MONOPOLY PRICING UNDER A HETEROGENEOUS BOYCOTT

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Abstract

This note studies monopoly pricing when boycott participation is heterogeneous over consumer valuations. A boycott is modeled as a valuation-dependent survival function $s(v; \beta)$ that maps boycott intensity $\beta \in [0, 1]$ into the probability that a buyer with willingness-to-pay v remains active in the market. The key mechanism is selection: when low-valuation consumers exit more strongly than high-valuation consumers, residual demand becomes less elastic at relevant prices, and the monopoly-optimal markup can rise even while quantity falls. We derive the pricing condition, show the comparative-static channel, and provide a reproducible numerical pipeline with scenario sweeps and publication-quality figures.

1 Motivation and Setup

In standard monopoly problems, a demand shift that lowers quantity typically lowers profit and may move price up or down depending on elasticity. A boycott introduces a specific form of demand shift: it can remove buyers non-uniformly across the valuation distribution. If boycott exposure is concentrated among low-WTP consumers, then the remaining buyer pool is relatively less price-sensitive. This composition effect is distinct from a uniform outward or inward demand shift.

Let the market have size $M > 0$. Consumer valuations satisfy

$$v \sim F, \qquad f(v) = F'(v), \qquad v \in [0, \infty).$$

A monopolist with constant marginal cost $c \geq 0$ posts a single price p.

1.1 Baseline Demand and Profit

Without a boycott, quantity demanded is

$$Q(p) = M \ \Pr(v \geq p) = M\big(1 - F(p)\big), \tag{1}$$

and profit is

$$\pi(p) = (p - c)Q(p). \tag{2}$$

1.2 Heterogeneous Boycott

Let $s(v; \beta) \in [0, 1]$ be the probability that valuation type v remains in the market under boycott intensity $\beta \in [0, 1]$. Residual demand is

$$Q_B(p; \beta) = M \int_p^\infty s(v; \beta) f(v) \, dv, \tag{3}$$

with boycott profit

$$\pi_B(p; \beta) = (p - c)Q_B(p; \beta). \tag{4}$$

Three canonical selection shapes are useful in practice:

1. **Left-tail selective**: low-WTP consumers are more likely to exit.

2. **Right-tail selective**: high-WTP consumers are more likely to exit.

3. **Middle-band selective**: middle valuations are most exposed.

2 Pricing Condition and Elasticity Channel

Differentiating profit with respect to price:

$$\frac{d\pi_B}{dp} = Q_B(p;\beta) + (p-c)\frac{dQ_B}{dp}(p;\beta), \tag{5}$$

where by Leibniz rule

$$\frac{dQ_B}{dp}(p;\beta) = -Ms(p;\beta)f(p). \tag{6}$$

At an interior optimum $p_B^*(\beta)$,

$$\frac{p_B^*(\beta)-c}{p_B^*(\beta)} = \frac{1}{|\varepsilon_B(p_B^*(\beta);\beta)|}, \tag{7}$$

with residual demand elasticity

$$\varepsilon_B(p;\beta) \equiv \frac{dQ_B}{dp}\frac{p}{Q_B} = -\frac{Ms(p;\beta)f(p)}{Q_B(p;\beta)}p. \tag{8}$$

Hence lower elasticity in absolute value (more inelastic residual demand) implies a higher optimal markup.

2.1 Comparative-Static Direction

Define the first-order condition as

$$\Phi(p,\beta) \equiv Q_B(p;\beta) + (p-c)Q_{B,p}(p;\beta) = 0.$$

If regularity conditions hold and $\Phi_p < 0$ at the optimum, then by implicit differentiation

$$\frac{dp_B^*}{d\beta} = -\frac{\Phi_\beta}{\Phi_p} = -\frac{Q_{B,\beta} + (p_B^*-c)Q_{B,p\beta}}{\Phi_p}. \tag{9}$$

When boycott intensity mainly removes low-valuation buyers, two forces tend to raise p_B^*:

1. The residual pool tilts toward high valuations, making demand locally less elastic.
2. The extensive margin near the current price thins among price-sensitive types.

The opposite holds for right-tail selection.

3 Computational Design

The accompanying code uses a lognormal valuation distribution with baseline parameters

$$\mu = 0.0, \quad \sigma = 0.7, \quad M = 1.0, \quad c = 0.4.$$

Boycott selection is represented by smooth logistic functions in $\log v$ for numerical stability and interpretable shape control. The solver proceeds as follows:

1. Build a dense valuation grid and compute a tail-integral lookup for $Q_B(p; \beta)$.

2. Solve for p^* using a grid search plus bounded local refinement.

3. Sweep over $\beta \in [0, 0.9]$ and store $(p^*, Q^*, \pi^*, \varepsilon^*)$.

4. Produce CSV outputs and publication-style figures.

4 Quantitative Results

Table 1 reports a benchmark at $\beta = 0.60$.

