# Deep Learning for Econometrics

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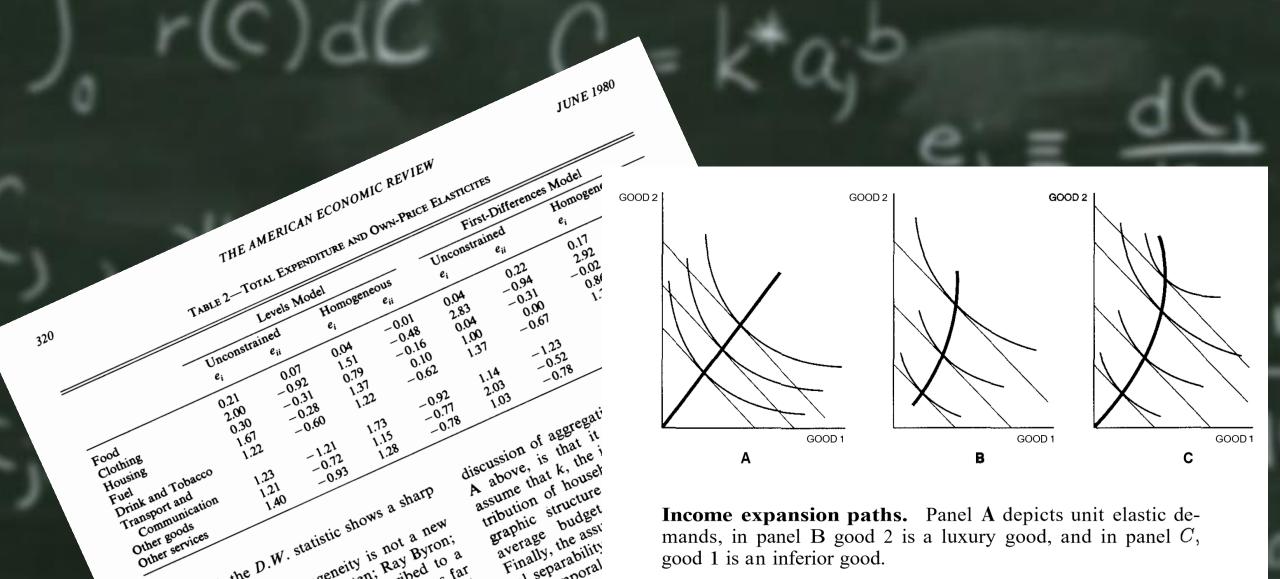
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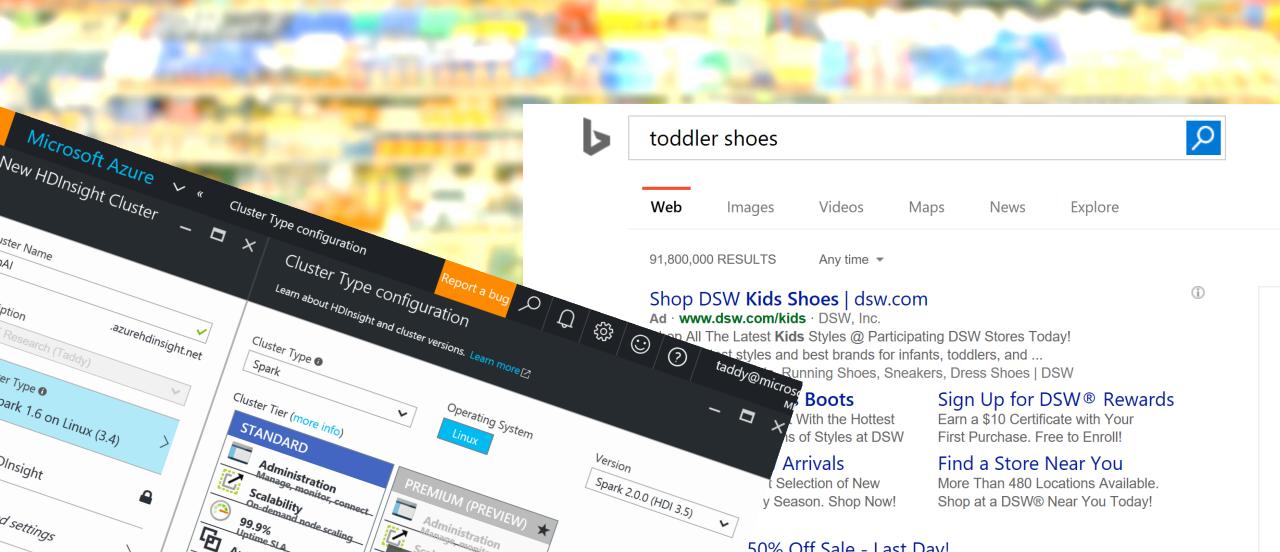
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# What do economists do?



