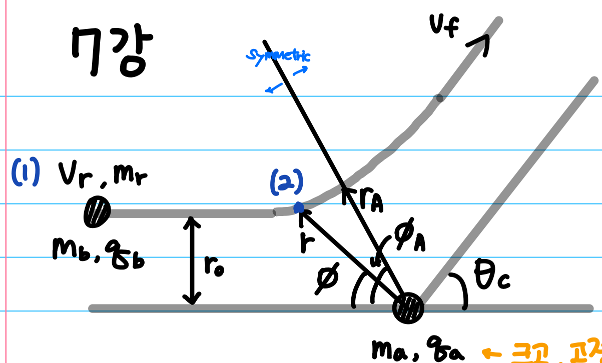
7강



← 크고, 고정된 것

Alfvén : guiding center motion

Cavendish lab
(Cambridge univ)

1871 Maxwell	1954 Mott
1879 Rayleigh	G.P. Thompson
1884 J.J Thomson	Chadwick
1919 Rutherford	Compton
	Wilson
1938 Bragg	Richardson

원자핵의 발견
↳ Rutherfold Scattering

r_0 vs θ_c

→ r_0와 θ_c의 관계를 구해보자.
↳ 반비례할 것으로 추론

· **Energy conservation**

$$E = \underbrace{\frac{1}{2}m_r v_r^2}_{(1)} = \frac{1}{2}m_r v_f^2 = \underbrace{\frac{1}{2}m_r(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{q_a q_b}{4\pi\varepsilon_0 r}}_{(2)}$$

↳ $v_f = v_r$

· **Angular momentum conservation**

$$m_r r^2\dot{\phi} = m_r v_r r_0 = m_r v_f r_0' \Rightarrow \underbrace{\dot{\phi} = \frac{v_r r_0}{r^2}}_{(3)}$$

↳ $r_0' = r_0$

· $2K \equiv \dfrac{q_a q_b / 4\pi\varepsilon_0}{\frac{1}{2}m_r v_r^2} = \text{const}$ (물리적 의미 : $\theta_c = \frac{\pi}{2}$ 일 때의 r_0값 $= K$)

(1) = (2) 에 (3) 대입 　　　(3)을 미분

근의공식 사용

$$\dot{r} = \mp v_r\left(1 - \frac{2k}{r} - \frac{r_0^2}{r^2}\right)^{1/2}, \quad \frac{d\phi}{dr} = \frac{\dot{\phi}}{\dot{r}} \quad \left(\because \frac{d\phi}{dr} = \frac{d\phi}{dt}\cdot\frac{dt}{dr}\right)$$

(+,−)는 가까워지고,
(+,+),(−,−)는 멀어진다

$$\frac{d\phi}{dr} = \mp\frac{r_0}{r^2}\left(1 - \frac{2k}{r} - \frac{r_0^2}{r^2}\right)^{-1/2} \quad (\text{경계조건 } r\to\infty \,;\, \phi = 0)$$

$$\phi = \cos^{-1}\left[\frac{r_0}{\sqrt{r_0^2+k^2}}\left(1 - \frac{2k}{r} - \frac{r_0^2}{r^2}\right)^{1/2}\right] - \cos^{-1}\left(\frac{r_0}{\sqrt{r_0^2+k^2}}\right) \quad \text{HW}$$

가장 가까울 때
(최소값)

$$(r = r_A, \; \dot{r}_A = 0) \rightarrow 1 - \frac{2k}{r_A} - \frac{r_0^2}{r_A^2} = 0$$

