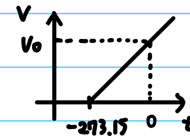
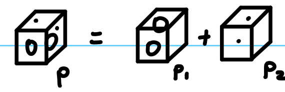
$1m^3$에 $10^{20}$개의 입자가 있고, 이들의 로렌츠힘을 하나하나 계산하면
너무 많은 시간이 걸려 불가능함 ⇒ 전체적인 관점에서 살펴보자.

# 8강

## < kinetic theory of gas > : 기체분자 운동론



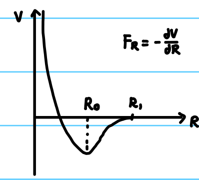

- **Boyle's law** : $PV = \text{const}$ at const $T$
- **Charle's law** : $\frac{V}{T} = \frac{V_0}{273.15^\circ C}$ at const $P$, $T(K) = 273.15^\circ C + t(^\circ C)$
- **equation of state for a gas** : $PV = qRT$ ($q$ mol, $R = 8.31\,J/K\cdot mol$)
- **Joule's law** : The energy content of a gas is independent of its volume.
- **Dolton's law of partial pressures** $p = p_1 + p_2$ at same T, V.



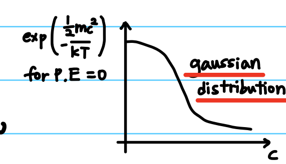

- **Avogadro's hypothesis** :
  at the same P, V, T ⇒ same # of molecules.

### * Assumptions

1. Nature of the molecules
   - identical molecules (mass)
   - spherical molecules
   - negligible intermolecular forces




2. Uniform distribution of the molecules in space (continuous density)
3. Continuous motion of molecules.
4. Isotropy of velocities ; 등방성 (mechanical equilibrium)
5. Maxwell-Boltzmann distribution of velocities in thermal equilibrium.

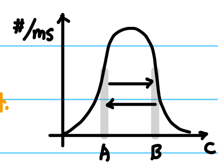



velocity space에서 입자가 발견될 확률 → $dn = A\exp\left(-\frac{\frac{1}{2}mc^2 + P.E}{kT}\right)du\,dv\,dw = A\exp\left(-\frac{E}{kT}\right)du\,dv\,dw = A\exp(-\beta E)\,du\,dv\,dw$

$(c^2 = u^2 + v^2 + w^2)$ velocity space 의 미소체적

Boltzmann factor

6. Equipartition of energy (degree of freedom) ; $\frac{1}{2}kT \to \frac{3}{2}kT$ ($x, y, z$에서 에너지가 각각 $\frac{1}{2}kT$ 씩 나눠 가져서 총 $\frac{3}{2}kT$ 가 된다)
7. Detailed balancing

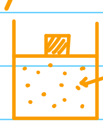



in steady-state equilibrium

A가 충돌해서 B로 갈 확률과,
B가 충돌해서 A로 갈 확률은 서로 같다.

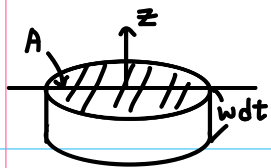
## < Gas theory >

**D. Bernoulli** (1738)



기체를 입자로 바라본 최초의 사람
: Kinetic theory의 시발점.

| | | |
|---|---|---|
| Fourier | | |
| Joule | → Maxwell | → Gibbs |
| Meyer | Boltzmann | Enskog |
| Helmholtz | vs Mach | Chapman |




$\vec{c} = (u, v, w)$

$w$ 성분을 가지는 밀도 : $n_w$

- # of collisions on A : $n_w A w dt$ ( ∵ 밀도 × 부피 : 이 안에 있는 친구들 다 때림)
- rate of collisions on A : $n_w A w$ ( ∵ $\frac{n_w A w dt}{dt} = n_w A w$ )

* molecule partial current

$$J_+ = (n_1 w_1 + n_2 w_2 + \cdots) A \leftarrow \left( \frac{n_1 w_1 + n_2 w_2 + \cdots}{n_1 + n_2 + \cdots} = \frac{n_1 w_1 + n_2 w_2 + \cdots}{n_+} = \overline{w_+} \right)$$

; $w>0$ 인 것들의 평균

$n_+$ : z+방향으로 가는 놈들의 평균

$$= n_+ \overline{w_+} A \leftarrow \left( n_+ = n_- = \frac{1}{2} n, \ \overline{w_+} = \frac{1}{2} \bar{c} \right)$$

받아들이기

$$= \frac{1}{4} n \bar{c} A$$

$$\boxed{J_+ = \frac{1}{4} n \bar{c} A}$$

* momentum partial current

$$\underbrace{n_w A w}_{\text{rate}} \underbrace{m w}_{\text{momentum}} = n_w A m w^2$$

$$\dot{P}_+ = (n_1 w_1^2 + n_2 w_2^2 + \cdots) A m \leftarrow \left( \frac{n_1 w_1^2 + n_2 w_2^2 + \cdots}{n_1 + n_2 + \cdots} = \frac{n_1 w_1^2 + n_2 w_2^2 + \cdots}{n_+} = \overline{w_+^2} \right)$$

