Fusion Plasma Theory 2

Lecture 20-21 : Chapman - Enskog - Braginskii

① Chapman - Enskog process

- Asymptotic expansions of the Boltzmann kinetic equation
- follows a hierarchy of temporal and spatial scales
  and solves the kinetic equation order by order in the scale separations.

- Time-scale ordering by Chapman - Enskog

$$\frac{Df}{Dt} \equiv \frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla} f + \vec{a}\cdot\frac{\partial f}{\partial \vec{v}} = C[f]$$

ordering by Knudsen number $K_n \equiv \frac{\lambda_{mfp}}{L} \equiv \epsilon \ll 1$

and $\nu \gg v_t/L \gg D/L^2 \quad (\Leftrightarrow \; t_0 \ll t_1 \ll t_2)$

$$\begin{cases} t_0 \sim \nu^{-1} & : \text{collisional transport} \\ t_1 \sim L/v_t \sim (L/\lambda_{mfp})(\lambda_{mfp}/v_t) \sim (L/\lambda_{mfp})\,\nu^{-1} \sim t_0/\epsilon & : \text{non-dissipative time-scale (MHD)} \\ t_2 \sim D/L^2 \sim (L^2/\lambda_{mfp}^2)\,\nu^{-1} \sim t_0/\epsilon^2 & : \text{dissipative and transport time scale.} \end{cases}$$

- Multi-scale analysis in Chapman - Enskog

$$\left(\frac{D}{Dt_0} + \epsilon\frac{D}{Dt_1} + \epsilon^2\frac{D}{Dt_2}\right)\left(f_0 + \epsilon f_1 + \epsilon^2 f_2 + \cdots\right) = C\left[f_0 + \epsilon f_1 + \epsilon^2 f_2 + \cdots\right]$$

i) Zeroth-order

$$\frac{Df_0}{Dt_0} = C[f_0]$$

← We will drop this (D/Dt_0) by assuming kinetic equilibrium.
(this happens instantaneously in the subsidiary time scales)

To go for next-orders, consider the collisional operator expansion first

$$C[f] = C\left[f_0 + \epsilon f_1 + \epsilon^2 f_2 + \cdots,\; f_0 + \epsilon f_1 + \epsilon^2 f_2 + \cdots\right]$$

$$= \underbrace{C[f_0,f_0]}_{C[f_0]} + \epsilon\Big(\underbrace{C[f_0,f_1] + C[f_1,f_0]}_{\hat{C}f_1}\Big) + \epsilon^2\Big(\underbrace{C[f_0,f_2] + C[f_2,f_0]}_{\hat{C}f_2} + \underbrace{C[f_1,f_1]}_{C[f_1]}\Big)$$

Resulting equations :

$$\begin{cases} 0 = C[f_0] \\ \frac{Df_0}{Dt_1} = \hat{C} f_1 \\ \frac{Df_0}{Dt_2} + \frac{Df_1}{Dt_1} = \hat{C} f_2 + C[f_1] \end{cases}$$

$0 = C[f_0]$ gives local Maxwellian :

$$f_M(\vec{x},t) = n(\vec{x},t)\left(\frac{m}{2\pi T(\vec{x},t)}\right)^{3/2} \exp\left(-m\frac{\vec{x}-\vec{u}(\vec{x},t)}{2T(\vec{x},t)}\right)$$

An important assumption is that $n, \vec{u}, T$ are true values, which are consistent with the total distribution function.

Let's put $f_0 = f_M$ to the first-order equation : $\dfrac{Df_0}{Dt_1} = \hat{C}f_1$

one will have the homogeneous solutions : $\hat{C}f_1^{homo} = 0$.

(1) $\underline{f_1^{homo} = (1, \vec{v}, v^2) f_M}$. (or $f_1^{homo} = (A + \vec{B}\cdot\vec{v} + Cv^2) f_M$)

The linear combinations can be used to adjust $n', \vec{u}', T'$ from f_0 to be consistent with true $n, \vec{u}, T$.

(2) To have a solution, the equation must meet the solvability conditions by Fredholm alternative theorem in the first place.

$$\frac{Df_0}{Dt_1} = \hat{C}f_1 \Rightarrow \underline{\left\langle (1,\vec{v},v^2) f_M, \frac{Df_M}{Dt_1} \right\rangle = \int (1,\vec{v},v^2) \frac{Df_M}{Dt_1} d\vec{v} = 0}$$

↳ These are just particle, momentum, energy conservation. of Maxwellian.

*note

: Fredholm alternative theorem

Solution f_1 이 존재하기 위해서는, $\hat{C}f_1$ 의 null space 와 $\dfrac{Df_0}{Dt_1}$ 가 수직이어야 한다.

$\Rightarrow \left\langle (1,\vec{v},v^2) f_M, \dfrac{Df_M}{Dt} \right\rangle = \int (1,\vec{v},v^2) \dfrac{Df_M}{Dt} d\vec{v} = 0$ 성립! by conservation.

- Second-order solvability condition

The same solvability condition is applied. Both linearized $\hat{C}$ and C operators have the solution of $(1,\vec{v},v^2) f_M$. To have the solution order f_2, it must meet the solvability condition :

$$\int d\vec{v}\,(1,\vec{v},v^2)\left(\frac{Df_M}{Dt_2} + \frac{Df_1}{Dt_1}\right) = 0$$

i) $\int d\vec{v}\,(1,\vec{v},v^2)\,\frac{Df_M}{Dt_2}=0$ → Euler (conservation) equation

ii) $\int d\vec{v}\,(1,\vec{v},v^2)\,\frac{Df_1}{Dt}=\int d\vec{v}\,(1,\vec{v},v^2)\left(\cancel{\frac{df_1}{dt}}+\vec{v}\cdot\vec{\nabla}f_1+\cancel{\vec{a}\cdot\frac{df_1}{d\vec{v}}}\right)=\vec{\nabla}\cdot\int d\vec{v}\,(1,\vec{v},v^2)\,\vec{v}f_1$

$$\left(\vec{a}\cdot\frac{df_1}{d\vec{v}}=\frac{d}{d\vec{v}}(\vec{a}f_1)-\left(\frac{d}{d\vec{v}}\cdot\vec{a}\right)f_1=\underbrace{\cancel{\frac{d}{d\vec{v}}(\vec{a}f_1)}}_{S_\infty}+\frac{q}{m}\cancel{\frac{d}{d\vec{v}}(\vec{v}\times\vec{B})}^{0}=0\ ,\quad \frac{d}{dt}=0\right)$$

⇒ Systematic conclusion of Chapman – Enskog kinetic asymptotic expansion :

$$\underbrace{\int d\vec{v}\,(1,\vec{v},v^2)\,\frac{Df_M}{Dt}}_{\text{Nondissipative part}}=\underbrace{-\vec{\nabla}\cdot\int d\vec{v}\,(1,\vec{v},v^2)\,\vec{v}f_1}_{\text{dissipative flux}}$$

Solvability condition returns the transport equation

② Preface for Braginskii

- Braginskii vs. Chapman – Enskog

  (1) Magnetization brings anisotropy (2) Two components creates frictional exchange even in Maxwellian.

- Assumption : $K_n=\epsilon\ll1$, $\rho/L\sim O(\epsilon)$, and high-flow ordering $\vec{u}_{i,e}\sim v_{ti}\ll v_{te}$.

③ Transformation of kinetic equation on a moving frame

$$\vec{w}=\vec{v}-\vec{u}(\vec{x},t)\quad\Rightarrow\quad f(\vec{x},\vec{v},t)\rightarrow f(\vec{x},\vec{w},t)$$

$$\left(\begin{aligned}\frac{d}{dt}\Big|_{\vec{v}}&=\frac{d}{dt}\Big|_{\vec{w}}+\frac{d\vec{w}}{dt}\Big|_{\vec{v}}\cdot\frac{d}{d\vec{w}}=\frac{d}{dt}\Big|_{\vec{w}}-\frac{d\vec{u}}{dt}\cdot\frac{d}{d\vec{w}}\\ \frac{d}{d\vec{x}}\Big|_{\vec{v}}&=\frac{d}{d\vec{x}}\Big|_{\vec{w}}+\frac{d\vec{w}}{d\vec{x}}\Big|_{\vec{v}}\cdot\frac{d}{d\vec{w}}=\frac{d}{d\vec{x}}\Big|_{\vec{w}}-\vec{\nabla}\vec{u}\cdot\frac{d}{d\vec{w}}\end{aligned}\right)$$

New kinetic equation becomes :

$$\frac{Df}{Dt}=\frac{df}{dt}+\vec{v}\cdot\frac{df}{d\vec{x}}+\vec{a}\cdot\frac{df}{d\vec{v}}=C[f]$$

$$\frac{df}{dt}+\vec{u}\cdot\vec{\nabla}f+\vec{w}\cdot\vec{\nabla}f-\vec{w}\cdot\vec{\nabla}\vec{u}\,\frac{df}{d\vec{w}}+\left[\frac{q}{m}\left(\vec{E}'+\vec{w}\times\vec{B}\right)-\frac{d\vec{u}}{dt}\right]\cdot\frac{df}{d\vec{w}}=C[f]$$

$$\Rightarrow\ \underline{\frac{df}{dt}+\vec{w}\cdot\vec{\nabla}f+\left[\frac{q}{m}\left(\vec{E}'+\vec{w}\times\vec{B}\right)-\frac{d\vec{u}}{dt}\right]\cdot\frac{df}{d\vec{w}}-\vec{w}\cdot\vec{\nabla}\vec{u}\,\frac{df}{d\vec{w}}=C[f]}$$

where $\vec{E}'=\vec{E}+\vec{u}\times\vec{B}$ and $\frac{d}{dt}=\frac{d}{dt}+\vec{u}\cdot\vec{\nabla}$

④ Electron kinetic equation.

