CS61B

Lecture 25: Advanced Trees

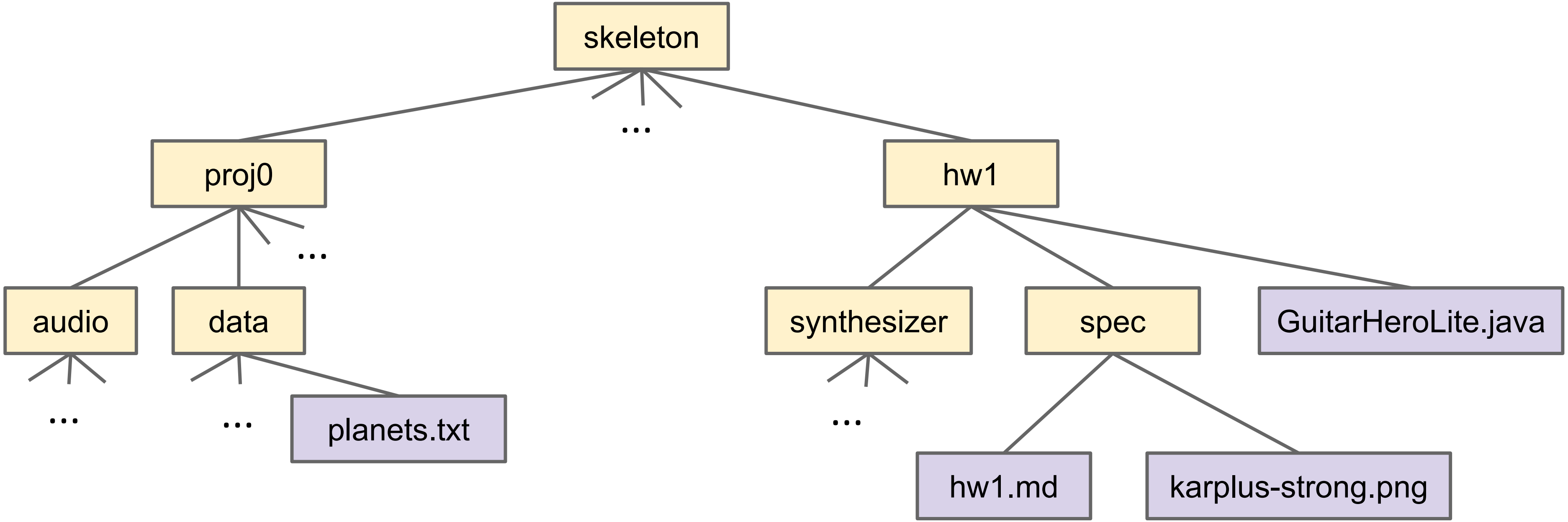
- Tree Traversals
- Level Order Traversal
- Range Finding
- Spatial (a.k.a. Geometric) Trees
- Tree Iterators (Extra)

Traversals

Rooted Trees

We’ve used BSTS to build Maps and Sets, and Heaps to build a PQ.

