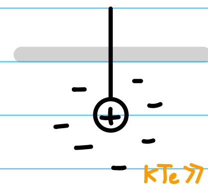
3강

- **Plasma** [quasi neutral
  (shield 바깥은 neutral 처럼 느껴짐)
  collective behavior
  (전체적으로 몰려가는 것)

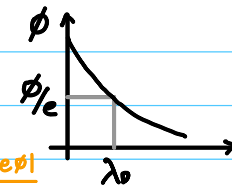
  $$\begin{cases} L \gg \lambda_D \\ N_D = n\frac{4}{3}\pi\lambda_D^3 \gg 1 \\ \omega\tau \gg 1 \end{cases}$$

$$\lambda_D = \sqrt{\frac{\varepsilon k T_e}{ne^2}} = 69\sqrt{\frac{T_e}{n}} \quad (T_e \text{ in K})$$

$$= 7430\sqrt{\frac{kT_e}{n}} \quad (T_e \text{ in eV})$$



$kT_e \gg |e\phi|$



ex) KSTAR

$kT_e = 10\,\text{keV}$, $n = 10^{20}\ \#/m^3$

$$\lambda_D = 7430\sqrt{\frac{10\times10^3}{10^{20}}} \sim 10^4 \cdot \frac{1}{10^8} = 0.1\,\text{mm}$$

과제 조사 (magnetosphere
ionosphere
Van Allen belt
(전리층)
우주선 방어 / 통신에 사용

- KSTAR

$n = 10^{20}\ \#/m^3$ $\rightarrow N = 10^{21}\ \#$ (KSTAR의 입자수)

$3\,\text{GHz} = 3\times10^9\ \#/s$

$= 3\times10^9 \times \underbrace{3.2\times10^7}_{1\text{year}/s}\ \#/\text{year} \simeq 10^{17}\ \#/\text{year}$

∴ 입자 하나하나 세는데만 ~10^4 year이 걸림. → 불가능

- **Single particle Motions**

Assumption

- 입자의 움직임에 의한 부차적인 현상 전기장/자기장 무시
- Uniform $\vec{E}$ and $\vec{B}$ Equation of Motions

★ $v_\parallel, v_\perp$: 모두 자기장 기준

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v}\times\vec{B}) \leftarrow \text{Lorentz Force}$$

1. $\vec{B} = 0$, $\vec{E} = E_x\hat{x} + E_z\hat{z}$ $\rightarrow m\frac{d\vec{v}}{dt} = q\vec{E}$

$$\begin{cases} m\dot{v}_x = qE_x \\ m\dot{v}_y = 0 \\ m\dot{v}_z = qE_z \end{cases} \Rightarrow \begin{cases} v_x = \frac{q}{m}E_x t + v_{x0} \\ v_y = v_{y0} \\ v_z = \frac{q}{m}E_z t + v_{z0} \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2}\frac{q}{m}E_x t^2 + v_{x0}t + x_0 \\ y = v_{y0}t + y_0 \\ z = \frac{1}{2}\frac{q}{m}E_z t^2 + v_{z0}t + z_0 \end{cases}$$

2. $\vec{E} = 0$, $\vec{B} = B\hat{z}$ $\rightarrow m\frac{d\vec{v}}{dt} = q(\vec{v}\times\vec{B})$

$$m\dot{\vec{v}} = q(\vec{v}\times\vec{B})$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ 0 & 0 & B \end{vmatrix} \Rightarrow \begin{array}{l} m\dot{v}_x = qBv_y \\ m\dot{v}_y = -qBv_x \\ m\dot{v}_z = 0 \end{array} \Rightarrow \begin{array}{l} m\ddot{v}_x = qB\dot{v}_y = qB\left(-\frac{qB}{m}\right)v_x \rightarrow \ddot{v}_x = -\left(\frac{qB}{m}\right)^2 v_x = -\omega_c^2 v_x \\ m\ddot{v}_y = qB\dot{v}_x = -qB\left(\frac{qB}{m}\right)v_y \rightarrow \ddot{v}_y = -\left(\frac{qB}{m}\right)^2 v_y = -\omega_c^2 v_y \end{array}$$

$$\boxed{\therefore \omega_c = \frac{|q|B}{m}}$$

$m\dot{v}_x = qBv_y$

$v_y = \frac{m}{qB}\dot{v}_x$

$$\Rightarrow \begin{cases} v_x = A\cos\omega_c t + B\sin\omega_c t \\ \quad = \underline{v_\perp \cos(\omega_c t+\delta)} \\ v_y = \frac{m}{qB}\left[-v_\perp \omega_c \sin(\omega_c t+\delta)\right] \\ \quad = \pm\frac{1}{\omega_c}\left[-v_\perp \omega_c \sin(\omega_c t+\delta)\right] \\ \quad = \underline{\mp v_\perp \sin(\omega_c t+\delta)} \\ v_z = v_{z0} \end{cases}$$

