

5. Guiding Center Lagrangian

p.2 Lagrangian vs. Hamiltonian

- Euler-Lagrange equation by Lagrangian $L(\vec{q}, \dot{\vec{q}}, t)$

$$\frac{d}{dt} \left(\frac{dL}{dq_i} \right) - \frac{dL}{dq_i} = 0$$

- covariant feature : E-L equation do not change for any generalized coordinates

cf) In Newton's, $F_i = m\ddot{x}_i \rightarrow F_j \neq m\ddot{x}_j$
 $x_i = x_i(q_j, t)$

- Hamiltonian (by Legendre transformation) : $(q_i, \dot{q}_i) \rightarrow (q_i, p_i)$

$$L(q_i, \dot{q}_i, t) \rightarrow p_i = \frac{dL}{d\dot{q}_i}$$

$$H(q_i, p_i, t) = p_i \dot{q}_i - L \quad \leftarrow \text{Hamilton E.o.M}$$

\therefore Law of motion by Hamiltonian $H(q_i, p_i, t)$:

$$\dot{q}_i = \frac{dp_i}{dt}, \quad p_i = -\frac{dH}{dq_i}$$

- Liouville theorem : phase-space volume is conserved.

p.3 Lagrangian and Hamiltonian with EM fields

If one define, $L(\vec{x}, \dot{\vec{x}}, t) = \frac{1}{2}m|\dot{\vec{x}}|^2 + e\vec{A}(\vec{x}, t) \cdot \dot{\vec{x}} - e\phi(\vec{x}, t)$

$$(T = \frac{1}{2}m|\dot{\vec{x}}|^2, V = e\phi - e\vec{A} \cdot \dot{\vec{x}} \Rightarrow L = T - V)$$

(E-L eq.)

$$\frac{d}{dt} \left(\frac{dL}{d\dot{x}} \right) = \frac{d}{dt} (m\dot{\vec{x}} + e\vec{A}) = m\ddot{\vec{x}} + e\frac{d\vec{A}}{dt} + e(\vec{x} \cdot \vec{\nabla})\vec{A} = \frac{dL}{d\vec{x}} = e\vec{\nabla}\vec{A} \cdot \dot{\vec{x}} - e\vec{\nabla}\phi$$

$$\ddot{m\vec{x}} = -e(\vec{\nabla}\phi + \frac{d\vec{A}}{dt}) + e\vec{\nabla}\vec{A} \cdot \dot{\vec{x}} - (\vec{x} \cdot \vec{\nabla})\vec{A} = e\vec{E} + e(\vec{x} \times (\vec{\nabla} \times \vec{A}))$$

$$(\text{Newton}) = e(\vec{E} + \vec{x} \times \vec{B})$$

Hamiltonian,

$$\begin{aligned} H(\vec{x}, \vec{p}, t) &= \vec{p} \cdot \dot{\vec{x}} - L \quad (\text{where } \vec{p} = \frac{dL}{d\dot{\vec{x}}} = m\dot{\vec{x}} + e\vec{A}(\vec{x}, t)) \\ &= m\dot{\vec{x}}^2 + e\vec{A}(\vec{x}, t) \cdot \dot{\vec{x}} - \left(\frac{1}{2}m|\dot{\vec{x}}|^2 + e\vec{A}(\vec{x}, t) \cdot \dot{\vec{x}} - e\phi(\vec{x}, t) \right) \\ &= \frac{1}{2}m|\dot{\vec{x}}|^2 + e\phi(\vec{x}, t) \end{aligned}$$

$$\therefore H(\vec{x}, \vec{p}, t) = \frac{1}{2m} |\vec{p} - e\vec{A}(\vec{x}, t)|^2 + e\phi(\vec{x}, t)$$

Here, (\vec{x}, \vec{p}) are canonical variables : they satisfy Hamiltonian E.o.M.

P.4 Phase-space Lagrangian

- (Issue in L : It does not provide energy or phase space volume conservation explicitly
(Issue in H : It requires canonical transformation and canonical variables are often unphysical.