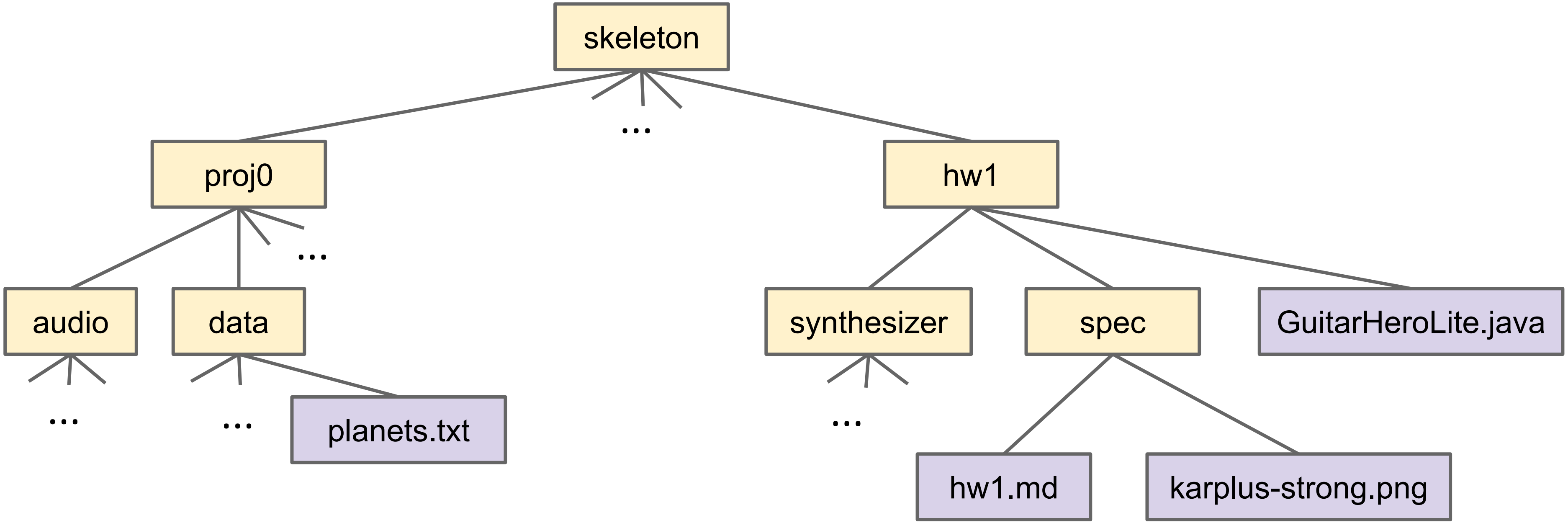
… but trees are a more general concept.



Rooted Trees

Given such a tree, find how much disk space all the files use.

- What one might call “tree iteration” is usually called “tree traversal.”
- Unlike lists, there are many natural orderings.



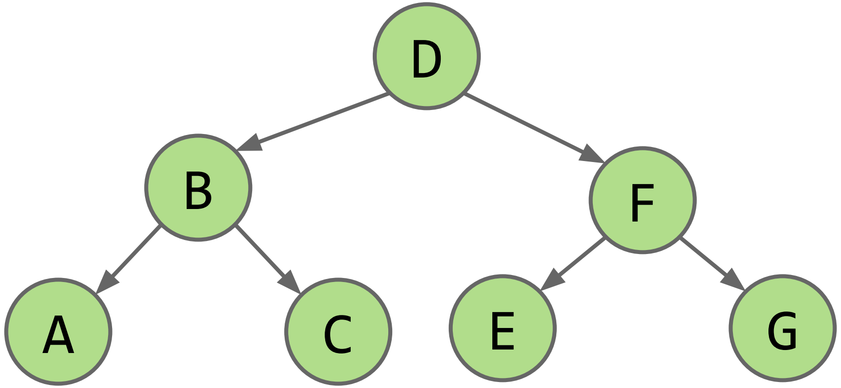
Tree Traversal

Level Order

- Traverse top-to-bottom, left-to-right (like reading in English):
- We say that the nodes are 'visited' in the given order.

Depth First Traversals

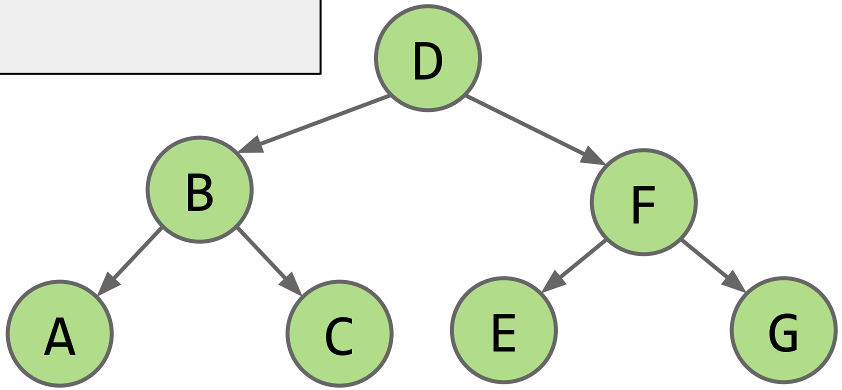
- Preorder, Inorder, Postorder
- Basic (rough) idea: Traverse "deep nodes" (e.g. A) before shallow ones (e.g. F).