Table 1: Optimal outcomes at boycott intensity $\beta = 0.60$

Scenario	p^*	Q^*	π^*	Elasticity at p^*
Baseline (no boycott)	1.2654	0.3683	0.3188	-1.4622
Left-tail selective	1.3658	0.3033	0.2929	-1.4140
Middle-band selective	1.4255	0.2529	0.2593	-1.3903
Right-tail selective	1.0698	0.2379	0.1594	-1.5970

Relative to baseline at $\beta = 0.60$:

1. Left-tail selection raises price by about 7.9% and lowers quantity by about 17.7%.

2. Middle-band selection raises price by about 12.7% and lowers quantity by about 31.3%.

3. Right-tail selection lowers price by about 15.5% and lowers quantity by about 35.4%.

These patterns match the elasticity mechanism: left-tail and middle-band selection reduce elasticity in absolute value near the optimum, while right-tail selection makes residual demand more elastic.

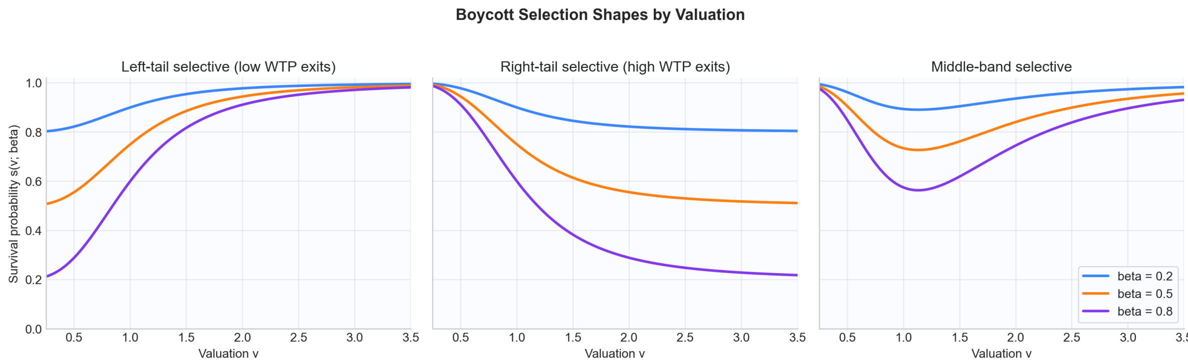

Figure 1: Boycott survival profiles $s(v; \beta)$ across valuation types.

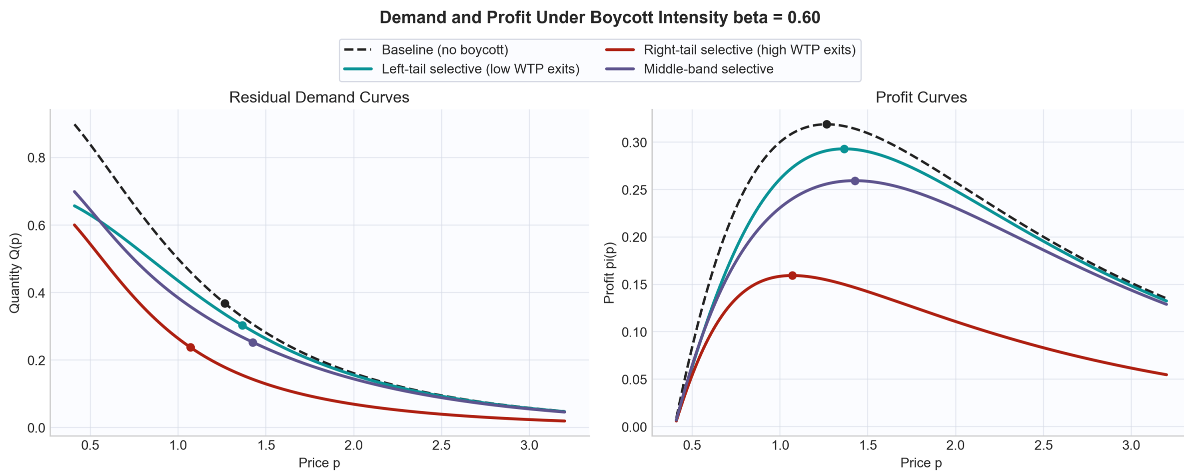

Figure 2: Residual demand and profit curves at $\beta = 0.60$ with scenario-specific optima marked.