good 1 is an inferior good.

# What do they need to do today?



#### What can we (AI) do to help?

The dimension of the economist's problem space has exploded

We can develop ML to navigate this space: stay safe and automate

We can also build new econometrics via deep structure

#### Example: Demand System

Suppose that you have transactions 't' on products 'j'. Write the quantity bought 'q' as

$$q_{tj} = \alpha_j(\boldsymbol{d}_t) + \gamma_j \log p_{tj} + e_{tj}$$

a function of utility we can  $(\alpha_j(\mathbf{d}_t))$  and can't  $(\mathbf{e}_{tj})$  see, plus price  $p_{tj}$ .

You need to have a model like this to target customers or set prices.

### But it's a system!

For example: There many different products

Demand for *j* depends on substitutes and complements

Or: where does price come from?

$$\log p_{tj} = \varphi_t(\mathbf{c}_j) + \psi_j q_{tj}^* + \nu_{tj}$$

and the *demand system* is in equilibrium when  $q_{tj}^{\star} = q_{tj}$ 

This equilibrium introduces `price endogeneity':  $\mathbb{E}[p_{tj}e_{tj}] \neq 0$ 

#### Sometimes it's just regression

If we treat  $\varepsilon_{tj}$  as independent this is a prediction problem e.g., model store transactions with covariates  $x_{tj}$  as

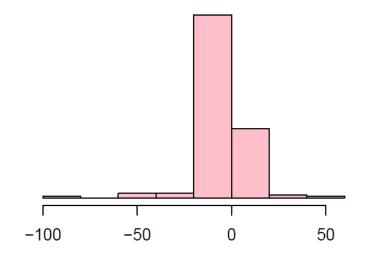
$$\mathbb{E} \log q_{tj} = \mathbf{x}'_{tj} \mathbf{\beta} + \log p_{tj} \mathbf{x}'_{tj} \mathbf{\gamma}$$

#### **Elasticities:**

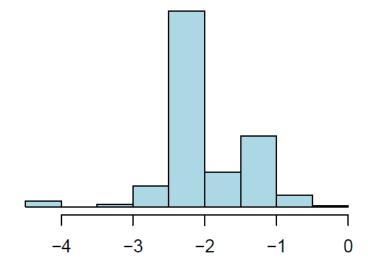
one shared:  $x_{ti} = 1$ 

$$\frac{dq}{dp}\frac{p}{q} = -0.23$$

brand-specific:  $x_{tjk} = \mathbb{1}_{[k=j]}$ 



 $x_{ti}$  = featurized description



#### Moving inside a demand system (AIDS)

It's *almost* ideal:

$$s_t = \alpha + \Gamma \log(p_t) + \beta \log \frac{e_t}{\phi_t} + \varepsilon_t$$

 $s_{tj}$  is the budget share for product j in basket t and  $e_t$  is the budget

$$(e_t = \sum_j \$_{tj} \text{ and } s_{tj} = \$_{tj}/e_t)$$

 $\phi_t$  is the translog price index  $\sum_j \log p_{tj} \left[ \alpha_j + \sum_k \gamma_{jk}^* \log p_{tk} \right]$  (which we will replace with a plug-in for estimation)

