TAXING THE FUEL OF GROWTH

Carbon Pricing, Capital Dynamics, and Energy Dependence

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All errors are my own

Code, and additional materials are available at:
`https://github.com/TadhgTM/Energy-Taxation-Project`

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Abstract

I study optimal carbon taxation in a dynamic general equilibrium model calibrated to the Canadian economy, where firms choose among oil, gas, and clean energy inputs and the economy adjusts over time through investment and capital reallocation. I solve for the welfare-maximizing carbon tax by evaluating present-value utility along the full transition path implied by competitive within-period equilibria linked by capital accumulation and household consumption smoothing.

In the baseline calibration, the optimal tax is $\tau^* \approx 0.06$ in model units. Using an emissions-intensity mapping based on Canadian energy-related CO_2 per unit of real GDP, this corresponds to roughly $400 CAD per tonne of CO_2. Relative to the no-tax benchmark, the optimal policy reduces cumulative emissions by more than 45%, while output and consumption decline smoothly over time rather than discretely, reflecting intertemporal adjustment through investment and consumption smoothing.

Two sensitivities are central. Increasing the marginal damage weight raises τ^* in a predictable way because investment reallocation remains an active margin even when aggregate output falls. By contrast, the optimal tax is highly sensitive to the economy's energy share in production: when energy is a small input, carbon pricing mainly reshapes the energy mix with limited macroeconomic disruption, while in more energy-intensive economies the same wedge functions like a large distortion to a core input and can trigger sharp output losses. These results imply that carbon pricing is most effective when complemented by policies that expand low-carbon capacity and reduce adjustment frictions in energy-intensive economies.

"Climate change is the result of the greatest market failure the world has seen. The evidence on the seriousness of the risks from inaction or delayed action is now overwhelming."[Stern, 2007]

- Nicholas Stern

1 Introduction

Climate change poses a fundamental challenge for economies that rely heavily on energy-intensive production. Carbon pricing is widely viewed as the canonical policy instrument for addressing this externality; yet, there remains substantial disagreement about the appropriate level of carbon taxes and the economic costs they impose. Much of this disagreement reflects the fact that climate policy operates over long horizons and interacts with investment, capital accumulation, and structural change. Static analysis, while analytically transparent, abstracts from these dynamic mechanisms and therefore provides an incomplete guide for policy design.

Beyond environmental risk, climate change is also a macroeconomic problem: physical damages, adaptation costs, and policy-driven transition risks can all reduce effective productivity and measured output over long horizons. For an energy-producing and trade-exposed economy such as Canada, the key policy question is not whether carbon should be priced, but how aggressively it can be priced once investment, capital turnover, and technology substitution respond endogenously over time.

This paper studies optimal carbon taxation in a dynamic general equilibrium framework calibrated to the structure of an energy-intensive economy. The model features heterogeneous energy technologies, emissions-driven welfare damages, and forward-looking households that choose consumption and saving subject to capital accumulation. Carbon taxes affect production costs directly through energy prices, but also indirectly by altering investment incentives and the future capital stock. By explicitly modelling these channels, the paper quantifies how optimal policy depends on both the valuation of climate damages and the economy's capacity to substitute away from emissions-intensive energy.

The core question is straightforward: *what level of carbon taxation maximizes social welfare once the dynamic adjustment of the economy is taken seriously?* Answering this question requires moving beyond static Pigouvian logic. Consider the canonical static Pigouvian benchmark: with a fixed production structure and a flow externality $D(Z)$, the efficient per-unit emissions tax equals marginal damages, $\tau^P = D'(Z)$, evaluated at the efficient allocation. In that environment, the tax mainly induces a within-period reallocation of inputs and a contemporaneous reduction in emissions. The present paper departs from this benchmark by allowing policy to change the intertemporal path of capital and the composition of energy production, so that the welfare effects of taxation depend on transition dynamics rather than on a single-period comparison. In a dynamic economy, emissions reductions are achieved gradually through changes in energy use and capital allocation. The welfare cost of carbon pricing therefore depends not only on contemporaneous output losses but also on how policy reshapes the transition path of consumption, investment, and production.

The analysis delivers three main components. First, it provides a fully articulated dynamic general equilibrium model in which carbon taxation, energy choice, and capital accumulation are jointly determined. This allows welfare to be evaluated as a present value over the entire transition path rather than at a single point in time. Second, the paper characterizes how the optimal carbon tax responds to uncertainty about climate damages and to uncertainty about the economy's structural reliance on energy. These comparative statics reveal that uncertainty about production structure can matter as much as, or more than, uncertainty about the social cost of carbon. Third, the model yields policy-relevant benchmarks by mapping the optimal tax into real-world carbon prices in a transparent and internally consistent manner.

The key results can be summarized succinctly. In the baseline calibration, the optimal carbon tax is positive and economically large when expressed in policy units. Higher valuations of climate damages lead to systematically higher optimal taxes and lower cumulative emissions, reflecting the forward-looking nature of welfare maximization. At the same time, the optimal policy is highly sensitive to the energy share in production. Economies that are more energy intensive face sharper trade-offs between emissions reduction and economic activity, limiting the scope for aggressive taxation in the absence of complementary policies that ease substitution.

These findings have direct implications for climate policy design. They suggest that debates centered solely on estimating the social cost of carbon miss a crucial dimension of the problem. The effectiveness and desirability of carbon pricing depend critically on the economy's technological and structural characteristics, particularly its ability to reallocate capital toward clean energy over time. Policies that expand this capacity can substantially reduce the welfare cost of decarbonization.

The remainder of the paper proceeds as follows. Section 1.1 reviews the related literature. Section 2 presents the dynamic general equilibrium model. Section 3 describes the calibration strategy and numerical solution method. Section 4 reports the baseline results and characterizes the transition-path effects of carbon taxation. Section 5 examines robustness with respect to climate damages and energy dependence. Section 6 benchmarks the model-implied tax against observed carbon prices. Section 7 discusses limitations and extensions. Section 8 concludes.

1.1 Related Literature

This paper builds from several strands of the climate economics literature. The foundational work on dynamic carbon taxation includes [Nordhaus, 1991], who developed the DICE model linking economic growth and climate change, and [Golosov et al., 2014], who derived closed-form expressions for optimal carbon taxes in a dynamic stochastic general equilibrium framework. My approach differs by embedding a rich energy sector with heterogeneous technologies, allowing the model to capture substitution dynamics that are absent in more aggregate treatments.

The analysis also relates to the literature on directed technical change and the energy transition. [Acemoglu et al., 2012] show that optimal policy combines carbon taxes with research subsidies to redirect innovation toward clean technologies. [Fried, 2018] studies how carbon taxes interact with investment in energy-producing capital. My model abstracts from endogenous innovation but incorporates capital reallocation across energy technologies, capturing an important margin of adjustment that static models miss.

Recent work has emphasized the role of transition dynamics in climate policy. [Barrage, 2020] characterizes optimal carbon taxes when climate damages affect both utility and productivity, finding that the optimal tax depends critically on the damage specification. [Hassler et al., 2021] study the macroeconomic effects of carbon taxation in a model with directed technical change and capital adjustment costs. This papers goal is to quantify how optimal policy depends on energy intensity in a tractable dynamic framework calibrated to Canadian data.

Finally, this paper connects to the empirical literature on carbon pricing. [Metcalf, 2019] surveys the economic effects of carbon taxes, finding that well-designed policies can reduce emissions without large GDP losses. [Cullenward and Victor, 2020] documents the gap between carbon prices implied by integrated assessment models and actual policy levels. My benchmarking exercise links to this discussion by mapping model-implied taxes to real-world units in a transparent manner.

