11강

$f(x, y, z, v_x, v_y, v_z, t)$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v}\cdot\nabla f + \frac{q(\vec{E}+\vec{v}\times\vec{B})}{m}\cdot\frac{\partial f}{\partial \vec{v}} = 0 ,$$

velocity space 에서 적분 ↓ ; 평균속도 (thermal motion은 isotropic 해서 모두 사라짐) ↓

$$\int_{\vec{v}} f\cdot d\vec{v} = n , \quad \int_{\vec{v}} \vec{v} f\, d\vec{v} = n\vec{u}$$

$$\left(\underset{\text{thermal motion}}{\vec{v} = \vec{w}} + \underset{\text{평균속도}}{\vec{u}} , \quad \frac{1}{2}m\bar{w}^2 = \frac{3}{2}kT \right)$$

7차원 함수를 velocity space 에서 적분하면, 측정, 이해할 수 있는 '속도', '밀도'를 알 수 있다.

[1] Continuity equation

$$\int_{\vec{v}}\Big[\underbrace{\frac{\partial f}{\partial t}}_{①} + \underbrace{\vec{v}\cdot\nabla f}_{②} + \underbrace{\frac{q\vec{E}}{m}\cdot\frac{\partial f}{\partial \vec{v}}}_{③} + \underbrace{\frac{q(\vec{v}\times\vec{B})}{m}\cdot\frac{\partial f}{\partial \vec{v}}}_{④}\Big]d\vec{v} \Rightarrow \boxed{\frac{\partial n}{\partial t} + \nabla\cdot(n\vec{u}) = 0}$$

① : $\int_{\vec{v}} \frac{\partial f}{\partial t} d\vec{v} = \frac{\partial}{\partial t}\int_{\vec{v}} f\, d\vec{v} = \frac{\partial n}{\partial t}$

② : $\int_{\vec{v}} \vec{v}\cdot\nabla f\, d\vec{v} = \int_{\vec{v}} \underset{(x,y,z)}{\nabla}\cdot(f\underset{(v_x,v_y,v_z)}{\vec{v}})\, d\vec{v} - \int_{\vec{v}} f\, \underbrace{\nabla\cdot\vec{v}}_{\to 0\ \text{서로 독립변수}}\, dv \qquad (\nabla\cdot(\phi\vec{A}) = \phi\,\nabla\cdot A + \nabla\phi\cdot A)$

$= \nabla\cdot\int_{\vec{v}} f\vec{v}\, d\vec{v} - 0 \;=\; \nabla\cdot(n\vec{u})$

③ : $\frac{q}{m}\int_{\vec{v}} \vec{E}\cdot\frac{\partial f}{\partial \vec{v}} d\vec{v} = \frac{q}{m}\int_{\vec{v}} \underbrace{\frac{\partial}{\partial \vec{v}}\cdot(f\vec{E})}_{\text{velocity space 에서의 divergence}}\, d\vec{v}$

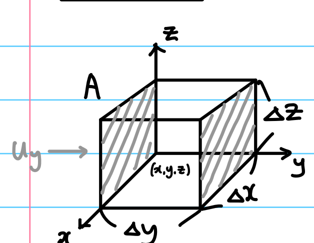
(발산정리) $= \frac{q}{m}\int_{S^\infty} f\vec{E}\cdot d\vec{s} \underset{\text{무한대의 surface 에서 f는 0 이다.}}{=} 0 \qquad \left(\because f \propto \exp\left(-\frac{\frac{1}{2}mv^2}{kT}\right)\right)$

④ : $\frac{q}{m}\int_{\vec{v}} (\vec{v}\times\vec{B})\cdot\frac{\partial f}{\partial \vec{v}}\cdot d\vec{v} = \frac{q}{m}\left[\int_{\vec{v}} \frac{\partial}{\partial \vec{v}}\cdot(f\vec{v}\times\vec{B})\, d\vec{v} - \int_{\vec{v}} f\, \frac{\partial}{\partial \vec{v}}(\vec{v}\times\vec{B})\, d\vec{v}\right]$

$= \frac{q}{m}\left[\int_{S^\infty} f\vec{v}\times\vec{B}\, d\vec{s} - 0\right] = 0$

$$\frac{\partial}{\partial \vec{v}}(\vec{v}\times\vec{B}) = \begin{vmatrix} \frac{\partial}{\partial v_x} & \frac{\partial}{\partial v_y} & \frac{\partial}{\partial v_z} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = 0$$

물리적 직관



$\Delta V = \Delta x \Delta y \Delta z$
mass density : ρ
$\vec{u} = (u_x, u_y, u_z)$

· y-direction

mass rate of fluid leaving A.

