15강

(복습) * Set of MHD equations

(i) $\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$, $\frac{\partial \sigma}{\partial t} + \nabla\cdot\vec{J} = 0$

(ii) $\rho\frac{d\vec{v}}{dt} = \sigma\vec{E} + \vec{J}\times\vec{B} - \nabla P$

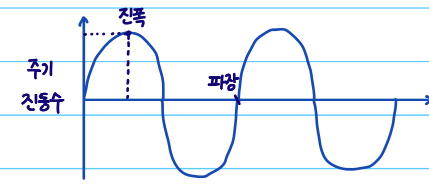
(iii) $\vec{E} + \vec{v}\times\vec{B} = \eta\vec{J} + \frac{\vec{J}\times\vec{B} - \nabla P_e}{ne}$

(iv) $\frac{d}{dt}\left(\frac{\rho^{\gamma}}{P}\right) = 0$

(v) $\varepsilon_0\nabla\cdot\vec{E} = \sigma$, $\nabla\cdot\vec{B} = 0$

$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$, $\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

(시작) * Waves in plasma



Wave equation : $\frac{\partial^2 u}{\partial t^2} - v^2\nabla^2 u = 0$

$\rightarrow u = A\cos\left(\frac{t}{c} + \delta\right)$, $\left(c = \frac{\omega}{k}\right)$

(3차원) · $\vec{u} = \vec{u_0}\exp i(\vec{k}\cdot\vec{r} - \omega t)$ $\vec{k}$: wave vector, $|\vec{k}|$: wave number

(1차원) · $u = \bar{u}\cos(kx - \omega t)$ $(k = 2\pi/\lambda)$, $\vec{u_0}$: 진폭의 크기와 방향을 포함

* Phase velocity

velocity of a point (ii) of constant phase (i) on the wave.

(i) $\frac{d}{dt}(kx - \omega t) = 0$, $k\frac{dx}{dt} - \omega = 0$, (ii) $\frac{dx}{dt} = \omega/k = u_p$

* Group velocity

$E_1 = E_0\cos[(k+\Delta k)x - (\omega+\Delta\omega)t]$

$E_2 = E_0\cos[(k-\Delta k)x - (\omega-\Delta\omega)t]$

put $kx - \omega t = a$, $\Delta kx - \Delta\omega t = b$

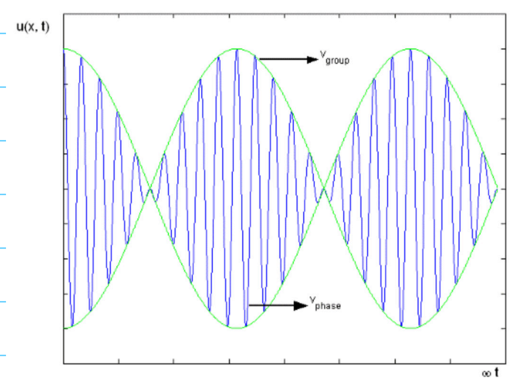
$E = E_1 + E_2 = E_0[\cos(a+b) + \cos(a-b)]$

$= E_0[\cos a\cos b - \cancel{\sin a\sin b} + \cos a\cos b + \cancel{\sin a\sin b}]$

$= 2E_0\cos a\cos b$

$= 2E_0\cos(\Delta kx - \Delta\omega t)\cos(kx - \omega t)$

$= \underbrace{E_0\cos(kx - \omega t)}_{\text{propagation}}\ \underbrace{2\cos(\Delta kx - \Delta\omega t)}_{\text{Amplitude}}$



; Amplitude의 phase velocity가 group velocity

$\therefore u_g = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$

(u_p는 빛의 속도보다 빠를 수 있지만, u_g는 빛의 속도보다 빠를 수 없다.)