⇒ Elegant way to overcome this issue : Phase-Space Lagrangian

$$L(\vec{q}, \dot{\vec{q}}, \vec{p}, \dot{\vec{p}}, t) = p_i \dot{q}_i - H(q_i, p_i, t)$$

It is said to be canonical, in a sense that E-L equation yields Hamiltonian equation.

$$\frac{d}{dt}\left(\frac{dL}{dq_i}\right) - \frac{dL}{dq_i} = 0 \rightarrow p_i = -\frac{dH}{dq_i}, \quad \frac{d}{dt}\left(\frac{dL}{dp_i}\right) - \frac{dL}{dp_i} = 0 \rightarrow \dot{q}_i = \frac{dH}{dp_i}$$

P.5 Non-canonical phase space Lagrangian with EM fields

$$L(\vec{x}, \dot{\vec{x}}, \vec{p}, \dot{\vec{p}}, t) = \vec{p} \cdot \dot{\vec{x}} - \frac{1}{2m} |\vec{p} - e\vec{A}(\vec{x}, t)|^2 - e\phi(\vec{x}, t)$$

Lagrangian works with
non-canonical coordinates

$$L(\vec{x}, \dot{\vec{x}}, \vec{v}, \dot{\vec{v}}, t) = (m\vec{v} + e\vec{A}(\vec{x}, t)) \cdot \dot{\vec{x}} - \frac{1}{2}m|\vec{v}|^2 - e\phi(\vec{x}, t)$$

(again, \vec{x}, \vec{v} are independent in terms of L)

$$\frac{d}{dt}\left(\frac{dL}{dv}\right) = \frac{dL}{d\dot{v}} \rightarrow 0 = m\dot{\vec{x}} - m\vec{v} \quad \therefore \dot{\vec{x}} = \vec{v}$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{\vec{x}}}\right) = \frac{dL}{d\vec{x}} \rightarrow \frac{d}{dt}(m\vec{v} + e\vec{A}) - e\vec{v}\vec{A} \cdot \dot{\vec{x}} + e\vec{v}\phi = 0$$

$$m\frac{d\vec{v}}{dt} + e\left(\frac{d\vec{A}}{dt} + (\vec{x} \cdot \vec{\nabla})\vec{A}\right) = e\vec{v}\vec{A} \cdot \dot{\vec{x}} - e\vec{v}\phi$$

$$\begin{aligned}\therefore m \frac{d\vec{v}}{dt} &= e(\vec{v} \cdot \vec{A} \cdot \dot{\vec{x}} - (\vec{x} \cdot \vec{v}) \vec{A}) - e(\vec{v} \phi + \frac{d\vec{A}}{dt}) \\ &= e(\dot{\vec{x}} \times (\vec{v} \times \vec{A})) - e(\vec{v} \phi + \frac{d\vec{A}}{dt}) \\ &= e(\vec{E} + \dot{\vec{x}} \times \vec{B})\end{aligned}$$

* Remember that E-L is not changed
when adding arbitrary total time derivative : $L \rightarrow L + \frac{dS}{dt}$
(gauge transformation)

$$I = \int_1^2 L dt, \quad I' = \int_1^2 (L + \frac{dS}{dt}) dt = \int_1^2 L dt + S(2) - S(1) = I + S(2) - S(1)$$

$$\oint I = 0 \iff \oint (I + S(2) - S(1)) = \oint I = 0 \Rightarrow \text{Same E-L equation}$$

(same actual path)

p.7 Small m/e expansion for guiding center motion

Let $L \rightarrow L/m$, $\epsilon = m/e$, then $L = (\vec{v} + \epsilon^{-1} \vec{A}) \cdot \dot{\vec{x}} - \frac{1}{2} v^2 - \epsilon^{-1} \phi$

$O(\epsilon')$ order : $L_1 = \epsilon^{-1} (\vec{A} \cdot \dot{\vec{x}} - \phi)$

$$\frac{d}{dt} \left(\frac{dL}{d\dot{\vec{x}}} \right) - \frac{dL}{d\vec{x}} = 0 \rightarrow \frac{d\vec{A}}{dt} = \vec{\nabla} \vec{A} \cdot \dot{\vec{x}} - \vec{\nabla} \phi \rightarrow \frac{d\vec{A}}{dt} + (\vec{x} \cdot \vec{\nabla}) \vec{A} - \vec{\nabla} \vec{A} \cdot \dot{\vec{x}} + \vec{\nabla} \phi = 0$$

$$\vec{\nabla} \phi + \frac{d\vec{A}}{dt} + \dot{\vec{x}} \times (\vec{v} \times \vec{A}) = 0 \rightarrow \underline{\vec{E} + \dot{\vec{x}} \times \vec{B} = 0}$$