(+: ion, −: electron)

$$\begin{cases} \cdot\ \underline{v_\perp = v_x^2 + v_y^2} \\ \cdot\ v_{x0} = v_\perp \cos\delta \\ \quad v_{y0} = \mp v_\perp \sin\delta \end{cases} \Rightarrow \underline{\tan\delta = \mp\frac{v_{y0}}{v_{x0}}}$$

$$\Rightarrow \begin{cases} x = \frac{v_\perp}{\omega_c}\sin(\omega_c t+\delta) + \underline{x_0} \\ y = \pm\frac{v_\perp}{\omega_c}\cos(\omega_c t+\delta) + y_0 \\ z = v_{z0}t + z_0 \end{cases}$$

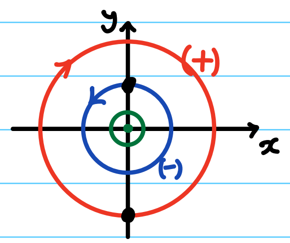
초기위치 x.
그냥 적분상수

$$\Rightarrow \underline{(x-x_0)^2 + (y-y_0)^2 = \left(\frac{v_\perp}{\omega_c}\right)^2}$$

$$\boxed{r_L = \frac{v_\perp}{\omega_c} = \frac{mv_\perp}{|q|B}\ \ ;\ \text{Larmor radius}}$$

↳ ion의 r_L이 훨씬 큼.

$x = r_L \sin(\omega_c t)$
$y = \pm r_L \cos(\omega_c t)$
생각하면 방향 알 수 있음



· 전자의 r_L < 이온의 r_L

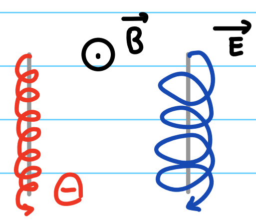
· Diamagnetic 특성
(자기장 약화시키는 방향으로의 모션)

시험기간에 다시 유도해보기.

3. $\vec{E} = E_x\hat{x} + E_z\hat{z}\ ,\ \vec{B} = B\hat{z}$

$$\begin{cases} v_x = v_\perp \cos(\omega_c t+\delta) \\ v_y = \mp v_\perp \sin(\omega_c t+\delta) - \frac{E_x}{B} \\ v_z = \frac{qE_z}{m}t + v_{z0} \end{cases} \Rightarrow \begin{cases} x = \frac{v_\perp}{\omega_c}\sin(\omega_c t+\delta) + x_0 \\ y = \pm\frac{v_\perp}{\omega_c}\cos(\omega_c t+\delta) - \frac{E_x}{B}t + y_0 \\ z = \frac{1}{2}\frac{qE_z}{m}t^2 + v_{z0}t + z_0 \end{cases}$$

$$\Rightarrow (x-x_0)^2 + \left(y-y_0+\frac{E_x}{B}t\right)^2 = \left(\frac{v_\perp}{\omega_c}\right)^2$$



Hannes Alfvén

guiding center가 $\frac{E_x}{B}$의 속도로 이동함.
(등속운동)

$$m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v}\times\vec{B})$$
$$0 = q(\vec{E} + \vec{v}\times\vec{B})\times\vec{B}$$
$$\vec{E}\times\vec{B} + (\vec{v}\times\vec{B})\times\vec{B}$$
$$= \vec{E}\times\vec{B} + \vec{B}\times(\vec{B}\times\vec{v})$$
$$= \vec{E}\times\vec{B} + \vec{B}(\cancel{\vec{B}\cdot\vec{v}}) - \vec{v}(B^2)$$

($\vec{v}$에서 $v_\perp$만 본다고 하면, $\vec{v}\cdot\vec{B} = 0$)

☆ 자기장에 수직인 속도만 나타내줌.
자기장에 평행한 초기속도 or 전기장이 있다면 이것도 고려해야함.

