

Webstudy Operation Team Presentation

How to write solutions

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Introduction

Target: JY, JH (Newbie!)

Goal: Share know-how for more efficient work

Today's example: Section 15.7 & 15.8 Migrate 16

Find the volume V of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 16z$.

$V =$

[해설 1. Cylindrical coordinate 활용]

- Step 1 구의 방정식 변형

$$\begin{aligned}x^2 + y^2 + z^2 &= 16z \\x^2 + y^2 + (z - 8)^2 &= 64\end{aligned}$$

- Step 2 원뿔과 구의 교점 찾기

$z = \sqrt{x^2 + y^2}$ 를 위의 변형된 구의 방정식에 대입 후 $r^2 = x^2 + y^2$ 로 치환

$$\begin{aligned}x^2 + y^2 + (z - 8)^2 &= 64 \\x^2 + y^2 + \left(\sqrt{x^2 + y^2} - 8\right)^2 &= 64 \\r^2 + (r - 8)^2 &= 64 \\r^2 + r^2 - 16r + 64 &= 64 \\2r^2 - 16r &= 0 \\r(r - 8) &= 0\end{aligned}$$

교선 : $r = 0, r = 8$

- Step 3 구와 원뿔 사이 부피 적분

$$\begin{aligned}\text{Volume} &= \int_0^{2\pi} \int_0^8 \left(\sqrt{64 - r^2} + 8 - r \right) r \, dr \, d\theta \\&= \int_0^{2\pi} \left(\frac{512}{2} \right) d\theta \\&= 512\pi\end{aligned}$$

First Step?

Obviously, solve problem.

Find the volume of the solid that lies above the cone

$z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 10z$.

Volume = π

! Question is missing t

Your last answer was interpreted as follows:

125

First Step!

Obviously, solve problem.

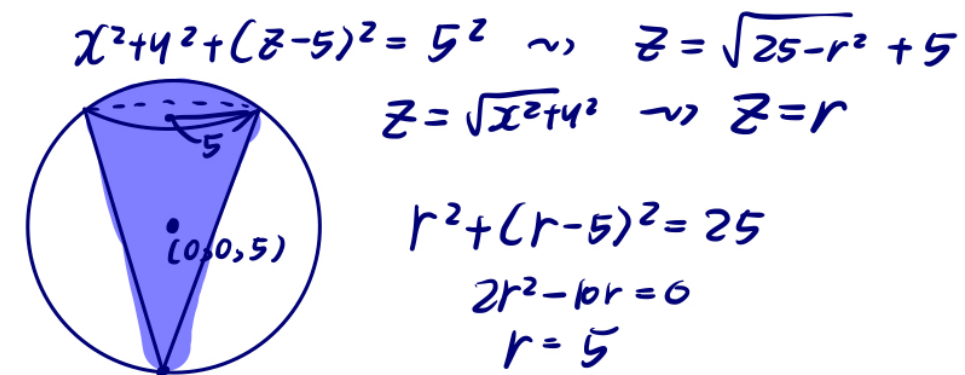
Find the volume of the solid that lies above the cone
 $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 10z$.

Volume = π

$$\rho \cos \phi = \sqrt{\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta}$$

Your last answer was interpreted as follows:

125



$$E = \left\{ (r, \theta, z) \mid \begin{array}{l} 0 \leq r \leq 5 \\ 0 \leq \theta \leq 2\pi \\ r \leq z \leq \sqrt{25 - r^2} + 5 \end{array} \right\}$$

$$\int_0^{2\pi} \int_0^5 \int_r^{\sqrt{25 - r^2} + 5} r \, dz \, dr \, d\theta$$

$$2\pi \int_0^5 r \sqrt{25 - r^2} + 5r - r^2 \, dr$$

$$2\pi \left[-\frac{1}{3} (25 - r^2)^{\frac{3}{2}} + \frac{5}{2} r^2 - \frac{r^3}{3} \right]_0^5$$

$$2\pi \cdot \left(\frac{125}{3} + \frac{125}{2} - \frac{125}{3} \right)$$

$$= 125\pi$$

First Step!

Then, consider random variables

Find the volume of the solid that lies above the cone
 $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 10z$.

Volume = π

$$\rho \cos \phi = \sqrt{\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta}$$

Your last answer was interpreted as follows:

125

Find the volume of the solid that lies above the cone
 $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 10z$.

Volume = π

$$\rho \cos \phi = \sqrt{\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta}$$

Your last answer was interpreted as follows:

