Deep Counterfactuals via IV

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Endogenous Errors

$$y = g(p, \boldsymbol{x}) + e \;\; \text{and} \;\; \mathbb{E}[\, p\, e\,] \neq 0$$

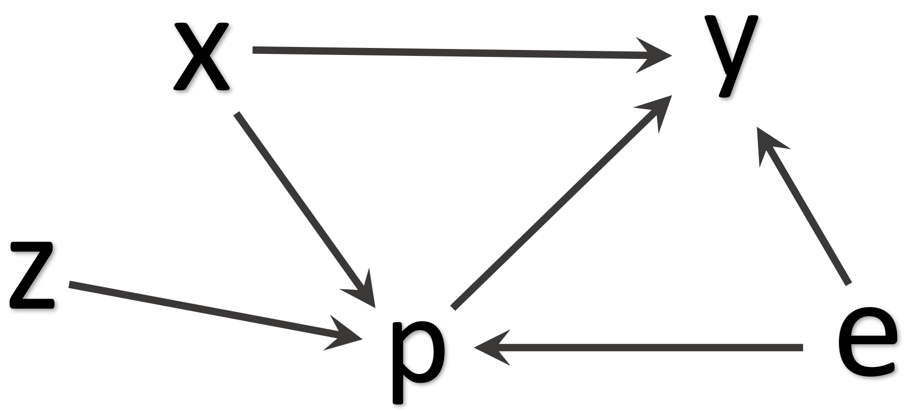
If you estimate this using naïve ML, you'll get

$$E[y|p, \boldsymbol{x}] = \; E_{e|p}[g(p, \boldsymbol{x}) + e] = g(p, \boldsymbol{x}) + E[e|p, \boldsymbol{x}]$$

This works for prediction. It doesn't work for counterfactual inference:

What happens if I change p independent of e?

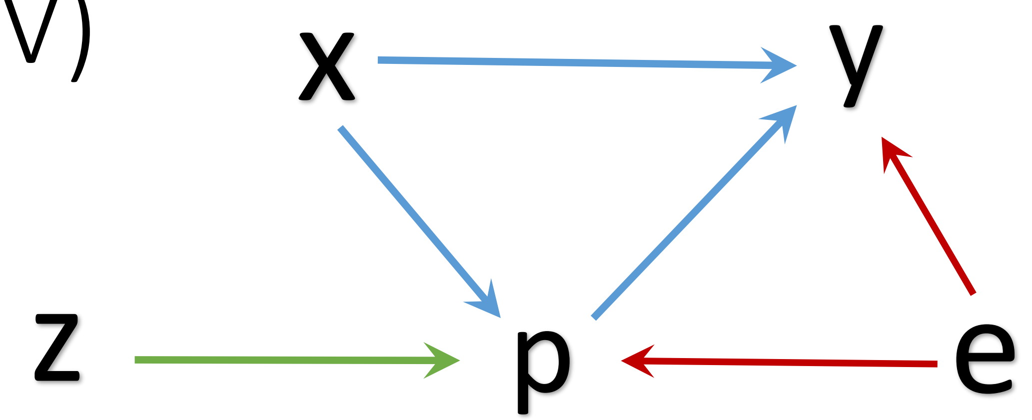
Instrumental Variables (IV)



In IV we have a special $z \perp e$ that influences policy p but not response y.

- Supplier costs that move price independent of demand (e.g., fish, oil)
- Any source of treatment randomization (intent to treat, AB tests, lottery)

Instrumental Variables (IV)



The *exclusion structure* implies

$$E[y|x,z] = E[g(p,x)|x,z] + E[e|x] = \int g(p,x)dF(p|x,z)$$

So to solve for *structural* $h(p,x)$ we have a new learning problem

$$\min_{g\in G} \sum \left(y_i - \int g(p,x_i)dF(p|x_i,z_i) \right)^2$$

$$\min_{g \in G} \sum \left(y_i - \int g(p, x_i) d\mathrm{P}(p|x_i, z_i) \right)^2$$

2SLS:

$p = \beta z + \nu$ and $g(p) = \tau p$ so that $\int g(p) d\mathrm{P}(p|z) = \tau\hat{p} = \tau\hat{\beta} z$

So you first regress p on z then regress y on $\hat{p}$ to recover $\hat{\tau}$.

This requires strict assumptions and homogeneous treatment effects.

$$\min_{g \in G} \sum \left(y_i - \int g(p, x_i) d\mathrm{P}(p | x_i, z_i) \right)^2$$

Or look to nonparametric 2SLS like in Newey and Powell:

$$g(p, x_i) \approx \sum_k \varphi_k(p, x_i) \text{ and } \varphi_k(p, x_i) \approx \sum_j \phi_{kj}(x_i, z_i)$$

But this requires careful crafting and will not scale with $\dim(x)$

$$\min_{g \in G} \sum \left(y_i - \int g(p, x_i) d\mathrm{F}(p|x_i, z_i) \right)^2$$