This is meaningful after aggregation, and we can actually estimate it

### Factorizing $\Gamma$

The price terms are key to finding complements and substitutes

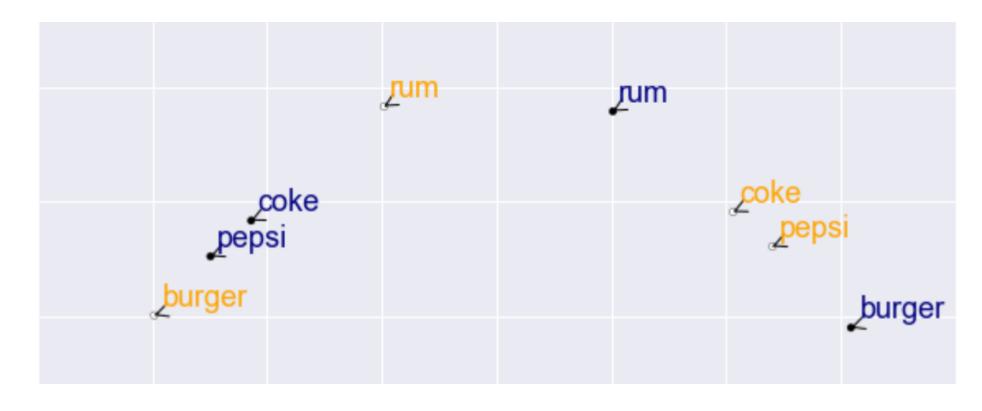
$$\mathbb{E}s_{tj} = \alpha_j + \sum_{k} \gamma_{jk} \log p_{tj} + \beta_j \frac{e_t}{\phi_t}$$

 $\Gamma$  is  $J \times J$ , so we need to reduce dimension if J is going to go big One option: square matrix factorizations from word/prod embedding

 $\Gamma = UV' + VU' + D$  where  $u_j, v_j$  are S-vectors and D is J-diagonal

(AIDS implies restrictions: 
$$\gamma_{jk} = \gamma_{kj}$$
,  $\sum_{j} \gamma_{jk} = \sum_{j} \gamma_{kj} = \sum_{j} \beta_{j} = 0$ )

#### Product Embeddings



substitutes (synonyms) are close in the same vector space complements (topical words) are close across vector spaces

#### Beer

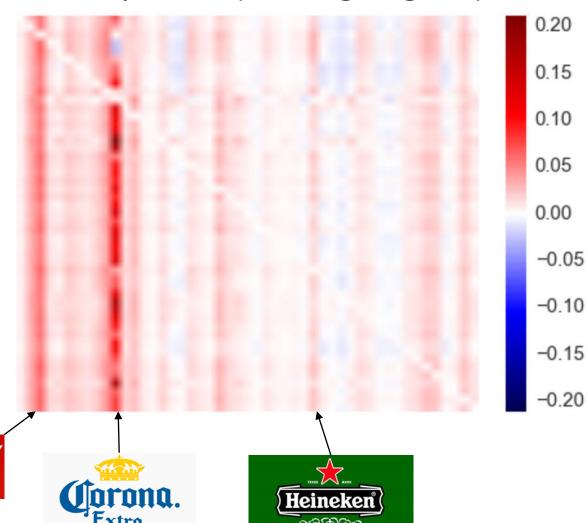
We fit on store-week totals.

Translate the  $\gamma_{jk}$  values into [compensated] elasticities as

$$\frac{\gamma_{jk}}{\overline{S_j}} - \overline{S_k} - \mathbb{1}_{[k=j]}$$

ludweiser

#### Elasticity matrix (omitting diagonal)



#### But wait... it's still a system

$$\mathbf{s}_t = \boldsymbol{\alpha} + \Gamma \log(\mathbf{p}_t) + \boldsymbol{\beta} \log \frac{e_t}{\phi_t} + \mathbf{e}_t$$

Recall: where does price come from?

$$\log p_{tj} = \varphi_t(\mathbf{c}_j) + \psi_j q_{tj}^* + \nu_{tj}$$

and the *demand system* is in equilibrium when  $q_{tj}^{\star}=q_{tj}$ 

This equilibrium introduces `price endogeneity':  $\mathbb{E}[p_{tj}e_{tj}] \neq 0$ 