* Plasma oscillation

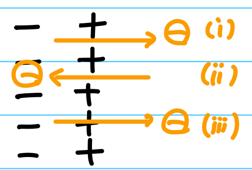
Assumption

① $\vec{B}=0$, ② no thermal motion $T=0$

③ fixed ion uniformly distributed

④ infinite plasma ⑤ only x-direction

· 우연히 순간 perturbation에 의해 (−)가 왼쪽에 몰린 상황



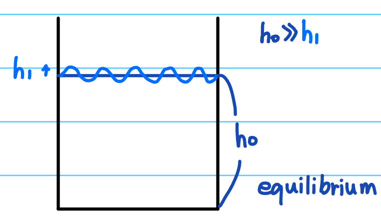
Overshoot

(전기장에 가속된 전자가 이온을 지나쳐버린다.)

$$\begin{cases} \dfrac{\partial n_e}{\partial t} + \nabla\cdot(n_e\vec{u_e}) = 0 , & \cancel{P_e = C_e(n_e m_e)^{\gamma}} \\ n_e m\left[\dfrac{\partial \vec{u_e}}{\partial t} + (\nabla\cdot\vec{u_e})\vec{u_e}\right] = -n_e e\left[\vec{E} + \vec{u_e}\times\cancel{\vec{B}}\right] - \cancel{\nabla P_e} + \cancel{R_{ei}} \\ \varepsilon_0 \nabla\cdot\vec{E} = \sigma , & \cancel{\nabla\cdot\vec{B}} = 0 \\ \nabla\times\vec{E} = \cancel{-\dfrac{\partial \vec{B}}{\partial t}} , & \cancel{\nabla\times\vec{B}} = \mu_0\vec{J} + \mu_0\varepsilon_0\dfrac{\partial \vec{E}}{\partial t} \end{cases}$$

$$\Rightarrow \begin{cases} (A) : \dfrac{\partial n_e}{\partial t} + \nabla\cdot(n_e\vec{u_e}) = 0 \\ (B) : n_e m\left[\dfrac{\partial \vec{u_e}}{\partial t} + (\nabla\cdot\vec{u_e})\vec{u_e}\right] = -n_e e\vec{E} \\ (C) : \varepsilon_0 \nabla\cdot\vec{E} = e(n_i - n_e) \end{cases}$$

Unknowns : ① ρ, ② $\vec{u_e}$, ③ $\vec{E}$



perturbation

① $n_e = n_0 + n_1$ (linearization)

② $\vec{u_e} = \vec{u_0} + \vec{u_1}$ $(a_0 \gg a_1)$

③ $\vec{E} = \vec{E_0} + \vec{E_1}$

Equilibrium → 초기상태 : $\vec{u_0} = 0$, $\vec{E_0} = 0$, $\nabla n_0 = 0$, $\dfrac{\partial n_0}{\partial t} = \dfrac{\partial \vec{u_0}}{\partial t} = \dfrac{\partial \vec{E_0}}{\partial t} = 0$

$n_1 = \bar{n}\exp i(kx - \omega t)$,

$\vec{u_1} = \bar{u_1}\exp i(kx - \omega t)$,

$\vec{E_1} = E_1\exp i(kx - \omega t)$, steady-state

$$\begin{cases} (A): \dfrac{d}{dt}(\cancel{n_0}+n_1) + \nabla\cdot\left[(n_0+n_1)(\cancel{\vec{u_0}}+\vec{u_1})\right] = 0 \\ (B): \cancel{(n_0+n_1)}\, m\left[\dfrac{d(\cancel{\vec{u_0}}+\vec{u_1})}{dt} + \left((\cancel{\vec{u_0}}+\vec{u_1})\cdot\nabla\right)(\cancel{\vec{u_0}}+\vec{u_1})\right] = -e\,\cancel{(n_0+n_1)}\,(\cancel{\vec{E_0}}+\vec{E_1}) \\ (C): \varepsilon_0 \nabla\cdot(\cancel{\vec{E_0}}+\vec{E_1}) = e\left[(\cancel{n_{i0}}+\cancel{n_{i1}}) - (\cancel{n_0}+n_1)\right] \end{cases}$$