; $w>0$ 인 것들의 평균

$n_+$ : z+방향으로 가는 놈들의 평균

$$= n_+ \overline{w_+^2} A m \leftarrow \left( \begin{array}{l} c^2 = u^2 + v^2 + w^2 \\ \overline{u^2} = \overline{v^2} = \overline{w^2} = \frac{1}{3} \overline{c^2}, \ \overline{w^2} = \overline{w_+^2} = \overline{w_-^2} \end{array} \right)$$

$$= \frac{1}{2} n \frac{1}{3} \overline{c^2} A m$$

$$= \frac{1}{6} n m \overline{c^2} A$$

질문 $\dot{P}_+ + \dot{P}_-$
( $\dot{P}_-$를 고려하는 이유는?)

$$P = (\dot{P}_+ + \dot{P}_-) / A = \frac{1}{3} n m \overline{c^2} \qquad \boxed{P = \frac{1}{3} n m \overline{c^2}}$$

← 역학적으로 얻어낸 결과

# 9강 ( 8강 증명 )

① eqn of state for a gas

$$\begin{cases} PV = \mathcal{g} RT \\ PV = \frac{1}{3} nV m \overline{c^2} = \frac{1}{3} N m \overline{c^2} \end{cases}$$

$PV = \mathcal{g} RT = \frac{1}{3} N m \overline{c^2}$ ← 이것이 아래 설명의 시발점

$$\frac{1}{2} m \overline{c^2} = \frac{3}{2N} \frac{1}{3} N m \overline{c^2} = \frac{3}{2N} \mathcal{g} RT = \frac{3}{2} \frac{1}{N} \frac{N}{N_A} RT = \frac{3}{2} \frac{R}{N_A} T = \frac{3}{2} kT$$

$$\boxed{\frac{1}{2} m \overline{c^2} = \frac{3}{2} kT}$$

( $R = 8.31\ \mathrm{J/mol \cdot K}$ , $N_A = 6.02 \times 10^{23} /\mathrm{mol}$ , $k = 1.38 \times 10^{-23}\ \mathrm{J/K}$ )

( 온도 = 입자의 평균운동에너지 )

② Avogadro's hypothesis

$$\begin{cases} PV = \mathcal{g} RT \\ PV = \frac{1}{3} nV m \overline{c^2} = \frac{1}{3} N m \overline{c^2} = \frac{1}{3} N 3kT = NkT \end{cases}$$

$PV = NkT$ , $\boxed{\frac{PV}{T} = Nk}$ at the same $P, V, T$ ⇒ same # of molecules.

③ Boyle's law

$PV = NkT$ = const

④ Charle's law

$\frac{V}{T} = \frac{Nk}{P}$ = const

⑤ Dolton's law of partial pressures

$PV = NkT$ , $P = \frac{N}{V} kT = nkT$

$P_1 = n_1 kT$ , $P_2 = n_2 kT$ → $P = P_1 + P_2 = (n_1 + n_2) kT$

## ( 물리량 감 찾기 )

- number densities

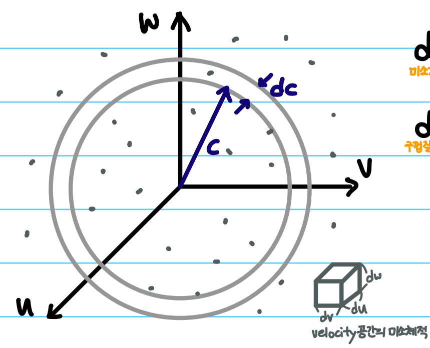
$p = nkT \rightarrow n = \frac{p}{kT}$

ex) 대기 : $2.45 \times 10^{25}\ p\,[atm]$ at $T = 300K$ , 우주 : $10^4\ \#/m^3$ , KSTAR $= 10^{20}\ \#/m^3$

- molecular speeds

$\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$   ex) $H_2 : 1.93 \times 10^3\ m/s$ ( @ 300K )

## * Maxwell's speed distribution ( Thermal Equilibrium = Mechanical Equilibrium )




$dn = A\exp\left(-\frac{\frac{1}{2}mc^2 + P.E}{kT}\right)du\,dv\,dw$ 미소체적

$dn = A\exp\left(-\frac{\frac{1}{2}mc^2 + P.E}{kT}\right)4\pi c^2 dc$ 구형껍질의 체적

$= nBc^2\exp\left(-\frac{\frac{1}{2}mc^2}{kT}\right)dc$

$= nf(c)dc$

$B = 4\pi A\exp\left(-\frac{P.E.}{kT}\right)$ , $n$은 왜 들어가는가? 질문

$f(c) = Bc^2\exp\left(-\frac{\frac{1}{2}mc^2}{kT}\right)$

확률분포함수 → $f(c)$ : Maxwell probability distribution function of molecular speeds. (속력)