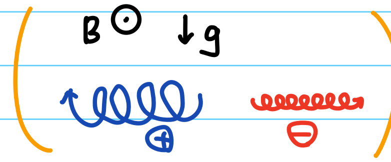
$$\boxed{\vec{v}_\perp = \frac{\vec{E}\times\vec{B}}{B^2}}\ ;\ \vec{E}\times\vec{B}\ \text{drift}$$

4강

* $E\times B$ drift $\quad \vec{V}_E = \dfrac{\vec{E}\times\vec{B}}{B^2} \Rightarrow \quad \vec{F} = q\vec{E}\ ;\ \vec{V}_E = \dfrac{\vec{E}\times\vec{B}}{B^2} = \dfrac{q\vec{E}\times\vec{B}}{qB^2} = \dfrac{\vec{F}_E\times\vec{B}}{qB^2}$

* general force $\quad \boxed{\vec{U}_f = \dfrac{\vec{F}\times\vec{B}}{qB^2}}$

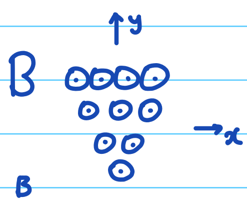
· gravitational force $\quad \boxed{\vec{U}_g = \dfrac{m\vec{g}\times\vec{B}}{qB^2}}$ (이온과 전자는 서로 반대방향으로 움직임)



* Non-uniform $\vec{B}$

1. ∇B

$\vec{B} = B(y)\hat{z}$

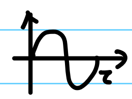


$\vec{F} = q(\vec{v}\times\vec{B})$

$r_L = \dfrac{m v_\perp}{|q|B}$ ← B가 변하므로 r_L도 변함
↳ drift motion 예상

(한 바퀴 돌 때, 이온이 받는 평균힘으로 $\vec{F}\times\vec{B}$ drift velocity를 유도)

$F_x = q v_y B$
$= \mp q v_\perp \sin(\omega_c t) B$

(i) $\overline{F_x} = \dfrac{1}{\tau}\displaystyle\int_0^\tau \mp q v_\perp B \sin\omega_c t\, dt = 0 \quad (\because$  $)$

(ii) $F_y = -q v_x B = -q(v_\perp \cos\omega_c t) B$

$= -q(v_\perp \cos\omega_c t)\left[B_0 + y\dfrac{dB}{dy}\Big|_{y=0}\right]$

$= -q v_\perp \cos\omega_c t\left(B_0 \pm r_L \cos\omega_c t \cdot \dfrac{dB}{dy}\Big|_{y=0}\right)$

$= -q v_\perp B_0 \cos\omega_c t \mp q v_\perp r_L \cos^2\omega_c t \cdot \dfrac{dB}{dy}$

테일러전개
$\left(\begin{array}{l} f(x) = f(0) + x\dfrac{df}{dx}\Big|_{x=0} + \cancel{\cdots} \\ B(y) = B(0) + y\dfrac{dB}{dy}\Big|_{y=0} + \cancel{\cdots} \end{array}\right)$

gradient 일정 가정
$\left(\dfrac{dB}{dy} = \dfrac{dB}{dy}\Big|_{y=0}\right)$

$\overline{F_y} = \dfrac{1}{\tau}\displaystyle\int_0^\tau F_y\, dt$

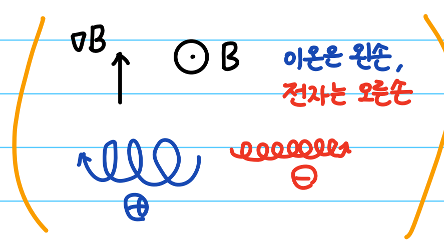
$= \mp \dfrac{1}{\tau} q v_\perp r_L \displaystyle\int_0^\tau \cos^2\omega_c t\, dt \cdot \dfrac{dB}{dy} = \mp q v_\perp r_L \dfrac{1}{2}\dfrac{dB}{dy}$

(i)+(ii) $\quad \vec{F} = \mp q v_\perp r_L \dfrac{1}{2}\dfrac{dB}{dy}\hat{y}$

$V_{\nabla B} = \dfrac{\vec{F}\times\vec{B}}{qB^2} = \dfrac{\mp \cancel{q} v_\perp r_L \frac{1}{2}\frac{dB}{dy}\hat{y}\times\vec{B}}{\cancel{q}B^2} = \dfrac{\mp v_\perp r_L \frac{1}{2}\nabla B\times\vec{B}}{B^2} \quad \left(\dfrac{dB}{dy}\hat{y} = \nabla B\right)$

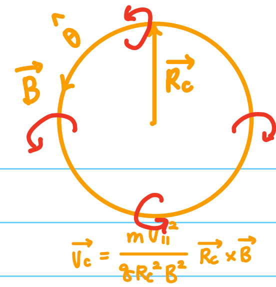
$\boxed{V_{\nabla B} = \pm\dfrac{1}{2}v_\perp r_L \dfrac{\vec{B}\times\nabla B}{B^2}}$

$r_L \ll L_B$
자기장의 변화는 Larmor radius에 비해 매우 작다는 가정이 깔려 있음.



