UNIVERSITY OF ZIMBABWE

DEPARTMENT OF MATHEMATICS AND COMPUTATIONAL SCIENCES LINEAR ALGEBRA ASSIGNMENT

27 Nov 2023
Time : 1.5 hours

Answer ALL questions carefully numbering them **A1** to **A4**.　　　[Total marks 60]

A1. (a) Prove that for any complex numbers z_1 and z_2,

(i)$\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$.　　　　(ii)$\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\bar{z_1}}{\bar{z_2}}$.　　　　[4,4]

(b) Given that $|z_1 + z_2| \leq |z_1| + |z_2|$ prove that for $z_1, z_2, ..., z_n \in \mathbb{C}$,
$|z_1 + z_2 + ... + z_n| \leq |z_1| + |z_n| + ... + |z_n|$.　　　　[5]

(c) Using De Moivre's Formula, prove the identity $\dfrac{\sin 5\theta}{\sin \theta} = 16\cos^4\theta - 12\cos^2\theta + 1$
if $\theta \neq n\pi$, $n = 0, 1, 2, ...$　　　　[8]

A2. (a) Let $z \in \mathbb{C}$, and let $w = \dfrac{z - i}{z + i}$.
　(i) Evaluate w when $z = 0$, and when $z = 1$.　　　　[2]
　(ii) Let $z = \beta$ where $\beta \in \mathbb{R}$. Show that for any such z the corresponding w
　　　always has unit modulus.　　　　[3]

(b) (i) Express the complex number $z = 24 + 7i$ in polar form.　　　　[2]
　(ii) Find the four values of $(24 + 7i)^{\frac{1}{4}}$ in exponential form, and plot them on an
　　　Argand diagram.　　　　[4]

A3. (a) Find a unit vector perpendicular to both $\mathbf{A} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{B} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$.　　[5]

(b) Show that $|\mathbf{A} \times \mathbf{B}|^2 + (\mathbf{A} \cdot \mathbf{B})^2 = |\mathbf{A}|^2|\mathbf{B}|^2$ for any vectors $\mathbf{A}$ and $\mathbf{B}$.　　[4]

(c) Find the area of the triangle having vertices at $P(1, 3, 2)$, $Q(2, -1, 1)$
and $R(-1, 2, 3)$.　　　　[5]

A4. (a) Find a set of parametric equations of the line that passes through the points
$(-2, 1, 0)$ and $(1, 3, 5)$.　　　　[4]

(b) Find the distance between the point $(0, 0, 0)$ and the plane $2x + 3y + z = 12$. [4]

(c) Find the inverse of $\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & 0 \\ 0 & 1 & 3 \end{pmatrix}$.　　　　[6]

END OF QUESTION PAPER