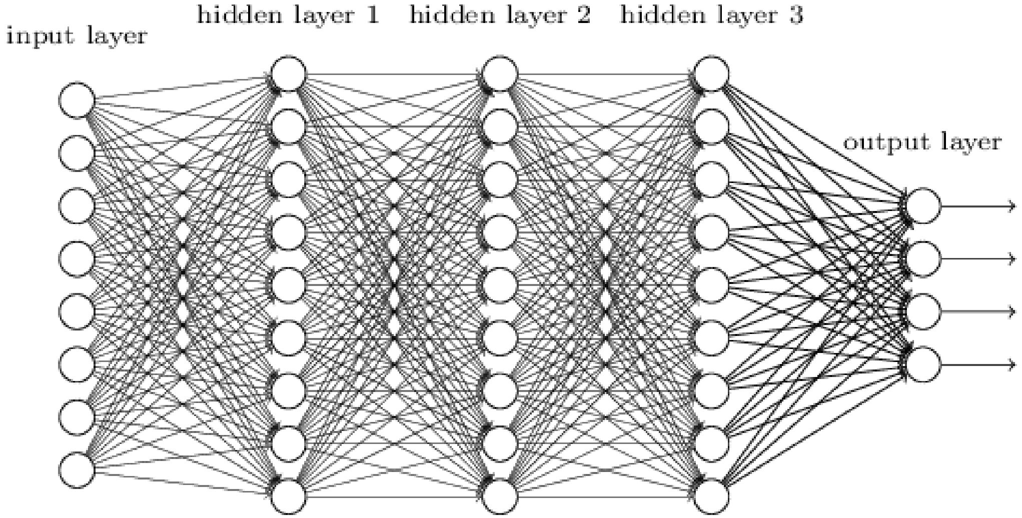
Instead, we propose to target the integral loss function directly

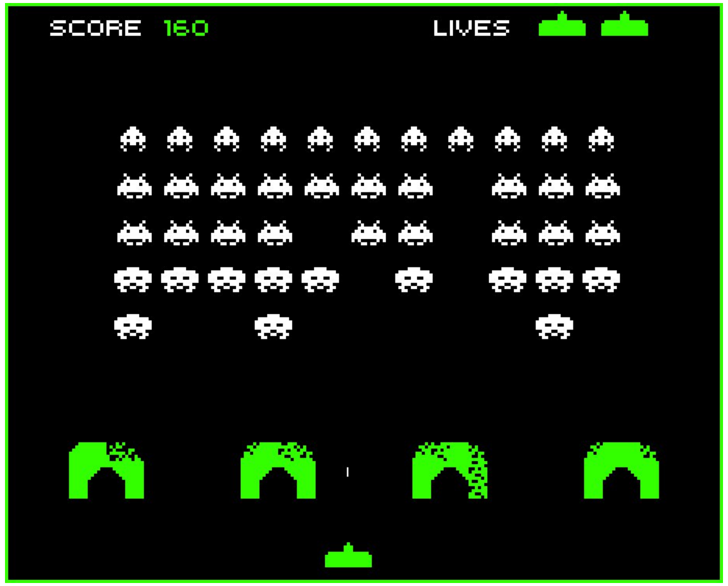
For discrete (or discretized) treatment

- Fit distributions $\hat{\mathrm{F}}(p|x_i, z_i)$ with probability masses $\hat{f}(p_b|x_i, z_i)$
- Train $\hat{g}$ to minimize $\left[\mathrm{y_i} - \sum_b g(\hat{p}_b, x_i) \hat{f}(p_b|x_i, z_i) \right]^2$

And you've turned IV into two *generic* machine learning tasks

Learning to love Deep Nets





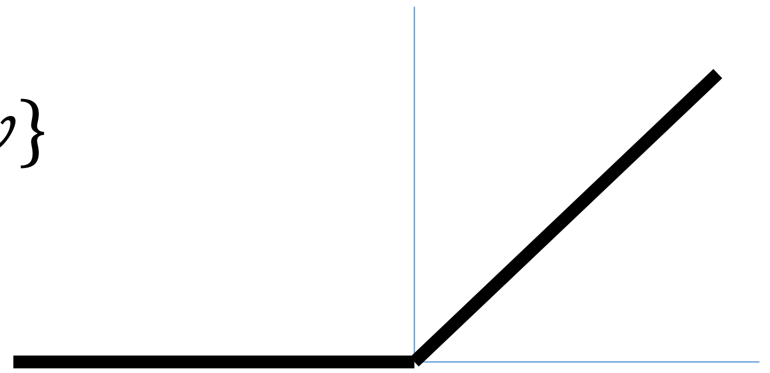
What is a deep net?

$$\hat{y}_i = \sum_k \eta_k(a_{ik}), \qquad a_{ik} = \sum_j w_{kj} z_j\,, \quad z_j = \sum_l h_l^1(b_{il})\,, \ldots$$

And so-on until you get down to a bottom layer $\{f_l(x_i)\}_l$

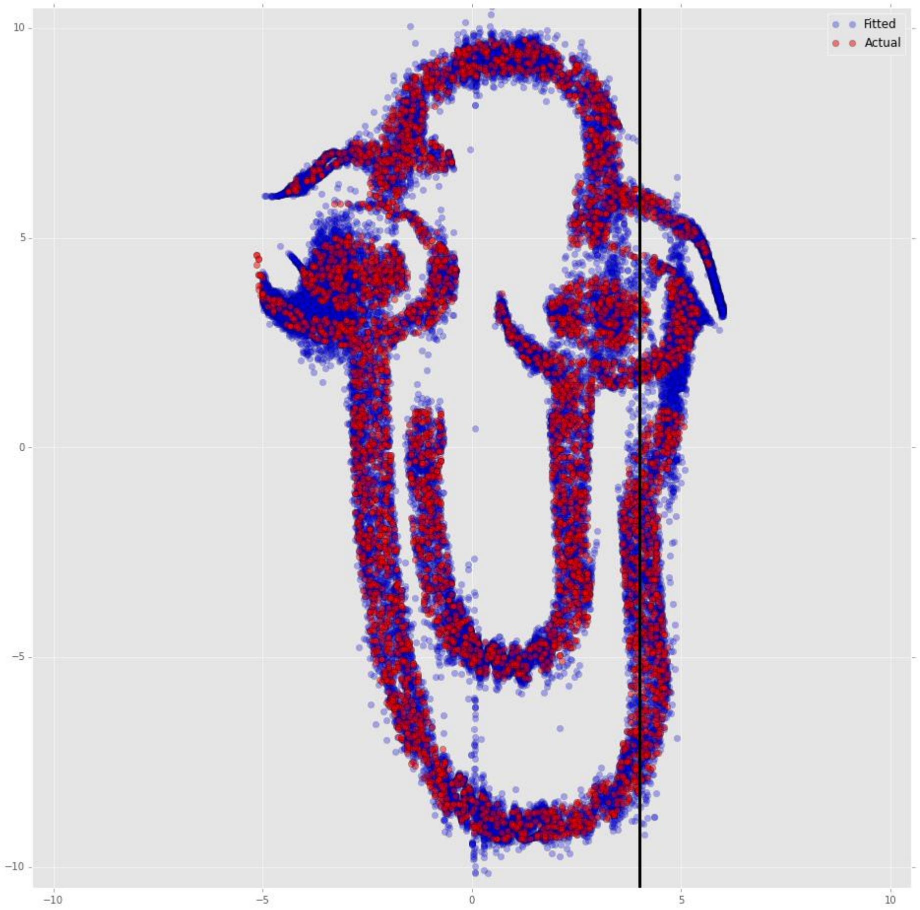
Many different variations here: recursive, convolutional, …

