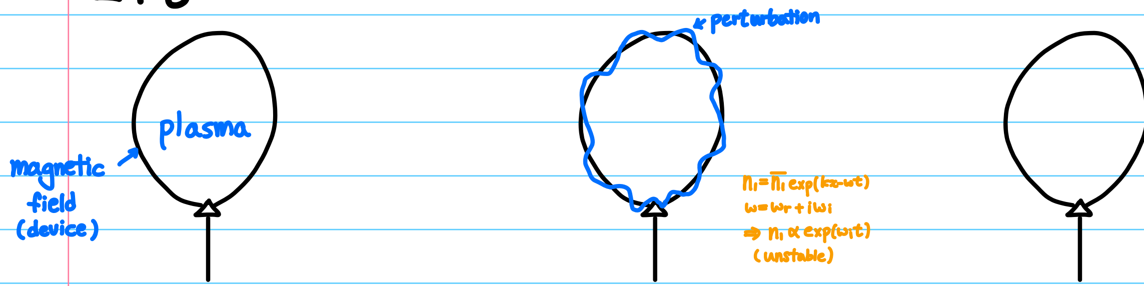
24강

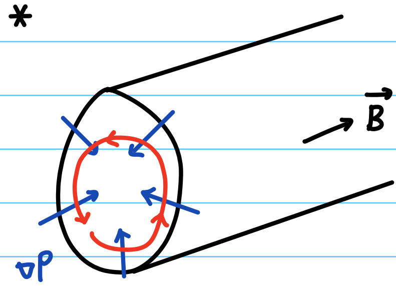


equilibrium (force balance)
내부 P, 외부 P, 텐션 고려

stability equilibrium but unstable

transport leakage and diffusion

*



$$\vec{U_D} = \frac{\vec{B}\times\nabla P}{nqB^2}$$

$$\vec{j_D} = n_i q_i \vec{u_D^i} + n_e q_e \vec{u_D^e} \qquad (n_i = n_e)$$

$$= \cancel{ne}\frac{\vec{B}\times\nabla P_i}{\cancel{ne}B^2} + \cancel{n(-e)}\frac{\vec{B}\times\nabla P_e}{\cancel{n(-e)}B^2} = \frac{\vec{B}\times\nabla P}{B^2} \quad (P = P_i + P_e)$$

recall

MHD equations

$$\cancel{\frac{d\rho}{dt}} + \nabla\cdot(\cancel{\rho\vec{u}}) = 0, \quad \cancel{\frac{d\sigma}{dt}} + \nabla\cdot\vec{j} = 0 \qquad \varepsilon_0\nabla\cdot\vec{E} = \sigma$$

$$\rho\cancel{\frac{d\vec{u}}{dt}} = \cancel{\sigma\vec{E}} + \vec{j}\times\vec{B} - \nabla P \qquad \nabla\cdot\vec{B} = 0$$

$$\vec{E} + \cancel{\vec{u}\times\vec{B}} = \eta\vec{j} + \cancel{\frac{\vec{j}\times\vec{B} - \nabla P_e}{ne}} \qquad \nabla\times\vec{E} = -\cancel{\frac{\partial\vec{B}}{\partial t}}$$

$$\nabla\times\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0\cancel{\frac{\partial\vec{E}}{\partial t}}$$

* : 매우 작아서 무시
* : Static plasma 가정 ($\frac{d}{dt}=0$, $\vec{u}=0$)

$$\Rightarrow \quad \vec{j}\times\vec{B} = \nabla P, \quad \nabla\cdot\vec{B} = 0, \quad \nabla\times\vec{B} = \mu_0\vec{j}, \quad \vec{E} = \eta\vec{j}$$

$$\boxed{\vec{j}\times\vec{B} = \nabla P}$$

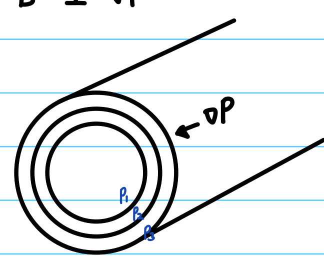
$$\vec{B}\times(\vec{j}\times\vec{B}) = \vec{B}\times\nabla P$$

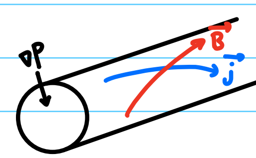
$$\vec{j}(\vec{B}\cdot\vec{B}) - \vec{B}(\vec{j}\cdot\vec{B}) = \vec{B}\times\nabla P$$

$$\perp : \vec{j_\perp}B^2 = \vec{B}\times\nabla P \qquad \therefore \vec{j_\perp} = \frac{\vec{B}\times\nabla P}{B^2}$$

(자기장을 걸어주면, $\vec{j_D}$가 유도되고 이 $\vec{j_D}\times\vec{B}$ 가 ∇P force와 평형을 이룬다.)