#### Endogenous Errors

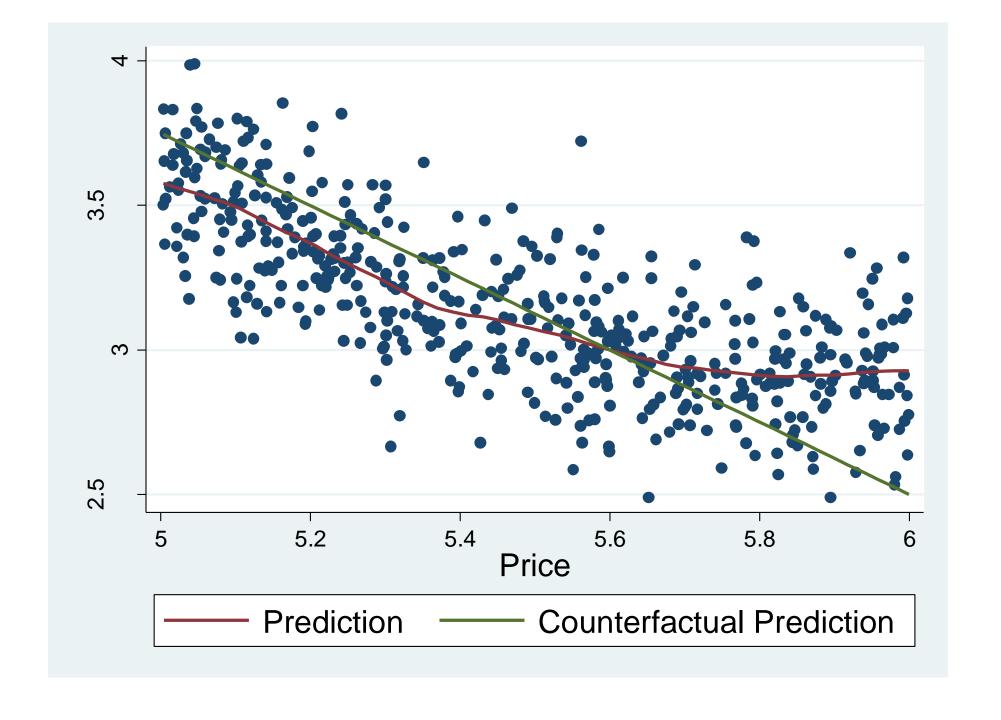
$$y = g(p, x) + e$$
 and  $\mathbb{E}[pe] \neq 0$ 

If you estimate this using naïve ML, you'll get

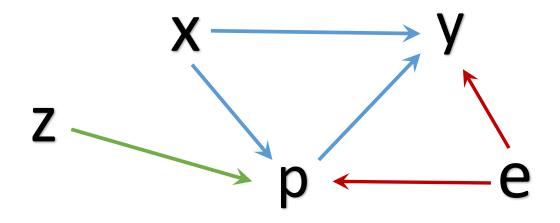
$$E[y|p,x] = E_{e|p}[g(p,x) + e] = g(p,x) + E[e|t,x]$$

This works for prediction. It doesn't work for counterfactual inference:

What happens if I change p independent of e?



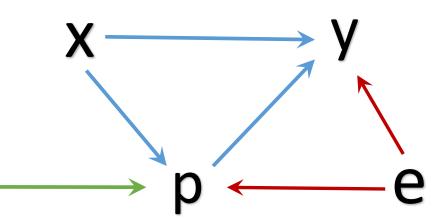
#### Instrumental Variables (IV)



In IV we have a special  $z \perp e$  that influences policy p but not response y.

- Supplier costs that move price independent of demand (e.g., fish, oil)
- Any source of treatment randomization (intent to treat, AB tests, lottery)

#### Instrumental Variables (IV)



The exclusion structure implies

$$E[y|x,z] = E[g(p,x) + e|x,z] = \int g(p,x)dP(p|x,z)$$

So to solve for structural g(p,x) we have a new learning problem

$$\min_{g \in G} \sum \left( y_i - \int g(p, x_i) dP(p|x_i, z_i) \right)^2$$

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You might have seen 2SLS:

$$p = \beta z + \nu$$
 and  $g(p) = \tau p$  so that  $\int g(p)dP(p|z) = \tau \hat{p} = \tau \hat{\beta} z$ 

So you first regress p on z then regress y on  $\hat{p}$  to recover  $\hat{\tau}$ .

This requires strict assumptions and homogeneous treatment effects.