quasi neutrality

↳ uniformly distributed

perturbation 끼리의 곱 → 매우 작아서 무시가능

$$\Rightarrow \begin{cases} (A'): \dfrac{dn_1}{dt} + \nabla\cdot(n_0\vec{u_1} + \cancel{n_1\vec{u_1}}) = 0 \\ (B'): m\left[\dfrac{d\vec{u_1}}{dt} + \cancel{(\vec{u_1}\cdot\nabla)\vec{u_1}}\right] = -e\vec{E_1} \\ (C'): \varepsilon_0 \nabla\cdot\vec{E_1} = -en_1 \end{cases}$$

put

$n_1 = \overline{n_1}\exp i(kx-\omega t)$

$u_1 = \overline{u_1}\exp i(kx-\omega t)$

$E_1 = \overline{E_1}\exp i(kx-\omega t)$

$\left(\dfrac{d}{dt} = -i\omega,\ \dfrac{d}{dx} = ik\right)$

$$\Rightarrow \begin{cases} (A''): -i\omega n_1 + ikn_0u_1 = 0 \\ (B''): -i\omega m u_1 = -eE_1 \\ (C''): ik\varepsilon_0 E_1 = -en_1 \end{cases}$$

연립 ; $-i\omega m u_1 = -e\left(\dfrac{-e}{i\cancel{k}\varepsilon_0}\right)\left(\dfrac{\cancel{i}\cancel{k}n_0}{\cancel{i}\omega}\right)u_1$

B를 기준으로 삼고, C,A를 도입

$$-i\omega m = \frac{e^2 n_0}{i\varepsilon_0\omega} \rightarrow \omega^2 = \frac{n_0e^2}{m\varepsilon_0}$$

equilibrium 상태의 density에 의존

$$\boxed{\omega_p^2 = \frac{n_0e^2}{m\varepsilon_0}}$$

<plasma frequency>

$$f_p = \frac{\omega_p}{2\pi} \sim 9\sqrt{n_0\ (\#/m^3)}$$

ex. K-STAR : $f_p = 9\times10^{10} = 90$ GHz

$$f_c \sim \frac{28}{B(T)}$$

$B \sim 0.3T \rightarrow f_p \approx f_c$

HW 플라즈마는 왜 wave가 아니라, oscillation 하는가?

(Chen 4.2절 참고)

연립 ; $(A)u_1 = (B)u_1$ $\quad A=B$

↳ $f(\omega,k)=0$: dispersion relation

17강

* plasma wave

$n, \vec{u_i}, \vec{E_i}$ 의 함수

$$\begin{cases} (A): \dfrac{\partial n_e}{\partial t} + \nabla\cdot(n_e \vec{u_e}) = 0 \\ (B): n_e m\left[\dfrac{\partial \vec{u_e}}{\partial t} + (\nabla\cdot\vec{u_e})\vec{u_e}\right] = -n_e e\vec{E} - \nabla P_e \\ (C): \varepsilon_0 \nabla\cdot\vec{E} = e(n_i - n_e) \end{cases}$$

∇P_e term을 추가하면, wave가 된다.

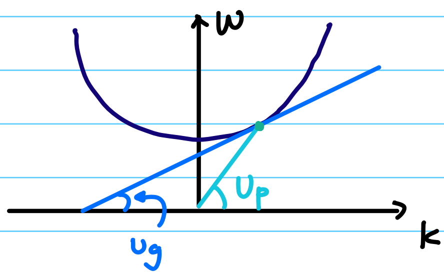
$\gamma k T_e \nabla n_e$

$$\left(\frac{\nabla P_e}{P_e} = \gamma \frac{\nabla n_e}{n_e} \qquad \nabla P_e = \gamma P_e \frac{\nabla n_e}{n_e} = \gamma \cancel{n_e} k T_e \frac{\nabla n_e}{\cancel{n_e}} = \underline{\gamma k T_e \nabla n_e} \right)$$

위와 같은 과정으로 계산 ...