125

$$\begin{aligned} x^2 + y^2 + (z-5)^2 &= 5^2 \leadsto z = \sqrt{25-r^2} + 5 \\ z &= \sqrt{x^2 + y^2} \leadsto z = r \\ r^2 + (r-5)^2 &= 25 \\ 2r^2 - 10r &= 0 \\ r &= 5 \\ E &= \{(r, \theta, z) \mid 0 \leq r \leq 5, 0 \leq \theta \leq 2\pi, r \leq z \leq \sqrt{25-r^2} + 5\} \\ \int_0^{2\pi} \int_0^5 \int_r^{\sqrt{25-r^2}+5} r \, dz \, dr \, d\theta \\ 2\pi \int_0^5 r \sqrt{25-r^2} + 5r - r^2 \, dr \\ 2\pi \left[-\frac{1}{3} (25-r^2)^{\frac{3}{2}} + \frac{5}{2} r^2 - \frac{r^3}{3} \right]_0^5 \\ 2\pi \cdot \left(\frac{125}{3} + \frac{125}{2} - \frac{125}{3} \right) \\ &= 125\pi \end{aligned}$$

$$\begin{aligned} x^2 + y^2 + (z-5)^2 &= 5^2 \leadsto z = \sqrt{25-r^2} + 5 \\ z &= \sqrt{x^2 + y^2} \leadsto z = r \\ r^2 + (r-5)^2 &= 25 \\ 2r^2 - 10r &= 0 \\ r &= 5 \\ E &= \{(r, \theta, z) \mid 0 \leq r \leq 5, 0 \leq \theta \leq 2\pi, r \leq z \leq \sqrt{25-r^2} + 5\} \\ \int_0^{2\pi} \int_0^5 \int_r^{\sqrt{25-r^2}+5} r \, dz \, dr \, d\theta \\ 2\pi \int_0^5 r \sqrt{25-r^2} + 5r - r^2 \, dr \\ 2\pi \left[-\frac{1}{3} (25-r^2)^{\frac{3}{2}} + \frac{5}{2} r^2 - \frac{r^3}{3} \right]_0^5 \\ 2\pi \cdot \left(\frac{125}{3} + \frac{125}{2} - \frac{125}{3} \right) \\ &= 125\pi \end{aligned}$$

Second Step

Write it with LaTeX

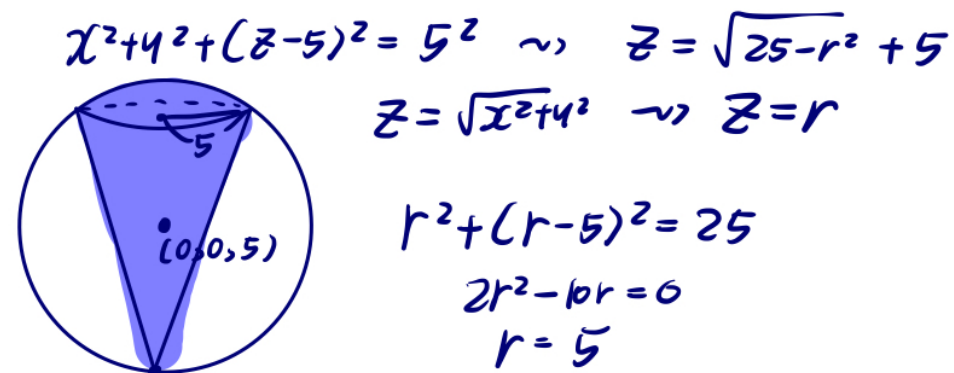
Find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 10z$.

Volume = 125 π

Your last answer was interpreted as follows:

125

$$\rho \cos \phi = \sqrt{\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta}$$



$$E = \left\{ (r, \theta, z) \mid \begin{array}{l} 0 \leq r \leq 5 \\ 0 \leq \theta \leq 2\pi \\ r \leq z \leq \sqrt{25 - r^2} + 5 \end{array} \right\}$$

$$\int_0^{2\pi} \int_0^5 \int_r^{\sqrt{25-r^2}+5} r \, dz \, dr \, d\theta$$

$$2\pi \int_0^5 r \sqrt{25-r^2} + 5r - r^2 \, dr$$

$$2\pi \left[-\frac{1}{3} (25-r^2)^{\frac{3}{2}} + \frac{5}{2} r^2 - \frac{r^3}{3} \right]_0^5$$

$$2\pi \cdot \left(\frac{125}{3} + \frac{125}{2} - \frac{125}{3} \right)$$

$$= 125\pi$$

L^AT_EX해설 시연

Jeon Yongjin

[해설 1. Cylindrical coordinate 활용]

Step 1 구의 방정식 변형

$$\begin{aligned} x^2 + y^2 + z^2 &= 5z \\ x^2 + y^2 + (z-5)^2 &= 25 \end{aligned}$$

Step 2 원뿔과 구의 교점 찾기

$z = \sqrt{x^2 + y^2}$ 를 위의 변형된 구의 방정식에 대입 후 $r^2 = x^2 + y^2$ 로 치환

$$\begin{aligned} x^2 + y^2 + (z-5)^2 &= 25 \\ x^2 + y^2 + (\sqrt{x^2 + y^2} - 5)^2 &= 25 \\ r^2 + (r-5)^2 &= 25 \\ r^2 + r^2 - 10r + 25 &= 25 \\ 2r^2 - 10r &= 0 \\ r(r-5) &= 0 \end{aligned}$$

교선 : $r = 0, r = 5$

Step 3 구와 원뿔 사이 부피 적분

$$\begin{aligned} Volume &= \int_0^{2\pi} \int_0^5 (\sqrt{25-r^2} + 5 - r) r \, dr \, d\theta \\ &= \int_0^{2\pi} \left(\frac{125}{2} \right) d\theta \\ &= 125\pi \end{aligned}$$

Now, It's coding