# Energy, Emissions, and Efficiency

## Optimal Energy Taxation in a Carbon-Constrained Economy

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# Objective

- **Objective:**
  Design an energy tax policy that makes firms pay the true social cost of emitting carbon.

- **How we model climate damage:**
  Energy use creates emissions, which cause damages

$$\text{Damage} = (\text{Harm from } CO_2) \times (CO_2).$$

**Under our approach:**

1. Each unit of fuel $j$ has an emissions factor $\phi_j$ and creates a per-unit damage $\omega\phi_j$.
2. Firms respond by substituting away from high-$\phi_j$ fuels.
3. Emissions fall.

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# Limitations & Modelling Choices

- **What our model *doesn't* do:**
  - Emissions don't affect productivity $A$ or damage output growth.
  - No "pollution hurts me" disutility term in household utility.
- **Why a linear loss function is useful:**
  - Tractability: every version of energy $j$ has an $\omega$, we can map taxes into welfare.
  - Transparency: $\omega$ is "dollars of climate harm per ton $CO_2$," so $\tau_j^* = \omega\phi_j$ is efficient.
  - Focus: we are isolating the energy channel—what is burned and how much.

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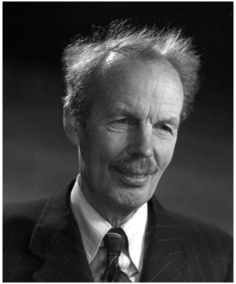
# Pigouvian Rule

- **Pigovian Principle:** We tax fuel at its marginal external damage.
- For each fuel $j$:

  tax per unit of fuel $j = (\$ \text{ harm per tn } CO_2) \times (\text{tn } CO_2 \text{ per unit})$.

- Efficiency - firms pay for the harm they cause.

$$\tau_j^* = \omega \, \phi_j$$



Arthur Cecil Pigou

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# Energy and Emissions

- The firm chooses fuel inputs $\{E_j\}_{j\in\mathcal{J}}$ (coal, gas, hydro, etc.).
- These inputs combine into an energy services bundle $E$ used in production.

$$E = \left(\sum_{j\in\mathcal{J}} \theta_j E_j^{\rho}\right)^{1/\rho}$$

- $\theta_j$: weight of fuel $j$ in $E$.

- Burning fuel $j$ emits $\phi_j$ tons of $CO_2$ per unit. Total emissions:

$$Z = \sum_{j\in\mathcal{J}} \phi_j E_j.$$

- Each ton of $CO_2$ imposes social damages valued at $\omega$ dollars per ton (the social cost of carbon). Total climate damage cost:

$$D = \omega Z.$$

# Why Substitution Matters

CES curvature parameter:

$$\rho = \frac{\sigma_E - 1}{\sigma_E}.$$

**High $\sigma_E$ (easy substitution):** Firms can swap away from dirty, high-$\phi_j$ fuels.

Output $Y$ and total energy $E$ remain unchanged.

**Low $\sigma_E$ (hard substitution):** If fuels are bad substitutes, they will use less energy overall.

$E$ and $Y$ decline.

This elasticity $\sigma_E$ is why our tax can cut emissions without killing production.

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# Behaviour Change, Not Revenue

## What the tax is for:

- Align private and social costs by setting $\tau_j^* = \omega\,\phi_j$, changing relative prices and the fuel mix $\{E_j\}$.
- Objective: reduce emissions $Z = \sum_j \phi_j E_j$ at least cost — not to raise funds.

## Revenue is a byproduct:

- Revenue is $T = \sum_j \tau_j E_j$, which is *endogenous* and tends to shrink as firms substitute away from high-$\phi_j$ fuels (especially when $\sigma_E$ is high).
- In our baseline, all $T$ is rebated (lump-sum or used to lower other distortions), so there is no net expansion of government spending. The resource constraint remains $C = Y - D$.

# Applicable

## Baseline travels

- What differs by country are *levels and shares*: $\{\theta_j\}$ (fuel mix), $\{\phi_j\}$ (emissions factors of delivered energy), prices $\{p_j\}$, $\sigma_E$ (substitution), and macro parameters $(A, \alpha, \beta)$.

## Developing vs. Developed:

- **Developed:** Typically higher $\sigma_E$ due to diversified supply (gas, interties, storage), dense infrastructure, and better access to capital $\Rightarrow$ easier fuel switching.
- **Developing:** Typically lower $\sigma_E$ due to demand, reliability constraints, credit frictions) $\Rightarrow$ bigger near-term output sensitivity.
- **Sector heterogeneity:** industry vs. power vs. transport have different $\sigma_E$.