$\int_0^\infty f(c)dc = 1$ , $f(c) = Bc^2\exp\left(-\frac{\frac{1}{2}mc^2}{kT}\right)dc$

↳ $B = \sqrt{\frac{2}{\pi}}\left(\frac{m}{kT}\right)^{3/2}$ (HW1)


- most probable speed : $C_\alpha = \sqrt{\frac{2kT}{m}}$ (HW2)
- average speed : $\bar{c} = \int_0^\infty c f(c)dc = \sqrt{\frac{8kT}{\pi m}}$ (HW3)
- rms speed : $\sqrt{\overline{c^2}} = \sqrt{\int_0^\infty c^2 f(c)dc} = \sqrt{\frac{3kT}{m}}$ (HW4)

( $\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT \rightarrow \sqrt{\overline{c^2}} = \sqrt{\frac{3kT}{m}}$ )

( molecule momentum partial current ) ↔ ( 맥스웰 확률분포함수 )
" 기체의 법칙

⇒ 같은 결과!! 물리의 아름다움.

HW : 적분하기 (아래 공식 이용)

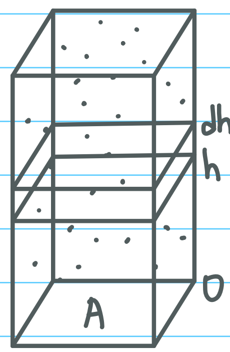
- $\int_0^\infty c^{2n}e^{-ac^2}dc = \frac{1\cdot3\cdot5\cdots(2n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}}$
- $\int_0^\infty c^{2n+1}e^{-ac^2}dc = \frac{n!}{2a^{n+1}}$   $(a>0)$

# 10강

## * Effect of Potential energy

### - Isothermal atmosphere

* kinetic theory로
$n=n_0 \exp\left(-\frac{mgh}{kT}\right)$ 유도




$$dn = A\exp\left(-\frac{\frac{1}{2}mc^2+P.E}{kT}\right)4\pi c^2 dc$$

$$= \underbrace{4\pi A \exp\left(-\frac{P.E}{kT}\right)}_{n} c^2 \exp\left(-\frac{\frac{1}{2}mc^2}{kT}\right)$$

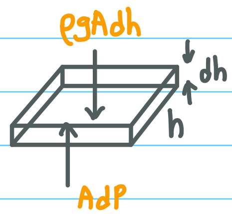
$$= \underbrace{n \quad B}\ c^2 \exp\left(-\frac{\frac{1}{2}mc^2}{kT}\right) dc$$

$$\Rightarrow n = \frac{4\pi A}{B}\exp\left(-\frac{P.E}{kT}\right) = \frac{4\pi A}{B}\exp\left(-\frac{mgh}{kT}\right)$$

$$h=0 \rightarrow n=n_0, \quad n_0 = \frac{4\pi A}{B} \quad \therefore \boxed{n=n_0\exp\left(-\frac{mgh}{kT}\right)}$$

밀도는 위로 갈수록 점점 줄어드는구나.

* mechanics로
$n=n_0 \exp\left(-\frac{mgh}{kT}\right)$ 유도




mass density

$$-\rho g A\, dh = A dP \quad (\rho = nm, \quad P = nkT)$$

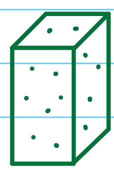
$$-nmgA\, dh = AkT\, dn$$

$$\frac{dn}{n} = -\frac{mg}{kT}dh, \quad \int_{n_0}^{n}\frac{dn}{n} = -\int_0^h \frac{mg}{kT}dh, \quad \ln\left(\frac{n}{n_0}\right) = -\frac{mgh}{kT}$$

$$\therefore \boxed{n = n_0\exp\left(-\frac{mgh}{kT}\right)} \quad \left(ex.\ 0^\circ C \quad n_0 \rightarrow \frac{n_0}{e} \ : 7900m\right)$$

에베레스트의 공기 입자는 $\frac{1}{e}$배 정도

1926. Novel Prize
Jean Baptiste Perrin
HW : 부력고려하여 계산하기

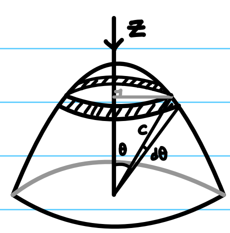


## * molecule partial current

$$J_+ = n_+ \overline{w_+} A = \frac{1}{4}n\bar{c}A$$

$$n_+ = \frac{1}{2}n, \quad \boxed{\overline{w_+} = \frac{1}{2}\bar{c}}$$

유도할거다!