2 Model

2.1 Overview

I study a finite-horizon dynamic general equilibrium economy with endogenous capital accumulation and an energy input that is produced using three technologies: oil, gas, and clean. In each period, I solve a static competitive equilibrium conditional on the period capital stock and a carbon tax τ. I then link periods through the law of motion for capital and an Euler equation for consumption, imposing a terminal capital condition at horizon T.

2.2 Agents and markets

Time is discrete, $t = 0, 1, \ldots, T-1$. There is a representative household, a competitive final-good firm, and three competitive energy producers indexed by $j \in \{\text{oil}, \text{gas}, \text{clean}\}$. The final good is the numeraire, meaning its price is normalized to one in every period, which pins down the overall price level and makes all other prices interpretable in units of the final good.

The state variable is the aggregate physical capital stock K_t. Labour is supplied elastically by the household and is allocated across the final-good firm and the energy sectors.

2.3 Final-good production

The final-good firm uses capital $K_{c,t}$, labour $L_{c,t}$, and a composite energy input E_t to produce output:

$$Y_t \;=\; A_{\text{final}}\, K_{c,t}^{a_c}\, L_{c,t}^{b_c}\, E_t^{e_c}, \tag{1}$$

with Cobb–Douglas shares satisfying $a_c + b_c + e_c = 1$. This restriction is enforced in the energy-share sensitivity exercises by setting

$$a_c \;=\; 1 - b_c - e_c. \tag{2}$$

2.4 Energy technologies

Each energy technology j produces energy $E_{j,t}$ using Cobb-Douglas capital and labour:

$$E_{j,t} \;=\; A_j\, K_{j,t}^{a_j}\, L_{j,t}^{1-a_j}, \qquad j \in \{\text{oil}, \text{gas}, \text{clean}\}. \tag{3}$$

2.5 Energy aggregation

The composite energy input is a CES aggregate of the three technology outputs:

$$E_t \;=\; \left(\sum_j \omega_{E,j}\, E_{j,t}^{\rho}\right)^{\frac{1}{\rho}}, \qquad \rho \equiv \frac{\eta - 1}{\eta}, \tag{4}$$

where $\eta > 0$ is the elasticity of substitution and $\omega_{E,j} > 0$ are share weights (normalized to sum to one in calibration). In the limiting Cobb–Douglas case $\eta \to 1$, the aggregator becomes

$$E_t \;=\; \exp\left(\sum_j \omega_{E,j} \log E_{j,t}\right). \tag{5}$$

Let $p_{j,t}$ denote the price of energy from technology j and $p_{E,t}$ the CES price index. The dual price index consistent with (4) is

$$p_{E,t} \;=\; \left(\sum_j \omega_{E,j}\, p_{j,t}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{6}$$

with the Cobb–Douglas limit $p_{E,t} = \exp(\sum_j \omega_{E,j} \log p_{j,t})$ when $\eta \to 1$.

2.6 Emissions and carbon taxation

Emissions are linear in technology outputs:

$$Z_t \;=\; \sum_j \phi_j\, E_{j,t}, \tag{7}$$

where $\phi_{\text{clean}} = 0$ and $\phi_{\text{oil}}, \phi_{\text{gas}} > 0$. I normalize $\phi_{\text{oil}} = 1$ and interpret ϕ_{gas} as relative carbon intensity.

A carbon tax τ is levied per unit of emissions. In the static energy-sector first-order conditions, the tax enters as a wedge in the net price received by each energy producer:

$$p_{j,t}^{\text{net}} \;=\; p_{j,t} - \tau\, \phi_j. \tag{8}$$

This makes clear that τ is a *unit wedge* (in the same units as prices), not an ad valorem "percent tax."

Tax revenues are rebated lump-sum to the household:

$$T_t \;=\; \tau\, Z_t. \tag{9}$$

2.7 Households and within-period equilibrium

Within each period t, given the capital stock K_t and policy (τ, ω), I solve a static competitive equilibrium. In the static equilibrium, the household consumes its factor income plus the lump-sum rebate:

$$C_t \;=\; w_t L_t + r_t K_t + T_t, \tag{10}$$

and supplies labour according to the intratemporal first-order condition

$$w_t \;=\; \chi\, C_t^{\gamma}\, L_t^{\sigma}, \tag{11}$$

where (γ, σ, χ) are preference parameters.

Period utility is CRRA in consumption net of a linear emissions penalty:

$$u(C_t, L_t, Z_t) \;=\; \frac{C_t^{1-\gamma}}{1-\gamma} \;-\; \chi\,\frac{L_t^{1+\sigma}}{1+\sigma} \;-\; \omega\, Z_t, \tag{12}$$

with the log case when $\gamma = 1$. The parameter ω is the marginal damage weight (a reduced-form proxy for the social cost of carbon) in the flow welfare criterion used to evaluate policy.

2.8 Capital accumulation and feasibility

Output is allocated between consumption and investment:

$$I_t \;=\; Y_t - C_t, \tag{13}$$

and I impose feasibility by requiring $I_t \geq 0$ (equivalently $C_t \leq Y_t$), implemented numerically by rejecting candidate paths that imply negative investment.

Capital evolves according to

$$K_{t+1} \;=\; (1-\delta)K_t + I_t, \tag{14}$$

with depreciation rate $\delta \in (0, 1)$.

2.9 Intertemporal consumption dynamics

To connect the sequence of static equilibria across time, I use a consumption Euler equation based on the competitive return to capital:

$$C_{t+1} \;=\; C_t \left[\beta\,(1 - \delta + r_{t+1})\right]^{\frac{1}{\gamma}}, \tag{15}$$

where $\beta \in (0, 1)$ is the discount factor and γ is the CRRA coefficient. Because r_{t+1} depends on K_{t+1} through the next period's static equilibrium, I compute C_{t+1} by iterating on (15) using the solved equilibrium return r_{t+1}, imposing the additional feasibility cap

$$C_{t+1} \;\leq\; 0.95\, Y_{t+1}. \tag{16}$$

2.10 Terminal condition and dynamic equilibrium

I impose a terminal capital target $K_T = \bar{K}$ and choose the initial consumption level C_0 to satisfy this terminal condition. Formally, given initial capital K_0 and policy (τ, ω), I define the terminal error

$$\varepsilon(C_0; \tau, \omega) \equiv K_T(C_0; \tau, \omega) - \bar{K}, \tag{17}$$

and I select C_0 such that $\varepsilon(C_0; \tau, \omega) = 0$ using a bracketed root-finding routine over $\log C_0$.

The resulting object is a dynamic equilibrium path

$$\{K_t, C_t, Y_t, E_t, Z_t, I_t\}_{t=0}^{T-1}$$

together with the associated sequence of within-period competitive equilibria.

2.11 Welfare criterion and optimal tax

For a given tax τ and damage weight ω, I evaluate policy using the present value of period utility along the equilibrium transition path:

$$W(\tau; \omega) = \sum_{t=0}^{T-1} \beta^t u(C_t, L_t, Z_t), \tag{18}$$

where $u(\cdot)$ is given in (12) and (C_t, L_t, Z_t) are the equilibrium outcomes under (τ, ω).