# Model Framework: Parameter Overview

**Household**

$C$ = consumption
$\ell$ = labour supplied
$K$ = capital owned by household
$w$ = wage (per unit of $\ell$)
$r$ = rental rate of capital
$\Pi$ = firm profits rebated to household
$T$ = lump-sum transfer
$\gamma$ = scale of labour disutility
$\sigma$ = labour curvature (inverse Frisch)

**Firm / Technology**

$Y$ = output
$A$ = TFP
$K, \ell$ = capital, labour inputs
$E$ = energy aggregate
$E_j$ = fuel (coal, gas, Hydro...)
$\alpha$ = capital share in $Y$
$\beta$ = labour share in $Y$
$\sigma_E$ = CES across fuels

$$\rho = \frac{\sigma_E - 1}{\sigma_E}$$

$\theta_j$ = weight of fuel $j$ in $E$
$p_j$ = pre-tax price of fuel $j$
$\tau_j$ = per-unit tax on fuel $j$

**Government & Environment**

$\phi_j$ = tons $CO_2$ per unit of fuel $j$

$$Z = \sum_j \phi_j E_j \quad \text{(total emissions)}$$

$\omega$ = \$ per ton $CO_2$ (social cost)
$D = \omega Z$ (climate damage cost)
$T$ = transfer financed by $\tau_j^*$
$\tau_j^* = \omega\phi_j$ Pigouvian tax rule

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# How the Pieces Fit Together

$$E_j \xrightarrow{\phi_j} Z \xrightarrow{\omega} D = \omega Z \xrightarrow{\tau_j = \omega\phi_j} \text{Fuel mix shifts} \rightarrow \downarrow Z, \text{ revenue } T \rightarrow \text{households}$$

Key point: since damages $D$ represent *resource drain*. From this, available consumption satisfies:

$$C = Y - D.$$

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# Equilibrium - Household

The representative household takes prices $\{w, r\}$, firm profits $\Pi$, and government transfer $T$ as given, and chooses aggregate consumption $C$ and labour supply $\ell$.

**Household Problem:**

$$\max_{C,\ell} \quad \underbrace{\frac{C^{1-\gamma}}{1-\gamma}}_{\text{utility from consumption}} - \underbrace{\frac{\ell^{1+\sigma}}{1+\sigma}}_{\text{disutility from labour}}$$

s.t.

$$C = w\,\ell + r\,K + \Pi + T.$$

**Labour Supply:**

$$\ell^{\sigma} = w\,C^{-\gamma} \quad \Longleftrightarrow \quad \ell = \left(\frac{w}{C^{\gamma}}\right)^{1/\sigma}.$$

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# Equilibrium - Firm

The firm chooses $K, \ell, \{E_j\}_{j\in\mathcal{J}}$ to solve:

## Firm Production

$$\max_{K,\ell,\{E_j\}_{j\in\mathcal{J}}} \underbrace{Y}_{\text{revenue}} - \underbrace{w\,\ell}_{\text{labour cost}} - \underbrace{r\,K}_{\text{capital cost}} - \underbrace{\sum_{j\in\mathcal{J}}(p_j+\tau_j)\,E_j}_{\text{energy user cost}}$$

s.t.

$$Y = A\,K^{\alpha}\ell^{\beta}E^{\,1-\alpha-\beta} \qquad \text{where} \qquad E = \left(\sum_{j\in\mathcal{J}}\theta_j E_j^{\rho}\right)^{1/\rho}, \quad \rho = \frac{\sigma_E - 1}{\sigma_E}.$$

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# Equilibrium - Government

The government chooses the transfer $T$ and fuel-specific taxes $\{\tau_j\}_{j\in\mathcal{J}}$, subject to:

**Government Budget Constraint:**

$$T = \sum_{j\in\mathcal{J}} \tau_j E_j$$

**Resource Constraint:**

$$C = Y - D \qquad \text{where} \qquad D = \omega \sum_{j\in\mathcal{J}} \phi_j E_j$$