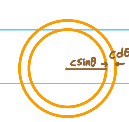

→ velocity space의 그림.
구 윗부분을 적분하면
molecule partial current
계산 가능

띠에 있는 입자수 비율

입체각의 정의

$$dn = nf(c)dc\,\frac{d\Omega}{4\pi}, \quad ds = 2\pi \cancel{c}\sin\theta\, \cancel{c}d\theta = \cancel{c^2}d\Omega$$

$$d\Omega = 2\pi\sin\theta\, d\theta$$




개수 　 z방향의 속도 　 면적

$$dJ_+ = dn\ c\cos\theta\ A$$ (둥근 띠에 있는 molecule partial current.)

$$= nf(c)dc\,\frac{1}{4\pi}(2\pi\sin\theta\, d\theta)\cdot c\cos\theta\ A$$

$$= \frac{1}{2}\sin\theta\cos\theta\, d\theta\, A\, c\, nf(c)dc$$

$$J_+ = \int_0^{\frac{\pi}{2}}\frac{A}{2}\sin\theta\cos\theta\left[\int_0^{\infty}\underbrace{c\,nf(c)dc}_{=n\bar{c}}\right]d\theta$$

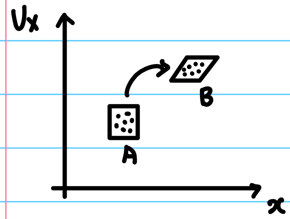
$$= \frac{A}{2}n\bar{c}\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta\, d\theta = \frac{A}{2}n\bar{c}\int_0^{\frac{\pi}{2}}\frac{1}{2}\sin 2\theta\, d\theta = \frac{1}{4}n\bar{c}A$$

(맥스웰 분포함수를 넣고 같은 결과를 얻음.)

* momentum partial current

$\dot{P}_+ = \frac{1}{6} n m \bar{c}^2 A$ ← 동일한 방식으로 증명 (HW)

* Boltzmann equation (플라즈마의 관심사는 분포함수가 어떻게 바뀌는가? 이다.)




phase space

$f(x, v_x, t)$

(1차원)

$/ \int f(\vec{v}) d^3v = n$ ← 플라즈마 쪽에서는 이렇게 사용

(cf. $\int f(c) dc = 1$ (in kinetic theory))

· phase는 변해도, 입자의 수는 변하지 않음.

$$0 = \frac{dN}{dt} = \frac{d}{dt} \int \underline{f(x, v_x, t)\, dx\, dv_x}$$

질문 Space와 Velocity로 적분하면 왜 입자개수가 되는가?

$$= \int \frac{df}{dt} dx\, dv_x$$

(전미분은 편미분의 합으로 표현된다.)

$$(1\text{-D}) \quad \frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial v_x} \cdot \frac{dv_x}{dt} = \frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + a_x \frac{\partial f}{\partial v_x} = 0$$

$$\Rightarrow \underline{\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f - \vec{a} \cdot \frac{\partial f}{\partial \vec{v}} = 0}$$

· $F = m \frac{d\vec{v}}{dt} = m\vec{a} = q(\vec{E} + \vec{v} \times \vec{B})$ , $f(x, y, z, v_x, v_y, v_z, t)$

<지배방정식>

$$\Rightarrow \boxed{\begin{array}{l} \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \frac{q(\vec{E} + \vec{v} \times \vec{B})}{m} \cdot \frac{\partial f}{\partial \vec{v}} = 0 \text{ or } \left(\frac{\partial f}{\partial t}\right)_c \\ \varepsilon_0 \nabla \cdot \vec{E} = \underline{\sigma} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu_0 \underline{\vec{J}} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \end{array}}$$

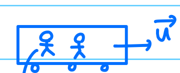
(위에서 유도)

(맥스웰 방정식)

$$\left( \begin{array}{ll} \sigma = \sum_j n_j q_j & \vec{J} = \sum_j n_j q_j \vec{u_j} \\ \quad = \sum_j q_j \int f_j dv_j & \quad = \sum_j q_j \int \vec{v_j} f_j d\vec{v_j} \end{array} \right)$$

(charge density)

($\vec{u}$ : 평균속도, $\vec{v}$ : 개별입자의 속도)




$\vec{v} = \vec{u} + \vec{w}$

↳ $\frac{1}{2} m \overline{w^2} = \frac{3}{2} kT$

$$\left(\frac{\partial f}{\partial t}\right)_c = 0$$ : Vlasov equation (온도 매우 높아서 collision 무시)

$$\left(\frac{\partial f}{\partial t}\right)_c = \frac{f_n - f}{\tau}$$ : Krook collision term (neutral들이 좀 있어서 플라즈마와 충돌하는 경우)

($f_n$ : distribution function of neutral atoms)