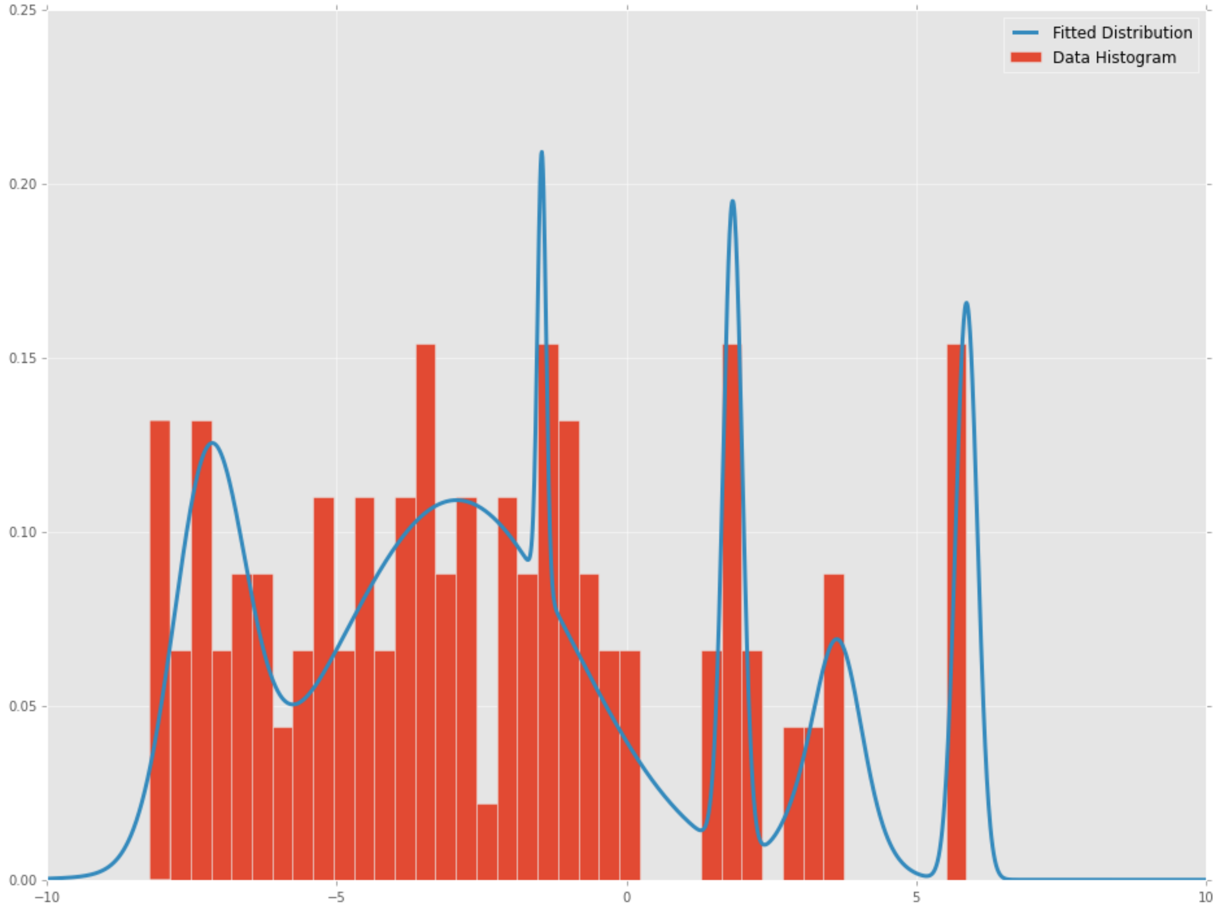
Apart from the bottom, usually $h(v) = \max\{0, v\}$



e.g., first-stage learning for $\mathrm{F}(p|x_i, z_i)$

Bishop 96: Final layer of network parametrizes a mixture of Gaussians





The second stage involves an integral loss function

If p is not discrete or can take many values, we can't just integrate

Brute force just samples from $\hat{\mathrm{F}}(p|x_i, z_i)$ and you take gradients on

$$\frac{1}{N}\sum_i \left(y_i - \frac{1}{B}\sum_b g(\dot{p}_{ib}, x_i; \theta)\right)^2, \quad \dot{p}_{ib} \sim \hat{\mathrm{F}}(p|x_i, z_i)$$