$\left(\begin{array}{l}\vec{j} \perp \nabla P \\ \vec{B} \perp \nabla P\end{array}, \ \vec{j}\times\vec{B} = \nabla P\right)$ 이므로 등압 실린더 위에서 $\vec{j}$와 $\vec{B}$값은 모두 그 크기가 같다.





* Concept of β

$$\nabla P = \vec{j}\times\vec{B}$$

$$= \frac{1}{\mu_0}(\nabla\times\vec{B})\times\vec{B} = \frac{1}{\mu_0}\left[(\vec{B}\cdot\nabla)\vec{B} - \frac{1}{2}\nabla B^2\right]$$

$\nabla\left(P+\frac{B^2}{2\mu_0}\right) = \frac{1}{\mu_0}\cancel{(\vec{B}\cdot\nabla)\vec{B}}$ 선형 장치 $\rightarrow \nabla\left(P+\frac{B^2}{2\mu_0}\right)=0 \quad \therefore P+\boxed{\frac{B^2}{2\mu_0}} = \text{const}$

magnetic field pressure

(무차원변수) $\beta \equiv \frac{P}{B^2/2\mu_0} = \frac{\text{Plasma particle pressure}}{\text{magnetic field pressure}}$ (태양 : $\beta \gg 1$)

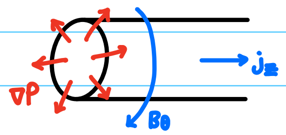
재질의 문제를 이 값으로 normalize 된다.

25강

Plasma Equilibrium

pinch의 방향은 current가 기준

* z-pinch



$\nabla\times\vec{B} = \mu_0\vec{j}$

$\vec{j}\times\vec{B} = \nabla P$

*참고 $\nabla\times\vec{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\theta} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & rA_\theta & A_z \end{vmatrix}$

$$\left(\nabla\times\vec{B} = \frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\theta} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ 0 & rB_\theta & 0 \end{vmatrix}, \quad \vec{j}\times\vec{B} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{z} \\ 0 & 0 & j_z \\ 0 & B_\theta & 0 \end{vmatrix} \right)$$

$$\frac{1}{r}\frac{d}{dr}(rB_\theta) = \mu_0 \underline{j_z}$$

$$-\underline{j_z}B_\theta = \frac{dP}{dr}$$

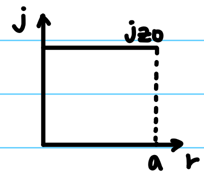
$$-\frac{B_\theta}{r\mu_0}\frac{d}{dr}(rB_\theta) = \frac{dP}{dr} \rightarrow -\frac{B_\theta}{r\mu_0}\left(B_\theta + r\frac{dB_\theta}{dr}\right) = \frac{dP}{dr}$$

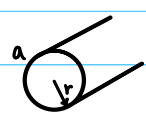
$$\rightarrow -\frac{B_\theta^2}{\mu_0 r} - \underbrace{\frac{B_\theta}{\mu_0}\frac{dB_\theta}{dr}}_{\hookrightarrow -\frac{1}{2\mu_0}\frac{dB_\theta^2}{dr}} = \frac{dP}{dr} \Rightarrow \frac{dP}{dr} + \frac{d}{dr}\left(\frac{B_\theta^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r} = 0$$

자기장의 tension force

$$\boxed{\frac{d}{dr}\left(P+\frac{B_\theta^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r} = 0}$$

ex)