**Pigouvian Policy:**

$$\tau_j^* = \omega\,\phi_j, \qquad \forall j \in \mathcal{J}.$$

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# Parameter Map

| Parameter | Economic meaning | Calibrated from |
|---|---|---|
| $\alpha$ | Capital share in output $Y$ | Canadian national accounts (Statistics Canada) |
| $\beta$ | Labour share in output $Y$ | Labour compensation share in GDP (Statistics Canada) |
| $\sigma_E$ | Elasticity of substitution across fuels in $E$ | Fuel-switching / energy substitution estimates (e.g. Fried 2017) |
| $\theta_j$ | Share weight of fuel $j$ in the energy bundle $E$ | Fuel expenditure shares (Statistics Canada; Natural Resources Canada Comprehensive Energy Use) |
| $\phi_j$ | Emissions factor: tons $CO_2$ per unit of fuel $j$ | Environment and Climate Change Canada greenhouse gas inventory |
| $\omega$ | Social cost of carbon: \$ per ton $CO_2$ | Integrated assessment models (DICE / RICE SCC) |

Fried, S. (2017), "Climate Policy and Innovation: A Quantitative Macroeconomic Analysis."

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# Central Planner Trade Off

## Emissions vs. activity:

- Raising fuel-specific taxes $\tau_j$ makes high-emissions fuels more expensive, so firms reduce use of dirty fuels (high $\phi_j$) and total emissions
- But higher $\tau_j$ also raises the user cost of energy $(p_j + \tau_j)$, which can lower energy $E$, output $Y$, and labour demand $\ell$.

## Government objective:

- The planner trades off:

$$\text{marginal climate benefit } = \omega \times (\text{reduced CO}_2)$$

vs.

$$\text{marginal economic cost } = \text{ lost consumption } C.$$

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# Social Cost of Carbon

- **Social cost of carbon (SCC)** is the present value of the marginal social damages from $CO_2$ emissions; it captures impacts on agriculture, health, infrastructure and ecosystems.
- SCC represents the shadow price for emissions reductions in benefit–cost analysis.
- SCC estimates rely on *integrated assessment models* combining climate processes and economic growth; the DICE model is a seminal example.

[3] Nordhaus, W. D. (2017). *Revisiting the Social Cost of Carbon.* Proceedings of the National Academy of Sciences. The DICE model links global economy and climate dynamics, computing the SCC ($\tilde{\$}31$/ton CO2 in 2015).

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# DICE Model

- **Dynamic Emissions:** The DICE model shows the social cost of carbon (SCC) by modelling the relationship between economic activity, carbon emissions, climate changes, and damage over a dynamic time frame

- **SCC Calculation:** The SCC is typically the derivative of the present value of consumption with respect to $CO_2$ emissions in a given year. This can be caluclated as the ratio of shadow prices in different climate scenarios.

Barrage et Nordhaus (2024), "Policies, projections, and the social cost of carbon: Results from the DICE-2023 model"

# Computable General Equilibrium Models

- **CGE models** provide economy wide representations of production, consumption and trade in a general equilibrium framework.
- They capture substitution possibilities between inputs and outputs and interactions between sectors, taxes and policies.
- CGE models are used to assess the impacts of energy taxes and environmental policies on output, welfare and emissions.

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# Conclusion

- **Welfare Efficient:** We let firms re-optimize the mix, while we return the revenue to households. It's efficient, transparent, and welfare-centred.
- **Consumer Subsidy:** The carbon tax raises revenue that we return to households, supporting consumption, while dirty energy becomes more expensive.
- **Progressive Output:** Our tax allows output to remain strong, and doesn't impede economic growth.

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# Energy, Emissions, and Efficiency

## Optimal Energy Taxation in a Carbon-Constrained Economy

Thank you for your attention!
Questions?

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# FOCs — Household

## Problem

$$\max_{C,\ell} \quad \frac{C^{1-\gamma}}{1-\gamma} - \frac{\ell^{1+\sigma}}{1+\sigma}$$
$$\text{s.t.} \quad C = w\,\ell + rK + \Pi + T.$$

## Lagrangian and FOCs

$$\mathcal{L} = \frac{C^{1-\gamma}}{1-\gamma} - \frac{\ell^{1+\sigma}}{1+\sigma} + \lambda\big(w\ell + rK + \Pi + T - C\big)$$
$$\frac{\partial \mathcal{L}}{\partial C}: C^{-\gamma} - \lambda = 0 \quad \Rightarrow \quad \lambda = C^{-\gamma}$$
$$\frac{\partial \mathcal{L}}{\partial \ell}: -\ell^{\sigma} + \lambda w = 0 \quad \Rightarrow \quad \ell^{\sigma} = \lambda w = w\,C^{-\gamma}.$$