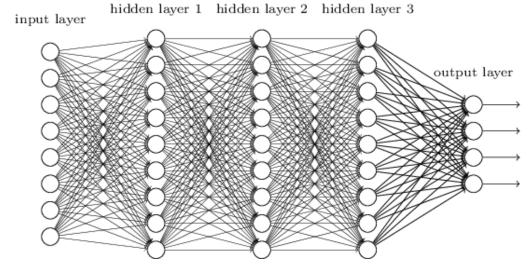
$$\min_{g \in G} \sum \left( y_i - \int g(p, x_i) dP(p|x_i, z_i) \right)^2$$

We can target this integral loss function directly with flexible g and P. Brute force version

- Fit conditional distributions  $\hat{P}(p|x_i,z_i)$ .
- Generate  $\{\hat{p}_{ib}\}_{b=1}^{B} \sim \hat{P}(p|x_i,z_i)$  for each i.
- Train  $\hat{g}$  to minimize  $[y_i B^{-1} \sum_b g(\hat{p}_{ib}, x_i)]^2$ .

Turns IV into two ML tasks: we can use DNNs for both  $\hat{P}$  and  $\hat{g}$ .

#### Learning to love Deep Nets

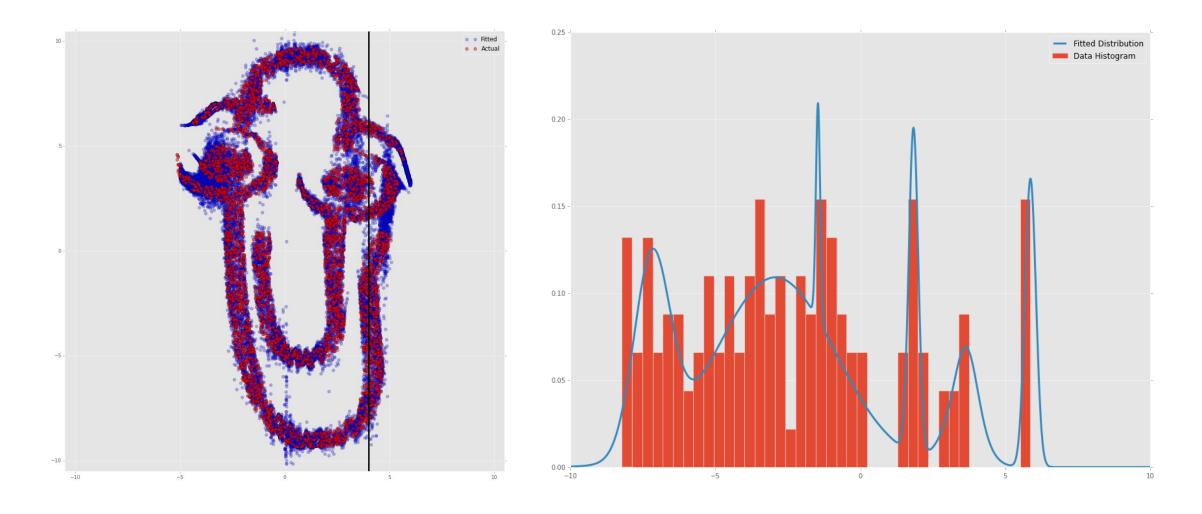






#### First Stage is out-of-the-box ML: learn $P(p|x_i,z_i)$

e.g., DNN fits distribution to maximize likelihood for a mixture of Gaussians.