$I = \pi a^2 j_{z0}$

$B_{\theta a} = \frac{\mu_0 I}{2\pi a}$

$P(a) = 0$

$\xrightarrow{\text{계산}}$

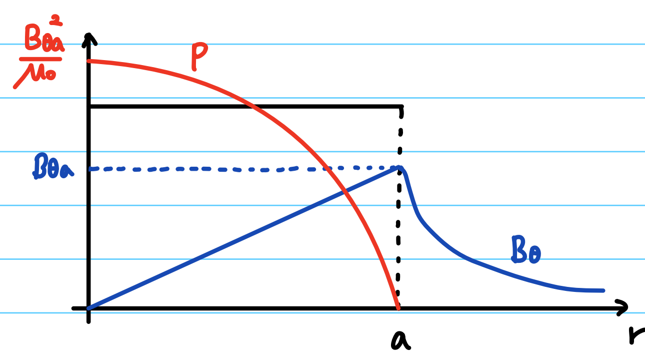
① $B_\theta(r) = B_{\theta a}\left(\frac{r}{a}\right)$

② $P(r) = \frac{B_{\theta a}^2}{\mu_0}\left(1-\frac{r^2}{a^2}\right)$

③ $\beta_\theta(r) = 2\left(\frac{a^2}{r^2}-1\right)$

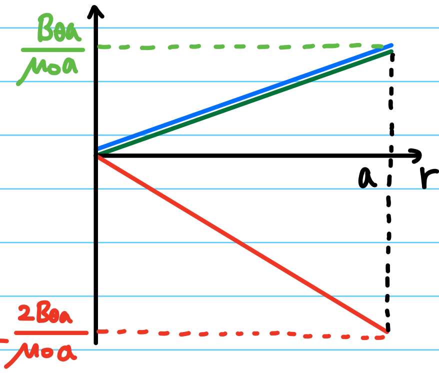
$$\langle\beta_\theta\rangle = \frac{\langle P\rangle}{B_{\theta a}^2/2\mu_0} = 1,$$

$$\langle P\rangle = \frac{\int_0^a 2\pi r P(r)\,dr}{\pi a^2}$$



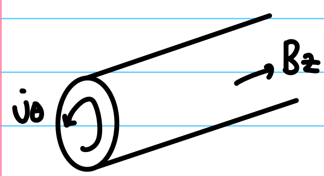
(z-pinch에서 평형을 만족하는 profile)

$$\frac{d}{dr}\left(P+\frac{B_\theta^2}{2\mu_0}\right)+\frac{B_\theta^2}{\mu_0 r}=0 \;\rightarrow\; \underbrace{\frac{dP}{dr}}_{①}+\underbrace{\frac{d}{dr}\left(\frac{B_\theta^2}{2\mu_0}\right)}_{②}+\underbrace{\frac{B_\theta^2}{\mu_0 r}}_{③}=0$$



magnetic pressure와 tension force의 합이 플라즈마의 팽창력을 그대로 balancing 해준다.

* θ-pinch



$$\nabla\times\vec{B}=\mu_0\vec{j}$$

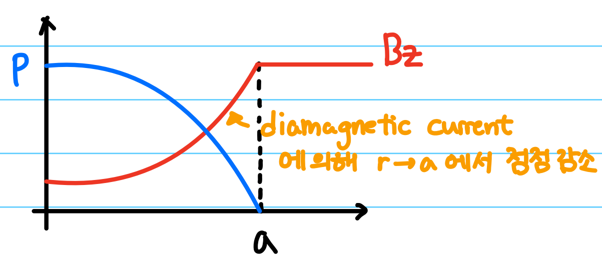
$$\vec{j}\times\vec{B}=\nabla P$$

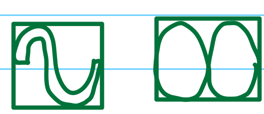
$$\left(\nabla\times\vec{B}=\frac{1}{r}\begin{vmatrix}\hat{r} & r\hat{\theta} & \hat{z}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z}\\ 0 & 0 & B_z\end{vmatrix},\quad \vec{j}\times\vec{B}=\begin{vmatrix}\hat{r} & \hat{\theta} & \hat{z}\\ 0 & j_\theta & 0\\ 0 & 0 & B_z\end{vmatrix}\right)$$

$$\left(\begin{aligned}-\frac{dB_z}{dr}&=\mu_0 \underline{j_\theta}\\ \underline{j_\theta} B_z&=\frac{dP}{dr}\end{aligned}\right)$$

$$-\frac{B_z}{\mu_0}\frac{dB_z}{dr}=\frac{dP}{dr} \;\rightarrow\; -\frac{d}{dr}\left(\frac{B_z^2}{2\mu_0}\right)=\frac{dP}{dr} \qquad \boxed{\therefore\ \frac{d}{dr}\left(P+\frac{B_z^2}{2\mu_0}\right)=0}$$