$$C_{ee}^{l}[f_e] + \nu_{ei}\mathcal{L}_{ei}[f_e] + \nu_{ei}\frac{m\vec{w}\cdot\vec{u}}{T_e} f_{Me}$$
$$= \frac{df_e}{dt} + \vec{w}\cdot\vec{\nabla} f_e - \left(\frac{e}{m}(\vec{E}' + \vec{w}\times\vec{B}) + \frac{d\vec{u}_e}{dt}\right)\cdot\frac{\partial f_e}{\partial \vec{w}} - \vec{w}\cdot\vec{\nabla}\vec{u}\cdot\frac{\partial f_e}{\partial \vec{w}}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} &C_{ee}^{l}[f_e] + \nu_{ei}\mathcal{L}_{ei}[f_e] + \frac{e}{m}\vec{w}\times\vec{B}\cdot\frac{\partial f_e}{\partial \vec{w}} \\ &= \frac{df_e}{dt} + \vec{w}\cdot\vec{\nabla} f_e - \left(\frac{e}{m}\vec{E}' + \frac{d\vec{u}_e}{dt}\right)\frac{\partial f_e}{\partial \vec{w}} - \vec{w}\cdot\vec{\nabla}\vec{u}\,\frac{\partial f_e}{\partial \vec{w}} - \nu_{ei}\frac{m\vec{w}\cdot\vec{u}}{T_e} f_{Me}\end{aligned}}$$

Expanding $f_e = f_{e0} + f_{e1} + \cdots$,

i) zeroth-order equation ($LHS : O(\epsilon^{-1})$, $RHS : O(1)$) $\therefore LHS \gg RHS$.

$$\underline{C_{ee}^{l}[f_e] + \nu_{ei}\mathcal{L}_{ei}[f_e] + \cancelto{0}{\frac{e}{m}\vec{w}\times\vec{B}\cdot\frac{\partial f_e}{\partial \vec{w}}} = 0} \quad \Rightarrow \quad \underline{f_{e0} = f_{Me} \ \text{(Maxwellian)}}$$

ii) next-order equation

*note
$$\ln f_{Me} = \text{const} + \ln n_e - \frac{3}{2}\ln T_e + \frac{mw^2}{2T_e}$$
$$\frac{d}{dt}f_m = f_m\frac{d}{dt}\ln f_m = f_m\left(\frac{d}{dt}\ln n_e - \frac{3}{2}\frac{d}{dt}\ln T_e + \frac{mw^2}{2T_e}\frac{d}{dt}\ln T_e\right) = f_m\left(\frac{d}{dt}\ln n_e + \left(\frac{mw^2}{2T_e} - \frac{3}{2}\right)\frac{d}{dt}\ln T_e\right)$$
$$\vec{w}\cdot\vec{\nabla} f_m = f_m\left(\vec{w}\cdot\vec{\nabla}\ln n_e + \left(\frac{mw^2}{2T_e} - \frac{3}{2}\right)\vec{w}\cdot\vec{\nabla}\ln T_e\right)$$
$$\frac{\partial f_m}{\partial \vec{w}} = -\frac{m\vec{w}}{T} f_m.$$

$$\boxed{\begin{aligned} &C_{ee}^{l}[f_{e1}] + \nu_{ei}\mathcal{L}_{ei}[f_{e1}] + \frac{e}{m}\vec{w}\times\vec{B}\cdot\frac{\partial f_{e1}}{\partial \vec{w}} \\ &= \Bigg[\frac{d}{dt}\ln n_e + \left(\frac{mw^2}{2T_e} - \frac{3}{2}\right)\frac{d}{dt}\ln T_e + \vec{w}\cdot\vec{\nabla}\ln n_e + \left(\frac{mw^2}{2T_e} - \frac{3}{2}\right)\vec{w}\cdot\vec{\nabla}\ln T_e \\ &\quad + \frac{m\vec{w}}{T_e}\left(\frac{e}{m}\vec{E}' + \frac{d\vec{u}_e}{dt}\right) + \frac{m\vec{w}\vec{w}}{T_e}\cdot\vec{\nabla}\vec{u}_e - \nu_{ei}\frac{m\vec{w}\cdot\vec{u}}{T_e}\Bigg] f_{Me}\end{aligned}}$$

⟨ Braginskii's correction equation ⟩

- Solvability condition of the first-order Chapman-Enskog expansion

$$\int d\vec{v}\,(1, \vec{w}, w^2) \times RHS = 0$$

*note that $\left(\int w_x f_{Me}(w)\, dw = 0 \right.$ (odd part) & $\int w_x w_y f_{Me}(w)\, dw = 0$ $\left.\right)$ (서로 다른 성분은 odd 번 곱하기)

[1] Particle conservation.

$$\int d\vec{w}\; 1 \cdot (RHS) = 0$$

i) $\int d\vec{w} \left[\frac{d\ln n_e}{dt} f_{Me} \right] = \underline{n_e \frac{d\ln n_e}{dt}}$

ii) $\int d\vec{w} \left[\left(\frac{mw^2}{2T_e} - \frac{3}{2} \right) \frac{d\ln T_e}{dt} f_{Me} \right] = \frac{d\ln T_e}{dt} \left(\frac{3p_e}{2T_e} - \frac{3}{2} n_e \right) = \underline{0}$

iii) $\int d\vec{w} \left[\vec{w}\cdot\vec{\nabla}\ln n_e + \left(\frac{mw^2}{2T_e} - \frac{3}{2} \right) \vec{w}\cdot\vec{\nabla}\ln T_e + \frac{m\vec{w}}{T_e}\left(\frac{e}{m}\vec{E}' + \frac{d\vec{u}_e}{dt} \right) - \nu_{ei} \frac{m\vec{w}\cdot\vec{u}}{T_e} \right] f_{Me} = \underline{0}$

iv) $\int d\vec{w} \left[\frac{m\vec{w}\vec{w}}{T_e} : \vec{\nabla}\vec{u}_e f_{Me} \right] = \frac{\overleftrightarrow{P}_e}{T_e} : \vec{\nabla}\vec{u}_e = \underline{n_e (\vec{\nabla}\cdot\vec{u}_e)}$

$\therefore$ (i) + (iv) $\Rightarrow$ $\boxed{\frac{d\ln n_e}{dt} = -\vec{\nabla}\cdot\vec{u}_e}$

[2] Momentum conservation

$\left(\text{*note } \int mw^2 f_{Me}\, dw = 3n_e T_e \right.$
$\left. \int (mw^2)^2 f_{Me}\, dw = 15 n_e T_e^2 \;/\; \int w^4 f_{Me}\, dw = 15 n_e \left(\frac{T_e}{m}\right)^2 \right)$

$$\int d\vec{w} \cdot \vec{w} \cdot (RHS) = 0$$

i) $\int d\vec{w} \left[\frac{d}{dt}\ln n_e + \left(\frac{mw^2}{2T_e} - \frac{3}{2} \right) \frac{d}{dt}\ln T_e + \frac{m\vec{w}\vec{w}}{T_e} \cdot \vec{\nabla}\vec{u}_e \right] \vec{w} f_{Me} = \underline{0}$

ii) $\int d\vec{w} \left[\vec{w}\cdot\vec{\nabla}\ln n_e \right] \vec{w} f_{Me} = \frac{\vec{\nabla} n_e}{n_e} \frac{p_e}{m_e} = \underline{\frac{T_e}{m_e} \vec{\nabla} n_e}$

iii) $\int d\vec{w} \left[\left(\frac{mw^2}{2T_e} - \frac{3}{2} \right) \vec{w}\cdot\vec{\nabla}\ln T_e \right] \vec{w} f_{Me} = \int d\vec{w} \left[\left(\frac{mw^2}{2T_e} - \frac{3}{2} \right) \vec{w}\vec{w} \cdot \frac{\vec{\nabla} T_e}{T_e} \right] f_{Me}$

$= \int d\vec{w} \frac{mw^2}{2T_e} \vec{w}\vec{w} \frac{\vec{\nabla} T_e}{T_e} f_{Me} - \frac{3p_e}{2m_e} \frac{\vec{\nabla} T_e}{T_e} = \frac{m\vec{\nabla} T_e}{2T_e^2} \int d\vec{w} \frac{1}{3} w^4 f_{Me} - \frac{3n_e}{2m_e} \vec{\nabla} T_e = \frac{m\vec{\nabla} T_e}{6\cancel{T_e^2}} \frac{15 n_e \cancel{T_e^2}}{m^2} - \frac{3n_e}{2m_e} \vec{\nabla} T_e$

$= \left(\frac{5}{2} - \frac{3}{2} \right) \frac{n_e}{m_e} \vec{\nabla} T_e = \underline{\frac{n_e \vec{\nabla} T_e}{m_e}}$

iv) $\int d\vec{w} \left[\frac{m\vec{w}}{T_e} \left(\frac{e}{m}\vec{E}' + \frac{d\vec{u}_e}{dt} \right) \right] \vec{w} f_{Me} = \frac{m}{T_e} \frac{n_e T_e}{m} \left(\frac{e}{m}\vec{E}' + \frac{d\vec{u}_e}{dt} \right) = \underline{n_e \left(\frac{e}{m}\vec{E}' + \frac{d\vec{u}_e}{dt} \right)}$

v) $\int d\vec{w} \left[-\nu_{ei} \frac{m\vec{w}\cdot\vec{u}}{T_e} \right] \vec{w} f_{Me} = -\nu_{ei} \frac{m_e}{T_e} \left(\frac{n_e T_e}{m} \right) \vec{u} = -n_e \nu_{ei} \vec{u} = \underline{-\frac{1}{m} \vec{R}_{ei}^{(0)}}$

