Hurst Exponent

TAEJUS YEE

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Procedure:

Given a time series X of length N (which we choose to be a power of 2 for convenience), divide it into a number of non-overlapping shorter time series of length n, with $n = N, \frac{N}{2}, \frac{N}{2^2}, \frac{N}{2^3}, ...$

Then, compute the average **rescaled range** for each value of n:

1. Calculate the mean and create a mean-adjusted series Y_t:

$$m = \frac{1}{n}\sum_{i=1}^{n} X_i, Y_t = X_t - m$$

   for $t = 1, 2, ..., n$

2. Calculate the **cumulative deviate** series Z_t:

$$Z_t = \sum_{i=1}^{t} Y_i$$

   for $t = 1, 2, ..., n$

3. Compute the range $R(n)$:

$$R(n) = \max(Z_1, Z_2, ..., Z_n) - \min(Z_1, Z_2, ..., Z_n)$$

4. Compute the **standard deviation** $S(n)$:

$$S(n) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - m)^2}$$

5. Calculate the rescaled range $(\frac{R(n)}{S(n)})$ and average over all the partial time series of length n.

The Hurst exponent is estimated by fitting the **power law** $\mathbb{E}[\frac{R(n)}{S(n)}] = Cn^H$ to the data (done by plotting $\log(R(n)/S(n))$ as a function of $\log(n)$ and fitting a straight line). The slope of the line gives H.