This determines the perpendicular drift to the leading order becomes $\vec{E} \times \vec{B}$ drift

$$\dot{\vec{x}}_L = \vec{v}_E = \frac{\vec{E} \times \vec{B}}{B^2}, \quad E_{||} \text{ must vanish } O(\epsilon^0) \text{ is consistent}$$

with this result

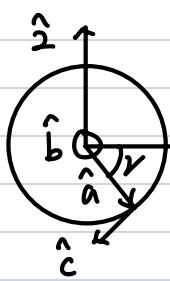
$O(\epsilon^0)$ order

In higher order expansion of Lagrangian, our actual goal is to find **new phase space coordinates** where the rapid oscillatory motion becomes ignorable.

$$(\vec{x}, \vec{v}) \rightarrow (\vec{X}, \vec{u}, \vec{w}, \gamma) \quad \text{Variable } b \rightarrow b, \quad \vec{p} = \vec{u}, \vec{w}, \gamma \text{로 표현가능}$$

where \vec{X} = guiding center, \vec{u} = parallel velocity, \vec{w} = perpendicular velocity,
 γ = gyrophase (ϵ which should be ignorable)

p.9 Phase space coordinates



$$\vec{v} = \hat{u}\hat{b} + \vec{w} + \vec{v}_E = \hat{u}\hat{b} + \hat{w}\hat{c} + \vec{v}_E$$

let rotating perpendicular velocity as, $\hat{c} = -\hat{i}\sin\gamma - \hat{j}\cos\gamma$

then, direction of gyrovector becomes, $\vec{p} = \hat{p}\hat{a}$

$$\hat{a} = \hat{b} \times \hat{c} = \hat{b} \times (-\hat{i}\sin\gamma - \hat{j}\cos\gamma) = \hat{i}\cos\gamma - \hat{j}\sin\gamma = -\frac{d\hat{c}}{d\gamma}$$

However, \vec{p} is the gyrovector in the frame drifting with $\vec{E} \times \vec{B}$ velocity,

thus, let's make the reference frame flexible upon $\vec{E} \times \vec{B}$ by using $\vec{d} = \vec{d}(\vec{x}, t)$

- $\vec{p} = \vec{p}_0 + \vec{d} = \frac{\hat{b} \times \vec{w}}{B} + \vec{d} \rightarrow \left(\frac{\hat{b} \times \vec{w} \hat{c}}{B} = \frac{w}{B} \hat{a}, \quad \frac{w}{B} \hat{a} \times \left(\frac{m}{e} \right) = \frac{mw}{eB} \hat{a} \right)$

$\therefore \vec{p}_0$ is the gyrovector in const \vec{B} and $\vec{E}=0$ to the leading order

$$\vec{x} = \vec{X} + \epsilon \vec{p}^{(0)}$$

- $\dot{\vec{x}} = \dot{\vec{X}} + \dot{\vec{p}}$ where $\vec{p} = \vec{p}(\vec{x}, \gamma, t)$ $\dot{\vec{x}} = \dot{\vec{X}} + \dot{\vec{p}}^{(0)}$

$$\dot{\vec{p}} = \dot{\gamma} \frac{d\vec{p}_0}{dt} + \epsilon \left(\frac{dp_0}{dt} + \vec{v} \cdot \vec{\nabla} p_0 + \vec{d} \right) = \frac{\vec{w}}{B} \dot{\gamma} + O(\epsilon)$$

$$\begin{cases} \vec{p} = \dot{\gamma} \frac{d\vec{p}}{d\gamma} + \epsilon \frac{d\vec{p}}{dt} = \gamma \frac{d\vec{p}}{d\gamma} + O(\epsilon) \\ \vec{p} = \frac{\hat{b} \times \hat{c}}{B} w = \frac{w}{B} \hat{a} = \frac{w}{B} (\hat{i}\cos\gamma - \hat{j}\sin\gamma) \\ \frac{d\vec{p}}{d\gamma} = \frac{w}{B} (\hat{i}\sin\gamma - \hat{j}\cos\gamma) = \frac{w}{B} \hat{c} \end{cases}$$

$$\dot{\vec{p}} = \left(\frac{\partial}{\partial \gamma} \cdot \dot{\gamma} \hat{t} + \frac{d}{dt} \right) \vec{p} = \left(\frac{d\vec{p}}{d\gamma} + \epsilon \frac{d\vec{p}}{dt} \right)$$

p.11 Expansion for guiding center Lagrangian

$$L = (\vec{v} + \epsilon^{-1} \vec{A}) \cdot \dot{\vec{x}} - \frac{1}{2} v^2 - \epsilon^{-1} \phi \quad (\text{Same as above})$$