$\therefore$ To sum-up, $\frac{T_e}{m_e}\vec{\nabla} n_e + \frac{n_e}{m_e}\vec{\nabla} T_e + n_e\left(\frac{e}{m}\vec{E}' + \frac{d\vec{u_e}}{dt}\right) - \frac{1}{m}\vec{R_{ei}}^{(0)} = 0$

$$\Rightarrow \boxed{\frac{d\vec{u_e}}{dt} = -\frac{e}{m_e}\vec{E}' + \frac{\vec{R_{ei}}^{(0)} - \vec{\nabla}(n_e T_e)}{m_e n_e}}$$

$\boxed{3}$ Energy conservation

$*$note $\int w^2 f_{Me}\, d\vec{w} = 3n_e\left(\frac{T_e}{m_e}\right)$, $\int w^4 f_{Me}\, d\vec{w} = 15 n_e\left(\frac{T_e}{m_e}\right)^2$

$$\int w^2 \cdot (\text{RHS})\, d\vec{w} = 0$$

i) $\int d\vec{w}\, w^2\left[\frac{d\ln n_e}{dt} f_{Me}\right] = \frac{3n_e T_e}{m}\frac{d\ln n_e}{dt}$

ii) $\int d\vec{w}\, w^2\left[\left(\frac{mw^2}{2T_e} - \frac{3}{2}\right)\frac{d\ln T_e}{dt} f_{Me}\right] = \frac{d\ln T_e}{dt}\left[\frac{m_e}{2T_e} 15 n_e\left(\frac{T_e}{m_e}\right)^2 - \frac{3}{2}\cdot 3n_e\left(\frac{T_e}{m_e}\right)\right] = 3\frac{n_e T_e}{m}\frac{d\ln T_e}{dt}$

iii) $\int d\vec{w}\, w^2\left[\frac{m\vec{w}\vec{w}}{T_e} : \vec{\nabla}\vec{u_e} f_{Me}\right] = \frac{m_e}{T_e}\cdot\int \frac{1}{3} w^4 \overleftrightarrow{I} : \vec{\nabla}\vec{u_e}\, f_{Me} = 5\frac{m_e}{T_e} n_e\left(\frac{T_e}{m_e}\right)^2 \vec{\nabla}\cdot\vec{u_e} = 5\frac{n_e T_e}{m_e}\vec{\nabla}\cdot\vec{u_e}$

iv) $\int d\vec{w}\cdot(\text{etc...}) = 0$ (odd-part)

$\therefore$ To sum-up, $\frac{d\ln n_e}{dt} + \frac{d\ln T_e}{dt} + \frac{5}{3}\vec{\nabla}\cdot\vec{u_e} = 0 \Rightarrow (-\vec{\nabla}\cdot\vec{u_e}) + \frac{d\ln T_e}{dt} + \frac{5}{3}\vec{\nabla}\cdot\vec{u_e} = 0$

$$\Rightarrow \boxed{\frac{d\ln T_e}{dt} = -\frac{2}{3}\vec{\nabla}\cdot\vec{u_e}}$$

– Inserting 3 conservation eqns to Braginskii's correction equation for electrons:

$$\left(\begin{aligned} & C_{ee}^{l}[f_{e1}] + \nu_{ei}\mathcal{L}_{ei}[f_{e1}] + \frac{e}{m}\vec{w}\times\vec{B}\cdot\frac{df_{e1}}{d\vec{w}} \\ & = \Big[\underbrace{\frac{d\ln n_e}{dt}}_{a} + \underbrace{\left(\frac{mw^2}{2T_e} - \frac{3}{2}\right)\frac{d\ln T_e}{dt}}_{b} + \underbrace{\vec{w}\cdot\vec{\nabla}\ln n_e}_{c} + \underbrace{\left(\frac{mw^2}{2T_e} - \frac{3}{2}\right)\vec{w}\cdot\vec{\nabla}\ln T_e}_{d} \\ & \quad + \underbrace{\frac{m\vec{w}}{T_e}\cdot\left(\frac{e}{m}\vec{E}' + \frac{d\vec{u_e}}{dt}\right)}_{e} + \underbrace{\frac{m_e\vec{w}\vec{w}}{T_e} : \vec{\nabla}\vec{u_e}}_{f} - \underbrace{\nu_{ei}\frac{m\vec{w}\cdot\vec{u}}{T_e}}_{g.}\Big] f_{Me} \end{aligned}\right)$$

$$(e) = \frac{m\vec{w}}{T_e}\cdot\left[\frac{\vec{R_{ei}}^{(0)}}{mn_e} - \frac{T_e\vec{\nabla}n_e + n_e\vec{\nabla}T_e}{mn_e}\right] = \frac{\vec{w}\cdot(mn\hat{\nu}_{ei}\vec{u})}{n_e T_e} - \vec{w}\cdot\vec{\nabla}\ln T_e - \vec{w}\cdot\vec{\nabla}\ln n_e$$

$\rightarrow (c) + (e)_3 = 0$

$\rightarrow (d) + (e)_2 = \left(\frac{mw^2}{2T_e} - \frac{5}{2}\right)\vec{w}\cdot\vec{\nabla}\ln T_e$

$\rightarrow (g) + (e)_1 = \frac{m}{T_e}(\hat{\nu}_{ei} - \nu_{ei})\vec{w}\cdot\vec{u}$

$$w_\perp = \frac{d}{d\gamma}(\vec{w_\perp}\times\hat{b}) = \frac{d}{d\gamma}(\sin\gamma\,\hat{x} - \cos\gamma\,\hat{y}) = \cos\gamma\,\hat{x} + \sin\gamma\,\hat{y}$$

[3] $$\vec{q_{\perp e}} = -T_e\int d\vec{w}\,\vec{w_\perp}\,L_1^{(3/2)}(x)\,\tilde{f}_{e1}^{(1)} = -T_e\int d\vec{w}\,\frac{d}{d\gamma}(\vec{w_\perp}\times\hat{b})\,L_1^{(3/2)}(x)\,\tilde{f}_{e1}^{(1)}$$

부분적분 $$= T_e\int d\vec{w}\,(\vec{w_\perp}\times\hat{b})\,L_1^{(3/2)}(x)\,\frac{d\tilde{f}_{e1}^{(1)}}{d\gamma}$$ ← 이제 우리는 $\tilde{f}_{e1}^{(1)}$이 아니라 $\frac{d\tilde{f}_{e1}^{(1)}}{d\gamma}$ 만 알면 됨.

(*note eq.(49) :

$$w_{ce}\frac{d\tilde{f}_{e1}^{(1)}}{d\gamma} = -C_{ee}^{l}[\tilde{f}_{e1}^{(0)}] - \nu_{ei}\mathcal{L}_{ei}[\tilde{f}_{e1}^{(0)}] + f_{Me}\frac{m(\hat{V}_{ei}-V_{ei})}{T_e}\vec{w_\perp}\cdot\vec{u}$$)

$$\vec{q_{\perp e}} = \frac{T_e}{w_{ce}}\int d\vec{w}\,(\vec{w_\perp}\times\hat{b})\,L_1^{(3/2)}(x)\,f_{Me}\left[1-\frac{3\sqrt{\pi}}{4}\left(\frac{V_{te}}{w}\right)^3\right]\frac{2\vec{w_\perp}\cdot(\vec{u_i}-\vec{u_e})}{V_{te}^2\,\tau_{ei}}$$
$$+\frac{T_e}{w_{ce}^2}\int d\vec{w}\,(\vec{w_\perp}\times\hat{b})\,L_1^{(3/2)}(x)\left[C_{ee}^{l}+\nu_{ei}\mathcal{L}_{ei}\right]\left(f_{Me}L_1^{(3/2)}(x)\,(\vec{w_\perp}\times\hat{b})\cdot\vec{\nabla}\ln T_e\right)$$

$$\boxed{\therefore\ \vec{q_{\perp e}} = \frac{3}{2}\frac{p_e}{\tau_{ei}w_{ce}}\hat{b}\times(\vec{u_i}-\vec{u_e}) - 4.66\frac{p_e}{m_e\tau_{ei}w_{ce}^2}\vec{\nabla}_\perp T_e}$$

⑧ Electron friction force

[1] $$R_{\parallel ei} = \frac{m_e n_e}{\tau_{ei}}(u_{\parallel i}-u_{\parallel e}) - \frac{3\sqrt{\pi}}{4\tau_{ei}}m_e V_{te}^3\int d\vec{w}\,\frac{w_\parallel}{w^3}\langle f_{e1}\rangle$$
$$= \frac{m_e n_e}{\tau_{ei}}(u_{\parallel i}-u_{\parallel e})\left\{1-\frac{3\sqrt{\pi}}{2}\frac{V_{te}}{n_e}\int d\vec{w}\,\frac{w_\parallel^2}{w^3}f_{Me}\left[0.284L_1^{(3/2)}(x)+0.032L_2^{(3/2)}(x)\right]\right\}$$
$$-\frac{5}{2}n_e\hat{b}\cdot\vec{\nabla}T_e\times\frac{3\sqrt{\pi}}{2}\frac{V_{te}}{n_e}\int d\vec{w}\,\frac{w_\parallel^2}{w^3}f_{Me}\left[0.506L_1^{(3/2)}(x)-0.253L_2^{(3/2)}(x)\right]$$
$$= \frac{m_e n_e}{\tau_{ei}}(u_{\parallel i}-u_{\parallel e})\left[1-0.284\int_0^\infty dx\,e^{-x}L_1^{(3/2)}(x)-0.032\int_0^\infty dx\,e^{-x}L_2^{(3/2)}(x)\right]$$
$$-\frac{5}{2}n_e\hat{b}\cdot\vec{\nabla}T_e\left[0.506\int_0^\infty dx\,e^{-x}L_1^{(3/2)}(x)-0.253\int_0^\infty dx\,e^{-x}L_2^{(3/2)}(x)\right]$$

$$\boxed{R_{\parallel ei} = 0.51\frac{m_e n_e}{\tau_{ei}}(u_{\parallel i}-u_{\parallel e}) - 0.71\,n_e\hat{b}\cdot\vec{\nabla}T_e}$$ ⟨Parallel friction force⟩