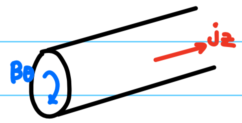
no tension force



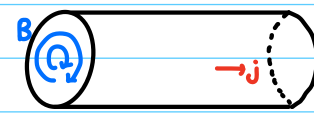
(예고) z-pinch에서  와 같은 불안정성 발생

26강

* z-pinch

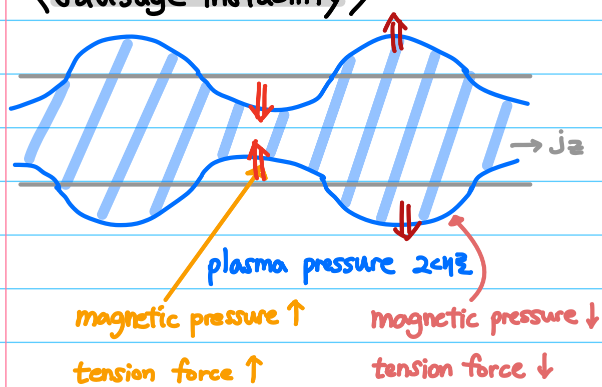


$$\frac{d}{dr}\left(p+\frac{B_\theta^2}{2\mu_0}\right)+\frac{B_\theta^2}{\mu_0 r}=0$$



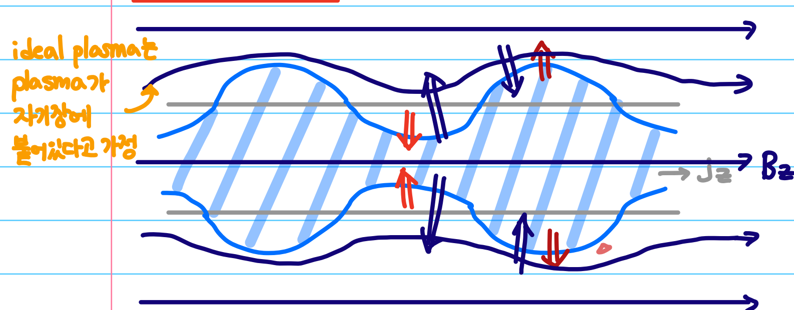
collision에의한 diffusion & 양쪽 끝으로 이동함으로 τ 가 결정될것으로 추측

<sausage instability>

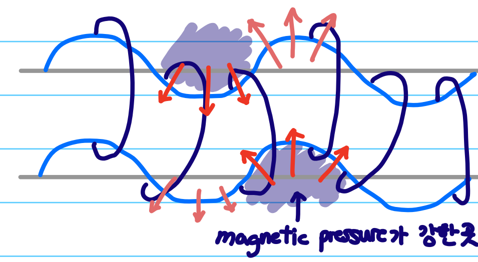


↓

축 방향의 자기장을 걸면, instability가 어느정도 해소가 됨.

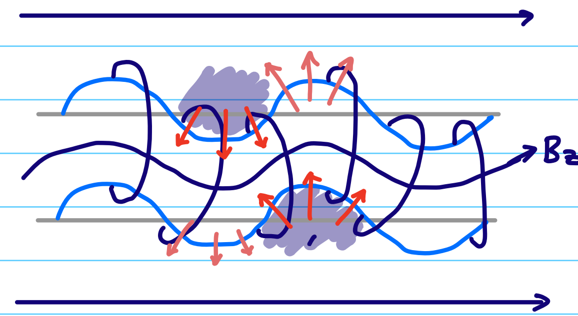


<kink instability>

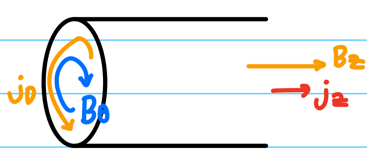


↓

축방향 자기장을 걸면, magnetic tension에 의해 instability의 grow가 약해진다.