But this is super inefficient

Stochastic Gradient Descent

You have loss $L(\boldsymbol{D},\ \theta)$ where $\boldsymbol{D} = [\boldsymbol{d}_1\ \dots \boldsymbol{d}_N]$

In the usual GD, you iteratively descend

$$\theta_t = \theta_{t-1} - \boldsymbol{C}_t \nabla L(\boldsymbol{D}, \theta_{t-1})$$

In SGD, you instead follow *noisy* but *unbiased* sample gradients

$$\theta_t = \theta_{t-1} - \boldsymbol{C}_t \nabla L(\{\boldsymbol{d}_{t_b}\}_{b=1}^{B}, \theta_{t-1})$$

SGD for integral loss functions

Our one-observation stochastic gradient is

$$\nabla L(d_i, \theta) = -2\left(y_i - \int g_\theta(p, x_i) d\hat{F}(p|x_i, z_i)\right) \int g_\theta{}'(p, x_i) d\hat{F}(p|x_i, z_i)$$

Do SGD by pairing each observation with *two independent* treatment draws

$$\nabla\hat{L}(d_i, \theta) = -2(y_i - g_\theta(\hat{p}_{i1}, x_i))\, g_\theta'(\hat{p}_{i2}, x_i), \qquad \hat{p}_{ib} \sim \hat{\mathrm{F}}(p|x_i, z_i)$$

So long as the draws are independent, $\mathbb{E}\nabla\hat{L}(d_i, \theta)$ = $\mathbb{E}\nabla L(d_i, \theta)$ = $L(\boldsymbol{D}, \theta)$

Validation and model tuning

We can do *causal validation* via two OOS loss functions

Leave-out deviance on first stage

$$\sum_{i \in LO} -\log \hat{f}(p|x_i, z_i)$$

Leave-out loss on second stage (constrained fit on $\mathbb{E}[y|xz]$)

$$\sum_{i \in LO} \left(y_i - g_\theta(\dot{p}_i, x_i)\right)^2, \;\; \dot{p}_i \sim \hat{\mathrm{F}}(p|x_i, z_i)$$

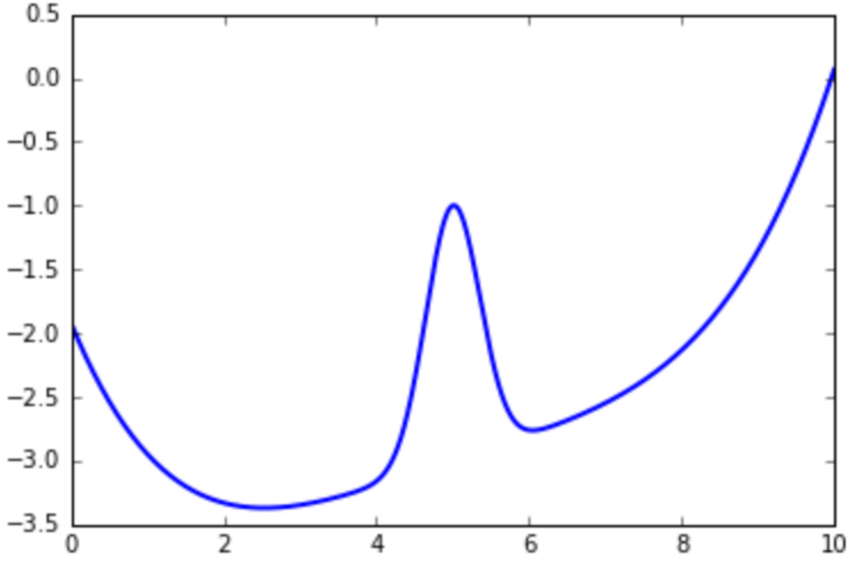
You want to minimize both of these (in order).

heterogeneous price effects

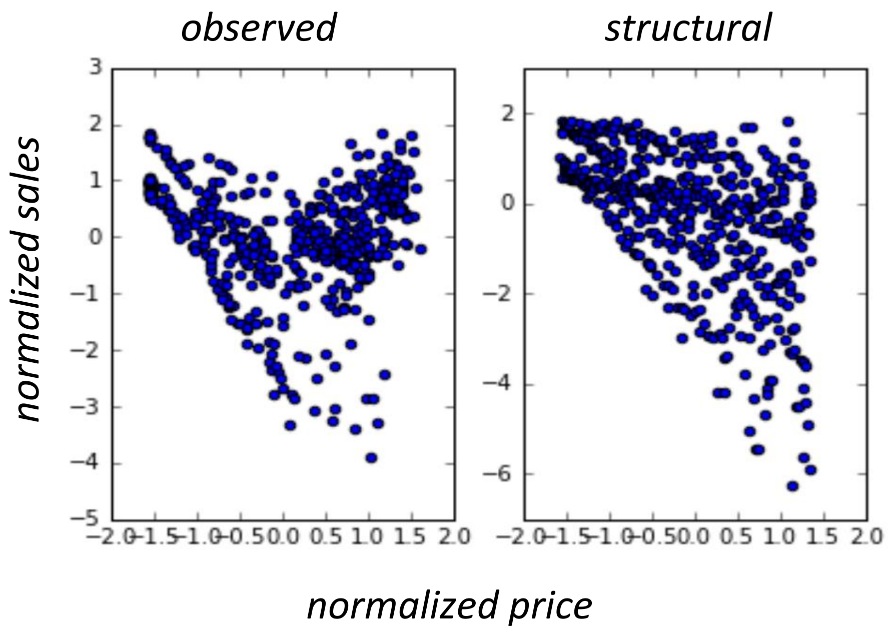
$$y = 100 + s\psi_t + (\psi_t - 2)p + e,$$