The welfare-maximizing carbon tax is

$$\tau^*(\omega) \in \arg\max_{\tau \in [\underline{\tau}, \overline{\tau}]} W(\tau; \omega), \tag{19}$$

which I compute numerically via a global grid search over $\tau \in [\underline{\tau}, \overline{\tau}]$ followed by a bounded local refinement using L-BFGS-B.

2.12 Equilibrium existence and numerical discipline

This model is solved as a sequence of within-period static competitive equilibria linked by capital accumulation and a consumption Euler equation. Because the policy experiments involve potentially large wedges (the carbon tax) and strong relative-price movements across energy technologies, it is important to clarify (i) when a within-period equilibrium exists and is well behaved, and (ii) how the numerical implementation enforces feasibility without imposing substantive economic assumptions.

Within-period equilibrium. Fix a period t, an inherited aggregate capital stock $K_t > 0$, and a policy (τ, ω). The within-period allocation solves a standard competitive equilibrium with constant-returns technologies and strictly positive share parameters. Under Cobb–Douglas production in the final-good sector and each energy sector, interior input demands exist whenever factor prices are positive and the net-of-tax energy prices received by producers are non-negative:

$$p_{j,t}^{\text{net}} = p_{j,t} - \tau\phi_j \geq 0 \qquad \text{for all } j \in \{\text{oil}, \text{gas}, \text{clean}\}.$$

Given $K_t > 0$ and positive productivity parameters $\{A_{\text{final}}, A_j\}$, profit maximization implies well-defined factor demands and energy supply, and the CES aggregator delivers a strictly positive composite energy input E_t and price index $p_{E,t}$ as long as $\omega_{E,j} > 0$ and $\eta > 0$. Market clearing for capital and labour then pins down (r_t, w_t) jointly with the household intratemporal condition. In the parameter region considered in the calibration and counterfactuals, the computed equilibria satisfy positivity of quantities and prices and achieve small residuals in the equilibrium system, indicating that the within-period equilibrium is well behaved.

Dynamic feasibility and intertemporal linkage. Across periods, feasibility requires non-negative investment and a non-degenerate capital stock:

$$I_t = Y_t - C_t \geq 0, \qquad K_{t+1} = (1-\delta)K_t + I_t \geq 0.$$

Because the dynamic path is constructed by combining the Euler equation with period-by-period equilibrium returns, certain candidate initial consumption values C_0 can generate infeasible paths (e.g., $C_t > Y_t$ implying $I_t < 0$). Such candidates are treated as inadmissible and excluded from the welfare evaluation. This is not an additional economic restriction; it is the enforcement of the model's accounting identities and non-negativity constraints on capital accumulation.

Terminal condition and uniqueness of the computed path. To rule out explosive or degenerate accumulation paths in a finite-horizon implementation, I impose a terminal capital target $K_T = \bar{K}$ and choose the initial consumption C_0 to satisfy it. Given (τ, ω), the implied terminal capital $K_T(C_0; \tau, \omega)$ is monotone in C_0 over the admissible region in the sense that higher initial consumption reduces investment and therefore lowers terminal capital. This monotonicity makes the terminal condition a well-posed scalar root-finding problem. In practice, the root is obtained via bracketed search over $\log C_0$, which guarantees convergence to a solution whenever the terminal error changes sign over the bracket.

Numerical guards and interpretation. Two implementation details are used to improve numerical stability in regions where equilibrium objects become ill-conditioned.

First, I parameterize strictly positive choice variables (e.g., sectoral factor allocations) in logs. This prevents the solver from proposing negative capital or labour allocations during iteration and reduces spurious failures due to boundary violations. Log-parameterizing enforces positivity and improves conditioning of the nonlinear solver. It does not "normalize" the variance of simulated outcomes; it only changes the parameterization of the search space to reduce boundary pathologies (e.g., negative factor allocations during iteration).

Second, I impose a mild consumption feasibility cap,

$$C_t \leq \varphi Y_t, \qquad \varphi \in (0,1),$$

when iterating on the Euler equation. This cap is a numerical regularization that prevents the algorithm from stepping into regions where $I_t < 0$ and the implied return schedule becomes discontinuous due to feasibility rejection. Economically, it simply restricts attention to the admissible set where the accounting identities hold and capital remains non-degenerate. In the reported results, the optimal policies and qualitative comparative statics are not driven by the cap binding systematically; rather, it improves robustness of the path-construction routine in high-tax or high-energy-dependence experiments.

Scope. The analysis therefore focuses on the subset of (τ, ω) for which a competitive equilibrium exists period-by-period, the induced dynamic path respects feasibility, and equilibrium residuals are small. Welfare comparisons are computed only over this admissible policy set, ensuring that the reported optima reflect economically coherent equilibria rather than numerical extrapolation outside the model's feasible region.

3 Calibration and Solution

This section describes how the model is disciplined by data and how equilibrium outcomes and optimal policy are computed. The goal is to make transparent which features of the Canadian economy are built into the model and how dynamic welfare comparisons are conducted, without relying on implementation-specific details.

3.1 Calibration strategy

I calibrate the model to capture three core features of the Canadian economy: (i) aggregate production structure, (ii) energy use and composition, and (iii) capital accumulation.

The final good is normalized as the numeraire, and aggregate labour supply is normalized to one. The initial capital stock and terminal capital target are set equal, implying a stationary environment absent policy intervention. This normalization ensures that welfare comparisons reflect transitional dynamics rather than arbitrary differences in scale.

Final-good production follows a Cobb–Douglas structure with factor shares

$$a_c = 0.247, \qquad b_c = 0.655, \qquad e_c = 0.098,$$

so that energy accounts for just under ten percent of gross output[Natural Resources Canada, 2025]. These shares match Canadian income and expenditure data and determine how strongly energy prices transmit into production costs.

Energy intensity is disciplined by targeting the ratio of energy use to output,

$$\frac{E}{Y} = 6.9,$$

which corresponds to approximately 6.9 megajoules of energy per unit of real GDP[Natural Resources Canada, 2025]. Together with the energy share e_c, this target pins down the scale of energy prices in equilibrium.

3.2 Energy technologies and emissions

The energy sector consists of three technologies: oil, gas, and clean. Each technology uses capital and labour to produce energy, and these outputs are aggregated into a composite energy input through a CES function. The elasticity of substitution across technologies is set to $\eta = 2.0$, allowing for meaningful but imperfect substitution when relative prices change.

Technology-specific productivity parameters are chosen so that, in the absence of policy, the model reproduces Canada's observed energy supply mix[Statistics Canada, 2025b]:

$$52.4\% \text{ oil}, \qquad 39.6\% \text{ gas}, \qquad 8.0\% \text{ clean}.$$

Emissions are proportional to energy production:

$$Z_t = \sum_j \phi_j E_{j,t}.$$

I normalize oil emissions intensity to one, set gas emissions intensity relative to oil, and assign zero emissions to clean energy. This normalization is purely internal and does not affect behavior; it provides a convenient reference for mapping the model tax into physical units later.

3.3 Preferences and climate damages

Households have constant-relative-risk-aversion preferences over consumption and disutility from labour. Climate damages enter welfare directly as a linear penalty proportional to aggregate emissions:

$$u(C_t, L_t, Z_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\sigma}}{1+\sigma} - \omega Z_t.$$

The parameter ω governs the marginal welfare cost of emissions and serves as a reduced-form proxy for the social cost of carbon. Importantly, damages affect welfare but do not alter production possibilities directly. This separation allows me to isolate how valuation of damages interacts with economic adjustment costs.