$$L = \epsilon^{-1} (\vec{A} + \epsilon (\vec{p} \cdot \vec{\nabla}) \vec{A}) \cdot (\dot{\vec{x}} + \dot{\vec{p}}) - \epsilon^{-1} (\phi + \epsilon (\vec{p} \cdot \vec{\nabla}) \phi)$$

$$+ \vec{v} \cdot (\dot{\vec{x}} + \dot{\vec{p}}) - \frac{1}{2} v^2 + O(\epsilon)$$

$$= \underline{\epsilon^{-1} \vec{A} \cdot \dot{\vec{x}}} + \epsilon^{-1} \vec{A} \cdot \dot{\vec{p}} + (\vec{p} \cdot \vec{\nabla}) \vec{A} \cdot \dot{\vec{x}} + (\vec{p} \cdot \vec{\nabla}) \vec{A} \cdot \dot{\vec{p}}$$

$$- \underline{\epsilon^{-1} \phi} - (\vec{p} \cdot \vec{\nabla}) \phi + \vec{v} \cdot (\dot{\vec{x}} + \dot{\vec{p}}) - \frac{1}{2} v^2 + O(\epsilon)$$

Note that $L \rightarrow L + \frac{dS}{dt}$ removing $\frac{dS}{dt}$ is possible.

- (a) $\epsilon^{-1} \vec{A} \cdot \dot{\vec{p}} = \frac{d}{dt} (\vec{A} \cdot \vec{p}) - \left(\frac{d\vec{A}}{dt} + \vec{X} \cdot \vec{\nabla} \vec{A} \right) \cdot \vec{p}$

$$\frac{d}{dt} (\vec{A} \cdot \vec{p}) = \vec{A} \cdot \dot{\vec{p}} + \vec{A} \cdot \vec{p} \rightarrow \frac{d}{dt} (\vec{A} \cdot \vec{p}) = \vec{A} \cdot \dot{\vec{p}} + \epsilon^{-1} \vec{A} \cdot \dot{\vec{p}}$$

$$\begin{aligned} \bullet (a)+(b)+(d) &= -\frac{d\vec{A}}{dt} \cdot \vec{p} - \vec{x} \cdot \vec{\nabla} \vec{A} \cdot \vec{p} + (\vec{p} \cdot \vec{\nabla}) \vec{A} \cdot \vec{x} - (\vec{p} \cdot \vec{\nabla}) \phi \\ &= \vec{p} \cdot \vec{E} + \vec{p} \cdot (\vec{\nabla} \vec{A} \cdot \vec{x} - \vec{x} \cdot \vec{\nabla} \vec{A}) = \vec{p} \cdot (\vec{E} + \vec{x} \times \vec{B}) \end{aligned}$$

$$\bullet (c) (\vec{p} \cdot \vec{\nabla}) \vec{A} \cdot \vec{p} = \in \frac{d}{dt} (\vec{p} \cdot \vec{\nabla} \vec{A} \cdot \vec{p}) - \vec{p} \cdot \vec{\nabla} \vec{A} \cdot \vec{p} - \cancel{\in \vec{p} \cdot \frac{d\vec{A}}{dt} \cdot \vec{p}}$$

$$\begin{aligned} \cancel{\in \frac{d}{dt} (\vec{p} \cdot \vec{\nabla} \vec{A} \cdot \vec{p})} &= \cancel{\in \vec{p} \cdot \vec{\nabla} \vec{A} \cdot \vec{p}} + \cancel{\in \vec{p} \cdot \frac{d}{dt} (\vec{\nabla} \vec{A}) \cdot \vec{p}} + \cancel{\in (\vec{p} \cdot \vec{\nabla}) \vec{A} \cdot \vec{p}} \\ &= \frac{1}{2} \vec{p} \cdot (\vec{\nabla} \vec{A} \cdot \vec{p} - \vec{p} \cdot \vec{\nabla} \vec{A}) = \frac{1}{2} \vec{p} \cdot (\vec{p} \times (\vec{\nabla} \times \vec{A})) = \frac{1}{2} \vec{p} \cdot (\vec{p} \times \vec{B}) \end{aligned}$$

$$\Rightarrow L = \in^{-1}(\vec{A} \cdot \vec{x} - \phi) + \vec{p} \cdot (\vec{E} + \vec{x} \times \vec{B}) + \frac{1}{2} \vec{p} \cdot (\vec{p} \times \vec{B}) + \vec{v} \cdot (\vec{x} + \vec{p}) - \frac{1}{2} v^2 + O(\in)$$