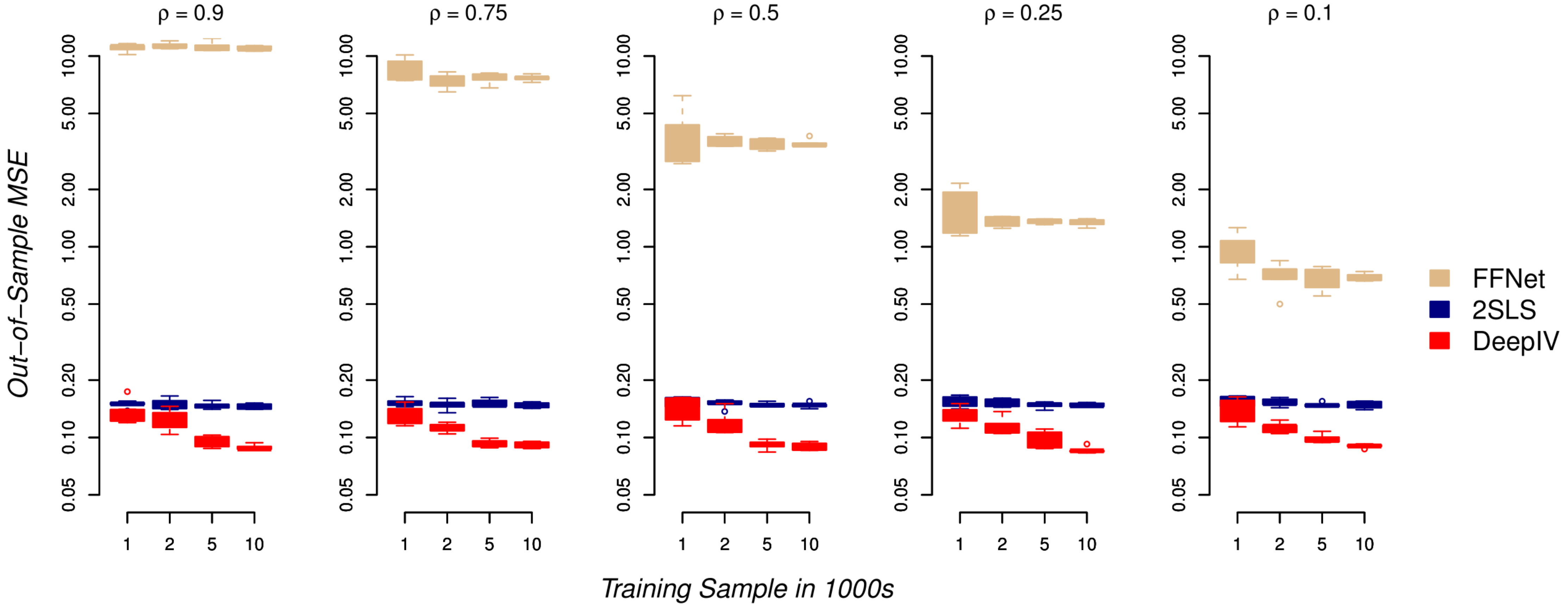
$$p = 25 + (z+3)\psi_t + v$$

$$z,\ v \sim \mathrm{N}(0,1) \ \text{ and } \ e \sim \mathrm{N}(\rho v, 1-\rho^2),$$



- 'time' dependent prices and elasticity
- 7 different customer types multiply discounts, sensitivities, and demand




ρ = 0.9
ρ = 0.75
ρ = 0.5
ρ = 0.25
ρ = 0.1
Out-of-Sample MSE
0.05
0.10
0.20
0.50
1.00
2.00
5.00
10.00
1
2
5
10
Training Sample in 1000s
FFNet
2SLS
DeepIV

Inference? Good question

Data split! Get top node values and averages on left-out data:

$$\bar{\eta}_{ik} = E_{\hat{F}(\dot{p}|x_i,z_i)}\eta_k(x_i,\dot{p}) \quad and\ \eta_{ik} = \eta_k(x_i,p_i)$$

Stack as instruments $\overline{\mathrm{H}} = [\bar{\eta}_1 \cdots \bar{\eta}_L]'$ and treatments $\mathrm{H} = [\eta_1 \cdots \eta_L]'$

Then the treatment effect is $\hat{\beta} = (\overline{H}'H)^{-1}\overline{H}'y$ with usual variance and

$$\mathrm{var}\left(\hat{h}(x,p)\right) = \eta'(x,p)\, V_\beta\, \eta(x,p).$$

Inference? Good question

Or Bayes: recall the deep net

$$\hat{y}_i = \sum_k \eta_k(a_{ik}), \qquad a_{ik} = \sum_j w_{kj} z_j\,, \quad z_j = \sum_l h_l^1(b_{il})\,, \ldots$$

When training with SGD, we actually use dropout for regularization

At each update, calculate gradients against $\mathrm{W}_l = \Xi_l \Omega_l$ at layer l where

$$\Xi_l = \mathrm{diag}(\, \xi_{l1} \ \ldots \xi_{lK_l}), \qquad \xi_{kj} \sim \mathrm{Bern}(c)$$

Variational Bayesian inference via dropout

VB minimizes $\mathbb{E}_{\mathrm{q}}[\,-\log p(\boldsymbol{D}|\boldsymbol{W}) \; - \log p(\boldsymbol{W}) \; + \; \log q(\boldsymbol{W})]$

With $q(\boldsymbol{W}) = \prod_l \prod_k (c1_{[W_{lk}=\Omega_{lk}]} + (1-c)\,1_{[W_{lk}=0]})$ and normal prior, this objective becomes

$$\mathbb{E}_{q(c,\boldsymbol{\Omega})}\ell(D|W) + \sum_{l=1}^{L} c\lambda\|\Omega_l\|^2 + \sum_{l=1}^{L} K_{l-1}\,[c\log(c) + (1-c)\log(1-c)]\,.$$

So dropout is VB!