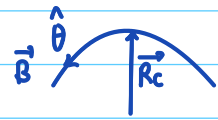
이온은 왼손,
전자는 오른손

2. Curvature



drift 방향은 $\hat{z}$ 이지만, 처음부터 있던 $\vec{v_{\parallel}}$을 함께 고려하면 hellical 한 운동을 생각할 수 있다.

$\vec{B} = B_\theta(r)\,\hat{\theta}$



$$\vec{F} = \frac{mv_{\parallel}^2}{R_c}\hat{r} = \frac{mv_{\parallel}^2}{R_c^2}\vec{R_c}$$

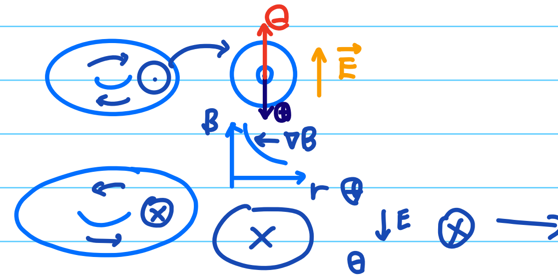
$$\vec{v_c} = \frac{\vec{F_c}\times\vec{B}}{qB^2} = \frac{\frac{mv_{\parallel}^2}{R_c^2}\vec{R_c}\times\vec{B}}{qB^2} = \frac{mv_{\parallel}^2}{qR_c^2B^2}\vec{R_c}\times\vec{B}$$

$$\boxed{\vec{v_c} = \frac{mv_{\parallel}^2}{q}\frac{\vec{R_c}\times\vec{B}}{R_c^2\cdot B^2}}$$

$r_L \ll R_c$ 가 가정되었음.

3. ∇B + curvature

HW: curvature가 있으면 항상 ∇B가 존재한다. 왜?



5강

자기장 방향에 수직인 것 → · ∇B

$$v_{\nabla B} = \pm\frac{1}{2}v_\perp r_L\frac{\vec{B}\times\nabla B}{B^2} = \pm\frac{1}{2}v_\perp\frac{mv_\perp}{|q|B}\frac{\vec{B}}{B}\times\frac{\nabla B}{B} \leftarrow \left(\frac{\nabla B}{B} = -\frac{\vec{R_c}}{R_c^2}\right)$$

$$= \frac{\frac{1}{2}mv_\perp^2}{qB^2}\frac{\vec{B}\times(-\vec{R_c})}{R_c^2} = \frac{\frac{1}{2}mv_\perp^2\,\vec{R_c}\times\vec{B}}{qB^2R_c^2}$$

· curvature

· 자기장 방향과 나란한 것

$$v_c = \frac{mv_{\parallel}^2\,\vec{R_c}\times\vec{B}}{qB^2R_c^2}$$

$$\boxed{v_{\nabla B + c} = \frac{m}{q}\frac{\vec{R_c}\times\vec{B}}{R_c^2B^2}\left(v_{\parallel}^2 + \frac{1}{2}v_\perp^2\right)}$$

· ∇B가 생긴다는 말은 뭔가 $\vec{F}$ 처럼 생각할 수 있는 무언가가 있는게 아닐까? →

$$\cdot\ \vec{v}_{\nabla B} = \pm\frac{1}{2}v_\perp r_L\frac{\vec{B}\times\nabla B}{B^2} = \frac{\vec{F}\times\vec{B}}{qB^2}$$

$$\rightarrow\ -\frac{1}{2}v_\perp\frac{mv_\perp}{\cancel{q}B}\frac{\nabla B\times\vec{B}}{\cancel{B^2}} = \frac{\vec{F}\times\vec{B}}{\cancel{qB^2}} \Rightarrow \vec{F_\perp} = -\frac{\frac{1}{2}mv_\perp^2}{B}\nabla B = -\mu\nabla_\perp B$$

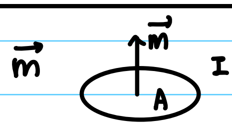
이게 무슨 의미..? ∇B의 수직 방향은 평면인데?