Same as Spitzer-Harm : 0.51

[2] $$\overrightarrow{q_{\wedge e}} = -T_e \int d\vec{w}\, \overrightarrow{w_\perp} L_1^{(3/2)}(x) \tilde{f}_{e1}^{(1)} = \int d\vec{w}\, \overrightarrow{w_\perp} \left[L_1^{(3/2)}(x)\right]^2 f_{Me} \frac{\overrightarrow{w_\perp}\times\hat{b}}{w_{ce}} \cdot \vec{\nabla} T_e$$

$$= \frac{1}{w_{ce}} \int_{-1}^{1} d\xi \int_0^\infty dw\, w^2 \int_0^{2\pi} d\gamma \left(L_1^{(3/2)}\right)^2 f_{Me}\, \overrightarrow{w_\perp}\,\overrightarrow{w_\perp}\times\hat{b}\cdot\vec{\nabla} T_e$$

$$\left(\int_0^{2\pi} \overrightarrow{w_\perp}\,\overrightarrow{w_\perp}\, d\gamma = \pi w_\perp^2 \left(\overleftrightarrow{I} - \hat{b}\hat{b}\right) \right)$$

$$\overrightarrow{q_{\wedge e}} = \frac{\pi}{w_{ce}} \int_{-1}^{1} d\xi \int_0^\infty dw\, w^2 w_\perp^2 \left(L_1^{(3/2)}\right)^2 f_{Me} \left(\overleftrightarrow{I} - \hat{b}\hat{b}\right) \times \hat{b}\cdot\vec{\nabla} T_e$$

$$\left(w_\perp^2 = w^2 - w_\parallel^2 = (1-\xi^2) w^2 , \quad \int_{-1}^{1} (1-\xi^2) d\xi = 2 - \frac{2}{3} = \frac{4}{3} \right)$$

$$\overrightarrow{q_{\wedge e}} = \frac{\pi}{w_{ce}} \frac{4}{3} \int_0^\infty dx\, x^{3/2} \left(\frac{v_{te}}{2}\right)^5 \left(L_1^{(3/2)}\right)^2 \frac{n_e}{\pi^{3/2} v_{te}^3} e^{-x} \left(\overleftrightarrow{I} - \hat{b}\hat{b}\right) \times \hat{b}\cdot\vec{\nabla} T_e$$

$$= \frac{4 n_e}{3 w_{ce}\sqrt{\pi}} \underbrace{\frac{v_{te}^2}{2}}_{\frac{T_e}{M}} \underbrace{\int_0^\infty e^{-x} x^{3/2} \left(L_1^{(3/2)}\right)^2 dx}_{15\sqrt{\pi}/8} \cdot \underbrace{\left(\overleftrightarrow{I} - \hat{b}\hat{b}\right)\times\hat{b}\cdot\vec{\nabla} T_e}$$

$$\left(\overleftrightarrow{I} - \hat{b}\hat{b}\right)\times\hat{b}\cdot\vec{\nabla} T_e = (\hat{x}\hat{x} + \hat{y}\hat{y}) \times \hat{z}\cdot\vec{\nabla} T_e = -\hat{x}\hat{y}\cdot\vec{\nabla} T_e + \hat{y}\hat{x}\cdot\vec{\nabla} T_e = -\hat{x}\frac{dT_e}{dy} + \hat{y}\frac{dT_e}{dx} = \hat{b}\times\vec{\nabla} T_e$$

$$\therefore \overrightarrow{q_{\wedge e}} = \frac{4}{3\sqrt{\pi}} \frac{p_e}{m_e w_{ce}} \frac{15\sqrt{\pi}}{8} \hat{b}\times\vec{\nabla} T_e = \frac{5}{2}\frac{p_e}{m_e w_{ce}} \hat{b}\times\vec{\nabla} T_e \qquad \therefore \boxed{\overrightarrow{q_{\wedge e}} = \frac{5}{2}\frac{p_e}{m_e \omega_{ce}} \hat{b}\times\vec{\nabla} T_e}$$

- Friction force $\vec{R_{ei}} = -m_e \int d\vec{w}\, \nu_s^{ei}\, \vec{w}\,(f_{e0} + f_{e1})$

$$\vec{R_{ei}} = \frac{m_e n_e}{\tau_{ei}}(\vec{u_i} - \vec{u_e}) - \frac{3\sqrt{\pi}}{4\tau_{ei}} m_e V_{te}^3 \int d\vec{w}\, \frac{\vec{w}}{w^3}\left(\langle f_{e1}\rangle + \tilde{f}_{e1}^{(0)}\right) \equiv \vec{R_{\parallel ei}} + \vec{R_{\perp ei}}$$

⑰ Electron heat flux

[1] $\vec{q_{\parallel e}} = -T_e \int d\vec{w}\, \vec{w}\, L_1^{(3/2)}(x) \langle f_{e1}\rangle = -T_e \int d\vec{w}\, w_\parallel \hat{b}\, L_1^{(3/2)}(x)\langle f_{e1}\rangle$

$\left(\int d\vec{w} = 2\pi \int_{-1}^{1} d\xi \int_0^\infty w^2 dw, \quad w_\parallel = w\xi\right)$

$$\vec{q_{\parallel e}} = -T_e \cdot 2\pi \int_{-1}^{1} d\xi \int_0^\infty dw \cdot w^2 (w\xi) L_1^{(3/2)}(x)$$
$$\times \left\{ w\xi\, f_{Me}\, L_1^{(3/2)}(x) \left[0.506 \frac{5}{2}\tau_{ei} \hat{b}\hat{b}\cdot\frac{\vec{\nabla} T_e}{T_e} + 0.284 \frac{m_e(\vec{u_{i\parallel}} - \vec{u_{e\parallel}})}{T_e}\right]\right.$$
$$= -\frac{4\pi}{3}\int_0^\infty dw\, w^4 \left[L_1^{(3/2)}(x)\right]^2 f_{Me}\left[0.506\frac{5}{2}\tau_{ei}\hat{b}\hat{b}\cdot\vec{\nabla}T_e + 0.284\, m_e(\vec{u_{i\parallel}} - \vec{u_{e\parallel}})\right]$$

$$\left(x = \frac{w^2}{V_{te}^2}, \quad 2w\,dw = V_{te}^2 dx \to w^4 dw = V_{te}^4 x^2 \cdot \frac{V_{te}^2}{2w} dx = \frac{1}{2} V_{te}^5 x^{3/2} dx\right)$$
$$f_{Me} = n_e\left(\frac{1}{\pi V_{te}^2}\right)^{3/2} \exp\left(-\frac{w^2}{V_{te}^2}\right) = \frac{n_e}{\pi^{3/2} \cdot V_{te}^3}\exp(-x)$$

$$\vec{q_{\parallel e}} = -\frac{4\pi}{3}\frac{n_e}{\pi^{3/2}}\frac{V_{te}^2}{2}\left[0.506\frac{5}{2}\tau_{ei}\hat{b}\hat{b}\cdot\vec{\nabla}T_e + 0.284 m_e(\vec{u_{i\parallel}} - \vec{u_{e\parallel}})\right]\underbrace{\int_0^\infty e^{-x} x^{3/2}\left[L_1^{(3/2)}(x)\right]^2}_{=15\sqrt{\pi}/8}$$

$$\therefore \boxed{\vec{q_{\parallel e}} = -3.16\frac{p_e \tau_{ei}}{m_e}\hat{b}\hat{b}\cdot\vec{\nabla}T_e - 0.71\, p_e(\vec{u_{i\parallel}} - \vec{u_{e\parallel}})\hat{b}}$$

parallel conductivity friction.

one can obtain a_0, a_1 from (LHS) = (RHS), which gives $\langle f_{e1} \rangle$

In particular for $Z=1$.

$$\boxed{\begin{aligned}\langle f_{e1}\rangle = {} & w_{\parallel} f_{Me} L_1^{(3/2)}\left(\frac{w^2}{v_{te}^2}\right)\left[0.506 \times \frac{5}{2}\tau_{ei}\hat{b}\cdot\vec{\nabla}\ln T_e + 0.284 \times \frac{m_e(u_{i\parallel}-u_{e\parallel})}{T_e}\right] \\ & - w_{\parallel} f_{Me} L_2^{(3/2)}\left(\frac{w^2}{v_{te}^2}\right)\left[0.253 \times \frac{5}{2}\tau_{ei}\hat{b}\cdot\vec{\nabla}\ln T_e - 0.032 \times \frac{m_e(u_{i\parallel}-u_{e\parallel})}{T_e}\right]\end{aligned}}$$

where $m = m_e$ and $\vec{u} = \vec{u_i} - \vec{v_e}$.