(more complex argument in Gal and Ghahramani 2015)

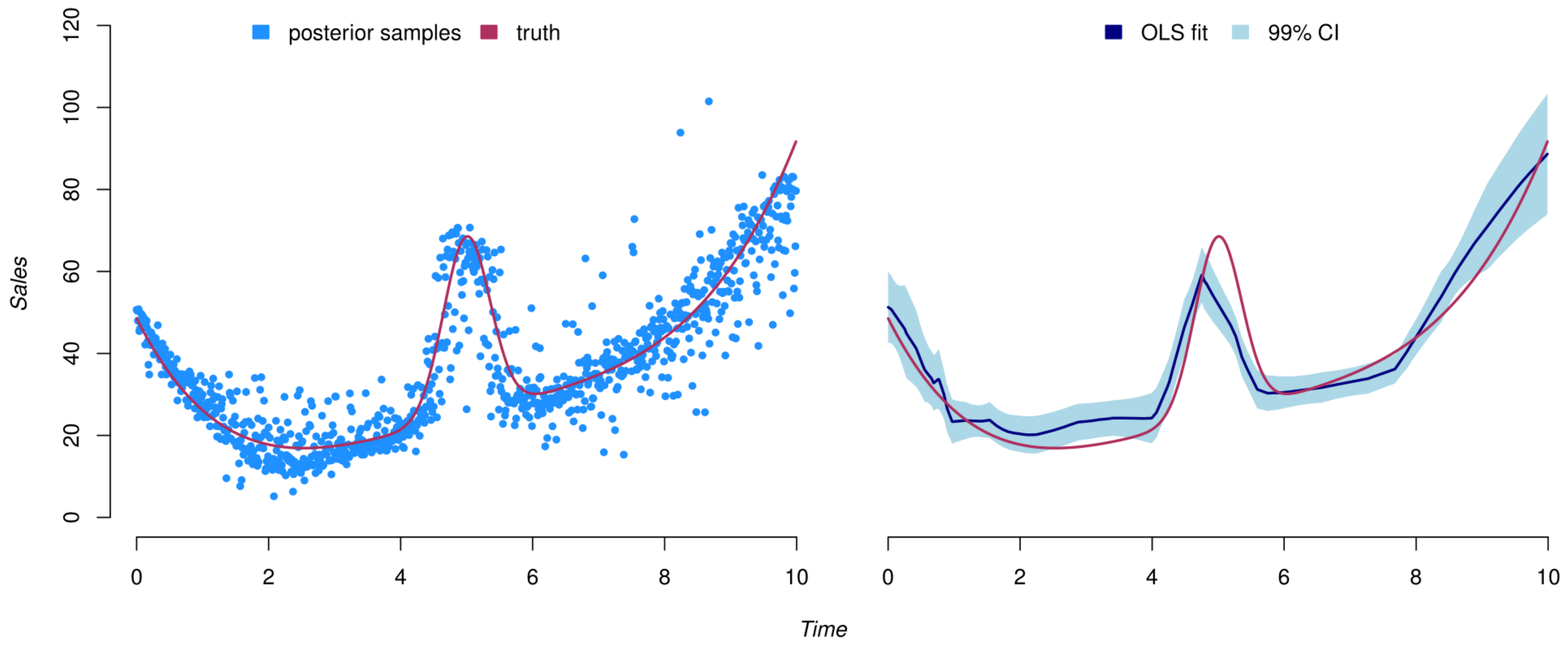
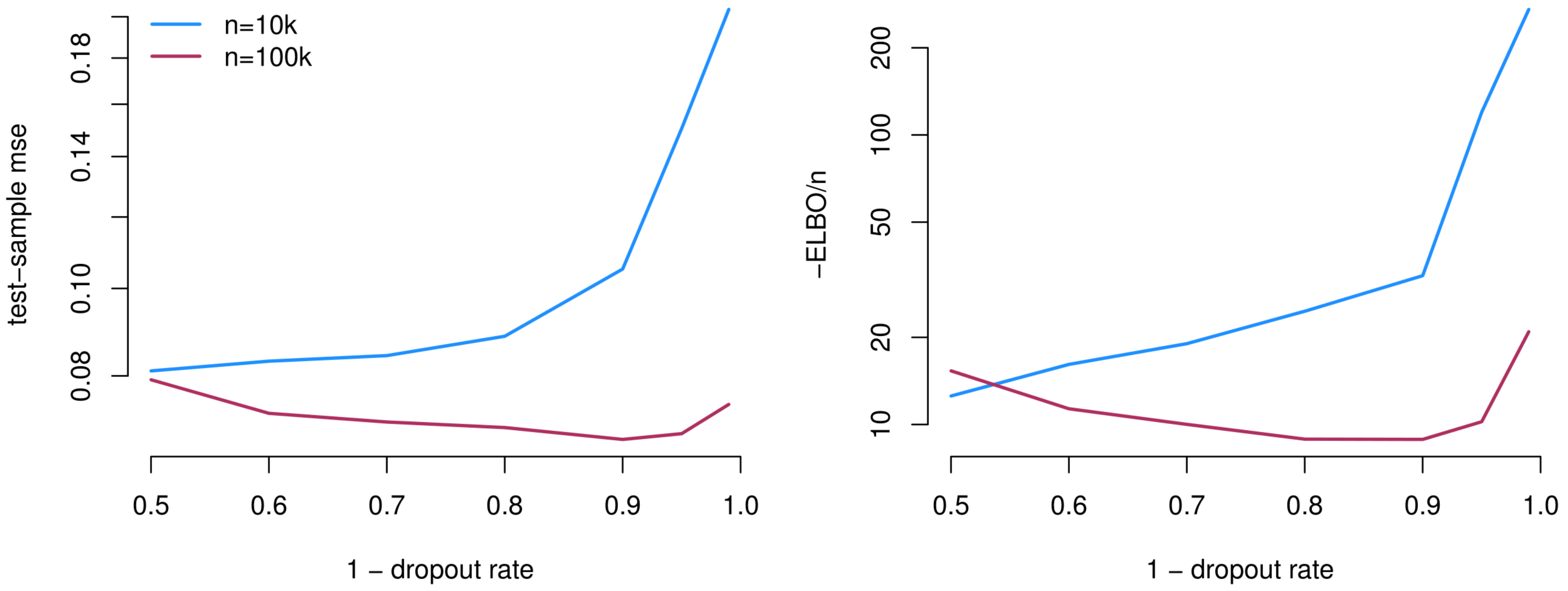