자기장에 수직방향 B의 수직방향

동일한 값

$\boxed{\mu:\ \text{magnetic moment}}$

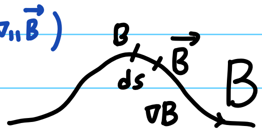
$$\boxed{\mu = \frac{\frac{1}{2}mv_\perp^2}{B}}\ \cdots\ (A)$$



$$m = IA = \frac{|q|}{\tau}\pi r_L^2 = |q|\cdot\frac{w_c}{2\cancel{\pi}}\cancel{\pi}r_L^2 = |\cancel{q}|\frac{|q|B}{2\cancel{m}}\frac{m^2v_\perp^2}{B^2\cancel{q^2}} = \frac{\frac{1}{2}mv_\perp^2}{B}$$

(τ: 주기 $= 2\pi/w_c$)

$(\vec{F_{\parallel}} = -\mu\nabla_{\parallel}\vec{B})$



자기장 선을 따라 가고 있는데, 자기장이 점점 강해지는 경우

$$\vec{F_{\parallel}} = m\frac{d\vec{v_{\parallel}}}{dt} = -\mu\nabla_{\parallel}B$$

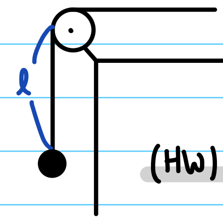
$$v_{\parallel}m\frac{dv_{\parallel}}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv_{\parallel}^2\right) = -\mu\underbrace{\frac{dB}{ds}}_{(\nabla_{\parallel}B)}\cdot\underbrace{\frac{ds}{dt}}_{(\vec{v_{\parallel}})} \approx -\mu\frac{dB}{dt}\ \cdots\ (B)$$

Energy Conservation

$$\frac{d}{dt}\left(\underbrace{\frac{1}{2}mv_{\parallel}^2}_{\text{kinetic E}}+\underbrace{\frac{1}{2}mv_{\perp}^2}_{\text{potential E}}\right)=0 \quad , \quad \frac{d}{dt}\left(\frac{1}{2}mv_{\parallel}^2\right)+\frac{d}{dt}\left(\frac{1}{2}mv_{\perp}^2\right)=0$$

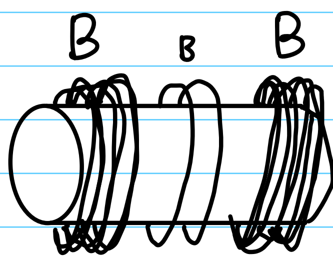
$$\underbrace{-\mu\frac{dB}{dt}}_{Ⓑ}+\underbrace{\frac{d}{dt}(\mu B)}_{Ⓐ}=0 \rightarrow \cancel{-\mu\frac{dB}{dt}}+\cancel{\mu\frac{dB}{dt}}+B\frac{d\mu}{dt}=0$$

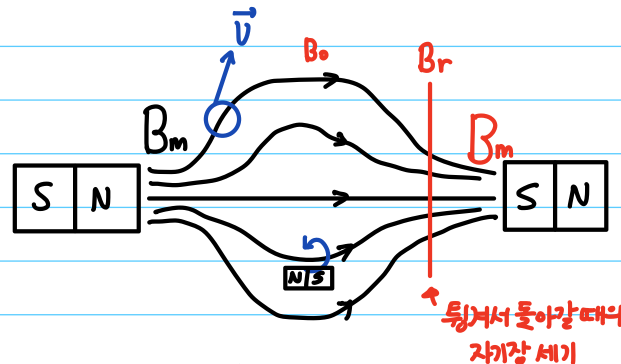
$$\boxed{\frac{d\mu}{dt}=0}$$ μ: adiabatic invariant

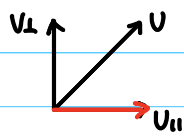


6강

· Magnetic mirror







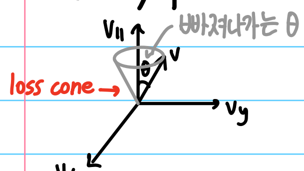
$$\vec{F_{\parallel}}=-\mu\nabla_{\parallel}B=-\frac{\frac{1}{2}mv_{\perp}^2}{B}\nabla B$$

(만약 $\vec{v}=\vec{v_{\parallel}}$ 이면, $\underbrace{\vec{F_{\parallel}}}_{\text{가둬주는 힘.}}=0$ 이므로 모두 빠져나감)

