EE 120 SIGNALS AND SYSTEMS, Fall 2019

Homework #1

1. Omitted.

2. (a) stable, causal, FIR;
   (b) unstable, causal, IIR;
   (c) stable, non-causal, IIR;
   (d) stable, causal, IIR.

3. (a) Omitted.
   (b) Omitted.
   (c) Omitted.
   (d) When $x[n]$ is the unit impulse, the response is

$$h[n] = 1 + \sum_{k=0}^{n} \delta[k], \ n > 0,$$

   and when the unit impulse is scaled by α, the response is

$$\hat{h}[n] = 1 + \alpha \sum_{k=0}^{n} \delta[k] \neq \alpha h[n].$$

   (d) Linear and time-invariant.

4. (a)

$$h[n] = \frac{1}{4}(\delta[n-1] + 2\delta[n] + \delta[n+1])$$

   (b)

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k]e^{-jk\omega} = \frac{1+\cos\omega}{2} = \cos^2\frac{\omega}{2}$$

   (c) Low-pass filter.

5. (a)

$$H(j\omega) = \frac{1}{1 + RC(j\omega)}$$

   (b)

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (RC\omega)^2}}$$

   (The plot is omitted.)

Homework #2

1.
$$\hat{a}_k = \frac{1}{jk\pi}[1-(-1)^k] = \begin{cases} 0, & k : \text{even} \\ \frac{2}{jk\pi}, & k : \text{odd}. \end{cases}$$

2. (a) Omitted.
   (b) Omitted.
   (c) Omitted.
   (d) Property (b) and (c).

3. (a) The matrix elements are $w_{mn} = e^{j(m-1)\frac{2\pi}{N}(n-1)}$, $m, n = 1, 2, \cdots, N$. Denote $w = e^{j2\pi/N}$, so that

$$W = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & w & \cdots & w^{N-1} \\ \vdots & \vdots & & \vdots \\ 1 & w^{N-1} & \cdots & w^{(N-1)^2} \end{bmatrix}.$$

   (b) $(x[0], x[1], x[2])^T = (1, 0, 0)^T$, $w = e^{j2\pi/3}$,

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \Rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \frac{w^3}{(w-1)^2(w+1)} \\ -\frac{w}{(1-w)^2} \\ \frac{1}{(w-1)^2(w+1)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

   (c)
$$W^{-1} = \frac{1}{N} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & w^{-1} & \cdots & w^{-(N-1)} \\ \vdots & \vdots & & \vdots \\ 1 & w^{-(N-1)} & \cdots & w^{-(N-1)^2} \end{bmatrix}.$$

   (d) Omitted.

4. Omitted.

5.
$$y(t) = s(t)\cos\varphi.$$

when $\varphi = 90°$,

$$y(t) \equiv 0.$$

Homework #3

1. (a) $h_k = h^{(k)}(t)$.

   (b)
   $$H_1(\omega) = j\omega e^{-\omega^2/2},\ |H_1(\omega)| = |\omega|\, e^{-\omega^2/2}.$$
   The plot of $|H_1(\omega)|$ is omitted.

   (c)
   $$H_{\text{diff}}(\omega) = j\omega,\ |H_{\text{diff}}(\omega)| = |\omega|\,.$$

2. (a) Omitted.

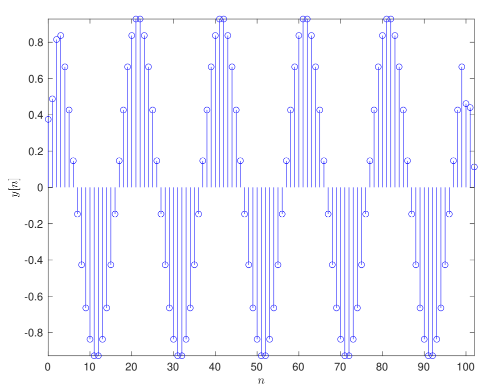
   (b)
   $$H(e^{j\omega}) = -\frac{[2a - (1+a^2)\cos\omega] - j(1-a^2)\sin\omega}{1 - 2a\cos\omega + a^2},$$
   $$\angle H(e^{j\omega}) = \pi + \arcsin[(1-a^2)\sin\omega].$$

   (c)
   $$y[n] = \cos\left(\frac{\pi}{6}n - \varphi\right) + \cos(n\pi - \pi),\ \varphi = \pi - \arcsin\frac{1}{3}.$$

   (d)
   $$y[n] - ay[n-1] = -ax[n] + x[n-1].$$

3. (a) $N = 103$.