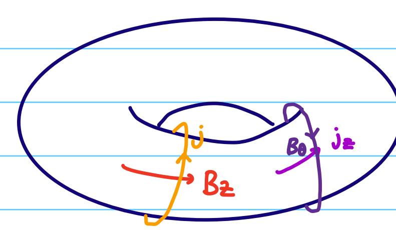


* 축방향 자기장을 고려한 z-pinch



but 양쪽 끝으로 새는 문제 발생 →

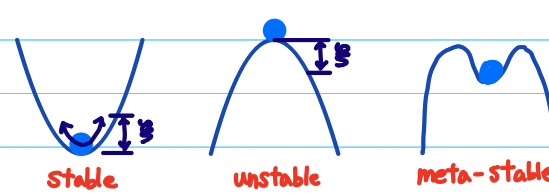
토카막 등장



Plasma instability

non-uniformity $\xrightarrow[\text{perturbation}]{}$ instability

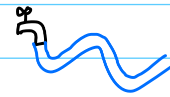
∇P, j



27강

Plasma instabilities

- Streaming instability : $\vec{j}$, fast particle , ∇P



- Rayleigh-Taylor instability : external force

물	$\downarrow g$	Plasma	$\odot \vec{B}$	Plasma	자기장 압력을 고려.	Vaccum	
기름		Vaccum	$\downarrow g$	Vaccum	$\uparrow \nabla B$	Plasma	$\uparrow \nabla B$

- Universal instability : ∇n, ∇p drift wave
- Kinetic instability : loss cone instability

Electron plasma wave

$\rightarrow$ dispersion relation $D(\omega, k) = 0$

$n_e = \underbrace{n_0}_{\text{equilibrium}} + \underbrace{n_1}_{\text{perturbation}}$, $\vec{u_e} = \vec{u_0} + \vec{u_1}$, $\vec{E} = \vec{E_0} + \vec{E_1}$

$$\begin{cases} \dfrac{dn_e}{dt} + \nabla\cdot(n_e \vec{u_e}) = 0 \\ n_e m\left[\dfrac{d\vec{u_e}}{dt} + (\vec{u_e}\cdot\nabla)\vec{u_e}\right] = -n_e e(\vec{E} + \vec{u_e}\times\vec{B}) - \gamma k T \nabla n_e \\ \varepsilon_0 \nabla\cdot\vec{E} = e(n_i - n_e) \end{cases}$$

$\Rightarrow \omega^2 = \omega_p^2 + \dfrac{\gamma k T}{m} k^2 > 0$, $\omega_p^2 = \dfrac{n_0 e^2}{m\varepsilon}$

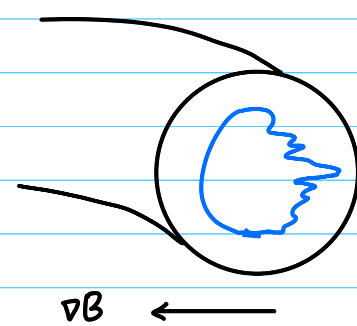
put $\omega = \omega_r + i\gamma$

$$\begin{aligned} n_1 &= \bar{n_1} \exp i(kx - \omega t) \\ &= \bar{n_1} \exp i(kx - \omega_r t)\exp(\gamma t) \\ &= \underline{\bar{n_1} \exp i(kx - \omega_r t) e^{\gamma t}} \end{aligned}$$

$\Rightarrow$ $\gamma > 0$: instability (unstable)

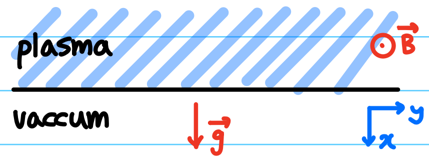
$\gamma < 0$: stable

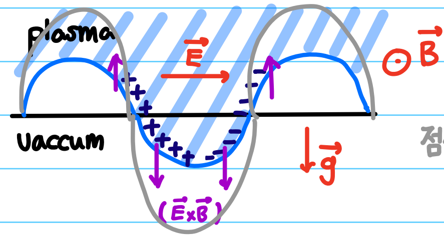
실제 토카막의 불안정성



28강

* Rayleigh-Taylor Instability





점점 instability가 커짐.

$$\boxed{\vec{u}_{i0} = \frac{M\vec{g}\times\vec{B}}{eB^2}, \quad \vec{u}_{e0} \simeq 0}$$