$$\underbrace{[(\rho u_y)_{y+\Delta y} - (\rho u_y)_y]\Delta x\Delta z\ (kg/s)}_{\text{단위시간당 변하는 질량}} = \frac{\Delta(\rho u_y)}{\Delta y}\Delta x\Delta y\Delta z = \frac{\Delta(\rho u_y)}{\Delta y}\Delta V$$

· Total loss rate of mass density in A.

$$\left[\frac{\Delta(\rho u_x)}{\Delta x} + \frac{\Delta(\rho u_y)}{\Delta y} + \frac{\Delta(\rho u_z)}{\Delta z}\right]\Delta V = \underbrace{-\frac{\partial \rho}{\partial t}\Delta V}_{\text{CV 내에서 변하는 질량}} \rightarrow$$

(Loss rate of mass density in A with the time rate of change of the mass density

$$\Delta x, \Delta y, \Delta z \to 0 \qquad \nabla\cdot(\rho\vec{v}) = -\frac{\partial \rho}{\partial t}$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0}$$

Continuity equation

if steady $\frac{\partial \rho}{\partial t} = 0$, $\nabla\cdot(\rho\vec{u}) = 0$

if uniform density $\nabla\cdot\vec{u} = 0$ (incompressible)

[2] Fluid equation of motion (momentum conservation equation)

입자 1개 : $m\frac{d\vec{v}}{dt} = q(\vec{E}+\vec{v}\times\vec{B})$

입자 n개 : $\underbrace{nm\frac{d\vec{u}}{dt} = nq(\vec{E}+\vec{u}\times\vec{B})}_{} \underbrace{-\nabla p}_{\text{압력힘}} \underbrace{-nm\frac{\vec{u}-\vec{u_0}}{\tau}}_{\text{중성유체와 충돌힘}}$ ($\vec{u_0}$: neutral fluid velocity)

(Lagrangian)

$nm\left(\frac{\partial\vec{u}}{\partial t}+(\nabla\cdot\vec{u})\vec{u}\right)$

Convective derivative

(Eulerian)

$\left(\frac{dT}{dt}=\frac{\partial T}{\partial t}+\vec{u}\cdot\nabla T\right)$

HW (i) $\int_{\vec{v}} m\vec{v}\left(\frac{df}{dt}\right)d\vec{v}$ 로 식유도

(momentum이 y방향으로 들어올때, x,y,z 방향 모두 가지고 들어올수 있다. → 9개)

(ii) Control Volume 으로 식유도

12강

연속방정식 $\int_{\vec{v}}\left(\frac{df}{dt}\right)d\vec{v}=0$, $\frac{\partial n}{\partial t}+\nabla\cdot(n\vec{u})=0$ ← 변수: $n, \vec{u}$ (2개) ;

Fluid eqn of motion

$m\frac{d\vec{v}}{dt} = q(\vec{E}+\vec{v}\times\vec{B})$

$nm\frac{d\vec{v}}{dt} = nq(\vec{E}+\vec{v}\times\vec{B}) \underbrace{-\nabla\cdot\overleftrightarrow{P}}_{\text{pressure tensor}} - \underbrace{\frac{nm(\vec{u}-u_0)}{\tau}}_{\text{neutral과 충돌}}$

$nm\left[\frac{\partial\vec{v}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}\right] = nq(\vec{E}+\vec{v}\times\vec{B}) - \nabla\cdot\overleftrightarrow{P} - \frac{nm(\vec{u}-u_0)}{\tau}$ ⋯(A)

1822 Navier
1823 Cauchy
1829 Poisson
1845 Stokes

$$\boxed{\rho\left[\frac{\partial\vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}\right] = -\nabla P + \rho\nu\nabla^2\vec{u}}$$

Navier-Stokes Equation

일반적인 유체 ⋯(A')

$nm\left[\frac{\partial\vec{v}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}\right] = -\nabla\cdot\overleftrightarrow{P} - \cancel{\frac{nm(\vec{u}-u_0)}{\tau}}$ 고려X

(∵ Navier-Stokes 방정식도 이 term을 무시했기 때문)

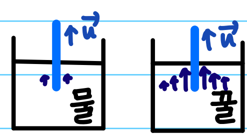
HW : 7 most important open problems in Mathematics

$-\nabla\cdot\overleftrightarrow{P} = -\nabla P + \rho\nu\nabla^2\vec{u}$

$\overleftrightarrow{P} = \begin{bmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{bmatrix}$ $\overleftrightarrow{P}$가 isotropic 이면, $\nabla\cdot\overleftrightarrow{P} = \nabla P$