Figure 3: Bayesian (left) and Frequentist (right) inference for a central slice of the counterfactual function, taken at the average price and in our 4^{th} customer category. Since the price effect for a given customer at a specific time is constant in (27), the curves here are a rescaling of the customer *price sensitivity* function.

Tuning the dropout rate is like treating it as a variational parameter



Ads Application

Taken from Goldman and Rao (2014)

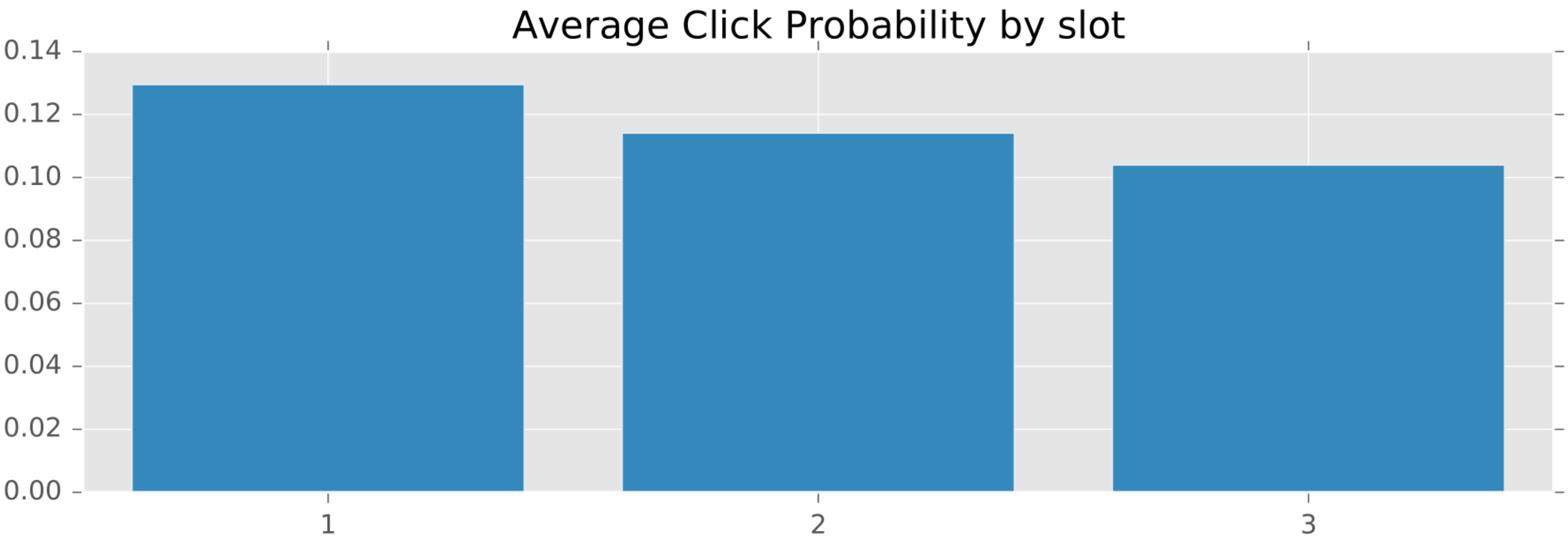
We have 74 mil click-rates over 4 hour increments for 10k search terms

Treatment: ad position 1-3

Instrument: background AB testing (bench of ~ 100 tests)