$B_r \le B_m$ 이어야 가둬짐.

$$\left(\begin{array}{l} \cdot\ \frac{1}{\cancel{2}}\cancel{m}v_{\parallel 0}^2+\frac{1}{\cancel{2}}\cancel{m}v_{\perp 0}^2=\cancel{\frac{1}{2}mv_{\parallel r}^2}+\frac{1}{\cancel{2}}\cancel{m}v_{\perp r}^2 \quad \cdots Ⓐ \\ \cdot\ \frac{\frac{1}{\cancel{2}}\cancel{m}v_{\perp 0}^2}{B_0}=\frac{\frac{1}{\cancel{2}}\cancel{m}v_{\perp r}^2}{B_r} \quad \cdots Ⓑ \end{array}\right) \Rightarrow \underset{Ⓑ}{\frac{v_{\perp 0}^2}{v_{\perp r}^2}=\frac{B_0}{B_r}} \Rightarrow \underset{Ⓐ}{\frac{v_{\perp 0}^2}{v_{\perp 0}^2+v_{\parallel 0}^2}=\frac{B_0}{B_r}}$$

<velocity space>



(v∥ 성분이 많으면 빠져나감.)

(θ : $\vec{v_{\parallel}}$ 와 $\vec{v}$ 가 이루는 각도)

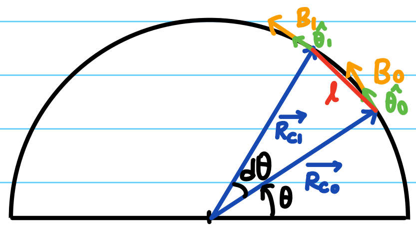
$$\left(\begin{array}{l} v\sin\theta=v_{\perp} \\ v\cos\theta=v_{\parallel} \end{array}\right) \Rightarrow \frac{v_{\perp 0}^2}{v_{\perp 0}^2+v_{\parallel 0}^2}=\frac{v^2\sin^2\theta}{v^2}=\sin^2\theta=\frac{B_0}{B_r}\ge\frac{B_0}{B_m}$$

$R_m=\frac{B_m}{B_0}$: mirror ratio, $\boxed{\sin^2\theta\ge\frac{1}{R_m}}$; trapping condition

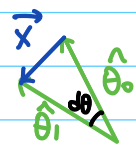
비율이 중요! $\left(\frac{3T}{1T}=\frac{3g}{1g}\right)$

(mobility 큰 electron 이 먼저 빠져나감.)

과제 2



①



$\Rightarrow$ $\hat{\theta}_1 - \hat{\theta}_0 = \vec{x}$, $|\vec{x}| = |\hat{\theta}\, d\theta| = d\theta$

$\vec{x}$의 방향 : $-\hat{r}$, $\vec{x}$의 크기 : $d\theta$

$$\frac{d\hat{\theta}}{d\theta} = \frac{\hat{\theta}_1 - \hat{\theta}_0}{d\theta} = \frac{-\hat{r}\, d\theta}{d\theta} = -\hat{r}$$

$\therefore \dfrac{d\hat{\theta}}{d\theta} = -\hat{r}$ ... ①

② $$\frac{d\hat{\theta}}{dl} = \frac{d\hat{\theta}}{R_c\, d\theta} = -\frac{\hat{r}}{R_c} = -\frac{\vec{R_c}}{R_c^2}$$

① 적용

$\therefore \dfrac{d\hat{\theta}}{dl} = -\dfrac{\vec{R_c}}{R_c^2}$ ... ②

③ $\nabla = \left(\frac{1}{r}\frac{d}{dr}, \frac{1}{r}\frac{d}{d\theta}, \frac{d}{dz}\right) \rightarrow \nabla\cdot\hat{\theta} = \frac{1}{r}\frac{d}{d\theta}$

$$\frac{\vec{R_c}}{R_c^2} = -\frac{d\hat{\theta}}{dl} = -\frac{d\hat{\theta}}{r\,d\theta} = -(\nabla\cdot\hat{\theta})\hat{\theta}$$

$\therefore \dfrac{\vec{R_c}}{R_c^2} = -(\nabla\cdot\hat{\theta})\hat{\theta}$ ...③

④ static state 가정 $\rightarrow$ $\nabla\times\vec{B} = 0$ ($\because$ Maxwell 방정식)

⑤ $(\nabla B)_\theta = \dfrac{dB}{dl} = B\dfrac{d\hat{\theta}}{dl} = -B\dfrac{\;}{R_c^2}$