   (b)




```
% matlab script
x = zeros(103, 1);
h = zeros(103, 1);
xRange = (0 : 99)';
x(1 : 100) = cos(0.1 * pi * xRange) + 0.5 * cos(pi * xRange);
h(1 : 4) = 1 / 4;
Y = fft(x) .* fft(h);
y = ifft(Y);
```

```
seriesPlot(y, 'b');
xlabel('$n$', Interpreter='latex');
ylabel('$y[n]$', Interpreter='latex');
axis([0, 102, -Inf, Inf]);
```

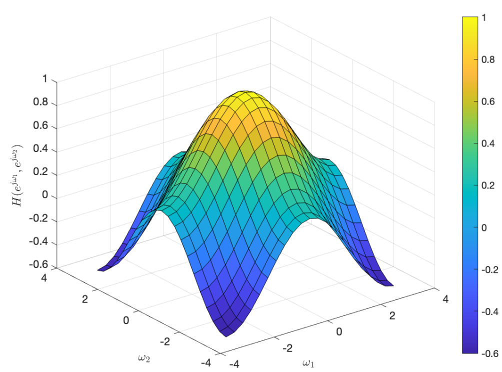
```
% matlab function
function seriesPlot(x, color)
    n = length(x);
    range = 0 : n - 1;
    plot(range, x, strcat(color, 'o'));

    for i = 1 : n
        hold on;
        plot([range(i), range(i)], [0, x(i)], strcat(color, '-'));
    end
end
```

4. (a) $h[n_1, n_2] =$

$$\frac{\delta[n_1, n_2] + \delta[n_1 - 1, n_2] + \delta[n_1 + 1, n_2] + \delta[n_1, n_2 - 1] + \delta[n_1, n_2 + 1]}{5}.$$

(b)

$$H(e^{j\omega_1}, e^{j\omega_2}) = \frac{1}{5}(1 + 2\cos\omega_1 + 2\cos\omega_2).$$



```
% matlab code
[omega1, omega2] = meshgrid(linspace(-pi, pi, 21));
H = (1 + 2*cos(omega1) + 2*cos(omega2)) / 5;
surf(omega1, omega2, H);
xlabel('$\omega_1$', Interpreter='latex');
ylabel('$\omega_2$', Interpreter='latex');
zlabel('$H(e^{j\omega_1}, e^{j\omega_2})$', Interpreter='latex');
```

5. Omitted.

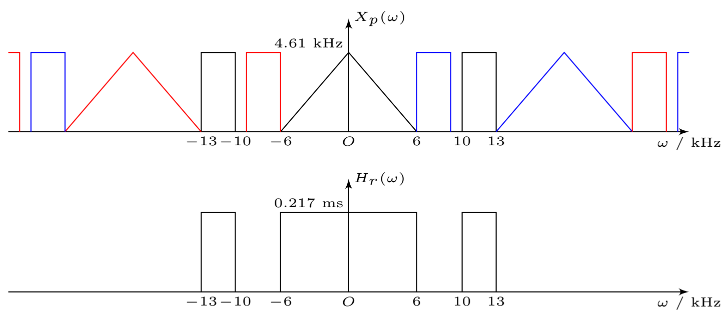
Homework #4

1. The angular velocity is

$$\omega = 12(3+4k)\pi \ \mathrm{s}^{-1},\ k = 0, 1, \cdots .$$

2. (a) 19 kHz.

   (b) The plots of $X_p(\omega)$ and the reconstruction filter's frequency response $H_r(\omega)$ are as below.



3. (a) Omitted.

   (b) $x_r(t) = 1$.

   (c) $\hat{x}_r(t) = -1 \neq x_r(t - \frac{1}{2})$.