그러나, not isotropic 이면, $\nabla\cdot\overleftrightarrow{P} = \nabla P - \underbrace{\rho\nu\nabla^2\vec{u}}_{}$ viscosity term

isotropic plasma 가정
↳ viscosity term 무시
$\nabla\cdot\overleftrightarrow{P} = \nabla P$



$\tau = \mu\frac{du}{dy}$ [F/A] shear stress (전단응력)

μ : dynamic viscosity → $\nu = \frac{\mu}{\rho}$ → ν : kinematic viscosity (momentum diffusivity)

· fluid equation of motion $\rightarrow n, \vec{u}, p$

$$\boxed{nm\left[\frac{d\vec{u}}{dt} + (\vec{u}\cdot\nabla)\vec{u}\right] = nq(\vec{E}+\vec{v}\times\vec{B}) - \nabla p}$$

· equation of continuity $\rightarrow n, \vec{u}$

$$\boxed{\frac{dn}{dt} + \nabla\cdot(n\vec{u}) = 0}$$

· equation of state $\rightarrow n, p$

$$p = c\rho^{\gamma} \quad \left(c : \text{const},\ \gamma = \frac{C_p}{C_v} : \text{ratio of specific heats}\right)$$

$$\nabla p = \nabla(c\rho^{\gamma}) = \nabla\left[c(nm)^{\gamma}\right] = cm^{\gamma}\nabla n^{\gamma} = cm^{\gamma}\gamma n^{\gamma-1}\nabla n$$

$$\frac{\nabla p}{p} = \frac{\cancel{c}\cancel{m^{\gamma}}\cdot\gamma n^{\cancel{\gamma-1}}\nabla n}{\cancel{c}\, n^{\gamma}\cancel{m^{\gamma}}} = \gamma\frac{\nabla n}{n} \quad \rightarrow \quad \boxed{\frac{\nabla p}{p} = \gamma\frac{\nabla n}{n}}$$

(for isothermal)

$$\nabla p = \nabla(nkT) = kT\nabla n\ , \quad \frac{\nabla p}{p} = \frac{kT\nabla n}{nkT} = \frac{\nabla n}{n} \rightarrow \gamma = 1$$

(for general)

$$\gamma = \frac{2+N}{N} \quad (N : \text{degree of freedom})$$

Summary

$$\begin{cases} \dfrac{dn_j}{dt} + \nabla\cdot(n_j\cdot\vec{u_j}) = 0 \\ n_j m_j\left[\dfrac{d\vec{u_j}}{dt} + (\vec{u_j}\cdot\nabla)\vec{u_j}\right] = n_j q_j(\vec{E}+\vec{u_j}\times\vec{B}) - \nabla P_j \\ P_j = C_j(n_j m_j)^{\gamma_j} \\ \varepsilon_0\nabla\cdot\vec{E} = \sum_j n_j q_j\ , \quad \nabla\times\vec{E} = -\dfrac{d\vec{B}}{dt} \\ \nabla\cdot\vec{B} = 0\ , \quad \nabla\times\vec{B} = \mu_0\sum_j n_j q_j\vec{u_j} + \mu_0\varepsilon_0\dfrac{d\vec{E}}{dt} \end{cases}$$

n : density

$\vec{u}$: velocity

P : pressure

$\rightarrow P = nkT$ (T : temperature)

(for $j = i, e$)

(플라즈마를 한강 물처럼 충분히 많은 충돌이 존재해서, Maxwell 분포를 따른다는 가정)

· Fick's law

$$\Gamma = -D\nabla n \ (= n\vec{u}) \quad \frac{dn}{dt} + \nabla(-D\nabla n) = 0 \rightarrow \underline{\frac{dn}{dt} - D^2\nabla^2 n = 0}$$

(이론을 모델링 할 때, 이렇게 n을 구할수도 있다.)

* Fluid drifts perpendicular to $\vec{B}$

$$nm\left[\cancelto{0}{\frac{d\vec{u}}{dt}} + \cancelto{0}{(\vec{u}\cdot\nabla)\vec{u}}\right] = nq(\vec{E}+\vec{u}\times\vec{B}) - \nabla P$$

(양변 $\times \vec{B}$)

$$nq\,\vec{E}\times\vec{B} + nq(\vec{u}\times\vec{B})\times\vec{B} - \nabla P\times\vec{B} = 0$$

$\left((\vec{u}\times\vec{B})\times\vec{B} = \vec{B}\times(\vec{B}\times\vec{u}) = \vec{B}(\vec{B}\cdot\vec{u}) - \vec{u}B^2 \right)$