⑥ Gyro-dependent electron distribution function

one can subtract the gyro-independent part from full equation

$$C_{ee}^{l}[\tilde{f}_{e1}] + \nu_{ei}\mathcal{L}[\tilde{f}_{e1}] + w_{ce}\frac{d\tilde{f}_{e1}}{d\gamma} = \left[\left(\frac{mw^2}{2T_e} - \frac{5}{2}\right)\vec{w_\perp}\cdot\vec{\nabla}\ln T_e + \frac{m(\hat{\nu}_{ei} - \nu_{ei})}{T_e}\vec{w_\perp}\cdot\vec{u}\right] f_{Me}$$

With $w_{ce} \gg \nu_{ei}$, one can make the subsequent ordering (last piece of Braginskii's electron correction equation)

$$\tilde{f}_{e1} = \tilde{f}_{e1}^{(0)} + \tilde{f}_{e1}^{(1)}$$

i) $$w_{ce}\frac{d\tilde{f}_{e1}^{(0)}}{d\gamma} = f_{Me}\left(\frac{mw^2}{2T_e} - \frac{5}{2}\right)\vec{w_\perp}\cdot\vec{\nabla}\ln T_e \qquad \text{(Lowest-order)}$$

Integrating over : $$\frac{d\tilde{f}_{e1}^{(0)}}{d\gamma} = \frac{f_{Me}}{w_{ce}}\left(\frac{mw^2}{2T_e} - \frac{5}{2}\right) w_\perp(\hat{x}\cos\gamma + \hat{y}\sin\gamma)\vec{\nabla}\ln T_e$$

$$\tilde{f}_{e1}^{(0)} = -\frac{f_{Me}}{w_{ce}} L_1^{(3/2)}(x)\cdot w_\perp(\hat{x}\sin\gamma - \hat{y}\cos\gamma)\vec{\nabla}\ln T_e$$

$\vec{w_\perp}\times\hat{b} = \hat{x}\sin\gamma - \hat{y}\cos\gamma$

$$\Rightarrow \tilde{f}_{e1}^{(0)} = -f_{Me}L_1^{(3/2)}(x)\frac{\vec{w_\perp}\times\hat{b}}{w_{ce}}\cdot\vec{\nabla}\ln T_e$$

ii) $$w_{ce}\frac{d\tilde{f}_{e1}^{(1)}}{d\gamma} = -C_{ee}^{l}[\tilde{f}_{e1}^{(0)}] - \nu_{ei}\mathcal{L}[\tilde{f}_{e1}^{(0)}] + f_{Me}\frac{m(\hat{\nu}_{ei} - \nu_{ei})}{T_e}\vec{w_\perp}\cdot\vec{u} \qquad \text{(Next-order)}$$

Now one has $f_{e1} = \langle f_{e1}\rangle + \tilde{f}_{e1}^{(0)} + \tilde{f}_{e1}^{(1)}$ and can evaluate electron heat flux

$$\vec{q_e} = \int d\vec{w}\,\vec{w}\left(\frac{1}{2}m_e w^2 - \frac{5}{2}T_e\right) f_{e1}$$

$$= -T_e\int d\vec{w}\,\vec{w}\,L_1^{(3/2)}(x)\left(\langle f_{e1}\rangle + \tilde{f}_{e1}^{(0)} + \tilde{f}_{e1}^{(1)}\right) \equiv \underbrace{\vec{q}_{\parallel e}}_{\text{parallel}} + \underbrace{\vec{q}_{\wedge e}}_{\text{diamagnetic}} + \underbrace{\vec{q}_{\perp e}}_{\text{perpendicular}}$$

This part can be solved in the same way as the Spitzer-Härm, using the associated Laguerre polynomial expansion.

$$\langle f_{e1}\rangle = \frac{2w_{\parallel}}{v_{te}^2} f_{Me} \sum_{k=1} a_k L_k^{(3/2)}(x). \quad \leftarrow k\geq 1 \text{ and } v_{\parallel}\to w_{\parallel} \text{ (notable difference)}$$

(i) LHS :
$$\int w_{\parallel} L_k^{(3/2)}(x)\left\{ C_{ee}^{l}[\langle f_{e1}\rangle] + \nu_{ei}\mathcal{L}_{ei}[\langle f_{e1}\rangle]\right\} d\vec{w}$$
$$= \boxed{-\frac{n_e}{\tau_{ei}Z}\begin{pmatrix} \sqrt{2}+\frac{13}{4}Z & \frac{3\sqrt{2}}{4}+\frac{69}{16}Z \\ \frac{3\sqrt{2}}{4}+\frac{69}{16}Z & \frac{45\sqrt{2}}{16}+\frac{433}{64}Z \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix}}$$
$\leftarrow$ From Eq(55) in Lec.19.

(ii) RHS :
$$\int d\vec{w}\, w_{\parallel} L_k^{(3/2)} f_{Me}\left\{\left(\frac{w^2}{v_{te}^2}-\frac{5}{2}\right) w_{\parallel}\hat{b}\cdot\vec{\nabla}\ln T_e + \left[1-\frac{3\sqrt{\pi}}{4}\left(\frac{v_{te}}{w}\right)^3\right]\frac{2w_{\parallel}u_{\parallel}}{v_{te}^2\tau_{ei}}\right\}$$

$w_{\parallel}^2 = w^2\xi^2$

$$= \int d\vec{w}\, w^2\xi^2 L_k^{(3/2)}(x) f_{Me}\left[\left(x-\frac{5}{2}\right)\hat{b}\cdot\vec{\nabla}\ln T_e + \left(1-\frac{3\sqrt{\pi}}{4x^{3/2}}\right)\frac{2u_{\parallel}}{v_{te}^2\tau_{ei}}\right]$$

$$\left(\begin{aligned} &{}^*\text{note } f_{Me} = n_e\left(\frac{m_e}{2\pi T_e}\right)^{3/2}\exp\left(-\frac{m_e w^2}{2T_e}\right) = \frac{n_e}{\pi^{3/2}}\frac{1}{v_{te}^3}\exp(-x) \\ &\int d\vec{w} = 2\pi\int_{-1}^{1}d\xi\int_0^\infty w^2 dw = \pi v_{te}^3\int_{-1}^{1}d\xi\int_0^\infty x^{1/2}dx, \quad x=\frac{w^2}{v_{te}^2}\end{aligned}\right)$$

$$= \frac{2n_e v_{te}^2}{3\sqrt{\pi}}\int_0^\infty dx\, x^{3/2}e^{-x}L_k^{(3/2)}(x)\left[-L_1^{(3/2)}(x)\hat{b}\cdot\vec{\nabla}\ln T_e + \left(1-\frac{3\sqrt{\pi}}{4x^{3/2}}\right)\frac{2u_{\parallel}}{v_{te}^2\tau_{ei}}\right]$$

$$= -\frac{4}{3\sqrt{\pi}}\frac{n_e}{m}\hat{b}\cdot\vec{\nabla}T_e \times \delta_{k1}\frac{15\sqrt{\pi}}{8} - \frac{n_e u_{\parallel}}{\tau_{ei}}\times\left(1-\delta_{k0}+\frac{3}{2}\delta_{k1}+\frac{15}{8}\delta_{k2}+\cdots\right)$$

$$\begin{array}{l} k=1 \to \\ k=2 \to \end{array} \boxed{-\frac{n_e}{\tau_e}\begin{pmatrix}\frac{5}{2}\frac{\tau_{ei}}{m}\hat{b}\cdot\vec{\nabla}T_e+\frac{3}{2}u_{\parallel} \\ \frac{15}{8}u_{\parallel}\end{pmatrix}}$$

$\left(\Gamma(z+1) = z\Gamma(z)\right)$

*note
$$\left\{\begin{aligned} &k=0 \quad \int_0^\infty dx\, e^{-x} = \Gamma(1) = 1 \\ &k=1 \quad \int_0^\infty dx\, e^{-x}\left(\frac{5}{2}-x\right) = \frac{5}{2}\Gamma(1)-\Gamma(2) = \frac{3}{2} \\ &k=2 \quad \int_0^\infty dx\, e^{-x}\left(\frac{35}{8}-\frac{7}{2}x+\frac{1}{2}x^2\right) = \frac{35}{8}\Gamma(1)-\frac{7}{2}\Gamma(2)+\frac{1}{2}\Gamma(3) = \frac{35}{8}-\frac{7}{2}+\frac{2}{2} = \frac{15}{8} \\ &\qquad\Downarrow \\ &\int_0^\infty dx\, x^{3/2}e^{-x}L_k^{(3/2)}(x)\left(1-\frac{3\sqrt{\pi}}{4x^{3/2}}\right) = \frac{3\sqrt{\pi}}{4} - \frac{3\sqrt{\pi}}{4}\left(\int_0^\infty dx\, e^{-x}L_k^{(3/2)}(x)\right) \\ &\qquad = \frac{3\sqrt{\pi}}{4}\left(1-\delta_{k0}+\frac{3}{2}\delta_{k1}+\frac{15}{8}\delta_{k2}+\cdots\right)\end{aligned}\right\}$$

$(a) = -\vec{\nabla}\cdot\vec{u_e}$

$(b) = \left(\frac{mw^2}{2T_e} - \frac{3}{2}\right)\left(-\frac{2}{3}\vec{\nabla}\cdot\vec{u_e}\right) = -\frac{mw^2}{3T_e}\vec{\nabla}\cdot\vec{u_e} + \vec{\nabla}\cdot\vec{u_e}$

$(f) = \frac{m}{T_e}\vec{w}\vec{w} : \vec{\nabla}\vec{u_e}$

$\rightarrow (a)+(b)+(f) = \frac{m_e}{T_e}\left(\vec{w}\vec{w} - \frac{w^2}{3}\overleftrightarrow{I}\right) : \vec{\nabla}\vec{u_e} = \frac{m}{2T_e}\left(\vec{w}\vec{w} - \frac{w^2}{3}\overleftrightarrow{I}\right) : \overleftrightarrow{W_e}$

where $\overleftrightarrow{W_e} = \vec{\nabla}\vec{u} + (\vec{\nabla}\vec{u})^T - \frac{2}{3}(\vec{\nabla}\cdot\vec{u})\overleftrightarrow{I}$ (rate of strain tensor)

$$\Rightarrow \boxed{\begin{aligned} &\hat{C}^{\ell}_{ee}[f_{e1}] + \nu_{ei}\mathcal{L}_{ei}[f_{e1}] + \frac{e}{m}\vec{w}\times\vec{B}\,\frac{\partial f_{e1}}{\partial\vec{w}} \\ &= \left[\left(\frac{mw^2}{2T_e} - \frac{5}{2}\right)\vec{w}\cdot\vec{\nabla}\ln T_e + \frac{m(\hat{\nu}_{ei} - \nu_{ei})}{T_e}\vec{w}\cdot\vec{u} + \frac{m}{2T_e}\left(\vec{w}\vec{w} - \frac{w^2}{3}\overleftrightarrow{I}\right) : \overleftrightarrow{W_e}\right] f_{Me} \end{aligned}}$$

where $\overleftrightarrow{W} = (\vec{\nabla}\vec{u}) + (\vec{\nabla}\vec{u})^T - \frac{2}{3}(\vec{\nabla}\cdot\vec{u})\overleftrightarrow{I}$ and $\nu_{ei}(v) = \frac{3\sqrt{\pi}}{4}\left(\frac{v_{te}}{w}\right)^3\hat{\nu}_{ei}$

(1) Thermodynamic force (2) friction force viscous force due to flow shear.