The second stage involves an integral loss function

Brute force just samples from  $\hat{P}(p|x_i,z_i)$  to evaluate

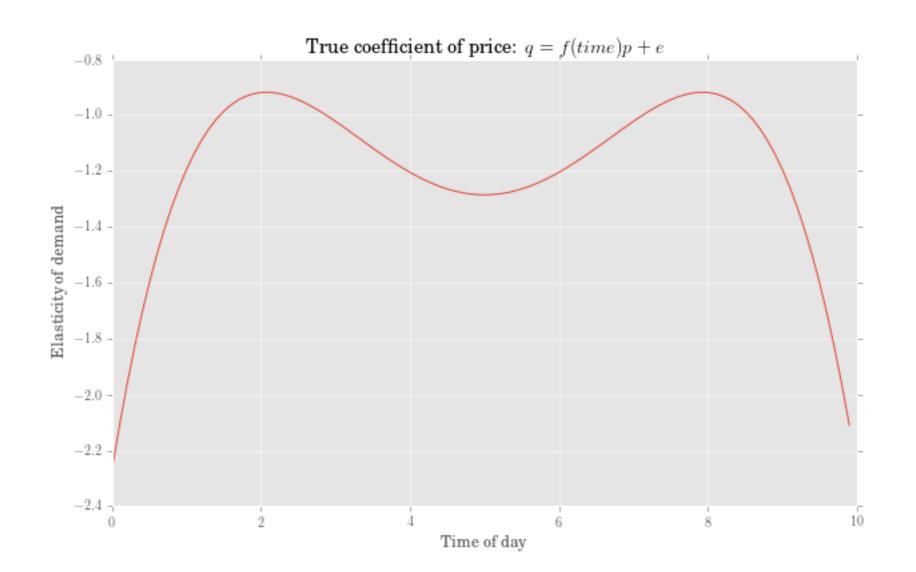
$$\min_{\theta \in \Theta} \frac{1}{N} \sum_{i} \left( y_i - \frac{1}{B} \sum_{b} g(\hat{p}_{ib}, x_i; \theta) \right)^2, \quad \hat{p}_{ib} \sim \hat{P}(p|x_i, z_i)$$

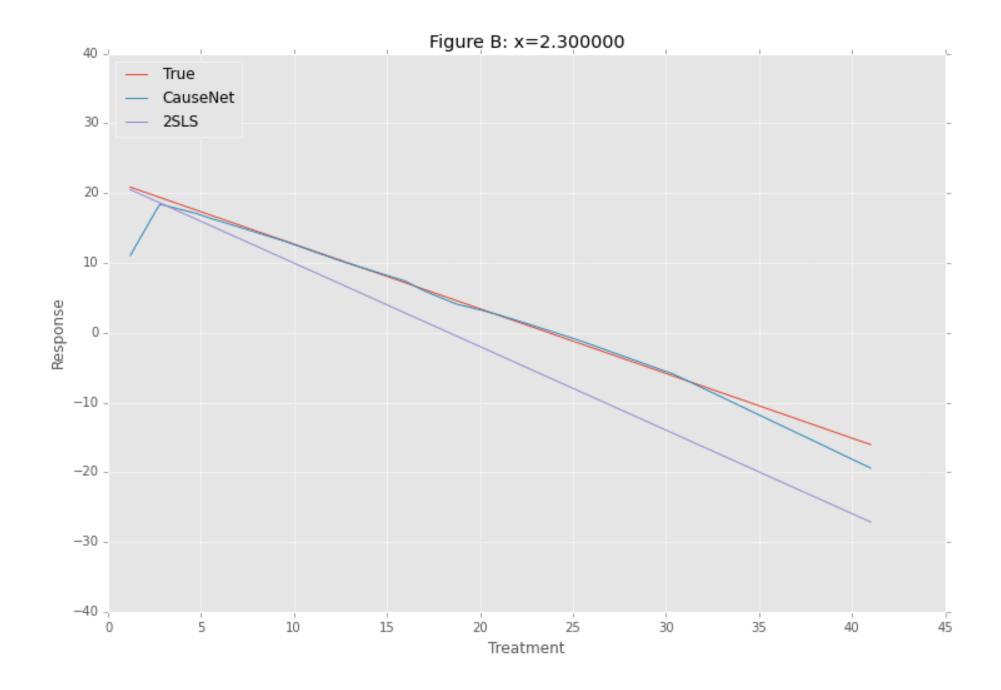
Instead, Stochastic Gradient Descent: optimize via unbiased gradient estimates based upon mini-batch sample of the full dataset.