- equilibrium

$$n_0 M\left[\cancel{\frac{d\vec{u}_{i0}}{dt}} + \cancel{(\vec{u}_{i0}\cdot\nabla)\vec{u}_{i0}}\right] = n_0 e\left(\overset{\text{초기에 전기장}=0}{\cancel{\vec{E}_0}} + \vec{u}_{i0}\times\vec{B}\right) + n_0 M g$$

$$\cancel{n_0} e\,\vec{u}_{i0}\times\vec{B} + \cancel{n_0} M g = 0 \rightarrow \underline{e\,\vec{u}_{i0}\times\vec{B} + Mg = 0} \quad (A)$$

- perturbation, ion

$$(n_0+n_1)M\left\{\frac{\partial}{\partial t}(\cancel{\vec{u}_0}+\vec{u}_1) + \left[(\vec{u}_0+\vec{u}_1)\cdot\nabla\right](\cancel{\vec{u}_0}+\vec{u}_1)\right\} = (n_0+n_1)e\left[\vec{E}_1 + (\vec{u}_0+\vec{u}_1)\times\vec{B}\right] + (n_0+n_1)M\vec{g}$$

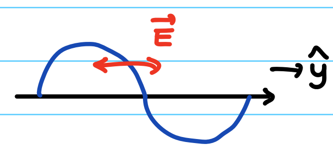
$$n_0 M\frac{d\vec{u}_1}{dt} + n_0 M\,\vec{u}_0\cdot\nabla\vec{u}_1 = n_0 e\vec{E}_1 + (n_0+n_1)e\,\vec{u}_1\times\vec{B} + \underbrace{(n_0+n_1)e\,\vec{u}_0\times\vec{B} + (n_0+n_1)M\vec{g}}_{\text{(A) 적용}} = 0$$

$$\cancel{n_0} M\frac{d\vec{u}_1}{dt} + \cancel{n_0} M\,\vec{u}_0\cdot\nabla\vec{u}_1 = \cancel{n_0} e\vec{E}_1 + \cancel{n_0} e\,\vec{u}_1\times\vec{B}$$

$$\underline{M\frac{d\vec{u}_1}{dt} + M\,\vec{u}_0\cdot\nabla\vec{u}_1 = e\vec{E}_1 + e\,\vec{u}_1\times\vec{B}} \qquad \left(\text{put } \vec{u}_1 \propto \exp i(ky-\omega t)\right)$$

$$-i\omega M u_1 + M ik u_0 u_1 = e(\vec{E}_1 + \vec{u}_1\times\vec{B})$$

$$\underline{-iM(\omega-ku_0)u_1 = e(\vec{E}_1 + \vec{u}_1\times\vec{B})}$$



$$\hat{x} : -iM(\omega-ku_0)u_{1x} = e(\cancel{E_{1x}} + u_{1y}B)$$

$$\hat{y} : -iM(\omega-ku_0)u_{1y} = eE_{1y} - eu_{1x}B$$

$$\begin{bmatrix} -iM(\omega-ku_0) & -eB \\ eB & -iM(\omega-ku_0) \end{bmatrix}\begin{bmatrix} u_{1x} \\ u_{1y} \end{bmatrix} = \begin{bmatrix} 0 \\ eE_{1y} \end{bmatrix}$$

$$\begin{bmatrix} u_{1x} \\ u_{1y} \end{bmatrix} = \frac{1}{e^2B^2 - \cancel{M^2(\omega-ku_0)^2}}\begin{bmatrix} -iM(\omega-ku_0) & eB \\ -eB & -iM(\omega-ku_0) \end{bmatrix}\begin{bmatrix} 0 \\ eE_{1y} \end{bmatrix}$$

<u>가정</u>: $e^2B^2 \gg M^2(\omega-ku_0)^2 \; : \; (\Omega_c)^2 \gg (\omega-ku_0)^2$

$$= \begin{bmatrix} \dfrac{\cancel{e^2}\cancel{B}E_{1y}}{\cancel{e^2}B^{\cancel{2}}} \\ -\dfrac{1}{e^{\cancel{2}}B^2}\, i\cancel{e}M(\omega-ku_0)E_{1y} \end{bmatrix} = \begin{bmatrix} \dfrac{E_{1y}}{B} \\ -i\dfrac{\omega-ku_0}{\Omega_c B}E_{1y} \end{bmatrix}$$

$$\Rightarrow \boxed{\begin{aligned} u_{i1x} &= \frac{E_{1y}}{B} \\ u_{i1y} &= -i\frac{\omega-ku_0}{\Omega_c B}E_{1y} \end{aligned}} \quad , \quad \boxed{\begin{aligned} u_{e1x} &= \frac{E_{1y}}{B} \\ u_{e1y} &= 0 \end{aligned}} \leftarrow \Omega_c = \frac{qB}{M},\ \omega_c = \frac{qB}{m}\ (M \gg m)$$

