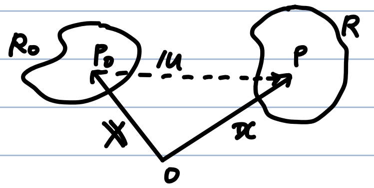
Ch.4 Particle kinematics

4.1 Bodies and their configuration



$\mathbf{x} = \mathbf{x}(\mathbf{X}, t) \rightarrow \underline{x_i} = x_i(\underline{X_R}, t) \quad (i, R = 1, 2, 3)$

Spatial coordinate (Eulerian) — Material coordinate (Lagrangian) : label to identify a particle.

$\mathbf{X} = \mathbf{X}(\mathbf{x}, t) \rightarrow X_R = X_R(x_i, t)$

4.2 Displacement and velocity

$\mathbf{u} = \mathbf{x} - \mathbf{X}$ or $\mathbf{u}(\mathbf{X}, t) = \mathbf{x}(\mathbf{X}, t) - \mathbf{X}$

$\mathbf{u}(\mathbf{x}, t) = \mathbf{x} - \mathbf{X}(\mathbf{x}, t)$

$\mathbf{v} = \frac{\partial}{\partial t}\mathbf{u}(\mathbf{X}, t) = \frac{\partial}{\partial t}\mathbf{x}(\mathbf{X}, t)$, $\quad v_i = \frac{\partial}{\partial t} x_i(X_R, t)$

4.3 Time rates of change

$\phi = G(X_R, t) = g(x_i, t)$

(i) $\frac{D\phi}{Dt} = \dot{\phi} = \frac{\partial}{\partial t} G(X_R, t)$: material derivative = convective derivative (입자하나 tracking)

(ii) $\phi = g(x_i(X_R, t), t)$

$\rightarrow \frac{D}{Dt}\phi = \frac{\partial g}{\partial x_i} \cdot \frac{\partial x_i}{\partial t} + \frac{\partial g}{\partial t} = v_i \cdot \frac{\partial g}{\partial x_i} + \frac{\partial g}{\partial t}$ or $\frac{D\phi}{Dt} = \mathbf{v} \cdot \nabla g + \frac{\partial g}{\partial t}$

$\rightarrow \delta\phi \approx g(x_i + v_i \delta t, t + \delta t) - g(x_i, t) \approx (\mathbf{v} \cdot \nabla g + \frac{\partial g}{\partial t})\delta t$

Instead of $G(X_R, t)$, $g(x_i, t)$, we will use $\phi(X_R, t)$, $\phi(x_i, t)$.

$\rightarrow \frac{D}{Dt}\phi = \frac{\partial}{\partial t}\phi(X_R, t) = v_j \cdot \frac{\partial \phi}{\partial x_j} + \frac{\partial \phi}{\partial t}$

(Example) $\phi = x_i$, $\frac{D}{Dt}\phi = ?$

$\frac{D}{Dt}\phi = v_j \cdot \frac{\partial \phi}{\partial x_j} + \frac{\partial \phi}{\partial t} = v_j \cdot \frac{\partial}{\partial x_j} x_i(x_k, t) + \frac{\partial}{\partial t} x_i(x_k, t) = v_i$

4.4 Acceleration

$x_i = x_i(X_R, t)$, $\quad v_i = \frac{\partial}{\partial t} x_i(X_R, t)$, $\quad f_i = \frac{\partial}{\partial t} v_i(X_R, t) = \frac{\partial^2}{\partial t^2} x_i(X_R, t)$

$(\mathbf{v} = \dot{\mathbf{x}}(\mathbf{X}, t)) \qquad (\mathbf{f} = \dot{\mathbf{v}}(\mathbf{X}, t) = \ddot{\mathbf{x}}(\mathbf{X}, t))$

$f_i = \frac{D}{Dt} v_i = \frac{\partial}{\partial t} v_i(x_j, t) + v_k \frac{\partial}{\partial x_k} v_i(x_j, t)$

$(\mathbf{f} = \mathbf{v} \cdot \nabla \mathbf{v} + \frac{\partial}{\partial t}\mathbf{v})$

(Example) $x_1 = X_1(1+a^2t^2)$, $x_2 = X_2$, $x_3 = X_3$

$v_1 = 2X_1a^2t$, $v_2 = v_3 = 0$ / $f_1 = 2X_1a^2t$

$$f_1 = \frac{Dv_1}{Dt} = \frac{d}{dt}\left(\frac{2x_1a^2t}{1+a^2t^2}\right) + \left(\frac{2x_1a^2t}{1+a^2t^2}\right)\cdot\left(\frac{2a^2t}{1+a^2t^2}\right) = \frac{2x_1a^2}{1+a^2t^2}$$

4.5 Steady motion . Particle paths & Streamlines

- motion is steady if $\mathbf{v} = \mathbf{v}(\mathbf{x})$ (not $\mathbf{v} = \mathbf{v}(\mathbf{X})$)

  ex) 자유낙하하는 물 (velocity at any point is independent of time)

- path of particle : $dx_i = v_i(X_R, t)dt$

  $= v_i(X_R(x_i,t), t)dt$

  $= v_i(x_j, t)dt$

- Streamline : $dx_i = v_i(x_j, t)d\tau$

  lines that are tangent to the velocity field.

⇒ In general, particle path ≠ Stream line

But if, the motion is steady, they are the same.