* Electron plasma wave

$$\boxed{\omega^2 = \omega_p^2 + \frac{\gamma k T_e}{m} k^2} = \frac{ne^2}{m\varepsilon_0} + \frac{\gamma k T_e}{m} k^2$$



$\left(\frac{1}{2} m v_{Th}^2 = kT_e \right)$

$\rightarrow \omega^2 = \omega_p^2 + \frac{1}{2}\gamma v_{Th}^2 k^2$

$2\omega d\omega = \gamma v_{Th}^2 2k\,dk$

$v_g = \frac{d\omega}{dk} = \frac{\gamma v_{Th}^2}{2} \cdot \frac{k}{\omega}$ $\qquad \therefore v_g = \frac{\gamma v_{Th}^2}{2 v_p}$

(i) $k = \infty$ ← propagation $\qquad$ (ii) $k = 0$ ← oscillation

$v_p = \sqrt{\frac{\gamma}{2}}\, v_{Th} = v_g$ $\qquad$ $v_g = 0$

$$\underbrace{\omega_p^2}_{\text{E field에 의한 oscillation}} + \underbrace{\frac{\gamma k T_e}{m} k^2}_{\text{Thermal motion에 의해 압축, 이완하며 전파}} = \omega^2$$

E field에 의한 oscillation

Thermal motion에 의해 압축, 이완하며 전파

→ <u>oscillation + propagation</u>

• $\frac{\gamma k T_e}{m}k^2$ term 뜯어보기 일반적인 gas의 상황 (only thermal motion)

$$\left(\begin{aligned} &\frac{d\rho}{dt}+\nabla\cdot(\rho\vec{u})=0 \\ &\rho\left[\frac{d\vec{u}}{dt}+(\vec{u}\cdot\nabla)\vec{u}\right] = \underline{-\nabla P} + \cancel{\rho\nu^2\nabla\vec{u}} \end{aligned}\right)$$

$\frac{\nabla P}{P}=\gamma\frac{\nabla\rho}{\rho}$ $\therefore \nabla P=\gamma P\frac{\nabla\rho}{\rho}$

viscosity term 무시

$$\left(\rho=\rho_0+\rho_1\,,\quad \vec{v}=\cancel{\vec{v}_0}^{\,0}+\vec{v}_1\right)$$

$$\left(\begin{aligned} &\frac{d\rho_1}{dt}+\rho_0\nabla\cdot\vec{v}_1=0 \\ &\rho_0\frac{d\vec{v}_1}{dt}=-\gamma\frac{P\nabla\rho}{\rho} \end{aligned}\right) \Rightarrow \left(\begin{aligned} &-i\omega\rho_1+\rho_0 ikv_1=0 \\ &-i\omega\rho_0 v_1=-\gamma\frac{P_0 ik\rho_1}{\rho_0} \end{aligned}\right)$$

$$\Rightarrow -i\omega\rho_0 v_1 = -\gamma\frac{P_0 ik}{\rho_0}\left(\frac{\rho_0 ik}{i\omega}\right)v_1\,,\quad \omega\rho_0=\frac{\gamma P_0 k^2}{\omega}\,,\quad \omega^2=\frac{\gamma P_0}{\rho_0}k^2$$

$$\therefore \left(\frac{\omega}{k}\right)^2=\frac{\gamma P_0}{\rho_0}=\frac{\gamma n_0 kT}{n_0 M}=\frac{\gamma kT}{M}=C_s^2$$ (C_s : sound speed)

18강 <electrostatic waves>

No thermal motion
<oscillation>

(i)
$$\begin{aligned} &\frac{dn_e}{dt}+\nabla\cdot(n_e\vec{u_e})=0 \\ &n_e m\left[\frac{d\vec{u_e}}{dt}+(\vec{u_e}\cdot\nabla)\vec{u_e}\right]=-n_e e\vec{E} \\ &\varepsilon_0\nabla\cdot\vec{E}=e(n_i-n_e) \end{aligned}$$

$$\left(\begin{aligned} &n_e=n_0+n_1 \\ &\vec{u_e}=\vec{u_1} \\ &\vec{E}=\vec{E_1} \end{aligned}\right)$$