Interpreting and disciplining ω. Because damages enter flow utility linearly as $-\omega Z_t$, the consumption-equivalent marginal damage (a model social cost of carbon) is

$$\mathrm{SCC}_t^{\mathrm{model}} \equiv \frac{\partial u/\partial Z_t}{\partial u/\partial C_t} = \frac{\omega}{u_C(C_t)} = \omega\, C_t^{\gamma} \quad \text{under CRRA.}$$

In the baseline calibration, consumption is normalized near one in the no-tax benchmark, so ω can be read as approximately the marginal damage in units of the final good per unit of emissions. I set $\omega = 0.08$ to represent a baseline damage valuation in this consumption-equivalent metric, and I report robustness over a wide range of ω to make transparent how the optimal tax scales with damage valuations.

3.4 Dynamic structure

The economy evolves over time through capital accumulation:

$$K_{t+1} = (1-\delta)K_t + I_t, \qquad I_t = Y_t - C_t.$$

In each period, given the inherited capital stock and the carbon tax τ, firms and households interact in competitive markets. These within-period equilibria determine output, factor prices, energy use, and emissions. Periods are then linked through the capital law of motion and household consumption smoothing.

Consumption evolves according to an Euler condition that balances current consumption against expected returns to capital. This intertemporal channel is central to the analysis: climate policy affects not only current production but also investment incentives and the pace of capital accumulation.

A terminal capital condition is imposed to rule out explosive or degenerate paths. Initial consumption is chosen so that the economy converges smoothly back to its long-run capital level by the end of the horizon.

3.5 Welfare evaluation

For a given policy (τ, ω), welfare is evaluated as the present value of period utility along the transition path:

$$W(\tau;\omega) = \sum_{t=0}^{T-1} \beta^t u(C_t, L_t, Z_t).$$

This criterion captures three forces simultaneously: reductions in emissions, distortions to production through higher energy prices, and intertemporal trade-offs arising from investment and consumption smoothing. As a result, the optimal carbon tax reflects both static efficiency considerations and dynamic adjustment costs.

3.6 Optimal tax computation

For each value of the damage parameter ω, I compute the welfare-maximizing carbon tax by searching over a bounded policy space:

$$\tau^*(\omega) = \arg \max_{\tau \in [0, \bar{\tau}]} W(\tau; \omega).$$

The optimal tax is identified by first evaluating welfare across a broad grid of candidate tax rates and then refining the solution locally around the best-performing policy. Only policies that generate well-behaved transition paths and satisfy feasibility constraints are considered admissible.

This procedure is repeated both for the baseline calibration and for counterfactual economies in which key structural parameters, such as the energy share in production, are altered.

4 Baseline Results

This section presents the baseline results from the calibrated dynamic model. I first report the welfare-maximizing carbon tax under the baseline damage specification, then describe the associated transition-path dynamics for output, consumption, investment, and emissions. I conclude by highlighting how these results differ from static general equilibrium predictions.

4.1 Baseline optimal carbon tax

Under the baseline calibration, with damage weight $\omega = 0.08$, the welfare-maximizing carbon tax is

$$\tau^* \approx 0.06.$$

This magnitude is economically meaningful and substantially larger than what would be implied by a static general equilibrium analysis. The optimal tax reflects the trade-off between three forces: reduced emissions, higher contemporaneous production costs, and the intertemporal response of investment and capital accumulation.

Unlike in static environments, the optimal policy is not pinned down by a near-flat welfare function around zero. Instead, welfare exhibits a clear interior maximum at a positive tax rate, indicating that emissions pricing generates net welfare gains once dynamic adjustment is taken into account.

4.2 Transition-path dynamics

The introduction of the optimal carbon tax induces a non-trivial transition path. Energy becomes more expensive for firms, raising production costs in the short run. However, this price signal triggers a gradual reallocation of capital toward cleaner energy technologies and away from emissions-intensive production.

Aggregate output declines modestly relative to the no-tax benchmark, reflecting higher energy costs. In the baseline calibration, average output along the transition path is approximately 10% below the no-tax level. This contraction is not abrupt; rather, it unfolds smoothly as capital adjusts.

Consumption responds differently from output. Because households smooth consumption intertemporally, part of the adjustment occurs through reduced investment rather than large immediate consumption losses. Average consumption declines by roughly 13% along the transition path, with the largest reductions occurring early in the transition when capital is being reallocated.

Investment dynamics are central to the policy's effects. The carbon tax lowers the after-tax return to capital in energy-intensive activities, reducing investment incentives in the short run. As a result, the average investment rate rises slightly, reflecting a shift toward cleaner capital rather than an overall collapse in accumulation. Capital converges back toward its long-run level by the terminal period, consistent with the imposed terminal condition.

4.3 Emissions and environmental outcomes

Figure 1 illustrates the transition dynamics by comparing paths under the optimal tax ($\tau^* \approx 0.06$) against the no-tax benchmark ($\tau = 0$). The six panels display output, consumption, capital, energy use, emissions, and investment over the model horizon. Several features are noteworthy. First, output and consumption decline smoothly rather than abruptly, reflecting the economy's ability to spread adjustment costs over time. Second, emissions fall dramatically under the optimal policy, by more than 45% cumulatively, while the output decline is modest in comparison. Third, capital converges to approximately the same terminal level under both policies. Finally, investment remains positive throughout, indicating that decarbonization is achieved through reallocation rather than disinvestment.