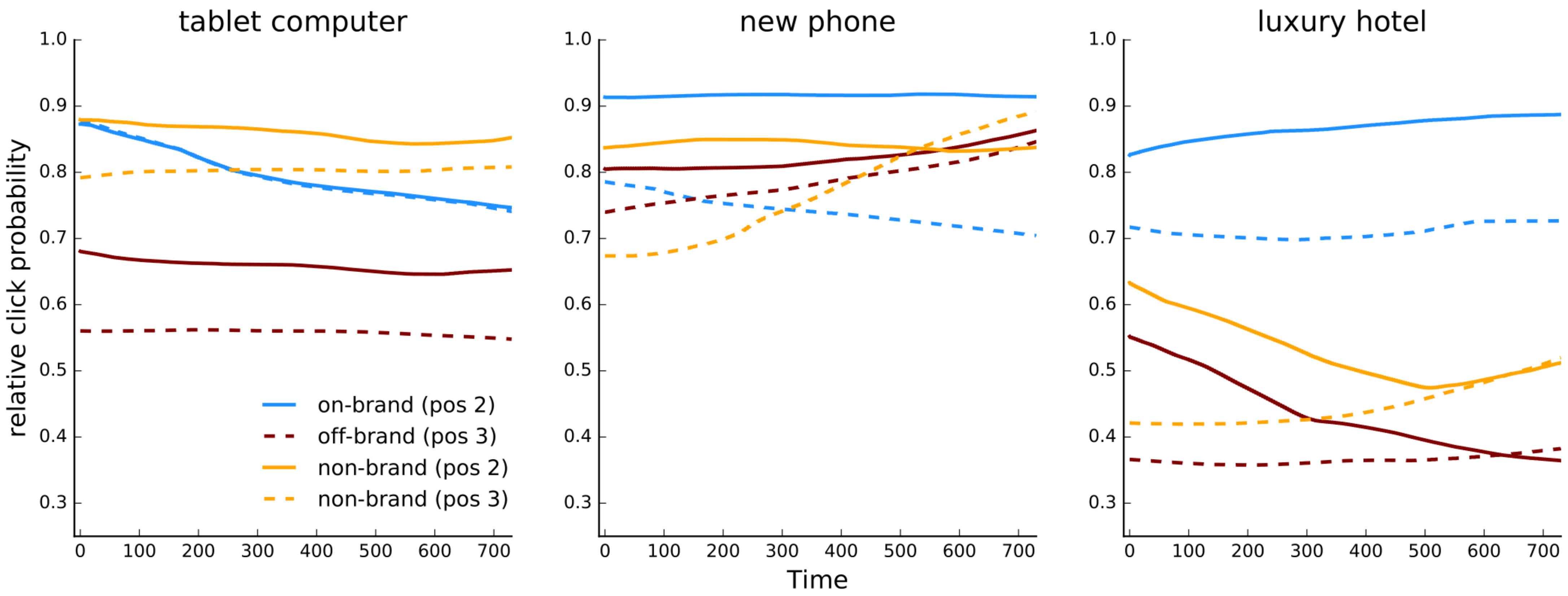
Covariates: advertiser id and ad properties, search text, time period

Average Treatment Effects



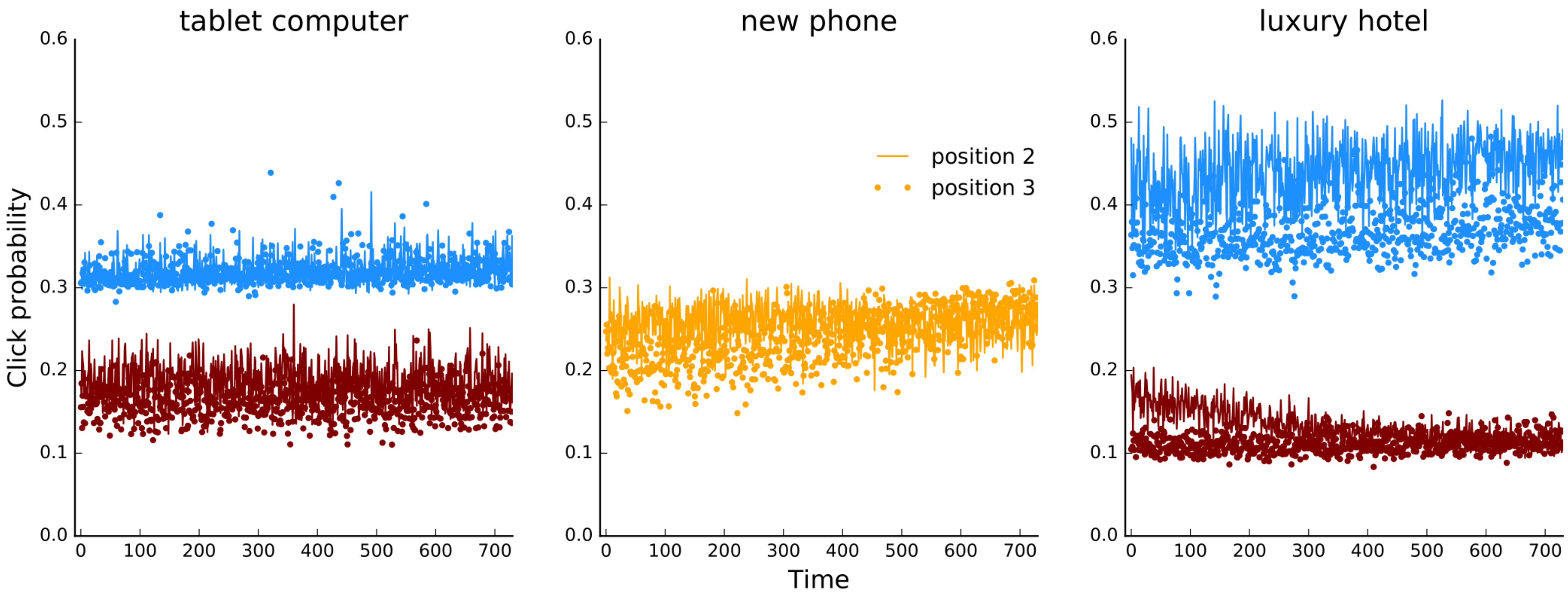
These compare to observed click probabilities of 0.33, 0.1, and 0.05.

Heterogeneous Treatment Effects



The *relative* click-probabilities can be interpreted *causally.*

Posterior Treatment Effect Samples



Each point/dash is an independent draw from the `posterior’

Automated Learning and Intelligence for Causation and Economics

We use economic theory to build systems of tasks that can be addressed with deep nets and other state-of-the-art ML.

This is the construction of systems for *Economic* AI