# FOCs — Household Cont.

## Labour Supply

$$\boxed{\ell^{\sigma} = w\,C^{-\gamma} \qquad \Longleftrightarrow \qquad \ell = \left(\frac{w}{C^{\gamma}}\right)^{1/\sigma}}$$

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# FOCs — Firm

## Profit

$$\max_{K,\ell,E} \; AK^{\alpha}\ell^{\beta}E^{1-\alpha-\beta} - rK - w\ell - P_E E,$$

## FOCs

$$\frac{\partial \Pi}{\partial K} = 0 \Rightarrow \boxed{r = \alpha A K^{\alpha-1}\ell^{\beta}E^{1-\alpha-\beta} = \alpha \frac{Y}{K}}$$

$$\frac{\partial \Pi}{\partial \ell} = 0 \Rightarrow \boxed{w = \beta A K^{\alpha}\ell^{\beta-1}E^{1-\alpha-\beta} = \beta \frac{Y}{\ell}}$$

$$\frac{\partial \Pi}{\partial E} = 0 \Rightarrow \boxed{P_E = (1-\alpha-\beta) A K^{\alpha}\ell^{\beta}E^{-\alpha-\beta} = (1-\alpha-\beta) \frac{Y}{E}}$$

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# FOCs — Energy

*Intuition:* Minimize the cost of delivering $E$ using fuels $E_j$ that are imperfect substitutes. The FOCs yield CES shares and the energy price index $P_E$. Let $c_j \equiv p_j + \tau_j$ be the user cost of fuel $j$.

## Cost Minimization

$$\min_{\{E_j\}_{j\in\mathcal{J}}} \sum_{j\in\mathcal{J}} c_j E_j \quad \text{s.t.} \quad E = \Big(\sum_{j\in\mathcal{J}} \theta_j E_j^{\rho}\Big)^{1/\rho}, \qquad \rho = \frac{\sigma_E - 1}{\sigma_E}.$$

## Lagrangian and Stationarity

$$\mathcal{L} = \sum_j c_j E_j + \lambda\Big[E - \Big(\sum_m \theta_m E_m^{\rho}\Big)^{1/\rho}\Big]$$

$$\frac{\partial \mathcal{L}}{\partial E_j} = 0 \;\Rightarrow\; c_j = \lambda\,\theta_j\, E^{1-\rho} E_j^{\rho-1}.$$

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# FOCs - Fuel by Fuel

The value of the marginal product of each fuel equals its user cost. This is equivalent to the CES share system.

## Direct FOC for $E_j$

With $Y = AK^{\alpha}\ell^{\beta}E^{1-\alpha-\beta}$ and $E = (\sum_m \theta_m E_m^{\rho})^{1/\rho}$,

$$\frac{\partial Y}{\partial E_j} = \frac{\partial Y}{\partial E}\cdot\frac{\partial E}{\partial E_j} = (1-\alpha-\beta)\frac{Y}{E}\cdot\left(\theta_j E^{1-\rho}E_j^{\rho-1}\right) = (1-\alpha-\beta)\,Y\,\theta_j\,E^{-\rho}\,E_j^{\rho-1}.$$

The firm's FOC sets value of marginal product to user cost:

$$\boxed{(1-\alpha-\beta)\,Y\,\theta_j\,E^{-\rho}\,E_j^{\rho-1} \;=\; p_j+\tau_j \;=\; c_j}$$

which is equivalent to the CES demand/share conditions on the previous slide.

# FOCs — Pigouvian Tax Rule

Social planner internalizes the marginal damage $\omega\phi_j$ from fuel $j$. Decentralization with a per-unit tax $\tau_j$ replicates the planner's FOC when $\tau_j = \omega\phi_j$.

## Planner vs. Firm

$$\textbf{Private FOC:} \quad (1-\alpha-\beta)\, Y\, \theta_j\, E^{-\rho}\, E_j^{\rho-1} \;=\; p_j + \tau_j.$$

$$\textbf{Social FOC:} \quad (1-\alpha-\beta)\, Y\, \theta_j\, E^{-\rho}\, E_j^{\rho-1} \;=\; p_j + \underbrace{\omega\,\phi_j}_{\text{marginal external damage}}.$$

## Pigouvian Tax (Decentralization)

$$\boxed{\tau_j^* \;=\; \omega\,\phi_j, \qquad \forall j \in \mathcal{J}}$$

This aligns private and social marginal costs fuel-by-fuel, ensuring the competitive allocation matches the planner's allocation.