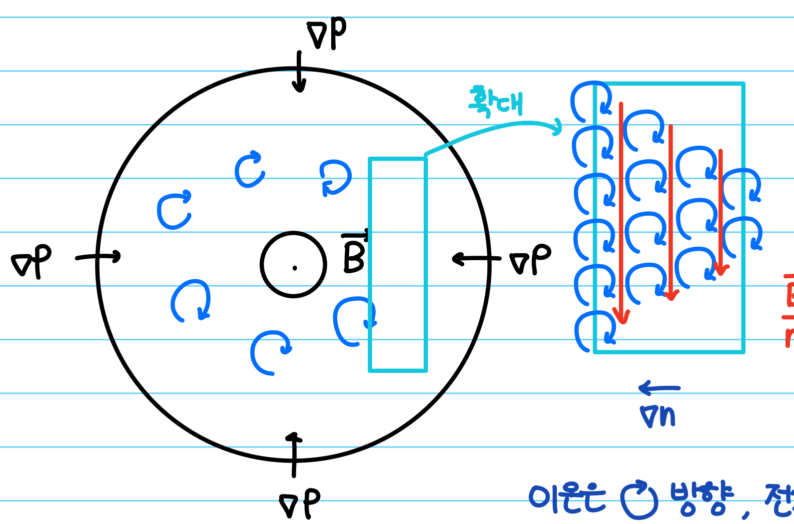
$$nq\,\vec{E}\times\vec{B} + nq\,\vec{B}(\vec{B}\cdot\vec{u}) - nq\,\vec{u}B^2 - \nabla P\times\vec{B} = 0$$

($\vec{u}$ is perpendicular to $\vec{B} \to u_\perp$)

$$nq\,\vec{E}\times\vec{B} - nq\,\vec{u}_\perp B^2 - \nabla P\times\vec{B} = 0$$

$$\vec{u}_\perp = \frac{nq}{nqB^2}(\vec{E}\times\vec{B}) - \frac{\nabla P\times\vec{B}}{nqB^2} = \frac{\vec{E}\times\vec{B}}{B^2} + \frac{\vec{B}\times\nabla P}{nqB^2}$$

$$\boxed{\therefore\ \vec{u}_\perp = \underbrace{\frac{\vec{E}\times\vec{B}}{B^2}}_{\substack{E\times B \text{ drift} \\ (\vec{v}_E)}} + \underbrace{\frac{\vec{B}\times\nabla P}{nqB^2}}_{\substack{\text{Diamagnetic drift} \\ (\vec{v}_D)}}}$$



$\frac{\vec{B}\times\nabla P}{nqB^2}$ 의 이해도 (이온)

이온은 ↻ 방향, 전자는 ↺ 방향

→ 전류는 ↻ 방향

→ 이 전류가 만드는 자기장 ⊗ 방향

(자기장을 줄이는 방향)

(diamagnetic)

⇒ diamagnetic drift $\left(\vec{J}_D = \sum_{j=i,e} n_j q_j \vec{v}_D \right)$

13강

(복습) * $\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial f}{\partial z}\cdot\frac{dz}{dt} + \frac{\partial f}{\partial v_x}\cdot\frac{dv_x}{dt} + \frac{\partial f}{\partial v_y}\cdot\frac{dv_y}{dt} + \frac{\partial f}{\partial v_z}\cdot\frac{dv_z}{dt}$

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial T}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial T}{\partial z}\cdot\frac{dz}{dt} = \frac{\partial T}{\partial t} + \vec{v}\cdot\nabla T$$

$\left(\nu = \frac{\mu}{\rho}\ ,\ \tau = \mu\frac{d\vec{u}}{dy}\ [F/A]\right)$

* $nm\left[\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u}\right] = nq(\vec{E} + \vec{u}\times\vec{B}) - \nabla P$, $\rho\left[\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u}\right] = -\nabla p + \underline{\rho\nu\nabla^2\vec{u}}$ ← ∇P와 같은 단위

Fluid equation of motion — Navier-Stokes eqn

* $P = c\rho^{\gamma}$ $\left(\begin{array}{l}\gamma = 1 \ :\ \text{isothermal} \\ \gamma = \frac{2+N}{N} \ :\ \text{adiabatic compression}\end{array}\right)$

* Complete set of fluid equations

$$\frac{\partial n_j}{\partial t} + \nabla\cdot(n_j\cdot\vec{u_j}) = 0$$

이온과 전자가 주고받는 모멘텀.