One can obtain (from $\vec{p} = \frac{\hat{b} \times \hat{w} \hat{c}}{B} + \vec{d}$, $\vec{p} = \frac{\hat{w}}{B} \dot{\gamma}$)

$$\bullet \vec{p} \cdot \vec{E} = \frac{\hat{b} \times \hat{w}}{B} \cdot \vec{E} + \vec{d} \cdot \vec{E} = \hat{w} \cdot \frac{\vec{E} \times \hat{b}}{B} + \vec{d} \cdot \vec{E} = \underline{\hat{w} \cdot \vec{v}_E + \vec{d} \cdot \vec{E}}$$

$$\begin{aligned} \bullet \vec{p} \cdot (\vec{x} \times \vec{B}) &= \frac{\hat{w} \hat{a}}{B} \cdot (\vec{x} \times \vec{B} \hat{b}) + \vec{d} \cdot (\vec{x} \times \vec{B}) = \vec{x} \cdot (\hat{w} \hat{b} \times \hat{a}) + \vec{x} \cdot (\vec{B} \times \vec{d}) \\ &= \underline{-\vec{x} \cdot \hat{w} + \vec{x} \cdot (\vec{B} \times \vec{d})} \end{aligned}$$

$$\begin{aligned} \bullet \frac{1}{2} \vec{p} \cdot (\vec{p} \times \vec{B}) &= \frac{1}{2} \frac{\hat{w} \hat{a}}{B} \cdot \left(\frac{\hat{w} \hat{c}}{B} \dot{\gamma} \times \vec{B} \hat{b} \right) + \frac{1}{2} \vec{d} \cdot \left(\frac{\hat{w} \hat{c}}{B} \dot{\gamma} \times \vec{B} \hat{b} \right) = -\frac{\hat{w}^2}{2B} \dot{\gamma} - \frac{\hat{w} \dot{\gamma}}{2} \hat{a} \cdot \vec{d} \\ &= \underline{-\frac{\hat{w}^2}{2B} \dot{\gamma} - \frac{B}{2} \vec{d} \cdot \vec{p} \cdot \dot{\gamma}} \quad (\vec{p} = \frac{\hat{b} \times \hat{w} \hat{c}}{B} = \frac{\hat{w}}{B} \hat{a} \rightarrow \hat{w} \hat{a} = \vec{p} \cdot \vec{B}) \end{aligned}$$

$$\bullet \vec{v} \cdot \vec{p} = (\vec{u} + \vec{w} + \vec{v}_E) \cdot \vec{p} = \frac{\hat{w}^2}{B} \dot{\gamma} + \frac{\vec{v}_E \cdot \hat{w}}{B} \dot{\gamma}$$

$$\bullet \frac{1}{2} v^2 = \frac{1}{2} u^2 + \frac{1}{2} w^2 + \frac{1}{2} v_E^2 + \vec{w} \cdot \vec{v}_E$$

Altogether one obtains:

$$\begin{aligned} L &= \in^{-1}(\vec{A} \cdot \vec{x} - \phi) + \vec{w} \cdot \vec{v}_E + \vec{d} \cdot \vec{E} - \vec{x} \cdot (\vec{w} - \vec{B} \times \vec{d}) - \dot{\gamma} \left(\frac{\hat{w}^2}{2B} + \frac{B \vec{d} \cdot \vec{p}_o}{2} - \frac{\hat{w}^2}{B} - \frac{\vec{v}_E \cdot \hat{w}}{B} \right) \\ &\quad + (\vec{u} + \vec{w} + \vec{v}_E) \cdot \vec{x} - \frac{1}{2} u^2 - \frac{1}{2} w^2 - \frac{1}{2} v_E^2 - \vec{v} \cdot \vec{v}_E \\ &= \underline{u \hat{b} \cdot \vec{x} + \hat{w} \cdot \vec{x} + \vec{v}_E \cdot \vec{x}} \end{aligned}$$

$$L = \underbrace{(\epsilon^{-1} \vec{A} + u \hat{b}) \cdot \dot{\vec{X}}}_{+ \dot{\vec{X}} (\vec{v}_E + \vec{B} \times \vec{d})} + \underbrace{\left(\frac{\omega^2}{2B} + \frac{\vec{v}_E \cdot \vec{\omega}}{B} - \frac{\vec{B} \vec{d} \cdot \vec{p}_0}{2} \right) \dot{y}}_{+ \left(\vec{d} \cdot \vec{E} - \frac{v_E^2}{2} \right) - \left(\epsilon^{-1} \phi + \frac{u^2}{2} + \frac{\omega^2}{2} - \frac{v_E^2}{2} \right) + O(\epsilon)} \dots (38)$$

p.14 Guiding center Lagrangian by Littlejohn and Cary

let's choose $\vec{d} = \frac{\vec{B} \times \vec{v}_E}{B^2}$: guiding-center polarization displacement
in the rotating $\vec{E} \times \vec{B}$ frame