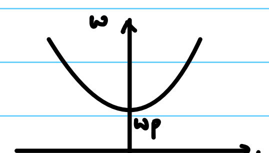
− +
− +
− +
− +

$$\omega^2=\omega_p^2=\frac{ne^2}{m\varepsilon_0}$$

Thermal
motion
<propagation>

(ii)
$$\begin{aligned} &\frac{dn_e}{dt}+\nabla\cdot(n_e\vec{u_e})=0 \\ &n_e m\left[\frac{d\vec{u_e}}{dt}+(\vec{u_e}\cdot\nabla)\vec{u_e}\right]=-n_e e\vec{E}\,\underline{-\nabla P_e} \\ &\varepsilon_0\nabla\cdot\vec{E}=e(n_i-n_e) \end{aligned}$$

$= -\gamma kT_e\nabla n_e$



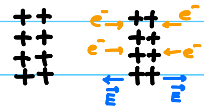
$$\omega^2=\omega_p^2+\frac{\gamma kT_e}{m}k^2$$

$$\begin{aligned} &\frac{d\rho}{dt}+\nabla\cdot(\rho\vec{v})=0 \\ &\rho\left[\frac{d\vec{v}}{dt}+(\vec{v}\cdot\nabla)\vec{v}\right]=-\nabla P \\ &\omega^2=\left(\frac{\gamma kT}{M}\right)k^2 \\ &C_s=\left(\frac{\gamma kT}{M}\right)^{1/2} \end{aligned}$$

⟨ electromagnetic waves ⟩

ion의 perturbation

* Ion wave (Acoustic wave)



$\vec{E}$, ∇P 이 두가지 이유로 모여있는 이온은 퍼져나가려고함.

(i) 전자의 thermal motion이 없다면, 전자가 이온을 shielding하고, $\vec{E}$에 의한 영향은 존재하지 않음. 오직 ∇P에 의한 영향만 존재

→ ∇P

(ii) 전자의 thermal motion이 있다면, 완벽한 shielding이 불가능해서 $\vec{E}$도 영향을 주게 된다.

→ $\vec{E}$, ∇P

$$\left(\frac{\omega}{k}\right)^2 = \frac{\gamma kT}{M} = C_s^2$$

$$\frac{dn_i}{dt} + \nabla\cdot(n_i\vec{u_i}) = 0 \rightarrow \frac{dn_1}{dt} + n_0\nabla\cdot\vec{u_1} = 0 \rightarrow \boxed{-i\omega n_1 + n_0 iku_1 = 0}$$

$$n_i M\left[\frac{d\vec{u_i}}{dt} + (\vec{u_i}\cdot\nabla)\vec{u_i}\right] = n_i e\vec{E} - \nabla P_i \rightarrow n_0 M\frac{d\vec{u_1}}{dt} = -n_0 e\nabla\phi_1 - \gamma kT_i\nabla n_1 \rightarrow \boxed{-n_0 M i\omega u_1 = -n_0 eik\phi_1 - \gamma kT_i ikn_1}$$

이온의 움직임에 따라, 전자는 즉각적으로 따라오며 quasi-neutrality를 만족한다고 생각하기.

$$\varepsilon_0\nabla\cdot\vec{E} = \underbrace{e(n_i - n_e) = 0}_{\text{Boltzmann relation을 완벽히 따를 때.}} \rightarrow \vec{E} = -\nabla\phi$$

$$n_i = n_0 + n_1, \quad \vec{u_i} = \vec{u_1}, \quad \phi = \phi_1$$

Boltzmann relation : $n_e = n_0\exp\left(\frac{e\phi}{kT_e}\right) = n_0 + n_0\frac{e\phi}{kT_e} \rightarrow \boxed{n_1 = n_0\frac{e\phi_1}{kT_e}}$

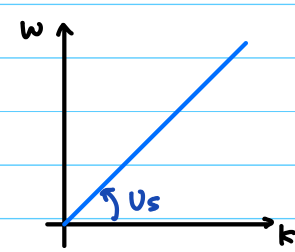
(전자가 충분히 매우 빠를 때)

$kT \gg e\phi$ 인 영역을 살펴보겠다.