$$n_j m_j\left[\frac{\partial\vec{u_j}}{\partial t} + (\vec{u_j}\cdot\nabla)\vec{u_j}\right] = n_j q_j(\vec{E} + \vec{u_j}\times\vec{B}) - \nabla P_j + R_{ji}$$

$$P_j = C_j(n_j m_j)^{\gamma_j}$$

$$\varepsilon_0\nabla\cdot\vec{E} = \sum_j n_j q_j \quad , \quad \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\cdot\vec{B} = 0 \quad , \quad \nabla\times\vec{B} = \mu_0\sum_j n_j q_j\vec{u_j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \quad (j = \text{ion, electron})$$

< Plasma approximation >

$n_i \approx n_e \approx n$

<u>but</u> $ne + n(-e) \neq 0$

* fluid drifts perpendicular to $\vec{B}$

$$\vec{u_\perp} = \frac{\vec{E}\times\vec{B}}{B^2} + \frac{\vec{B}\times\nabla P}{nqB^2} \quad (= \vec{U_E} + \vec{U_D})$$

(본격)

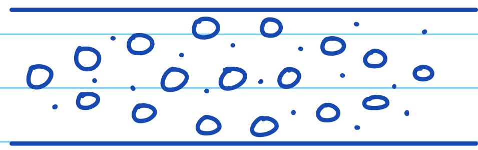
*fluid drifts parallel to $\vec{B}$

$$n_j m_j\left[\frac{d\vec{u_j}}{dt}+(\vec{u_j}\cdot\nabla)\vec{u_j}\right]=n_j q_{\sigma_j}(\vec{E}+\vec{u_j}\times\vec{B})-\nabla P_j+R_{ji}$$

(i) ∇n 존재

(ans) Boltzmann relation

· e　　o ion



$\vec{B}$

$\hat{z}$

∇n $\longrightarrow$ $\longleftarrow$　　$T=\text{const}$

∇P $\longrightarrow$ $\longleftarrow$

F_p $\longleftarrow$ $\longrightarrow$

(전자가 훨씬 빠르게 이동) ϕ $\ominus$ $\oplus$ $\ominus$

$\vec{E}$ $\longleftarrow$ $\longrightarrow$

(전자가 받는 힘) $\vec{F_e}$ $\longrightarrow$ $\longleftarrow$

$$n_j m_j\left[\frac{d\vec{u_j}}{dt}+\overbrace{\cancel{(\vec{u_j}\cdot\nabla)\vec{u_j}}}^{\text{무시}}\right]=n_j q_{\sigma_j}(\vec{E}+\vec{u_j}\times\vec{B})-\nabla P_j+\overbrace{\cancel{R_{ji}}}^{\text{무시}}$$

(isothermal, z-direction, for electrons ($n_e=n$)) + grad(n) 존재 에서의 parallel motion

$$nm\frac{du_e}{dt}=-neE_z-\overbrace{kT\frac{dn}{dz}}^{\left(\frac{dP}{dz}=kT\frac{dn}{dz}\right)}$$

$$\frac{du_e}{dt}=-\frac{e}{m}E_z-\frac{kT}{nm}\frac{dn}{dz}=0$$

((i) force balance 상황
(ii) neglect electron's inertia)

$$\frac{e}{\cancel{m}}\frac{d\phi}{dz}-\frac{kT}{n\cancel{m}}\frac{dn}{dz}=0\rightarrow\frac{e}{kT}\frac{d\phi}{dz}-\frac{1}{n}\frac{dn}{dz}=0\rightarrow\frac{e\phi}{kT}-\ln n=\text{const}$$

$$\rightarrow\ln n=\text{const}+\frac{e\phi}{kT}$$

$\therefore n=n_0\exp\left(\frac{e\phi}{kT}\right)$ ($n=n_0$ at $\phi=0$) (Debye length 구할 때 사용함)

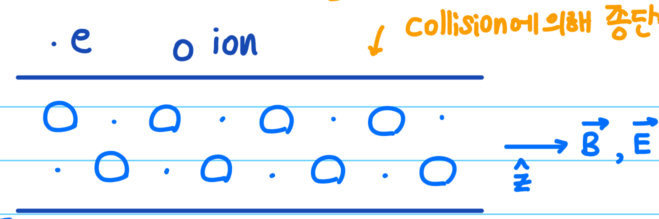
전자는 Boltzmann relation 을 만족함.