Then, eq. (38) Simplifies to $\left(\frac{B \vec{B} \times \vec{v}_E}{2B^2} \cdot \vec{p}_0 = \frac{\vec{B} \vec{B} \times \vec{v}_E}{2B} \cdot \frac{\omega}{B} \hat{a} = \frac{\vec{v}_E \cdot \vec{\omega}}{2B} \right)$

$$L = (\epsilon^{-1} \vec{A} + u \hat{b}) \cdot \dot{\vec{X}} + \left(\frac{\omega^2}{2B} + \frac{\vec{v}_E \cdot \vec{\omega}}{2B} \right) \dot{y} - \left(\epsilon^{-1} \phi + \frac{u^2}{2} + \frac{\omega^2}{2} - \frac{v_E^2}{2} \right) + O(\epsilon)$$

$$(*) \left(\vec{v}_E \cdot \frac{\vec{\omega}}{2B} \right) \dot{y} = \frac{\epsilon}{2} \frac{d}{dt} (\vec{v}_E \cdot \vec{p}_0) - \frac{\epsilon}{2} \left(\frac{d \vec{v}_E}{dt} \right) \cdot \vec{p}_0$$

$$\epsilon \frac{d}{dt} (\vec{v}_E \cdot \vec{p}_0) = \epsilon \frac{d}{dt} \left(\frac{d \vec{v}_E}{dt} \right) \vec{p}_0 + \vec{v}_E \cdot \frac{d \vec{p}_0}{dt} \quad \left(\frac{d \vec{p}_0}{dt} = \frac{\vec{\omega}}{B} \dot{y} \right)$$

$$\therefore L = (\epsilon^{-1} \vec{A} + u \hat{b}) \cdot \dot{\vec{X}} + \frac{\omega^2}{2B} \dot{y} - \left(\epsilon^{-1} \phi + \frac{u^2}{2} + \frac{\omega^2}{2} - \frac{v_E^2}{2} \right) + O(\epsilon)$$

put back $\epsilon = \frac{m}{e}$, $L \rightarrow mL$, $\dot{y} \rightarrow e\dot{y}$, with definition of $\mu \equiv \frac{mw^2}{2B}$

$$L = m \left(\frac{e}{m} \vec{A} + u \hat{b} \right) \cdot \dot{\vec{X}} + \frac{mw^2}{2B} e \dot{y} - m \left(\frac{e}{m} \phi + \frac{u^2}{2} + \frac{\omega^2}{2} - \frac{v_E^2}{2} \right)$$

$$\therefore L_{gc} \equiv (m u \hat{b} + e \vec{A}) \cdot \dot{\vec{X}} + \frac{m}{e} \mu \dot{y} - H_{gc}$$

$$H_{gc} \equiv \frac{1}{2} m u^2 + \mu B + e \phi - \frac{1}{2} m v_E^2$$

note that 'y' is a cyclic coordinate

Littlejohn's G.C. Lagrangian can be obtained by $v_E = 0$

The addition of v_E was done later by Cary.

p.16 Modified potentials and EM fields

Littlejohn's L_{gc} is often written :

$$L_{gc} = e \vec{A}^* \cdot \dot{\vec{x}} + \epsilon \mu \dot{y} - \left(\frac{1}{2} m \dot{u}^2 + \mu B + e \phi^* \right)$$

with $\begin{cases} \text{modified vector potential} & \vec{A}^* \equiv \vec{A} + \epsilon u \hat{b} \\ \text{modified electric potential} & \phi^* \equiv \phi - \frac{1}{2} \epsilon u^2 \end{cases}$

Accordingly modified EM fields, seen by guiding center becomes

$$\vec{B}^* = \vec{\nabla} \times \vec{A}^*$$

$$\vec{E}^* = -\vec{\nabla} \phi^* - \frac{d \vec{A}^*}{dt}$$