4. (a)

$$h_d[n] = \begin{cases} \frac{1}{nT}, & n \neq 0, \\ \frac{\pi j}{T}, & n = 0. \end{cases}$$

   (b) ?

$$\hat{H}_d(e^{j\Omega}) = \frac{\alpha j}{T}(\pi - 2\sin\Omega),\ \alpha = -\frac{1}{2}.$$

   (c) ?

$$y_d[n] = -\frac{1}{2T}(j\pi x[n] - x[n+1] + x[n-1]).$$

5. (a)

$$X_p(e^{j\omega_1}, e^{j\omega_2}) = \frac{1}{N_1 N_2} \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} X(e^{j(\omega_1 - k_1 \frac{2\pi}{N_1})}, e^{j(\omega_2 - k_2 \frac{2\pi}{N_2})}).$$

   (b) $\frac{2\pi}{N_1} > 2\omega_{M1}$ and $\frac{2\pi}{N_2} > 2\omega_{M2}$, where ω_{M1}, ω_{M2} satisfy

$$X(e^{j\omega_1}, e^{j\omega_2}) = 0,\ \omega_{M1} < |\omega_1| \leq \pi,\ \omega_{M2} < |\omega_2| \leq \pi.$$

   (c)

$$h_r[n_1, n_2] = \mathrm{sinc}\left(\frac{n_1}{N_1}\right) \mathrm{sinc}\left(\frac{n_1}{N_2}\right).$$

Homework #5

1. (a)
$$X(s) = \frac{s+1}{s^2+2s+5}, \;\; \mathrm{Re}\, s > -1.$$
   (b)
$$X(s) = -\frac{1}{s^2+1}, \;\; \mathrm{Re}\, s < 0.$$
   (c)
$$X(s) = \frac{s^2-1}{s^2+1}, \;\; \mathrm{Re}\, s > 0.$$
   (d)
$$X(s) = e^{-s}, \;\; s \in \mathbb{C}.$$
2. (a)
$$x(t) = e^{-2t} \cos t u(t).$$
   (b)
$$x(t) = \cos(t-1)u(t-1).$$
   (c)
$$x(t) = \frac{1}{2} t \sin t u(t).$$
   (d)
$$x(t) = (-e^{-3t} + 2e^{-4t})u(t).$$
3. (a) Omitted.
   (b) (c) If the input $x(t) = \cos \omega_n t u(t)$, which is bounded, then
$$Y(s) = H(s)X(s) = \frac{\omega_n^2}{s^2+\omega_n^2} \cdot \frac{s}{s^2+\omega_n^2} = \frac{\omega_n^2 s}{(s^2+\omega_n^2)^2}.$$
   Use the results in 2(c), and we can get an unbounded output:
$$y(t) = \frac{1}{2} \omega_n t \sin \omega_n t u(t).$$
   (d) Omitted.
4. See Figure 1.
5. (a)
$$H(s) = \frac{b_0 + b_1 s}{a_0 + a_1 s + s^2}.$$
   (b)
$$H(s) = \frac{b_0 + b_1 s}{a_0 + a_1 s + s^2}.$$

6. Denote $y(t) \risingdotseq Y(p)$, then

$$\dot{y}(t) \risingdotseq pY(p) - \frac{5}{2},\ \ddot{y}(t) \risingdotseq p^2Y(p) - \frac{5}{2}p + \frac{9}{2},\ x(t) \risingdotseq \frac{1}{p+3},$$

and

$$Y(p) = \frac{\frac{5}{2}p^2 + \frac{21}{2}p + 10}{(p+1)(p+2)(p+3)} = \frac{1}{p+1} + \frac{1}{p+2} + \frac{1/2}{p+3}.$$