⑤ Gyro-averaged electron distribution function

Now use $\vec{w} = w_\parallel\hat{b} + w_\perp(\hat{x}\cos\gamma + \hat{y}\sin\gamma) \rightarrow \frac{e}{m}\vec{w}\times\vec{B}\cdot\frac{\partial f_{e1}}{\partial\vec{w}} = W_{ce}\frac{\partial f}{\partial\gamma}$

$\left(\frac{e}{m}\vec{w}\times\vec{B}\cdot\frac{\partial f_{e1}}{\partial\vec{w}} = \frac{eB}{m}\left[w_\perp(\sin\gamma\,\hat{x} - \cos\gamma\,\hat{y})\right]\cdot\left[\hat{x}\frac{\partial f_{e1}}{\partial w_x} + \hat{y}\frac{\partial f_{e1}}{\partial w_y}\right] = \frac{eB}{m}w_\perp\left(\sin\gamma\frac{\partial f}{\partial w_x} - \cos\gamma\frac{\partial f}{\partial w_y}\right) = W_{ce}\frac{\partial f_{e1}}{\partial\gamma}\right.$

$\left.\quad *\text{note } \frac{\partial f}{\partial\gamma} = \frac{\partial w_x}{\partial\gamma}\frac{\partial f}{\partial w_x} + \frac{\partial w_y}{\partial\gamma}\frac{\partial f}{\partial w_y} = w_\perp\left(-\sin\gamma\frac{\partial f}{\partial w_x} + \cos\gamma\frac{\partial f}{\partial w_y}\right)\right)$

Compared to the hydrodynamic Chapman-Enskog, a magnetized plasma contains the fast gyromotion making kinetic response strongly anisotropic. ⇒ Let's perform gyro-averaging to isolate parallel & perpendicular directions

$\underline{f_{e1} = \langle f_{e1}\rangle + \tilde{f}_{e1}}$ where $\langle f_{e1}\rangle = \frac{1}{2\pi}\int f_{e1}\,d\gamma$

This is what Braginskii did.

From $\langle \hat{C}^{\ell}_{ee}[f_{e1}]\rangle = \hat{C}_{ee}[\langle f_{e1}\rangle]$, $\langle\mathcal{L}[f_{e1}]\rangle = \mathcal{L}[\langle f_{e1}\rangle]$

$\langle\vec{w}\rangle = w_\parallel\hat{b}$, $\langle\vec{w}\vec{w}\rangle = w_\parallel^2\hat{b}\hat{b} + \frac{1}{2}w_\perp^2(\overleftrightarrow{I} - \hat{b}\hat{b})$, and $\overleftrightarrow{W} \simeq 0$ (viscous-term)

$$\Rightarrow \boxed{\hat{C}^{\ell}_{ee}[\langle f_{e1}\rangle] + \nu_{ei}\mathcal{L}_{ei}[\langle f_{e1}\rangle] = \left[\left(\frac{mw^2}{2T_e} - \frac{5}{2}\right)w_\parallel\hat{b}\cdot\vec{\nabla}\ln T_e + \frac{m(\hat{\nu}_{ei} - \nu_{ei})}{T_e}w_\parallel u_\parallel\right] f_{Me}}$$

<gyro-averaged part>

$\boxed{2}$ $$\vec{R}_{\perp ei} = \frac{m_e n_e}{\tau_{ei}}(\vec{u_{\perp i}} - \vec{u_{\perp e}}) - \frac{3\sqrt{\pi}}{4\tau_{ei}} m_e v_{te}^3 \int d\vec{w} \frac{\vec{w_\perp}}{w^3} \tilde{f}_{ei}^{(0)}$$

$$= \frac{m_e n_e}{\tau_{ei}}(\vec{u_{\perp i}} - \vec{u_{\perp e}}) + \frac{3\sqrt{\pi}}{4w_{ce}\tau_{ei}} m_e v_{te}^3 \int d\vec{w} \frac{\vec{w_\perp}}{w^3} f_{Me} L_1^{(3/2)}(x)\, \vec{w_\perp}\cdot\hat{b}\times\vec{\nabla}\ln T_e$$

$$= \frac{m_e n_e}{\tau_{ei}}(\vec{u_{\perp i}} - \vec{u_{\perp e}}) + \frac{3n_e}{4w_{ce}\tau_{ei}} \int_{-1}^{1} d\xi (1-\xi^2) \int_0^\infty dx\, e^{-x} L_1^{(3/2)}(x)\, \hat{b}\times\vec{\nabla}T_e$$

$$= \frac{m_e n_e}{\tau_{ei}}(\vec{u_{\perp i}} - \vec{u_{\perp e}}) + \frac{3}{2}\frac{n_e}{w_{ce}\tau_{ei}}\hat{b}\times\vec{\nabla}T_e$$

⑨ Summary: Electron transport coefficients by Braginskii

$$q_{\parallel e} = -\kappa_{\parallel e}\nabla_\parallel T_e - 0.71 p_e (u_{i\parallel} - u_{e\parallel}) \qquad \left(\kappa_{\parallel e} \equiv 3.16\frac{p_e\tau_{ei}}{m_e}\right)$$

$$\vec{q}_{\wedge e} = \frac{5}{2}\frac{p_e}{m_e w_{ce}}\hat{b}\times\vec{\nabla}T_e$$

$$\vec{q}_{\perp e} = -\kappa_{\perp e}\vec{\nabla}_\perp T_e + \frac{3}{2}\frac{p_e}{\tau_{ei} w_{ce}}\hat{b}\times(\vec{u_{i\perp}} - \vec{u_{\perp e}}) \qquad \left(\kappa_{\perp e} = 4.66\frac{p_e}{m_e\tau_{ei}w_{ce}^2}\right)$$

$$\overleftrightarrow{\Pi}_e \simeq 0$$

$$R_{\parallel ei} = \eta_\parallel e^2 n_e^2 (u_{i\parallel} - u_{\parallel e}) - 0.71 n_e \nabla_\parallel T_e \qquad \left(\eta_\parallel \equiv 0.51\frac{m_e}{n_e e^2 \tau_{ei}}\right)$$

$$\vec{R}_{\perp ei} = \eta_\perp e^2 n_e^2 (\vec{u_{\perp i}} - \vec{u_{\perp e}}) + \frac{3}{2}\frac{n_e}{w_{ce}\tau_{ei}}\hat{b}\times\vec{\nabla}T_e \qquad \left(\eta_\perp \equiv \frac{m_e}{n_e e^2\tau_{ei}}\right)$$

$$Q_{ie} = \frac{3m_e n_e}{m_i\tau_{ei}}(T_e - T_i)$$

⑩ Ion kinetic equation

Assume: ion-ion collisions are dominant. (neglect all e-i terms in electron eqn).

$$\Rightarrow \boxed{C_{ii}^{l}[f_{i1}] + w_{ci}\frac{\partial f_{i1}}{\partial \gamma} = \left[\left(\frac{m_i w^2}{2T_i} - \frac{5}{2}\right)\vec{w}\cdot\vec{\nabla}\ln T_i + \frac{m_i}{2T_i}\left(\vec{w}\vec{w} - \frac{w^2}{3}\overleftrightarrow{I}\right):\overleftrightarrow{W_i}\right] f_{Mi}}$$

However, we cannot neglect ion viscosity W_i (unlike electron viscosity $\overleftrightarrow{W_e} \simeq 0$)

$$f_{i1} = \langle f_{i1}\rangle + \tilde{f}_{i1}$$

$$\Rightarrow C_{ii}^{l}[\langle f_{i1}\rangle] = \left[\left(\frac{m_i w^2}{2T_i} - \frac{5}{2}\right) w_{\parallel}\hat{b}\cdot\vec{\nabla}\ln T_i + \frac{m_i}{2T_i}\left(w_{\parallel}^2 - \frac{w_{\perp}^2}{2}\right)\left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right):\overleftrightarrow{W_i}\right] f_{Mi}$$

$$\left(\begin{aligned} &\text{*note}\quad \langle\vec{w}\vec{w}\rangle - \frac{w^2}{3}\overleftrightarrow{I} = w_{\parallel}^2\hat{b}\hat{b} + \frac{w_{\perp}^2}{2}(\overleftrightarrow{I} - \hat{b}\hat{b}) - \left(\frac{w_{\parallel}^2}{3} + \frac{w_{\perp}^2}{3}\right)\overleftrightarrow{I} \\ &\qquad = w_{\parallel}^2\left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right) - \frac{w_{\perp}^2}{2}\left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right) = \left(w_{\parallel}^2 - \frac{w_{\perp}^2}{2}\right)\left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right)\end{aligned}\right)$$

⑪ Associated Laguerre polynomials for angular distribution

Expansion for ion kinetic equation:

$$\boxed{\langle f_{i1}\rangle = \underbrace{\frac{2w_{\parallel}}{v_{ti}^2} f_{Mi}\sum_{k=1} a_k L_k^{(3/2)}(x)}_{\equiv \langle f_{i1}\rangle_a} + \underbrace{\frac{2}{3}\left(\frac{w_{\parallel}^2 - w_{\perp}^2/2}{p_i v_{ti}^2}\right) f_{Mi}\sum_{k=0} b_k L_k^{(5/2)}(x)}_{\equiv \langle f_{i1}\rangle_b}}$$

one can see that $\langle f_{i1}\rangle_b$ is closely related to viscous tensor.