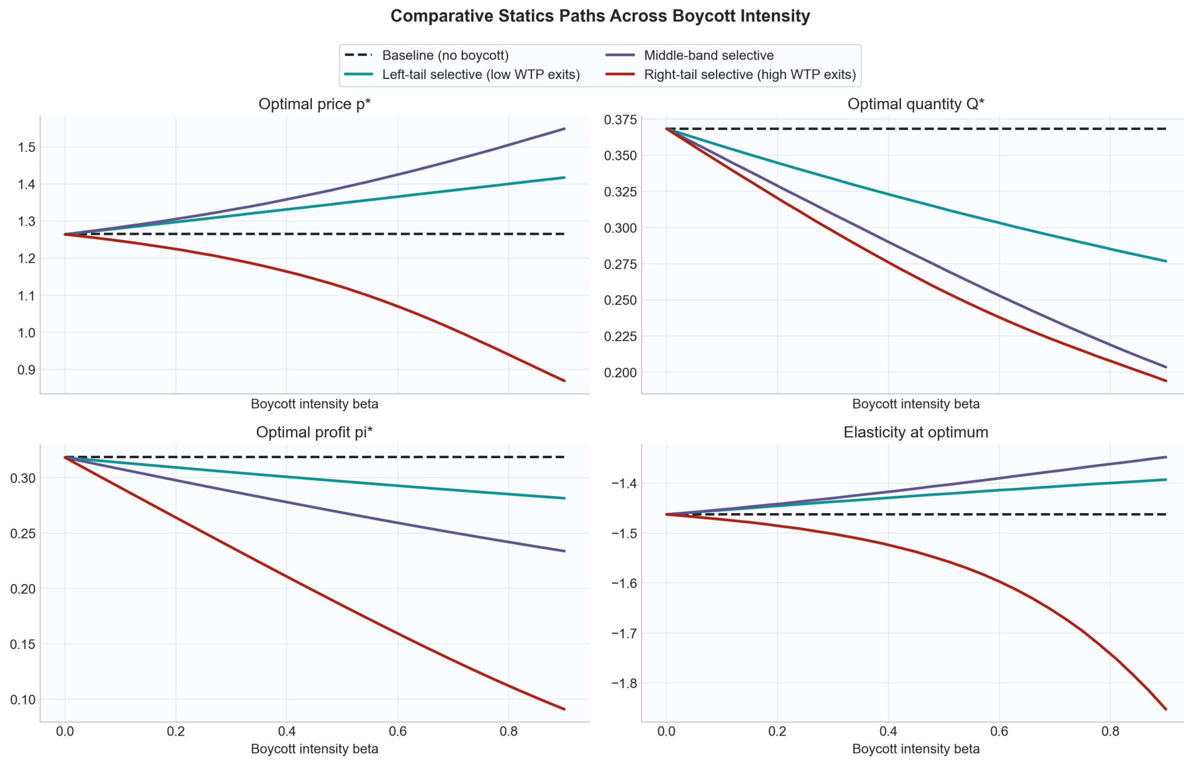

Figure 3: Comparative statics of optimal price, quantity, profit, and elasticity across boycott intensity.

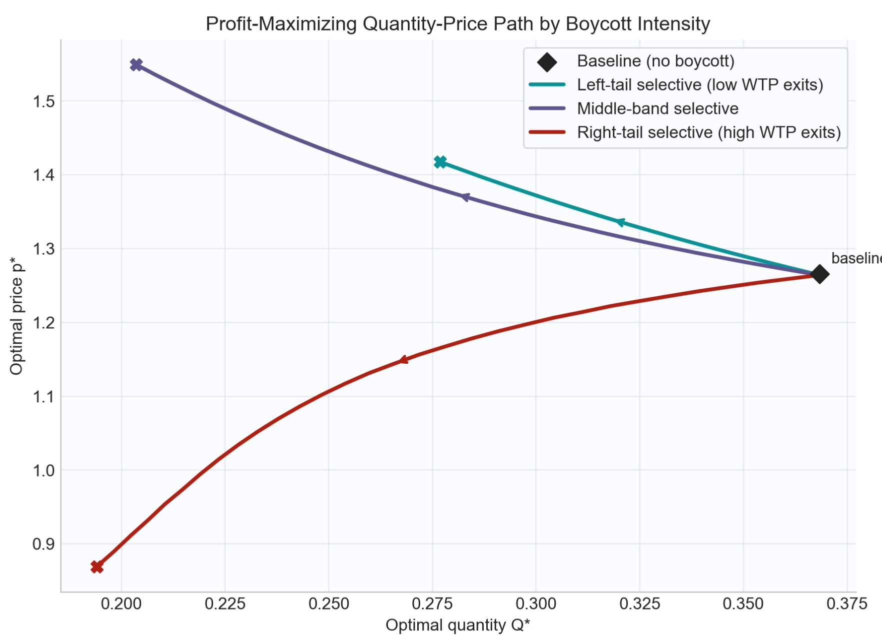

Figure 4: Profit-maximizing quantity-price path $(Q^*(\beta), p^*(\beta))$ by boycott type.

At high intensity ($\beta = 0.90$), the pattern strengthens: left-tail selection yields p^* about

12.0% above baseline, middle-band about 22.4% above baseline, and right-tail about 31.3% below baseline. Quantity and profit decline in all cases, but the decline is steepest under right-tail selection.

5 Interpretation for Boycott Episodes

The results separate two ideas that are often conflated in applied discussions:

1. **Participation effect**: boycott intensity reduces total demand.
2. **Composition effect**: who leaves the market changes residual elasticity.

Even when total demand falls, equilibrium price can increase if the composition effect dominates and makes the residual customer base less elastic. This is most likely when activists disproportionately mobilize low-valuation, highly price-sensitive segments while high-valuation segments remain attached to the product.