$\left(n=n_0\exp\left(-\frac{mgh}{kT}\right),\ n=n_0\exp\left(-\frac{P.E.}{kT}\right),\ \cdots\right)$

(ii) homogeneous

(ans) resistivity 존재

$\vec{E}$에의해 가속되지만, collision에의해 종단속도로 도달



무시

$$n_j m_j\left[\frac{d\vec{u}_j}{dt} + \cancel{(\vec{u}_j\cdot\nabla)\vec{u}_j}\right] = n_j q_j(\vec{E}+\vec{u}_j\times\vec{B}) - \cancel{\nabla p_j}^{\,0} + \underline{R_{ji}}$$

(이온과 전자의 momentum exchange)

(이동을 방해하는 저항으로 나타남)

(for electrons)

$$nm\frac{du_{ez}}{dt} = -neE_z + \underline{R_{eiz}} = -neE_z - nm\nu_{ei}(\vec{u}_e - \vec{u}_i) = 0$$

((i) force balance 상황
(ii) neglect elecron's inertia)

이온과 전자의 collision frequency

$$\cdot\ \vec{R}_{ei} = -nem\nu_{ei}(\vec{u}_{ez} - \vec{u}_{iz})$$

(ion에 의해 electron이 얻는 momentum)

$$-neE_z - nm\nu_{ei}(\vec{u}_{ez} - \vec{u}_{iz}) = 0$$

$$E_z = -\frac{m\nu_{ei}}{e}(\vec{u}_{ez} - \vec{u}_{iz}), \quad J_z = ne(\vec{u}_{iz} - \vec{u}_{ez})$$

$$\rightarrow \underline{E_z = \frac{m\nu_{ei}}{ne^2}J_z = \eta J_z}$$

Simplified Ohm's law

$$\boxed{\eta = \frac{m\nu_{ei}}{ne^2}\ ;\ \text{resistivity}}$$

→ 플라즈마의 저항은 collision에 의해 생성됨.

$$\cdot\ \vec{R}_{ei} = -nm\nu_{ei}(\vec{u}_e - \vec{u}_i) = -n\cancel{m}\frac{ne^2}{\cancel{m}}\eta(\vec{u}_e - \vec{u}_i) = -n^2e^2\eta(\vec{n}_e - \vec{n}_i) = ne\eta\vec{J}$$

(+ 온도가 올라갈수록 이온과 전자의 충돌이 잘 안 일어남. Think! 러더퍼드 산란)

14강

(복습)

* Complete set of fluid equations

$$\frac{dn_j}{dt} + \nabla\cdot(n_j \cdot \vec{u_j}) = 0$$

이온과 전자가 주고받는 모멘텀.

$$n_j m_j\left[\frac{d\vec{u_j}}{dt} + (\vec{u_j}\cdot\nabla)\vec{u_j}\right] = n_j q_j(\vec{E} + \vec{u_j}\times\vec{B}) - \nabla P_j + R_{ji}$$

$$P_j = C_j (n_j m_j)^{\gamma_j}$$

$$\varepsilon_0 \nabla\cdot\vec{E} = \sum_j n_j q_j \quad , \quad \nabla\times\vec{E} = -\frac{d\vec{B}}{dt}$$

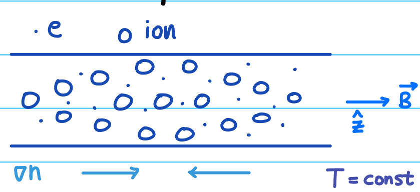
$$\nabla\cdot\vec{B} = 0 \quad , \quad \nabla\times\vec{B} = \mu_0 \sum_j n_j q_j \vec{u_j} + \mu_0\varepsilon_0 \frac{d\vec{E}}{dt} \quad (j = \text{ion}, \text{electron})$$

* fluid drifts perpendicular to $\vec{B}$

$$\vec{u_\perp} = \frac{\vec{E}\times\vec{B}}{B^2} + \frac{\vec{B}\times\nabla P}{nqB^2} \quad (= \vec{V_E} + \vec{V_D})$$

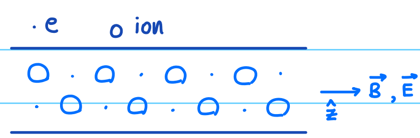
* fluid drifts parallel to $\vec{B}$

①



$n_e = n_0 \exp\left(\frac{e\phi}{kT}\right)$ ← Boltzmann response

②



Runaway electron

(계속 전자가 가속되는 상황)

$\vec{E} = \eta\vec{J}$ ← collision 에 의한 저항이 나타남.