(3) $\phi_1 = \frac{kT_e}{en_0}n_1$, (1) $n_1 = \frac{n_0 iku_1}{i\omega}$

(2) $-\cancel{n_0}M\cancel{i}\omega u_1 = -\cancel{n_0}\cancel{e}\cancel{i}k\frac{kT_e}{\cancel{en_0}}\frac{\cancel{n_0}\cancel{i}k}{\cancel{i}\omega}u_1 - \gamma kT_i\cancel{i}k\frac{\cancel{n_0}\cancel{i}k}{\cancel{i}\omega}u_1$

$$M\omega = \frac{k^2 kT_e}{\omega} + \frac{\gamma kT_i k^2}{\omega} \rightarrow \omega^2 = \left(\frac{kT_e}{M} + \frac{\gamma kT_i}{M}\right)k^2$$



$$\boxed{\left(\frac{\omega}{k}\right)^2 = \frac{kT_e + \gamma kT_i}{M} = v_s^2}, \quad \boxed{v_s = \sqrt{\frac{kT_e + \gamma kT_i}{M}}}$$

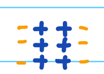
(i) ∇P (thermal motion)

(ii) $\vec{E}$ (potential) ← 전자의 thermal motion (다 달라붙지 않아서, 완전히 상쇄 X)

$$\left(\frac{\omega}{k}\right)^2 = \frac{kT_e + \gamma kT_i}{M} = v_s^2$$

4가지 상황

① $T_i = 0$, $T_e = 0$



(i) No thermal motion

(ii) 전자가 달라붙어 neutral을 만드므로 파는 전파되지 않는다.

② $T_i \neq 0$, $T_e = 0$

(i) 전기장의 효과는 전자가 다 상쇄

(ii) Thermal motion 에 의해서만 퍼져나감.

③ $T_i = 0$, $T_e \neq 0$

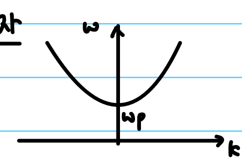
(i) 전자가 전기장을 상쇄하기는 하지만, 다 끌려오지는 않음.

(ii) No thermal motion

④ no electron HW

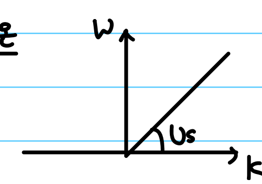
Summary (electrostatic)

전자



$$\omega^2 = \omega_p^2 + \frac{\gamma kT_e}{m}k^2$$

이온



$$\omega^2 = \frac{kT_e + \gamma kT_i}{M}k^2$$

* Electromagnetic waves with $B_0=0$, $B_1 \neq 0$

(진공)

$$\nabla\times\vec{E_1} = -\frac{\partial\vec{B_1}}{\partial t} = -\dot{\vec{B_1}}$$

$$c^2\nabla\times\vec{B_1} = \frac{\partial\vec{E_1}}{\partial t} = \dot{\vec{E_1}}$$

$$\rightarrow \quad c^2\nabla\times(\nabla\times\vec{B_1}) = \nabla\times\dot{\vec{E_1}}$$

$$c^2[\nabla(\cancel{\nabla\cdot\vec{B_1}})-\nabla^2\vec{B_1}] = -\ddot{\vec{B_1}}$$

$$\rightarrow -c^2(ik)^2B_1 = -(i\omega)^2B_1 \qquad \therefore c^2k^2=\omega^2 \qquad \rightarrow \left(\frac{\omega}{k}\right)^2 = c^2$$