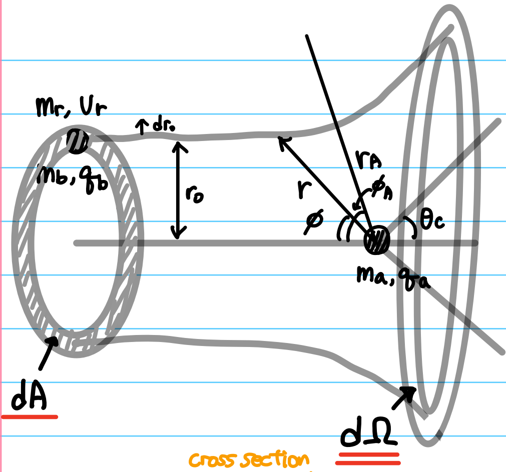
$$\phi_A = \cos^{-1}0 - \cos^{-1}\left(\frac{r_0}{\sqrt{r_0^2+k^2}}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{r_0}{\sqrt{r_0^2+k^2}}\right)$$

$$(\pi - 2\phi_A = \theta_c) \Rightarrow \theta_c = 2\cos^{-1}\left(\frac{r_0}{\sqrt{r_0^2+k^2}}\right) \rightarrow \cos\frac{\theta_c}{2} = \frac{r_0}{\sqrt{r_0^2+k^2}}, \quad \sin\frac{\theta_c}{2} = \frac{k}{\sqrt{r_0^2+k^2}}$$

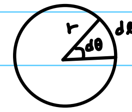
$$\frac{k}{r_0} = \tan\frac{\theta_c}{2} \;\text{ or }\; k = r_0\tan\frac{\theta_c}{2}, \quad 2k = \frac{q_a q_b/4\pi\varepsilon_0}{\frac{1}{2}m_r v_r^2}$$

(k의 물리적 의미 : $\theta_c = \frac{\pi}{2} \rightarrow k = r_0$)

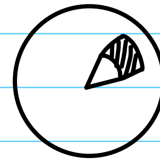
$\theta_c = \frac{\pi}{2} \rightarrow k = r_0\tan\frac{\pi}{4} = r_0$
즉, $\theta_c = \frac{\pi}{2}$ 일때의 r_0의 값



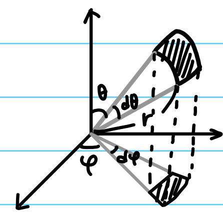
· solid angle (입체각)

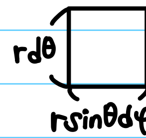


$dl = r\,d\theta$



$ds = r^2 d\Omega$





$\Rightarrow d\Omega = \frac{ds}{r^2} = \frac{r^2 \sin\theta\, d\theta\, d\varphi}{r^2}$

$\therefore d\Omega = \sin\theta\, d\theta\, d\varphi$

cross section = 미소면적

· $dA \equiv d\sigma_s = 2\pi r_0\, dr_0$, $d\Omega = 2\pi \sin\theta_c\, d\theta_c$

The differential scattering cross section for scattering into $(\theta_c, \theta_c + d\theta_c)$

$$d\sigma_s = \frac{d\sigma_s}{d\Omega} d\Omega \equiv \sigma_s'(\theta_c)\, d\Omega = \sigma_s'(\theta_c)\, 2\pi \sin\theta_c\, d\theta_c = 2\pi r_0\, dr_0$$

☆질문 (dΩ로 도달한 비율 중, dA에서 출발한 비율 →

= 과제

* Coulomb cross section (Rutherford scattering cross section) : $\sigma_s'(\theta_c) = \frac{d\sigma_s}{d\Omega}$

$$\sigma_s'(\theta_c) \equiv \frac{d\sigma_s}{d\Omega} = \frac{r_0\, dr_0}{\sin\theta_c\, d\theta_c}$$

$$= \frac{r_0}{\sin\theta_c}\left(\frac{r_0^2 \sec^2\frac{\theta_c}{2}}{2K}\right) \leftarrow \frac{K}{r_0} = \tan\frac{\theta_c}{2},\ -\frac{K}{r_0^2} dr_0 = \frac{1}{2}\sec^2\left(\frac{\theta_c}{2}\right) d\theta_c$$

