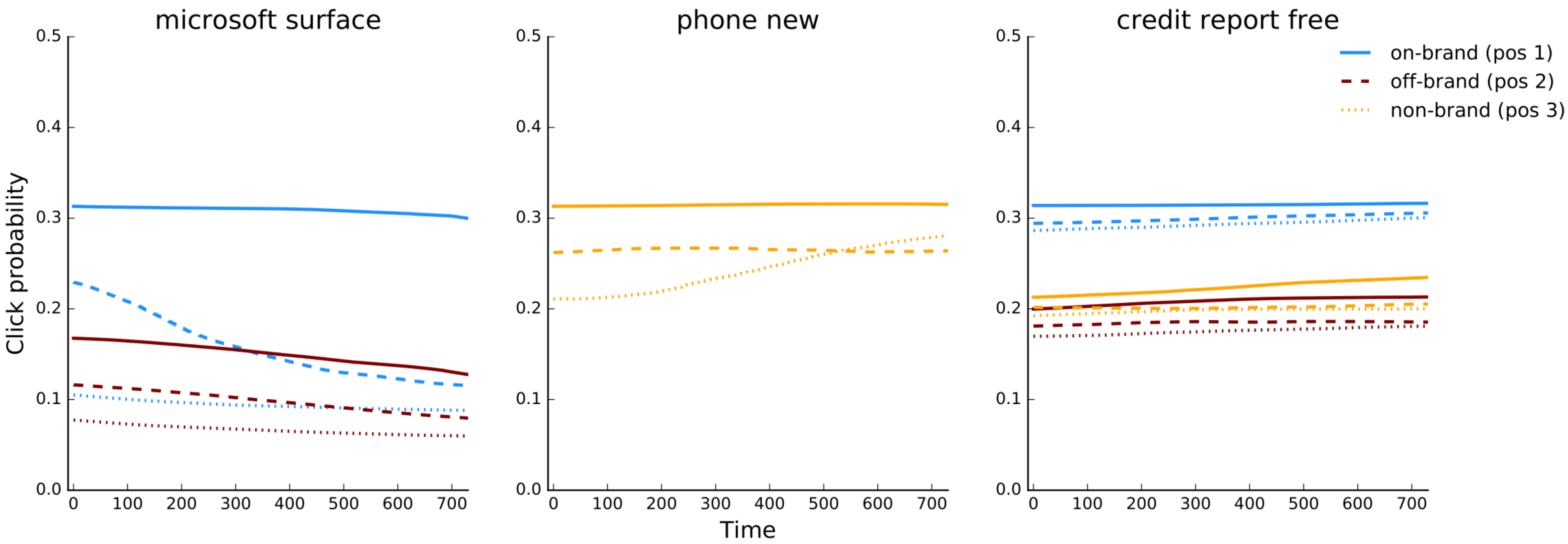
These compare to observed click probabilities of 0.33, 0.1, and 0.05.

Heterogeneous Treatment Effects



The *gaps* between ad-position lines can be interpreted *causally.*

Heterogeneous Treatment Effects



The *gaps* between ad-position lines can be interpreted *causally.*

Inference? Good question…

Recall the deep net

$$\hat{y}_i = \sum_k h_k^0(a_{ik}), \qquad a_{ik} = \sum_j w_{kj} z_j \,, \quad z_j = \sum_l h_l^1(b_{il}) \,, \ldots$$

When training with SGD, we actually use dropout for regularization

At each update, calculate gradients against $\tilde{\theta}$ where

$$\widetilde{w}_{kj} = w_{kj}\delta_{kj}, \qquad \delta_{kj} \sim \mathrm{Bern}(\pi)$$

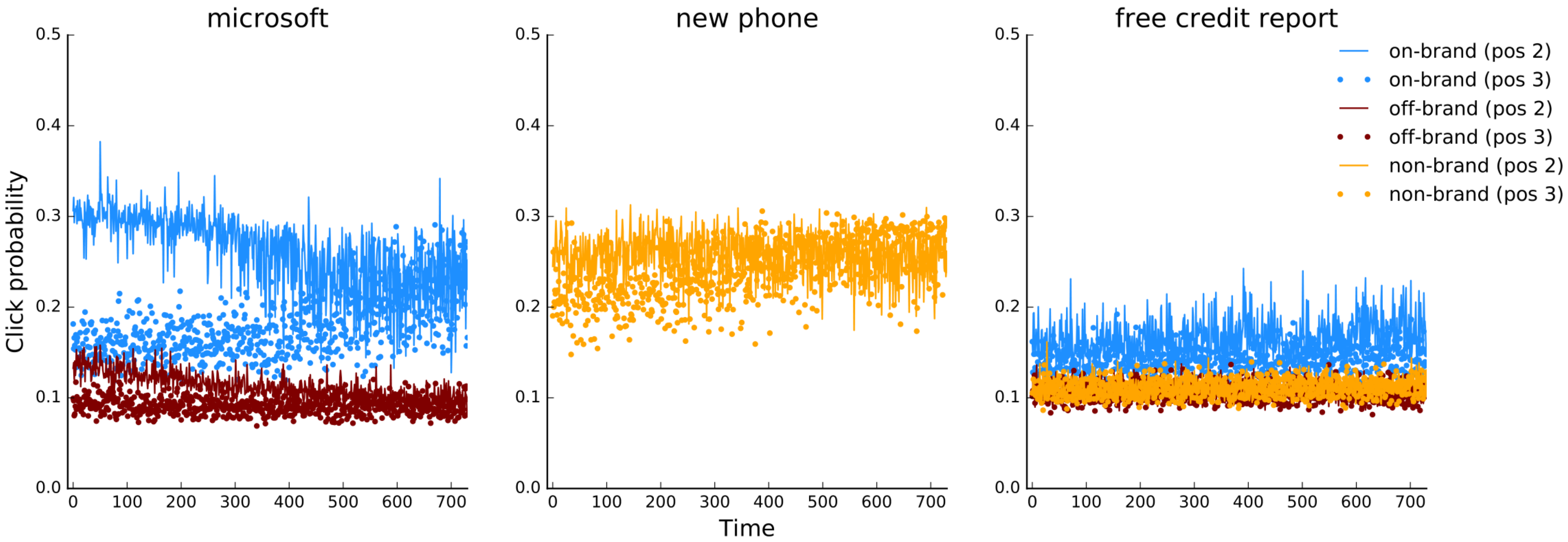
Inference? Good question…

Variational Bayes minimizes

$$KL(q \mid p) = \mathbb{E}_{\mathrm{q}}[\log q(y|x)] \; - \mathbb{E}[\log p(y|x, \boldsymbol{D})]$$

Gal and Ghahramani show that *dropout* SGD is optimizing variational distribution q, with uncertainty parametrized by the Bernoulli weights

Posterior Treatment Effect Samples



Each point/dash is an independent draw from the `posterior’

Economics and Artificial Intelligence

We have a track record pointing ML at questions of science + causation.
We’re going to replicate this success *at scale on unstructured data*

We use economic theory to build systems of tasks that can be addressed with deep nets and other state-of-the-art ML.

This is the construction of systems for *Economic* Artificial Intelligence.