· equilibrium continuity equation

$$\cancel{\frac{\partial n_0}{\partial t}} + \nabla\cdot(n_0\vec{u_0}) = 0 \quad \text{(A)}$$

· perturbation

$$\frac{\partial}{\partial t}(\cancel{n_0}+n_1) + \nabla\cdot\left[(n_0+n_1)(\vec{u_0}+\vec{u_1})\right] = 0$$

$$\frac{\partial n_1}{\partial t} + \underbrace{\cancel{\nabla\cdot(n_0\vec{u_0})}}_{(A)} + \underline{\nabla\cdot(n_0\vec{u_1}) + \nabla\cdot(n_1\vec{u_0})} + \cancel{\nabla\cdot(n_1\vec{u_1})} = 0$$

$$\hookrightarrow n_0\nabla\cdot\vec{u_1} + (\vec{u_1}\cdot\nabla)n_0 + n_1\cancel{\nabla\cdot\vec{u_0}} + (\vec{u_0}\cdot\nabla)n_0$$

$$\frac{\partial n_1}{\partial t} + n_0\nabla\cdot\vec{u_1} + (\vec{u_1}\cdot\nabla)n_0 + (\vec{u_0}\cdot\nabla)n_0 = 0 \quad \left(\text{put } \vec{u_1} \propto \exp i(ky-\omega t)\right)$$

$$-i\omega n_1 + ikn_0u_{1y} + \underbrace{u_{1x}n_0'}_{\left(u_{1x}\frac{\partial n_0}{\partial x} + u_{1y}\cancel{\frac{\partial n_0}{\partial y}}\right)} + iku_0n_1$$

ion : $-i\omega n_1 + ikn_0u_{i1y} + n_0'u_{i1x} + iku_{i0}n_1 = 0$

electron : $-i\omega n_1 + ikn_0\cancel{u_{e1y}}^{\,0} + n_0'u_{e1x} + ik\cancel{u_{e0}}^{\,0}n_1 = 0$

$$-i\omega n_1 + n_0'u_{e1x} = 0 \rightarrow -i\omega n_1 + n_0'\frac{E_{1y}}{B} = 0 \qquad \therefore\ \underline{n_1 = \frac{E_{1y}}{i\omega B}n_0'}$$

$$\cancel{-i\omega\frac{E_{1y}}{i\omega B}n_0'} + ikn_0\left(-i\frac{\omega-ku_0}{\Omega_c\cancel{B}}\cancel{E_{1y}}\right) + \cancel{n_0'\frac{E_{1y}}{B}} + iku_{i0}\frac{\cancel{E_{1y}}}{i\omega\cancel{B}}n_0' = 0$$

$$kn_0\frac{\omega-ku_0}{\Omega_c} + \frac{ku_{i0}n_0'}{\omega} = 0 \quad \left(\text{put } \vec{u_{i0}} = \frac{M\vec{g}\times\vec{B}}{eB^2} = -\frac{g}{\Omega_c}\hat{y}\right)$$

$$\cancel{k}n_0\frac{\omega-ku_0}{\cancel{\Omega_c}} + \frac{\cancel{k}}{\omega}\frac{g}{\cancel{\Omega_c}}n_0' = 0 \rightarrow n_0(\omega-ku_0) - \frac{gn_0'}{\omega} = 0$$

$$\omega^2 - ku_0\omega - g\frac{n_0'}{n_0} = 0$$

$$\boxed{\omega = \frac{ku_0 \pm \sqrt{k^2u_0^2 + 4gn_0'/n_0}}{2}}$$

if $\underline{k^2u_0^2 + 4gn_0'/n_0} < 0 \Rightarrow \omega$는 허수

$\uparrow n_0'$ $\downarrow \vec{g}$

$\vec{g}$와 n_0'의 부호가 반대라면 허수부분 등장가능

$(Im(\omega) > 0 \rightarrow$ instability$)$