(플라즈마)

$$\nabla\times\vec{E_1} = -\frac{\partial\vec{B_1}}{\partial t} = -\dot{\vec{B_1}}$$

$$c^2\nabla\times\vec{B_1} = \frac{\partial\vec{E_1}}{\partial t} + c^2\mu_0\vec{J} = \dot{\vec{E_1}} + c^2\mu_0\vec{J}$$

$$\left(\vec{J} = \sum_j n_j q_j \vec{u_j} = -n_e e\vec{u_e} + n_i e\vec{u_i} = -n_0 e\vec{u_1}\right)$$

ion은 거의 fix되었다. $\quad (-(n_0+n_1)e\vec{u_1})$

$\vec{J}$가 $\vec{u_1}$의 함수이고, 이 $\vec{u_1}$을 구하기 위함. →

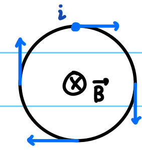
$$n_e m\left[\frac{\partial\vec{u_e}}{\partial t} + (\vec{u_e}\cdot\nabla)\vec{u_e}\right] = -n_e e(\vec{E} + \cancelto{0}{\vec{u_e}\times\vec{B}}) - \cancelto{0}{\nabla P_e}$$

$\because \vec{u_1}\times\vec{B_1}=0$ 편의상 전자의 압력등 무시

$\Rightarrow$ (연립)

$$\boxed{\omega^2 = c^2k^2 + \omega_p^2}$$

- $k\to 0$; cutoff 파의반사
  $(\omega=\omega_p^2 \rightarrow k=0 \rightarrow$ wave의 반사$)$
- $k\to\infty$; resonance
  $\left(-k^2 = \frac{-}{(\omega-\omega_c)^2}\right)$



cyclotron frequency (ω_c) = wave frequency (ω)

이온과 같은 방향으로 도는 wave → 이온 가열

전자와 같은 방향으로 도는 wave → 전자 가열

19강

(플라즈마)

① $\nabla\times\vec{E_1} = -\dot{\vec{B_1}}$

② $\nabla\times\vec{B_1} = \mu_0\vec{J_1} + \frac{1}{c^2}\dot{\vec{E_1}}$ $n_e = n_0+n_1$, $\vec{u_e} = u_1 + \cancel{u_0}$, $\vec{E} = \vec{E_1} + \cancel{E_0}$, $\vec{B} = \vec{B_1} + \cancel{B_0}$

↳ $\vec{J_1} = (n_i e\vec{u_i} - n_e e\vec{u_e}) = -(n_0+n_1)e\vec{u_1} = -n_0 e\vec{u_1}$

$$mn_e\left[\frac{\partial\vec{u_e}}{\partial t} + (\vec{u_e}\cdot\nabla)\vec{u_e}\right] = -\cancel{\nabla P_e} - n_e e(\vec{E} + \vec{u_e}\times\vec{B})$$

↳ $m(n_0+n_1)\left[\frac{\partial\vec{u_1}}{\partial t} + (\vec{u_1}\cdot\nabla)\vec{u_1}\right] = -(n_0+n_1)e(\vec{E_1} + \vec{u_1}\times\vec{B_1})$

③ $mn_0\frac{\partial\vec{u_1}}{\partial t} = -n_0 e\vec{E_1}$

(3) $-i\omega m\cancel{n_0}u_1 = -\cancel{n_0}eE_1$

(1) $\nabla\times(\nabla\times\vec{E_1}) = -\nabla\times\dot{\vec{B_1}} = -\frac{\partial}{\partial t}(\nabla\times\vec{B_1})$

↳ $\nabla(\nabla\cdot\vec{E_1}) - \nabla^2\vec{E_1} = -\mu_0\dot{\vec{J_1}} - \frac{1}{c^2}\ddot{\vec{E_1}} = \mu_0 n_0 e\dot{\vec{u_1}} - \frac{1}{c^2}\ddot{\vec{E_1}}$

$$i\vec{k}(i\vec{k}\cdot\vec{E_1}) - (ik)^2E_1 = -i\omega\mu_0 n_0 e u_1 - \frac{1}{c^2}(-i\omega)^2E_1$$