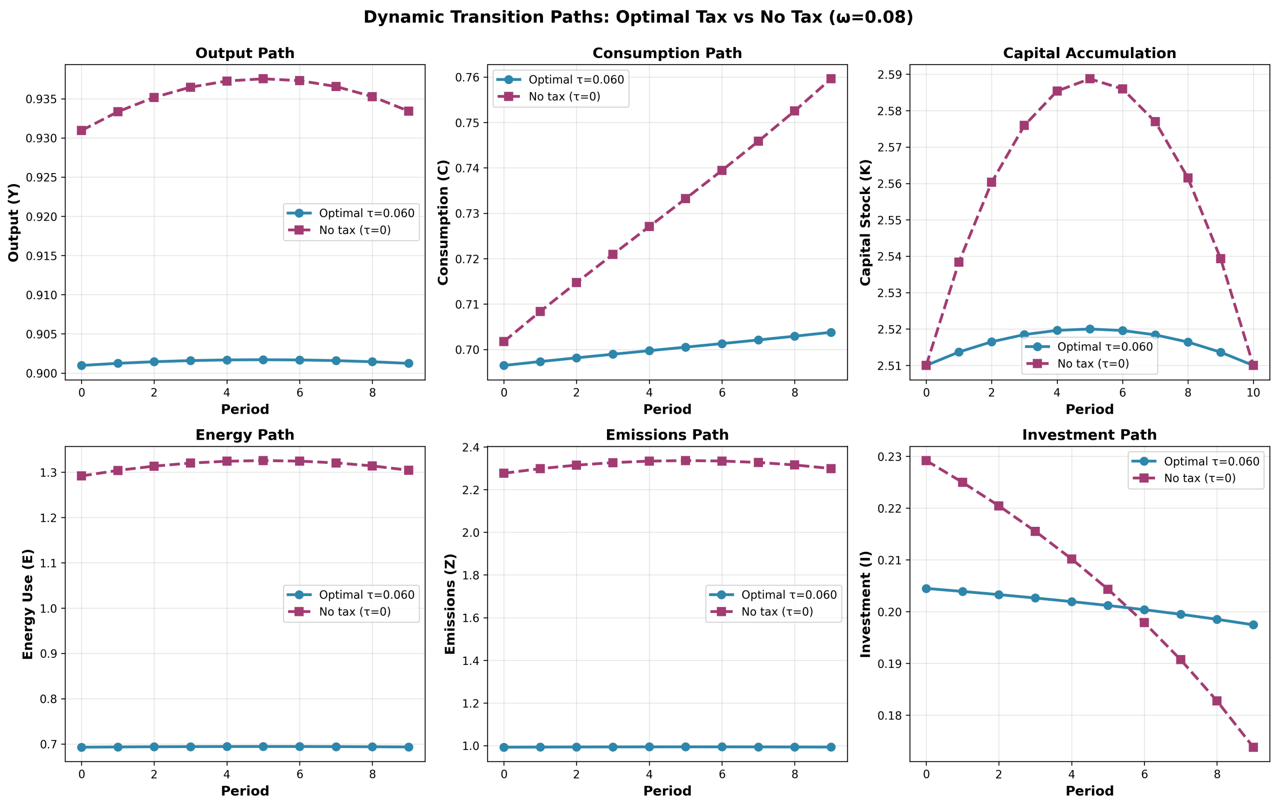

Figure 1: Transition paths under optimal carbon tax vs. no tax. The optimal tax ($\tau^* \approx 0.06$) reduces emissions substantially while generating modest output and consumption losses that are smoothed over time.

The environmental impact of the baseline policy is substantial. Aggregate emissions decline sharply relative to the no-tax equilibrium, with cumulative emissions over the transition path falling by more than 45%.

This reduction is achieved through two channels. First, firms substitute away from emissions-intensive energy inputs toward clean energy as relative prices change. Second, the modest contraction in economic activity reduces energy demand overall. Importantly, the bulk of emissions reductions comes from substitution rather than output collapse, indicating that the policy operates primarily through reallocation rather than deindustrialization.

4.4 Welfare decomposition and intuition

Despite lower output and consumption along the transition path, overall welfare increases relative to low or zero tax policies. The welfare gains arise because emissions reductions are valued at the margin more than the foregone consumption, particularly when future damages are discounted but persistent.

The dynamic structure is essential for this result. In a static model, higher energy prices translate almost one-for-one into consumption losses, leaving little room for welfare improvement unless damages are extreme. In contrast, the dynamic model allows the economy to adjust gradually through capital accumulation, spreading the costs of decarbonization over time.

As a result, the optimal tax balances near-term efficiency losses against long-run welfare gains from lower emissions, producing an interior optimum at a positive tax rate.

4.5 Comparison with static intuition

The baseline results highlight a key limitation of static general equilibrium analyses. Static models implicitly assume that capital and technology are fixed, forcing all adjustment to occur contemporaneously. Under those assumptions, optimal carbon taxes tend to be small and weakly identified.

By contrast, in the dynamic model, capital accumulation provides an additional margin of adjustment. Firms respond to the carbon tax by reallocating investment, and households absorb transitional costs through consumption smoothing. This intertemporal flexibility substantially raises the welfare payoff to emissions pricing and leads to a higher optimal tax.

In short, once dynamic adjustment is taken seriously, carbon taxation emerges as a quantitatively important policy instrument rather than a marginal correction.

5 Robustness

This section examines the robustness of the baseline results along two dimensions. First, I study sensitivity to the climate-damage parameter ω, which governs the marginal welfare cost of emissions. Second, I analyze sensitivity to the economy's structural dependence on energy, captured by the energy share in final production, e_c. These exercises distinguish uncertainty about the *valuation* of emissions damages from uncertainty about the *cost structure* of abatement.

5.1 Sensitivity to climate damages

5.1.1 Design

I vary the damage-weight parameter ω over a wide range, from 0.02 to 0.40. This range spans relatively low marginal damages as well as extremely high valuations consistent with catastrophic climate risk. For each value of ω, I recompute the welfare-maximizing carbon tax $\tau^*(\omega)$ using the same dynamic equilibrium procedure as in the baseline.

Changing ω affects welfare comparisons but does not alter production technologies or substitution possibilities. This exercise therefore isolates how the optimal policy responds to changes in the perceived benefits of emissions reduction, holding abatement costs fixed.

5.1.2 Results

Table 1 reports the results. The optimal carbon tax is increasing in the damage weight. When $\omega = 0.02$, the optimal tax is relatively small, $\tau^* \approx 0.014$. As ω rises to 0.30, the optimal tax increases steadily to approximately 0.095. At very high damage weights ($\omega = 0.40$), the optimal tax reaches the upper bound of the policy space.

Higher values of ω are associated with monotonic declines in welfare, output, and consumption, alongside large reductions in cumulative emissions. Across the full range considered, total emissions fall by more than 85%. Investment rates increase modestly with ω, reflecting stronger incentives to reallocate capital toward cleaner production despite declining aggregate output.

These patterns contrast sharply with static models in which the optimal tax may become insensitive to damages once technological limits bind. In the dynamic setting, capital accumulation and investment responses remain active margins of adjustment, allowing higher valuations of climate damages to justify increasingly aggressive policies.

Figure 2 visualizes these relationships by plotting the welfare function for four different damage weights. Each curve displays the characteristic concave shape expected from an interior optimum, with the peak shifting rightward as ω increases. The figure makes clear that higher damage valuations both reduce overall welfare (shifting curves downward) and justify higher optimal taxes (shifting peaks rightward). Importantly, the curves remain well-defined and smooth across the policy space, indicating that the welfare-maximizing tax is economically identified rather than a corner solution.

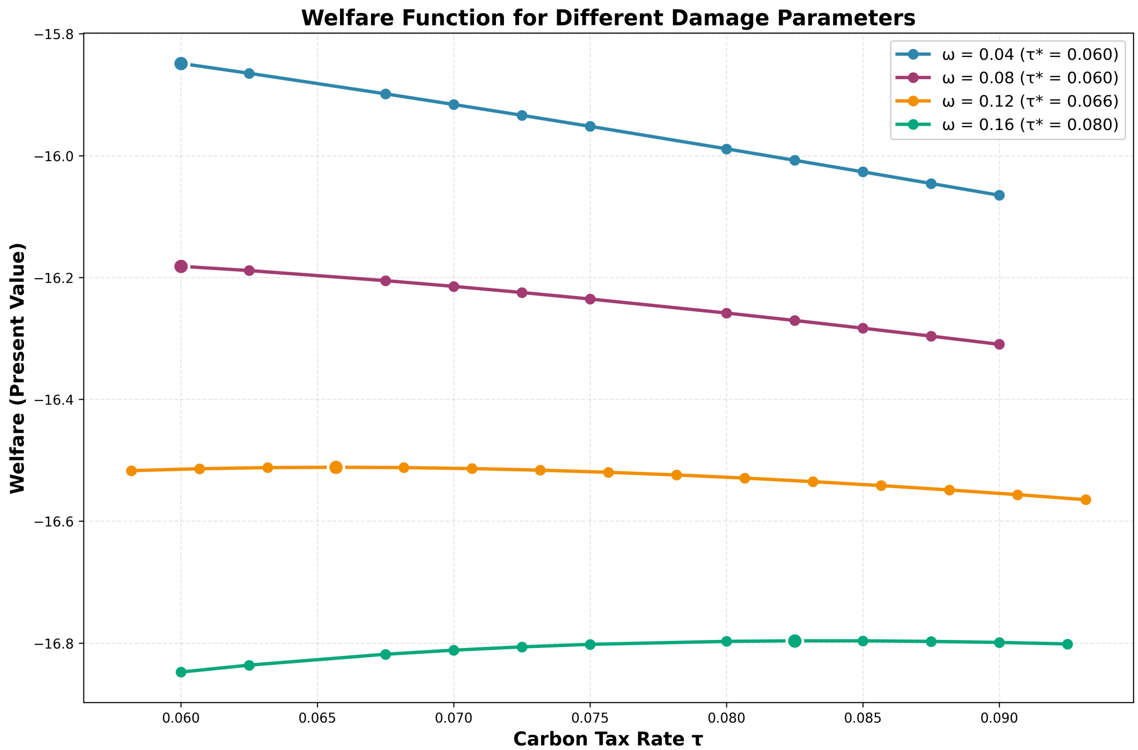

Figure 2: Welfare as a function of carbon tax rate for different damage parameters. Higher damage weights (ω) shift the welfare function downward and move the optimal tax rightward. All curves exhibit clear interior maxima.