$$\left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right)\int d\vec{w}\, m_i\left(w_{\parallel}^2 - \frac{w_{\perp}^2}{2}\right)\langle f_{i1}\rangle_b$$

$$= \left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right)\int d\vec{w}\, m_i\left(w_{\parallel}^2 - \frac{w_{\perp}^2}{2}\right)\frac{2}{3}\left(\frac{w_{\parallel}^2 - w_{\perp}^2/2}{p_i v_{ti}^2}\right) f_{Mi}\sum_{k=0} b_k L_k^{(5/2)}(x)$$

$$\left(\begin{aligned} &\text{*note}\quad \int d\vec{w} = 2\pi\int_{-1}^{1} d\xi\int_0^{\infty} w^2 dw = \pi v_{ti}^3\int_{-1}^{1} d\xi\int_0^{\infty} x^{1/2}dx, \quad w_{\parallel}^2 - \frac{w_{\perp}^2}{2} = w^2\left(\xi^2 - \frac{(1-\xi^2)}{2}\right) = w^2\left(\frac{3\xi^2 - 1}{2}\right) \\ &\qquad f_{Mi} = \frac{n_i}{\pi^{3/2} v_{ti}^3} e^{-x} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad = v_{ti}^2 x\left(\frac{3\xi^2 - 1}{2}\right)\end{aligned}\right)$$

$$= \left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right)\int_{-1}^{1} d\xi\int_0^{\infty}\pi v_{ti}^3 x^{1/2}dx\left(\frac{2}{3}\underbrace{\frac{m_i}{(n_i m_i v_{ti}^2/2)v_{ti}^2}}_{\frac{4}{3 n_i v_{ti}^4}}\right)\cdot\left(v_{ti}^4 x^2\left(\frac{3\xi^2 - 1}{2}\right)^2\right)$$

$$\times\left(\frac{\cancel{n_i}}{\pi^{3/2}\cancel{v_{ti}^3}} e^{-x}\right)\sum_{k=0} b_k L_k^{(5/2)}$$

$$= \left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right)\frac{4}{3\sqrt{\pi}}\int_{-1}^{1} d\xi\left(\frac{3\xi^2 - 1}{2}\right)^2\sum_k b_k\int_0^{\infty} x^{5/2}e^{-x} L_0^{(5/2)}(x)\, L_k^{(5/2)}(x)$$

$$= \left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right) b_0 \quad \Leftarrow \quad b_0 \ (\text{of } L_0^{(5/2)}) \text{ is parallel component of the viscous tensor.}$$

$\left(w_\parallel^2 - \frac{w_\perp^2}{2}\right) = w^2 P_2(\xi)$

Also, notice that $P_1(\xi) = \xi = \frac{w_\parallel}{w}$, $P_2(\xi) = \frac{1}{2}(3\xi^2 - 1)$: Legendre polynomial

Then, $$\boxed{\langle f_{i1}\rangle_a + \langle f_{i1}\rangle_b = 2\sqrt{x}\, P_1(\xi) f_{Mi} \sum_k \frac{a_k}{v_{ti}} L_k^{(3/2)}(x) + \frac{2}{3} x P_2(\xi) \sum_k \frac{b_k}{p_i} L_k^{(5/2)}(x)}$$

↳ one can separate the calculation $\langle f_{i1}\rangle_a$ and $\langle f_{i1}\rangle_b$ due to the orthogonality.

$$\left(\int_{-1}^{1} d\xi P_p(\xi) P_q(\xi) = \frac{2}{2p+1}\delta_{pq} \right)$$

⑫ Gyro-averaged ion distribution function.

이제 $\langle f_{i1}\rangle_a$ 와 $\langle f_{i1}\rangle_b$ 를 각각 따로 구해보자!

1. $$\int d\vec{w}\, w_\parallel L_k^{(3/2)} C_{ii}^{\ell}[\langle f_{i1}\rangle_a] = \frac{n_i}{\tau_{ii}} \begin{pmatrix} 1 & 3/4 \\ 3/4 & 45/16 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\int d\vec{w}\, w_\parallel L_k^{(3/2)}(x) f_{Mi} \left(x^2 - \frac{5}{2}\right) w_\parallel \hat{b}\cdot\vec{\nabla}\ln T_i = -\frac{5}{2}\frac{p_i}{m_i}(\hat{b}\cdot\vec{\nabla}\ln T_e)\delta_{k1}$$

$\Rightarrow$ $a_1 =$ ____, $a_2 =$ ____

$$\Rightarrow \langle f_{i1}\rangle_a = w_\parallel f_{Mi} \tau_{ii} (\hat{b}\cdot\vec{\nabla}\ln T_i)\left[\frac{25}{16} L_1^{(3/2)}(x) - \frac{5}{12} L_2^{(3/2)}(x)\right]$$

2. $$\int d\vec{w} \left(w_\parallel^2 - w_\perp^2/2\right) L_k^{(5/2)}(x) \cdot C_{ii}^{\ell}[\langle f_{i1}\rangle] = -\frac{6}{5}\frac{n_i}{\tau_{ii}} \begin{pmatrix} 1 & 3/4 \\ 3/4 & 205/48 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \end{pmatrix}$$

$$\int d\vec{w} \left(w_\parallel^2 - w_\perp^2/2\right) L_k^{(5/2)}(x) \frac{m_i}{2T_i}\left(w_\parallel^2 - w_\perp^2/2\right)\left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right) : \overleftrightarrow{W}_i f_{Mi}$$

$$= m_i n_i \int d\vec{w} \left(w_\parallel^2 - \frac{w_\perp^2}{2}\right)^2 L_k^{(5/2)}(x) \frac{f_{Mi}}{v_{ti}^2}\left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right) : \overleftrightarrow{W}_i = \frac{3}{2} p_i n_i \left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right) : \overleftrightarrow{W} : \delta_{k0}$$

$b_0 =$ ____, $b_1 =$ ____

$$\Rightarrow \langle f_{i1}\rangle_b = -\left(\frac{w_\parallel^2 - w_\perp^2/2}{v_{ti}^2}\right) f_{Mi} \tau_i \left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right) : \overleftrightarrow{W}_i \left[\frac{1025}{1068} L_0^{(5/2)}(x) - \frac{45}{267} L_1^{(5/2)}(x)\right]$$

⑬ Gyro-dependent ion distribution function

$$\vec{w}\vec{w} - \frac{w^2}{3}\overleftrightarrow{I} - \left(w_\parallel^2 - \frac{w_\perp^2}{2}\right)\left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right)$$

$$= \cancel{w_\parallel^2\hat{b}\hat{b}} + \underbrace{\vec{w}_\parallel\vec{w}_\perp + \vec{w}_\perp\vec{w}_\parallel + \vec{w}_\perp\vec{w}_\perp}_{} - \frac{w^2}{3}\overleftrightarrow{I} + \left(-\cancel{w_\parallel^2\hat{b}\hat{b}} + \frac{1}{3}w_\parallel^2\overleftrightarrow{I} + \frac{w_\perp^2}{2}\hat{b}\hat{b} - \frac{1}{6}w_\perp^2\overleftrightarrow{I}\right)$$

$$= (*) - \frac{w_\perp^2}{3}\overleftrightarrow{I} - \cancel{\frac{w_\parallel^2}{3}\overleftrightarrow{I}} + \cancel{\frac{w_\parallel^2}{3}\overleftrightarrow{I}} + \frac{w_\perp^2}{2}\left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right)$$

$$= \vec{w}_\parallel\vec{w}_\perp + \vec{w}_\perp\vec{w}_\parallel + \vec{w}_\perp\vec{w}_\perp + (w_\perp^2/2)\left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right)$$

$$\Rightarrow C_{ii}^{\ell}[\tilde{f}_{i1}] + w_{ci}\frac{\partial \tilde{f}_{i1}}{\partial \gamma} = f_{Mi}\left(\frac{w^2}{v_{ti}^2} - \frac{5}{2}\right)\vec{w}_\perp\cdot\vec{\nabla}\ln T_i + f_{Mi}\left[\frac{\vec{w}_\parallel\vec{w}_\perp + \vec{w}_\perp\vec{w}_\parallel + \vec{w}_\perp\vec{w}_\perp - (w_\perp^2/2)(\overleftrightarrow{I} - \hat{b}\hat{b})}{v_{ti}^2}\right] : \overleftrightarrow{W}_i$$

Similarly to electrons, take ordering $\tilde{f}_{i1} = \tilde{f}_{i1}^{(0)} + \tilde{f}_{i1}^{(1)}$

i) Lowest-order

$$w_{ci} \frac{d\tilde{f}_{i1}^{(0)}}{d\gamma} = f_{Mi} \left(\frac{w^2}{v_{ti}^2} - \frac{5}{2} \right) \vec{w}_\perp \cdot \vec{\nabla} \ln T + f_{Mi} \left[\frac{\vec{w}_\parallel \vec{w}_\perp + \vec{w}_\perp \vec{w}_\parallel + \vec{w}_\perp \vec{w}_\perp - (w_\perp^2/2)(\overleftrightarrow{I} - \hat{b}\hat{b})}{v_{ti}^2} \right] : \overleftrightarrow{W}_i$$