(본격)
물리량 정의

* Single fluid Equation

⟨Plasma approximation⟩
$n_i \approx n_e \approx n$ but $\sigma \neq 0$

· mass density : $\rho = n_i M + n_e m \approx n(M+m)$

· charge density : $\sigma = e(n_i - n_e)$

· velocity : $\vec{v} = \dfrac{n_i M \vec{u_i} + n_e m \vec{u_e}}{n_i M + n_e m} = \dfrac{n_i M \vec{u_i} + n_e m \vec{u_e}}{\rho} \approx \dfrac{M\vec{u_i} + m\vec{u_e}}{M+m}$

· current : $\vec{J} = n_i e \vec{u_i} - n_e e \vec{u_e} \approx ne(\vec{u_i} - \vec{u_e})$

· pressure : $P = P_i + P_e$

· $\vec{u_i}$를 $\vec{v}$와 $\vec{J}$로 표현 :

$$(M+m)\vec{v} = M\vec{u_i} + m\vec{u_e}, \quad \vec{u_e} = -\frac{M}{m}\vec{u_i} + \frac{M+m}{m}\vec{v}$$

$$\vec{u_i} = \vec{u_e} + \frac{\vec{J}}{ne} = -\frac{M}{m}\vec{u_i} + \frac{M+m}{m}\vec{v} + \frac{\vec{J}}{ne}$$

$$\frac{m+M}{m}\vec{u_i} = \frac{M+m}{m}\vec{v} + \frac{\vec{J}}{ne}$$

$$\therefore \vec{u_i} = \vec{v} + \frac{m}{m+M}\frac{\vec{J}}{ne} \approx \vec{v} + \frac{m}{M}\frac{\vec{J}}{ne}$$

· $\vec{u_e}$를 $\vec{v}$와 $\vec{J}$로 표현 :

$$(M+m)\vec{u} = M\vec{u_i} + m\vec{u_e}, \quad \vec{u_i} = -\frac{m}{M}\vec{u_e} + \frac{M+m}{M}\vec{v}$$

$$\vec{u_e} = \vec{u_i} - \frac{\vec{J}}{ne} = -\frac{m}{M}\vec{u_e} + \frac{M+m}{M}\vec{v} - \frac{\vec{J}}{ne}$$

$$\frac{M+m}{M}\vec{u_e} = \frac{M+m}{M}\vec{v} - \frac{\vec{J}}{ne}$$

$$\therefore \vec{u_e} = \vec{v} - \frac{M}{M+m}\cdot\frac{\vec{J}}{ne} \approx \vec{v} - \frac{\vec{J}}{ne}$$

Continuity Equation of mass

$$\frac{\partial}{\partial t} M n_i + \nabla\cdot(M n_i u_i) = 0$$

$$+ \quad \frac{\partial}{\partial t} m n_e + \nabla\cdot(m n_e u_e) = 0$$

$$\frac{\partial}{\partial t}(M n_i + m n_e) + \nabla\cdot(M n_i u_i + m u_e n_e) = 0 \Rightarrow \boxed{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0}$$

Continuity Equation of charge

$$\frac{\partial}{\partial t} e n_i + \nabla\cdot(e n_i u_i) = 0$$

$$- \quad \frac{\partial}{\partial t} e n_e + \nabla\cdot(e n_e u_e) = 0$$

$$\frac{\partial}{\partial t}(e(n_i - n_e)) + \nabla\cdot(e n_i \vec{u_i} - e n_e \vec{u_e}) = 0 \Rightarrow \boxed{\frac{\partial \sigma}{\partial t} + \nabla\cdot\vec{J} = 0}$$

Single fluid equation of motion

$$nM\frac{d\vec{u}_i}{dt} = n_i e(\vec{E}+\vec{u}_i\times\vec{B}) - \nabla P_i + \vec{R}_{ie}$$

$$+\quad nm\frac{d\vec{u}_e}{dt} = -n_e e(\vec{E}+\vec{u}_e\times\vec{B}) - \nabla P_e + \vec{R}_{ei}$$

n은 continuity equation에서 구해야 하므로, 즉 $\frac{d}{dt}$에 들어갈 수 없음.

$$n\frac{d}{dt}(M\vec{u}_i + m\vec{u}_e) = e\underbrace{(n_i - n_e)}_{\text{plasma approximation}}\vec{E} + e(n_i\vec{u}_i - n_e\vec{u}_e)\times\vec{B} - \nabla P$$

$$n(M+m)\frac{d\vec{v}}{dt} = \rho\frac{d\vec{v}}{dt} = \sigma\vec{E} + \vec{J}\times\vec{B} - \nabla P \quad\Rightarrow\quad \boxed{\rho\frac{d\vec{v}}{dt} = \sigma\vec{E} + \vec{J}\times\vec{B} - \nabla P}$$

(전자가 매우 빠르게 steady-state에 도달한다고 가정)