We can do SGD by pairing each observation with two treatment draws

$$\nabla g(\theta) \approx (y_i - g(\hat{p}_{i1}, x_i; \theta)) g'(\hat{p}_{i2}, x_i; \theta)$$

#### Linear Demand, Heterogeneous Effects





#### Ads Application

Taken from Goldman and Rao (2014)

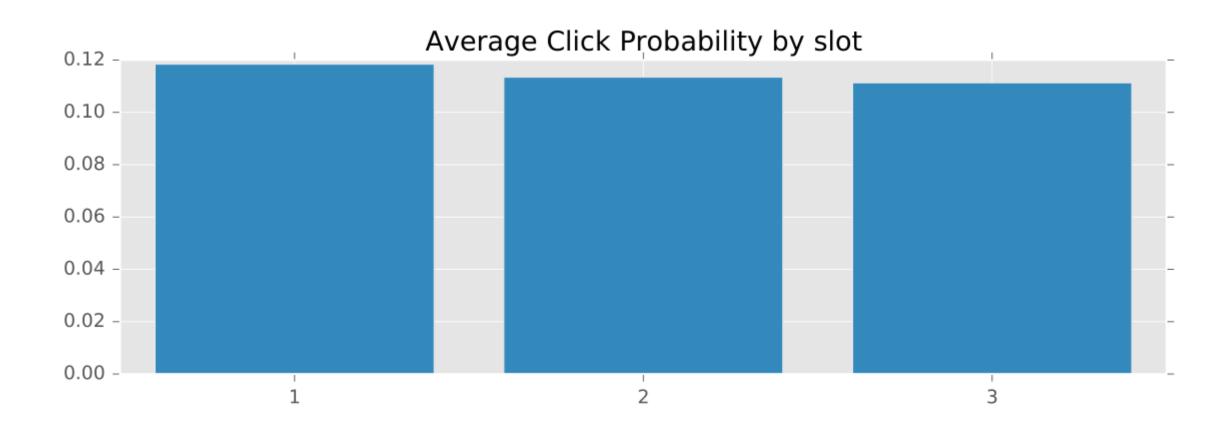
We have 74 mil click-rates over 4 hour increments for 10k search terms

Treatment: ad position 1-3

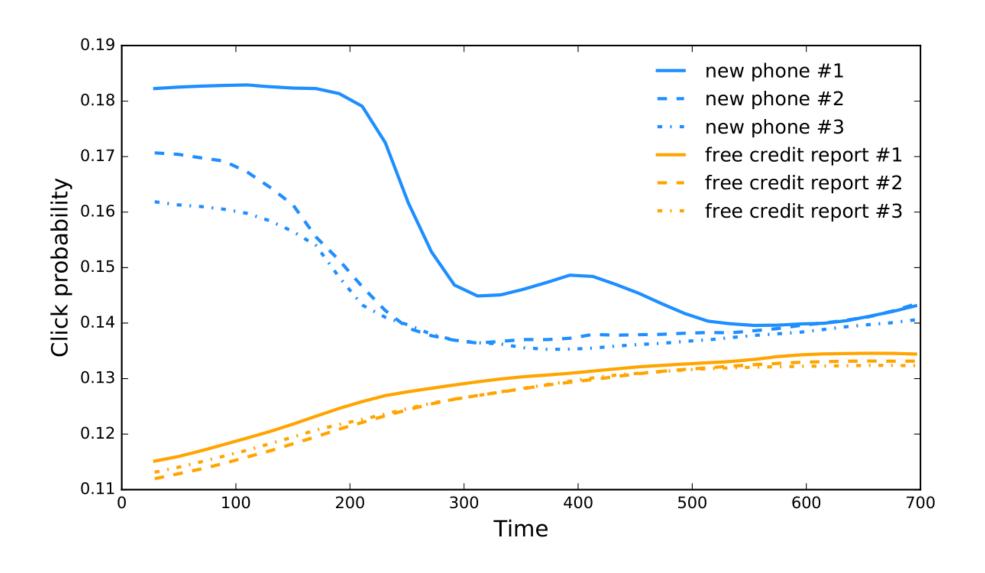
Instrument: background AB testing

Covariates: search text and time

### Average Treatment Effects



#### Heterogeneous Treatment Effects



#### Economics and Artificial Intelligence

We have a track record pointing ML at questions of science + causation. We're going to replicate this success at scale on unstructured data

We use economic theory to build systems of tasks that can be addressed with Deep nets and other state-of-the-art ML.

This is the construction of systems for Artificial *Economic* Intelligence.