$$\Rightarrow\ y(t) = \left(e^{-t} + e^{-2t} + \frac{1}{2}e^{-3t}\right)u(t).$$

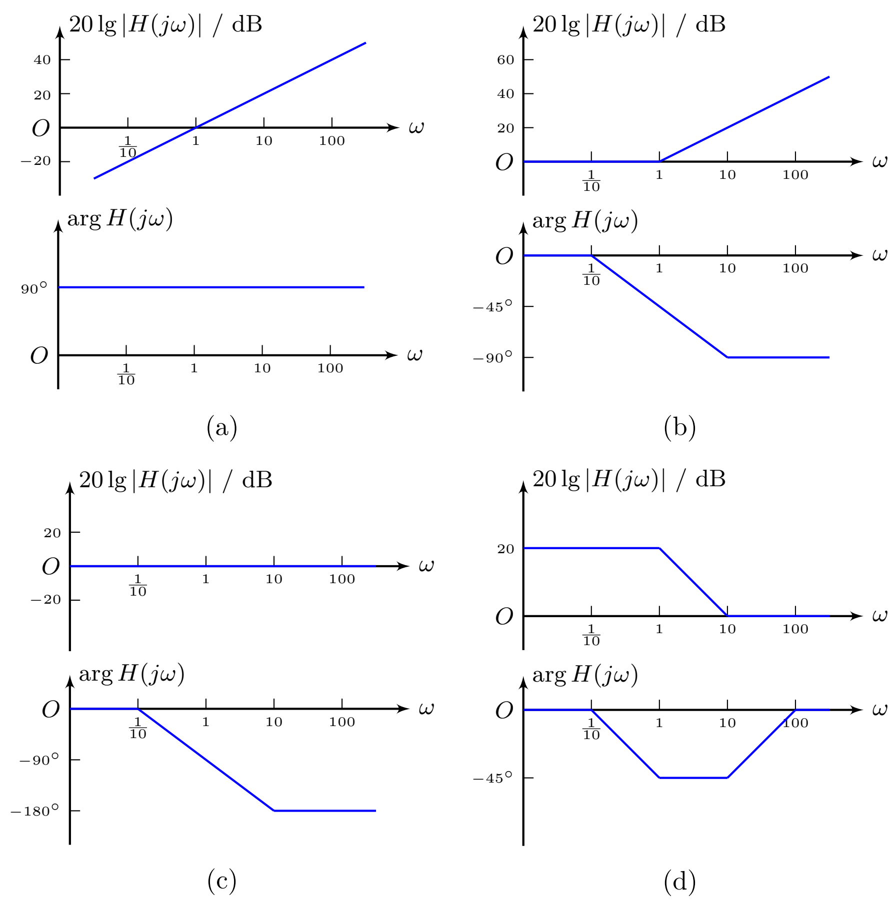


Figure 1: The Bode plots of problem 4

Homework #6

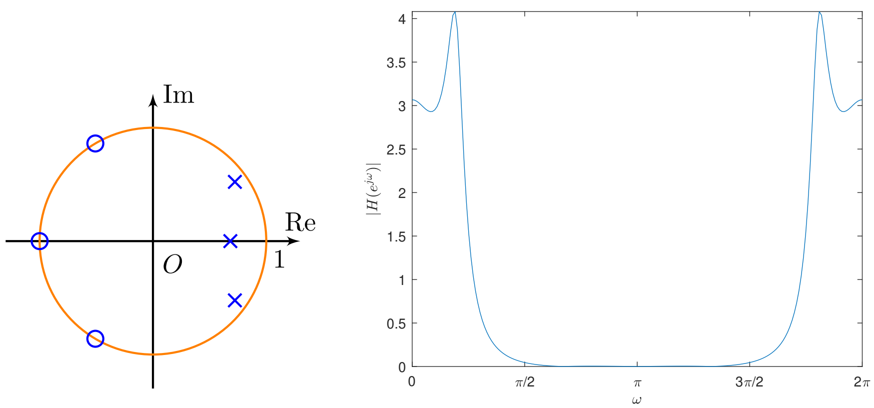
1.
$$x[n] = \left[-\frac{5}{3}\left(-\frac{1}{2}\right)^n + \frac{8}{3}(-2)^n\right]u[n].$$

2. (a)
$$H(z) = \frac{1}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}, \ |z| > 1.$$

   (b) Unstable.

   (c)
$$y[n] = \left[\left(\frac{1}{2}\right)^n + 2n\right]u[n].$$

3. (a) Stable. ($\times$ for poles and $\circ$ for zeros in the figure below.)



(a) (b)

4. (a) $\omega_0 = \arccos \beta$.

   (b) Omitted.

5. (a) Omitted.

   (b) $H_{\text{bp}}(e^{j0}) = H_{\text{bp}}(e^{j\pi}) = 0$, $H_{\text{bp}}(e^{j\omega_0}) = 1$.

6.
$$H(z) = \frac{(b_1 - a_1 b_0)z^{-1} + (b_2 - a_2 b_0)z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} + b_0.$$

7.
$$Y(z) = \frac{2 - \frac{3}{2}z^{-1}}{(1 - z^{-1})(1 - \frac{1}{2}z^{-1})} = \frac{1}{1 - z^{-1}} + \frac{1}{1 - \frac{1}{2}z^{-1}},$$
$$y[n] = 1 + \left(\frac{1}{2}\right)^n.$$