= electron's inertia neglected

$$nm\cancelto{0}{\frac{d\vec{u}_e}{dt}} = -n_e e(\vec{E}+\vec{u}_e\times\vec{B}) - \nabla P_e + \vec{R}_{ei}$$

$$\vec{E}+\vec{u}_e\times\vec{B} = -\frac{\nabla P_e}{ne} + \frac{\vec{R}_{ei}}{ne}$$

$$\vec{u}_e = \vec{v} - \frac{\vec{J}}{ne}$$

$$\vec{E}+\left(\vec{v}-\frac{\vec{J}}{ne}\right)\times\vec{B} = -\frac{\nabla P_e}{ne} + \frac{\vec{R}_{ei}}{ne}$$

$$\vec{R}_{ei} = -nm\nu_{ei}(\vec{u}_e - \vec{u}_i) = -n\eta n e^2(\vec{u}_e - \vec{u}_i) = \eta n e\vec{J}$$

$$\vec{E}+\vec{v}\times\vec{B} - \frac{\vec{J}\times\vec{B}}{ne} = -\frac{\nabla P_e}{ne} - \frac{nm\nu_{ei}(\vec{u}_e-\vec{u}_i)}{ne} = -\frac{\nabla P_e}{ne} + \frac{\eta n e\vec{J}}{ne}$$

$$\Rightarrow\quad \boxed{\vec{E}+\vec{v}\times\vec{B} = \eta\vec{J} + \frac{\vec{J}\times\vec{B} - \nabla P_e}{ne}}$$

by Hannes . Alfven (1942)

* Single fluid equation (Magneto Hydro Dynamics : MHD (자기유체역학))

(i) $\frac{d\rho}{dt} + \nabla\cdot(\rho\vec{v}) = 0, \quad \frac{d\sigma}{dt} + \nabla\cdot\vec{J} = 0$

(ii) $\rho\frac{d\vec{v}}{dt} = \sigma\vec{E} + \vec{J}\times\vec{B} - \nabla P$

(iii) $\underbrace{\vec{E}} + \vec{v}\times\vec{B} = \underbrace{\eta\vec{J}} + \frac{\vec{J}\times\vec{B} - \nabla P_e}{ne}$: generalized ohm's law

parallel 방향의 Ohm's law

(iv) $\frac{d}{dt}\left(\frac{\rho^\gamma}{P}\right) = 0$

(v) $\varepsilon_0 \nabla\cdot\vec{E} = \sigma, \quad \nabla\cdot\vec{B} = 0$

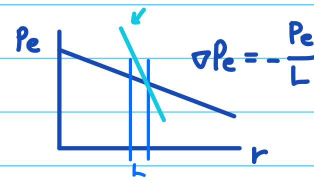
$\nabla\times\vec{E} = -\frac{d\vec{B}}{dt}, \quad \nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{d\vec{E}}{dt}$

압력이 급격하게 변하는 경우는 아래 결과가 성립하지 않는다.

(가정 : $\vec{v} = \vec{v_{ti}}$, $T_i = T_e$)

(이온의 thermal velocity)

$$\frac{\nabla P_e/ne}{\vec{v}\times\vec{B}} \underset{\text{term의 크기 비교}}{\approx} \frac{P_e/Lne}{v_{ti}B} \underset{\substack{k\text{ 생략}\\(\text{관계를 보는것이 목적})}}{\sim} \frac{\cancel{n}T_e/L\cancel{n}e}{v_{ti}B} \underset{1/2\text{ 생략}}{\sim} \frac{Mv_{ti}^2/Le}{v_{ti}B} \sim \frac{Mv_{ti}/Be}{L} \underset{\left(\frac{Mv_{ti}}{Be} = r_L\right)}{\sim} \frac{r_{Li}}{L} \ll 1$$



(characteristic length)
↳ gradient를 나타내는 scale length.

∇P가 너무 크지 않은 상황에서는 ∇P_e term 무시 가능
& $\vec{J}\times\vec{B}$도 비슷한 order라서 무시 가능 $\Rightarrow$ $\underline{\vec{E} + \vec{v}\times\vec{B} = \eta\vec{J}}$

* Ideal MHD eqn.

Ideal의 의미
↳ $\eta = 0$

(i) $\frac{d\rho}{dt} + \nabla\cdot(\rho\vec{v}) = 0, \quad \nabla\cdot\vec{J} = 0$

(ii) $\rho\frac{d\vec{v}}{dt} = \vec{J}\times\vec{B} - \nabla P$

(iii) $\vec{E} + \vec{v}\times\vec{B} = 0$

(iv) $\frac{d}{dt}\left(\frac{\rho^\gamma}{P}\right) = 0$

(v) $\nabla\cdot\vec{E} = 0, \quad \nabla\cdot\vec{B} = 0$

$\nabla\times\vec{E} = -\frac{d\vec{B}}{dt}, \quad \nabla\times\vec{B} = \mu_0\vec{J}$

charge density
and
displacement current
무시

<플라즈마 현상>
1) wave
2) transport
3) equilibrium
4) instability