5.1.3 Interpretation

The damage-sensitivity results highlight the importance of dynamics. Because emissions reductions occur gradually through capital reallocation and investment responses, higher valuations of climate damages translate directly into higher optimal taxes. Even when output and consumption fall, the welfare gains from avoided emissions justify more aggressive policy.

5.2 Sensitivity to energy dependence

5.2.1 Design

I next examine sensitivity to the energy share in final production, e_c, which measures the economy's structural reliance on energy inputs. I vary e_c from 0.02 to 0.16, spanning relatively flexible economies to highly energy-intensive ones.

Changing e_c alters the production technology itself. For each value, I adjust the capital share to preserve constant returns to scale and recompute the dynamic equilibrium and optimal policy. This ensures that

Table 1: Damage-Weight Sensitivity in the Dynamic Model

ω	τ^*	W_{PV}	Y_{avg}	C_{avg}	Z_{total}	K_T	$\bar{i}$
0.02	0.0139	-15.43	0.929	0.726	18.40	2.529	0.219
0.04	0.0600	-15.85	0.901	0.700	9.93	2.514	0.223
0.06	0.0600	-16.02	0.901	0.700	9.93	2.514	0.223
0.08	0.0600	-16.18	0.901	0.700	9.93	2.514	0.223
0.10	0.0600	-16.35	0.901	0.700	9.93	2.514	0.223
0.12	0.0657	-16.51	0.898	0.697	9.33	2.513	0.224
0.16	0.0800	-16.80	0.890	0.689	8.04	2.511	0.226
0.20	0.0900	-17.04	0.884	0.683	7.30	2.511	0.227
0.24	0.0900	-17.29	0.884	0.683	7.30	2.511	0.227
0.30	0.0945	-17.63	0.882	0.681	7.00	2.511	0.228
0.40	0.2500	-17.97	0.821	0.620	2.51	2.511	0.245

Notes: W_{PV} denotes present-value welfare. Y_{avg} and C_{avg} are averages over the transition path. Z_{total} is cumulative emissions. $\bar{i}$ denotes the average investment rate. At $\omega = 0.40$, the optimal tax reaches the upper bound of the policy space.

each counterfactual represents a coherent economy rather than a mechanical perturbation of the baseline.

5.2.2 Results

Table 2 shows that the relationship between energy dependence and optimal policy is highly non-monotonic. In low-energy-dependence economies ($e_c \leq 0.04$), the optimal tax is relatively high ($\tau^* \approx 0.05$), while output and consumption remain robust. In these cases, higher energy prices generate limited distortions because energy is a small input into production.

As e_c increases toward the baseline, output and consumption decline gradually and the optimal tax stabilizes near the baseline value. In more energy-intensive economies, however, output and consumption fall sharply even when the optimal tax does not increase substantially. In these cases, emissions reductions are achieved partly through contraction in economic activity rather than pure substitution.

Table 2: Energy-Share Sensitivity in the Dynamic Model

e_c	τ^*	W_{PV}	Y_{avg}	C_{avg}	E_{avg}	$\bar{i}$
0.02	0.050	-13.73	1.036	0.831	0.165	0.198
0.04	0.050	-14.49	0.991	0.787	0.322	0.206
0.06	0.010	-15.33	0.971	0.766	0.725	0.210
0.08	0.060	-15.72	0.923	0.722	0.574	0.219
0.10	0.060	-16.23	0.899	0.698	0.707	0.224
0.12	0.070	-16.73	0.872	0.671	0.776	0.230
0.14	0.020	-17.13	0.898	0.695	1.423	0.226
0.16	0.020	-17.44	0.890	0.687	1.611	0.228

Notes: E_{avg} denotes average energy use along the transition path. Higher e_c implies greater structural dependence on energy in production.

5.2.3 Interpretation

These results reveal a fundamental asymmetry. Uncertainty about climate damages leads to smooth and predictable changes in the optimal tax. In contrast, uncertainty about energy dependence generates large and sometimes unstable changes in economic outcomes.

In highly energy-intensive economies, carbon taxation acts like a negative productivity shock to a core input. When substitution possibilities are limited, emissions reductions are achieved through contraction in output and consumption rather than clean reallocation. By contrast, in more flexible economies, the same policy primarily reshapes the energy mix with modest aggregate losses.

Figure 3 illustrates the relationship between energy intensity and optimal policy. The left panel shows the optimal tax as a function of the energy share e_c. The relationship is generally positive, more energy-intensive economies warrant higher taxes to internalize larger emissions externalities, but exhibits notable non-monotonicities reflecting the interaction between substitution effects and output losses. The right panel plots welfare at the optimal tax, which declines monotonically with energy intensity as larger shares of output are affected by the carbon price.

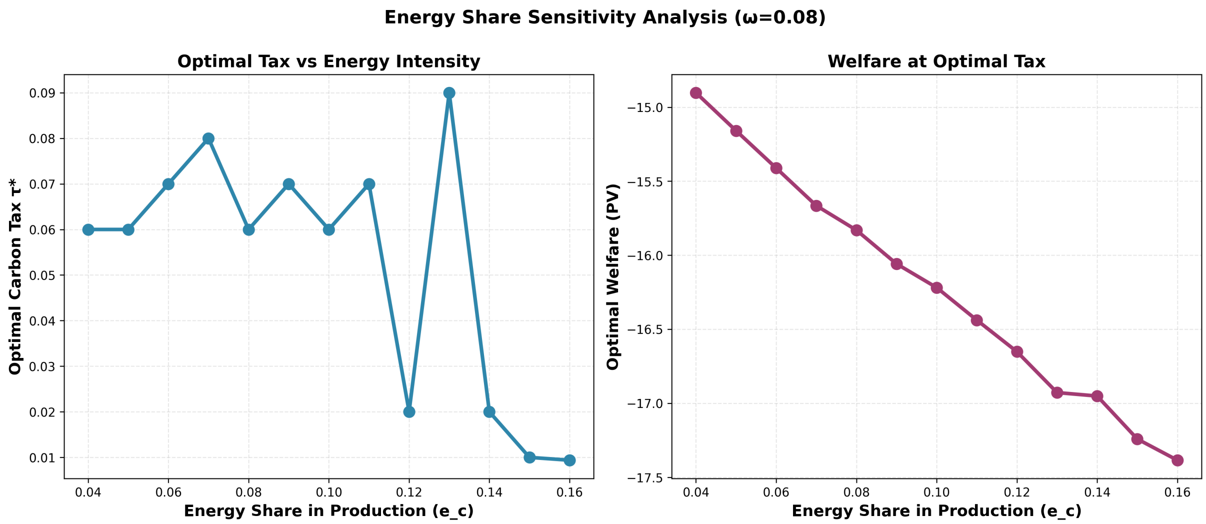

Figure 3: Energy share sensitivity analysis. Left: optimal carbon tax as a function of energy share in production. Right: welfare at the optimal tax. More energy-intensive economies face lower welfare even under optimal policy.