* $\boxed{\vec{k}\parallel\vec{E} \text{ 인 경우}}$ $\quad (ik(ikE_1) - (ik)^2E_1 = 0)$ → B_1이 있든, 없든 plasma frequency만 나타남.

$$0 = -i\omega\mu_0 n_0 e u_1 + \frac{1}{c^2}\omega^2E_1$$

$$0 = -\cancel{i\omega}\mu_0 n_0 e\frac{e}{\cancel{i\omega}m}\vec{E_1} + \frac{1}{c^2}\omega^2\vec{E_1} \rightarrow \therefore \omega^2 = c^2\mu_0\frac{n_0e^2}{m} = \frac{n_0e^2}{m\varepsilon_0}$$

(plasma frequency)

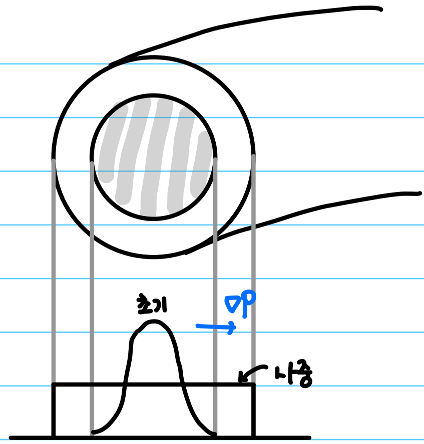
* $\boxed{\vec{k}\perp\vec{E_1} \text{ 인 경우}}$ $\quad (i\vec{k}(\cancelto{0}{i\vec{k}\cdot\vec{E_1}}) - (ik)^2E_1 = k^2E_1)$

$$k^2E_1 = -i\omega\mu_0 n_0 e u_1 + \frac{1}{c^2}\omega^2E_1$$

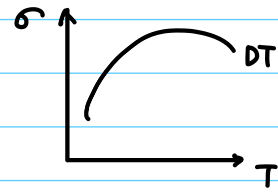
$$k^2E_1 = -\cancel{i\omega}\mu_0 n_0 e\frac{e}{\cancel{i\omega}m}\vec{E_1} + \frac{1}{c^2}\omega^2\vec{E_1} \rightarrow k^2 = -\frac{\omega_p^2}{c^2} + \frac{\omega^2}{c^2}$$

$$\boxed{\omega^2 = \omega_p^2 + c^2k^2}$$

플라즈마 특성   빛의 전파

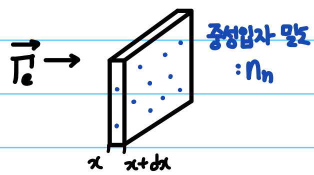


$P_{fusion} \propto n^2$



* Diffusion

Collision parameters



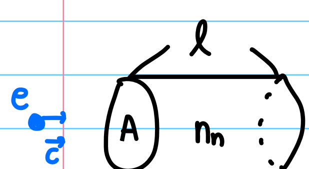
$$\Gamma(x+dx) = \Gamma(x) - \Gamma(x)\,\underbrace{n_n \cancel{A}\,dx}_{\text{전체입자수}}\,\underbrace{\frac{\sigma}{\cancel{A}}}_{\text{반응확률}}$$

$$\frac{\Gamma(x+dx)-\Gamma(x)}{dx} = -\Gamma(x)\,n_n\sigma$$

$$(dx \to 0)\quad \frac{d\Gamma}{dx} = -\Gamma n_n \sigma \,,\quad \frac{d\Gamma}{\Gamma} = -n_n\sigma dx \,,\quad \ln\Gamma = -n_n\sigma x + C$$

$$\therefore \Gamma = \Gamma_0 e^{-n_n\sigma x} = \Gamma_0 e^{-\frac{x}{\lambda}} \quad \left(\lambda = \frac{1}{n_n\sigma}\ ;\ \text{Mean free path}\right)$$

(충돌 간 평균거리)



$$MFP = \frac{l}{\text{충돌 횟수}} = \frac{l}{n_n A l} = \frac{1}{n_n A} = \frac{1}{n_n\sigma}$$

(A=σ)