$\Downarrow$ Integrate

$$\tilde{f}_{i1}^{(0)} = f_{Mi} L_1^{(3/2)}(x) \frac{\vec{w}_\perp \times \hat{b}}{w_{ci}} \cdot \vec{\nabla} \ln T_i$$

$$+ f_{Mi} \left[\frac{\vec{w}_\parallel \vec{w}_\perp \times \hat{b} + \vec{w}_\perp \vec{w}_\parallel \times \hat{b} + (w_\perp^2/4)\left[(\hat{x}\hat{x} - \hat{y}\hat{y}) \sin 2\gamma - (\hat{x}\hat{y} + \hat{y}\hat{x}) \cos 2\gamma \right]}{v_{ti}^2 w_{ci}} \right] : \overleftrightarrow{W}_i$$

ii) Next-order

$$w_{ci} \frac{d\tilde{f}_{i1}^{(1)}}{d\gamma} = - C_{ii}^{\ell} \left[\tilde{f}_{i1}^{(0)} \right]$$

← Once again, it's not necessary to integrate the equation

- Ion heat flux

$$\vec{q}_i = -T_i \int d\vec{w}\, \vec{w}\, L_1^{(3/2)}(x) \left(\langle f_{i1} \rangle + \tilde{f}_{i1}^{(0)} + \tilde{f}_{i1}^{(1)} \right) \equiv \vec{q}_{\parallel i} + \vec{q}_{\wedge i} + \vec{q}_{\perp i}$$

- Ion viscosity

$$\overleftrightarrow{\Pi} = \int d\vec{w}\, m_i \left(\vec{w}\vec{w} - \frac{w^2}{3} \overleftrightarrow{I} \right) \left(\langle f_{i1} \rangle + \tilde{f}_{i1}^{(0)} + \tilde{f}_{i1}^{(1)} \right)$$

$$= \left(\hat{b}\hat{b} - \frac{1}{3} \overleftrightarrow{I} \right) \int d\vec{w}\, m_i \left(w_\parallel^2 - \frac{w_\perp^2}{2} \right) \langle f_{i1} \rangle + \int d\vec{w}\, m_i \left(\vec{w}\vec{w} - \frac{w^2}{3} \overleftrightarrow{I} \right) \left(\tilde{f}_{i1}^{(0)} + \tilde{f}_{i1}^{(1)} \right)$$

$$= \Pi_{\parallel i} \left(\hat{b}\hat{b} - \frac{1}{3} \overleftrightarrow{I} \right) + \Pi_{\wedge i} + \Pi_{\perp i}$$

$\uparrow$ parallel viscosity $\uparrow$ gyro-viscosity $\uparrow$ perpendicular viscosity.

⑭ Ion transport coefficients by Braginskii

$$q_{T\parallel i} = -\kappa_{\parallel i} \nabla_\parallel T_i \qquad \left(\kappa_{\parallel i} \equiv 3.91 \frac{p_i \tau_{ii}}{m_i}\right)$$

$$\vec{q}_{T\wedge i} = \frac{5}{2} \frac{p_i}{m_i \omega_{ci}} \hat{b} \times \vec{\nabla} T_i$$

$$\vec{q}_{T\perp i} = -\kappa_{\perp i} \vec{\nabla}_\perp T_i \qquad \left(\kappa_{\perp i} \equiv 2 \frac{p_i}{m_i \tau_{ii} \omega_{ci}^2}\right)$$

$$\overleftrightarrow{\Pi}_{\parallel i} = -0.96 p_i \tau_{ii} \frac{3}{2} \left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right)\left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{I}\right) \overleftrightarrow{W}_i \quad \rightarrow \mu_{\parallel i} = 0.96 p_i \tau_{ii}$$

$$\overleftrightarrow{\Pi}_{\wedge i} = \frac{p_i}{4\omega_{ci}} \left[\hat{b} \times \overleftrightarrow{W}_i \cdot (\overleftrightarrow{I} + 3\hat{b}\hat{b}) - (\overleftrightarrow{I} + 3\hat{b}\hat{b}) \cdot \overleftrightarrow{W}_i \times \hat{b}\right]$$

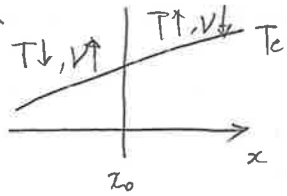
$$\overleftrightarrow{\Pi}_{\perp i} = -\frac{3}{10} \frac{p_i}{\tau_{ii} \omega_{ci}^2} \left[(\overleftrightarrow{I} - \hat{b}\hat{b}) \cdot \overleftrightarrow{W}_i \cdot (\overleftrightarrow{I} + 3\hat{b}\hat{b}) + (\overleftrightarrow{I} + 3\hat{b}\hat{b}) \cdot \overleftrightarrow{W}_i \cdot (\overleftrightarrow{I} - \hat{b}\hat{b})\right]$$

⑮ Summary.

(1) Parallel friction force.

$$R_{\parallel ei} = \eta_\parallel n_e^2 e^2 (u_{\parallel i} - u_{\parallel e}) - 0.71 n_e \nabla_\parallel T_e \qquad \left(\eta_\parallel \equiv 0.51 \frac{m_e}{n_e e^2 \tau_{ei}}\right)$$

friction to electrons by ions due to speed difference.



$\vec{\nabla} T_e$: ⟹ (From T↓ to T↑ particle will lose more momentum than from T↑ to T↓ particle due to difference of collision.)

(2) Parallel heat flux

$$q_{T\parallel e} = -\kappa_{\parallel e} \nabla_\parallel T_e - 0.71 p_e (u_{i\parallel} - u_{e\parallel}) \qquad \left(\kappa_{\parallel e} \equiv 3.16 \frac{p_e \tau_{ei}}{m_e}\right)$$

$$q_{T\parallel i} = -\kappa_{\parallel i} \nabla_\parallel T_i \qquad \left(\kappa_{\parallel i} \equiv 3.91 \frac{p_i \tau_{ii}}{m_i}\right)$$

$\Rightarrow \frac{\kappa_{\parallel e}}{\kappa_{\parallel i}} \sim O\left(\sqrt{\frac{m_i}{m_e}}\right)$

↳ electron heat flux is dominant.

(slow electrons are more likely to collide with ions and so are more apt to lose energy by flowing ions than fast electrons.)

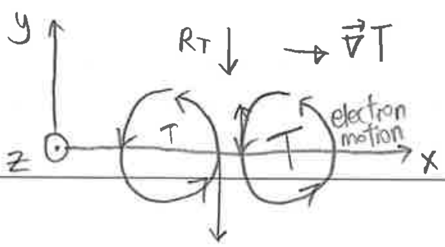
(3) Perpendicular friction force

*note $\omega_{ce} < 0$ ✩

$$\vec{R}_{\perp ei} = \eta_\perp e^2 n_e^2 (\vec{u}_{\perp i} - \vec{u}_{\perp e}) + \frac{3}{2} \frac{n_e}{\omega_{ce} \tau_{ei}} \hat{b} \times \vec{\nabla} T_e \qquad \left(\eta_\perp \equiv \frac{m_e}{n_e e^2 \tau_{ei}}\right)$$

gyro-motion plays a role to restrict the change in f.

$\Rightarrow \eta_\perp \simeq 2\eta_\parallel$



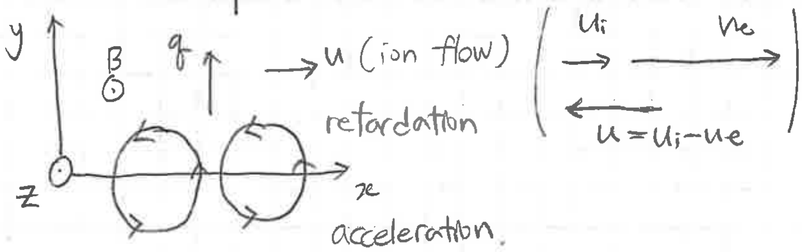
✩ (오른쪽이 더 빨라서 momentum을 덜 잃는다)

(4) Perpendicular heat flux

$$\vec{q}_{\wedge e} = \frac{5}{2}\frac{p_e}{m_e \omega_{ce}}\hat{b}\times\vec{\nabla}T_e , \quad \vec{q}_{\wedge i} = \frac{5}{2}\frac{p_i}{m_i \omega_{ci}}\hat{b}\times\vec{\nabla}T_i$$ ← diamagnetic

$$\vec{q}_{\perp e} = -\kappa_{\perp e}\vec{\nabla}_\perp T_e + \frac{3}{2}\frac{p_e}{\tau_{ei}\omega_{ce}}\hat{b}\times(\vec{u_{\perp i}} - \vec{u_{\perp e}}) \qquad \left(\kappa_{\perp e} \equiv 4.66\frac{p_e}{m_e \tau_{ei}\omega_{ce}^2}\right)$$

$$\vec{q}_{\perp i} = -\kappa_{\perp i}\vec{\nabla}_\perp T_i \qquad \left(\kappa_{\perp i} \equiv 2\frac{p_i}{m_i \tau_{ii}\omega_{ci}^2}\right)$$



(5) Further words

- $q_{\parallel e} \gg q_{\wedge e} \gg q_{\perp e}$, $\quad q_{\parallel e} : q_{\wedge e} : q_{\perp e} = 1 : \frac{1}{\tau_{ei}\omega_{ce}} : \frac{1}{(\tau_{ei}\omega_{ce})^2}$

$$\frac{\tau_{ii}\omega_{ci}}{\tau_{ei}\omega_{ce}} = O\left(\sqrt{\frac{m_e}{m_i}}\right)$$

- $q_{\parallel i} > q_{\wedge i} > q_{\perp i}$, $\quad q_{\parallel i} : q_{\wedge i} : q_{\perp i} = 1 : \frac{1}{\tau_{ii}\omega_{ci}} : \frac{1}{(\tau_{ii}\omega_{ci})^2}$

- $\underline{q_{\parallel e} \gg q_{\parallel i}}$, $q_{\wedge e} \sim q_{\wedge i}$, $\underline{q_{\perp e} \ll q_{\perp i}}$

- $\overleftrightarrow{\Pi}_i \gg \overleftrightarrow{\Pi}_e$