5.3 Policy implications

The robustness analysis delivers two key policy lessons. First, in a dynamic setting, higher valuations of climate damages robustly justify higher carbon taxes. Second, and more importantly, the economic cost of carbon taxation depends critically on the economy's structural capacity to substitute away from emissions-intensive energy.

This implies that carbon pricing is most effective when accompanied by policies that expand clean energy capacity and substitution possibilities. Without such complementary measures, aggressive carbon taxes in highly energy-dependent economies risk inducing large welfare losses through reduced economic scale rather than efficient decarbonization.

6 Policy Benchmarking

This section maps the model-implied optimal carbon tax τ^* into real-world units, expressed as Canadian dollars per tonne of CO_2. The objective is not to produce a headline-grabbing point estimate, but rather to provide a transparent and internally consistent translation from model units to policy-relevant magnitudes.

Because the model is dynamic and normalized, this mapping must be done carefully. The tax τ is levied per unit of emissions generated by energy production and is expressed in units of the final good.

Translating τ into CAD per tonne therefore requires anchoring the model to observed emissions intensity and output scales.

6.1 Conceptual mapping

In the model, the emissions tax enters firm optimization as a per-unit charge on emissions,

$$T_t = \tau \cdot Z_t, \tag{20}$$

where Z_t denotes aggregate emissions in period t. The tax rate τ therefore represents the marginal cost, in units of the final good, of one additional unit of emissions.

Let P_Y denote the price of the final good in CAD and let κ denote the emissions intensity of production, measured as tonnes of CO_2 per unit of real output. The implied carbon price in CAD per tonne, denoted $\mathcal{P}_{CO_2}$, is given by

$$\mathcal{P}_{CO_2} = \frac{\tau \cdot P_Y}{\kappa}. \tag{21}$$

This expression makes clear that the policy benchmark depends on two empirical objects: the price level used to value output and the emissions intensity of production.

6.2 Emissions intensity calibration

The emissions intensity κ is computed from Canadian data as the ratio of total energy-related CO_2 emissions to real GDP. Let Z^{data} denote annual emissions (in tonnes) and Y^{data} denote real GDP (in CAD). Then

$$\kappa = \frac{Z^{\text{data}}}{Y^{\text{data}}}. \tag{22}$$

Using recent Canadian data, this ratio implies that one unit of real GDP is associated with approximately $\kappa \approx 0.00015$ tonnes of CO_2. This value is used solely as a scaling device and does not affect any equilibrium outcomes in the model.

6.3 Benchmarking the optimal tax

In the baseline dynamic calibration with $\omega = 0.08$, the welfare-maximizing tax is approximately $\tau^* \approx 0.06$. Normalizing the price of output such that $P_Y = 1$ corresponds to one real Canadian dollar, the implied carbon price is

$$\mathcal{P}_{CO_2} \approx \frac{0.06}{0.00015} \approx 400 \text{ CAD per tonne}. \tag{23}$$

This value lies substantially above current Canadian carbon pricing levels. As of 2024, the federal carbon price stood at \$80 CAD per tonne, scheduled to rise to \$170 per tonne by 2030 under existing legislation[Environment and Climate Change Canada, 2024]. The model-implied optimal tax of approximately \$400 CAD per tonne is therefore 2–5 times higher than current or planned policy levels.

This gap admits several interpretations. First, actual policy may be constrained by political economy consideration, competitiveness concerns, distributional impacts, and electoral pressures, that the model abstracts from. Second, the model may overstate optimal taxes by omitting adjustment costs and sectoral heterogeneity that would reduce the welfare gains from aggressive pricing. Third, the gap may reflect genuine policy underpricing relative to the social cost of carbon when dynamic adjustment is taken into account.

Table 3 compares the model-implied benchmark against current and projected carbon prices in major jurisdictions. The model output falls within the range of prices implied by leading integrated assessment models (\$50–\$500 per tonne depending on discount rates and damage assumptions) but substantially exceeds implemented policies.

Table 3: Comparison of Carbon Prices

Jurisdiction / Benchmark	Price (CAD/tonne)	Year
Canada (current)	80	2024
Canada (scheduled)	170	2030
EU ETS (approximate)	100–120	2024
Model-implied (baseline)	400	—
Model-implied ($\omega = 0.02$)	93	—
Model-implied ($\omega = 0.40$)	1,670	—

6.4 Sensitivity and interpretation

The implied carbon price varies substantially across calibrations. As shown in the robustness analysis, higher damage weights ω push τ^* toward the upper bound of the policy space, implying carbon prices well in excess of this benchmark. Conversely, in more energy-intensive economies, optimal taxes are lower, reflecting the high economic cost of pricing a core production input.

Two caveats are worth emphasizing. First, the mapping depends linearly on emissions intensity. Economies with cleaner energy mixes or lower emissions per unit of output would imply lower CAD-per-tonne equivalents for the same τ^*. Second, the model abstracts from revenue recycling distortions, sectoral heterogeneity beyond energy production, and international leakage, all of which would affect real-world implementation.

6.5 Summary

The dynamic model implies optimal carbon prices that are economically meaningful and policy relevant when translated into real-world units. Crucially, these benchmarks emerge not from exogenous assumptions about the social cost of carbon, but from the interaction of emissions damages with capital accumulation, investment incentives, and long-run production structure. As a result, the implied carbon prices reflect both environmental benefits and the dynamic economic costs of adjustment.

7 Limitations and Extensions

The model developed in this paper makes several simplifying assumptions that should be acknowledged when interpreting the results.

International trade and carbon leakage. The model treats Canada as a closed economy, abstracting from international trade in goods, energy, and emissions permits. In practice, aggressive unilateral carbon pricing may induce carbon leakage, the relocation of emissions-intensive production to jurisdictions with weaker climate policy. This leakage would reduce the environmental effectiveness of carbon taxes and potentially justify lower optimal rates than the model implies. Extending the framework to an open-economy setting with trade in energy and final goods is an important direction for future work.

Technological change and innovation. Energy technologies are treated as fixed throughout the model horizon. In reality, carbon pricing induces directed technical change, shifting R&D effort toward clean technologies and potentially lowering the future cost of decarbonization [Acemoglu et al., 2012]. Incorporating endogenous innovation would likely raise the optimal carbon tax in early periods, as higher prices today accelerate the development of clean alternatives.

Uncertainty and learning. The model assumes perfect foresight about future policies and economic outcomes. Climate policy in practice operates under substantial uncertainty about damage functions, technology costs, and political constraints. A stochastic extension would allow analysis of how uncertainty about the social cost of carbon affects optimal policy design and the value of policy flexibility.

Distributional considerations. The representative-agent framework abstracts from heterogeneity in income, wealth, and energy consumption patterns. Carbon taxes are often regressive in their direct incidence, falling disproportionately on low-income households who spend larger shares of income on energy. The welfare analysis presented here does not capture these distributional effects, which may constrain the political feasibility of aggressive carbon pricing.