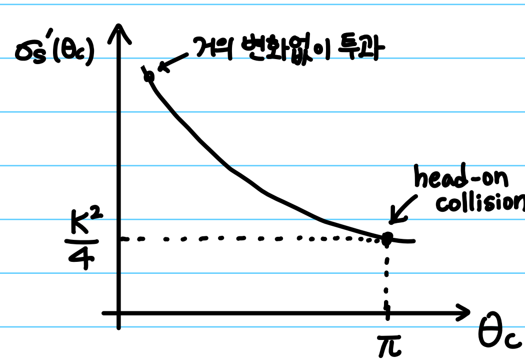
$$\left|\frac{dr_0}{d\theta_c}\right| = \frac{r_0^2}{2K}\sec^2\left(\frac{\theta_c}{2}\right)$$

$$= \frac{K^2}{4\sin^4\left(\frac{\theta_c}{2}\right)} \leftarrow \sin\theta_c = 2\sin\frac{\theta_c}{2}\cos\frac{\theta_c}{2}$$

$$= \frac{1}{4}\left(\frac{q_a q_b}{4\pi\varepsilon_0 m_r v_r^2}\right)^2 \frac{1}{\sin^2\left(\frac{\theta_c}{2}\right)}$$

$$\boxed{\sigma_s'(\theta_c) = \frac{K^2}{4\sin^4\left(\frac{\theta_c}{2}\right)}}$$

HW: physical meaning



Total scattering cross section

$$\sigma_s = \int \sigma_s'(\theta_c)\, d\Omega = 2\pi \int_{\theta_{min}}^{\pi} \left[\frac{k^2}{4\sin^4\left(\frac{\theta_c}{2}\right)} \right] \sin\theta_c\, d\theta_c$$

$$= \pi k^2 \int_{\theta_{min}}^{\pi} \frac{\cos(\theta_c/2)}{\sin^3(\theta_c/2)}\, d\theta_c = 2\pi k^2 \int_{\sin\frac{\theta_{min}}{2}}^{\sin\frac{\pi}{2}} \frac{d(\sin\theta)}{\sin^3\theta}$$

$$= \pi k^2 \left\{ \left[\sin\left(\frac{\theta_{min}}{2}\right) \right]^{-2} - 1 \right\}$$

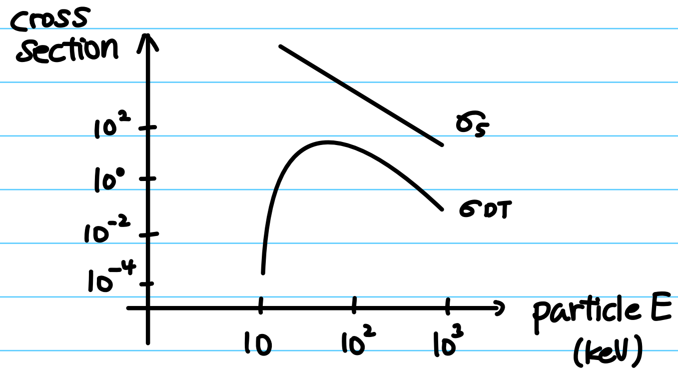
$$\begin{pmatrix} r_0^{max} = \lambda_D \\ \theta_{min} = 2\tan^{-1}\left(\frac{k}{\lambda_D}\right) \end{pmatrix}$$ ← 플라즈마의 조건 : collective behavior

θ_c를 0이 아니라, θ_{min}으로 적음.

→ $\theta_c = 0$ 이면, m_r 이 전기장을 전혀 느끼지 못하는 상황

⇒ θ_{min} 에 해당하는 r_0 = "Debye Length"

($r_0 > \lambda_D$ → quasi-neutrality ,
$r_0 < \lambda_D$ → 전기장을 느낌)



(Scattering cross section)

$\gg$

(fusion cross section)

∴ Beam Fusion 이 불가능한 이유를 설명

→ 한번에 쏘지말고, Confine 하는 이유.