Additional robustness dimensions. The robustness analysis focused on damage weights and energy shares. Additional sensitivities worth exploring include:

- **Discount rate (β):** Lower discount rates would place greater weight on future emissions damages, raising optimal taxes.
- **Depreciation rate (δ):** Higher depreciation speeds capital turnover, potentially facilitating faster reallocation toward clean technologies.
- **Substitution elasticity (η):** Higher elasticity makes it easier to substitute across energy types, reducing the output cost of carbon pricing.
- **Time horizon (T):** Longer horizons allow more capital adjustment, potentially justifying more aggressive near-term policy.

Despite these limitations, the model captures the essential trade-offs in climate policy design and provides a tractable framework for quantifying the dynamic effects of carbon taxation.

8 Conclusion

This paper studies optimal carbon taxation in a dynamic general equilibrium economy with capital accumulation, heterogeneous energy technologies, and emissions-driven welfare damages. By embedding energy use and emissions into a forward-looking production and investment environment, the analysis moves beyond static Pigouvian logic and highlights how climate policy interacts with capital formation, consumption smoothing, and long-run productive capacity.

The central finding is that optimal carbon policy is fundamentally dynamic. In the baseline calibration, the welfare-maximizing tax reflects not only the contemporaneous marginal damage of emissions, but also the way in which taxation reshapes investment incentives and the evolution of capital over time. Carbon pricing alters expected returns, shifts the composition of energy use, and affects the path of output and consumption throughout the transition. As a result, the optimal tax in the dynamic model differs both quantitatively and qualitatively from what would be inferred in a static setting.

The robustness analysis clarifies two distinct sources of policy uncertainty. First, uncertainty about the valuation of climate damages, captured by the damage weight ω, translates directly into uncertainty about the optimal tax. Higher damage valuations justify more aggressive carbon pricing, leading to lower cumulative emissions at the cost of reduced output and consumption along the transition path. Unlike static models in which the optimal tax may become insensitive to damages once technological limits bind, the dynamic framework preserves a strong link between damages and policy because investment and capital accumulation remain responsive to incentives.

Second, uncertainty about the economy's structural dependence on energy plays an even more consequential role. When energy is a small share of production, carbon taxation primarily induces substitution toward clean technologies with limited macroeconomic disruption. In contrast, in highly energy-intensive economies, the same policy acts as a severe distortion to a core input, generating sharp declines in output and consumption. These results imply that the effectiveness and desirability of carbon pricing depend critically on the availability of low-carbon substitutes and the flexibility of the production structure.

Taken together, the findings suggest that carbon pricing is a necessary but not sufficient instrument for efficient decarbonization. In economies with high energy dependence, complementary policies that expand clean energy capacity, reduce adjustment frictions, and lower the cost of substitution can substantially improve the welfare performance of carbon taxes. From a policy perspective, this shifts attention away from debates over the precise numerical value of the social cost of carbon and toward the structural conditions that determine how costly emissions reductions are to achieve.

More broadly, the dynamic framework developed in this paper highlights the importance of evaluating climate policy through the lens of transition dynamics rather than static efficiency alone. Climate policy is not a one-shot correction but a sequence of incentives that shape investment, technology adoption, and economic structure over time. As Figure 1 illustrates, the transition to a low-carbon economy unfolds gradually through capital reallocation rather than abrupt deindustrialization. Figure 2 demonstrates that welfare functions exhibit clear interior maxima, making the optimal tax economically well-identified. And Figure 3 reveals that structural energy dependence is a first-order determinant of policy effectiveness.

Models that abstract from these dynamics risk understating both the costs and the benefits of carbon pricing. By explicitly accounting for capital accumulation and forward-looking behavior, this analysis provides a more realistic benchmark for assessing climate policy in energy-intensive economies. The gap between model-implied optimal taxes (approximately \$400 CAD per tonne) and current policy levels (\$80 CAD per tonne) suggests substantial scope for more ambitious carbon pricing, though this conclusion must be tempered by the model's limitations regarding international trade, innovation, and distributional effects.

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A Calibrated Parameters

Table 4: Calibration Targets and Parameters

Parameter / Target	Value	Description / Role	Source / Notes
$\bar{L}$	1.0	Normalized labour endowment	Model normalization
GDP_{2024}	\$3.108551T	Canada nominal GDP, 2024	StatsCan Table 36-10-0221-01 [Statistics Canada, 2025c]
K_{2024}	\$7.8T	Net capital stock, 2024	StatsCan NBSA (2025-11-18) [Statistics Canada, 2025e]
K/Y	2.51	Capital–output ratio	From K_{2024}/GDP_{2024}
$\bar{K}$	2.51	Normalized capital endowment	$(K/Y) \times \bar{L}$
ϕ_{oil}	1.0	Emission intensity benchmark (oil = 1)	IEA Emission Factors [International Energy Agency, 2024]
ϕ_{gas}	0.7741	Emission intensity of gas (relative to oil)	56.9/73.5 gCO_2/MJ [International Energy Agency, 2024]
ϕ_{clean}	0.0	Zero operational emissions	Model assumption
a_{oil}	0.86	Capital share in oil production	Derived from LABEX/GDP [Statistics Canada, 2025a, Statistics Canada, 2025d]
a_{gas}	0.86	Capital share in gas production	Derived from LABEX/GDP [Statistics Canada, 2025a, Statistics Canada, 2025d]
a_{clean}	0.90	Capital share in clean-energy production	Derived from LABEX/GDP [Statistics Canada, 2025a, Statistics Canada, 2025d]
A_1	$A_{\text{oil}} = 2.0$	Initial TFP (oil)	Endogenously calibrated
A_2	$A_{\text{gas}} = 1.0$	Initial TFP (gas)	Endogenously calibrated
A_3	$A_{\text{clean}} = 2.5$	Initial TFP (clean)	Endogenously calibrated
b_c	0.6548	Labour share in final-good production	FRED LABSHPCAA156NRUG [Federal Reserve Economic Data, 2025]
e_c	0.098	Energy share of final GDP	Energy Fact Book 2025–26 [Natural Resources Canada, 2025]
a_c	0.2472	Capital share in final-good production	Residual $1 - b_c - e_c$
γ	2.0	CRRA curvature	Fried et al. (2022) [Fried et al., 2022]
σ	0.5	Frisch elasticity	Fried et al. (2022) [Fried et al., 2022]
χ	(calibrated)	Labour disutility scale	Ensures $L_s \approx 0.50$
τ	0.0	Carbon tax rate (baseline)	Pre-tax steady state
Target energy shares	$(0.524, 0.396, 0.080)$	Oil / gas / clean shares in primary energy	StatsCan; renormalised [Statistics Canada, 2025b]
E/Y target	6.9 MJ/CAD	Energy intensity of GDP	Canadian benchmark [Natural Resources Canada, 2025]
L_{TARGET}	0.50	Target labour supply as share of $\bar{L}$	Full-time work approx. 8/16
Y_{TARGET}	1.0	Output normalization	Baseline $Y \approx 1$
P_E^{TARGET}	$e_c/(E/Y)$	Implied energy price	Internal consistency
Bounds on γ_j	$[0.50, 0.99]$	Curvature bounds for energy tech	Ensures decreasing returns
Bounds on A_{scale}	$[0.01, 1.00]$	Bounds for energy TFP scale	Stability requirement
Bounds on χ	$[0.01, 2.0]$	Bounds for labour disutility	Ensures feasible L_s