Depth First Traversals

Preorder: “Visit” a node, then traverse its children: D B A C F E G

```
preOrder(BSTNode x) {
    if (x == null) return;
    print(x.key)
    preOrder(x.left)
    preOrder(x.right)
}
```



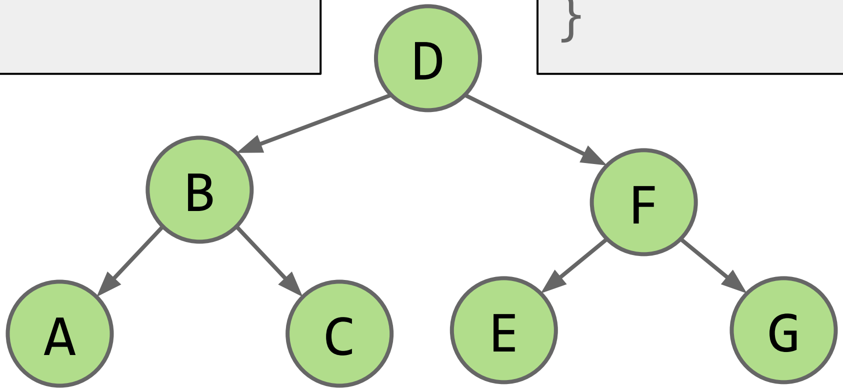
Depth First Traversals

Preorder traversal: “Visit” a node, then traverse its children: DBACFEG

Inorder traversal: Traverse left child, visit, then traverse right child: A B C D E F G

```
preOrder(BSTNode x) {
    if (x == null) return;
    print(x.key)
    preOrder(x.left)
    preOrder(x.right)
}
```

```
inOrder(BSTNode x) {
    if (x == null) return;
    inOrder(x.left)
    print(x.key)
    inOrder(x.right)
}
```



Depth First Traversals http://yellkey.com/top

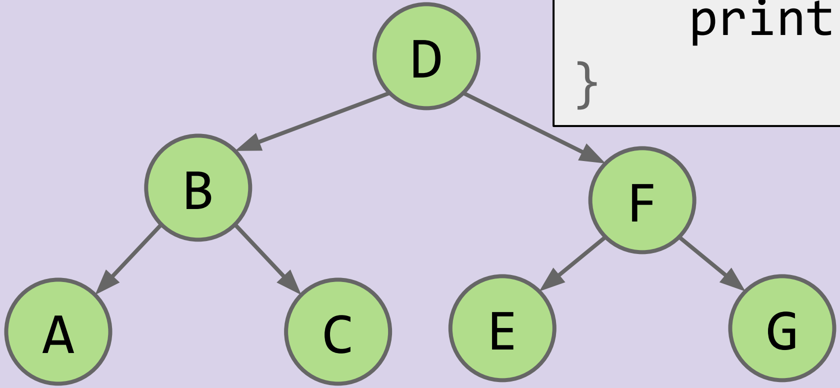
Preorder traversal: “Visit” a node, then traverse its children: DBACFEG

Inorder traversal: Traverse left child, visit, traverse right child: ABCDEFG

Postorder traversal: Traverse left, traverse right, then visit: ???????

1. DBACEFG
2. GFEDCBA
3. GEFCABD
4. ACBEGFD
5. ACBFEGD
6. Other

```
postOrder(BSTNode x) {
    if (x == null) return;
    postOrder(x.left)
    postOrder(x.right)
    print(x.key)
}
```



Depth First Traversals

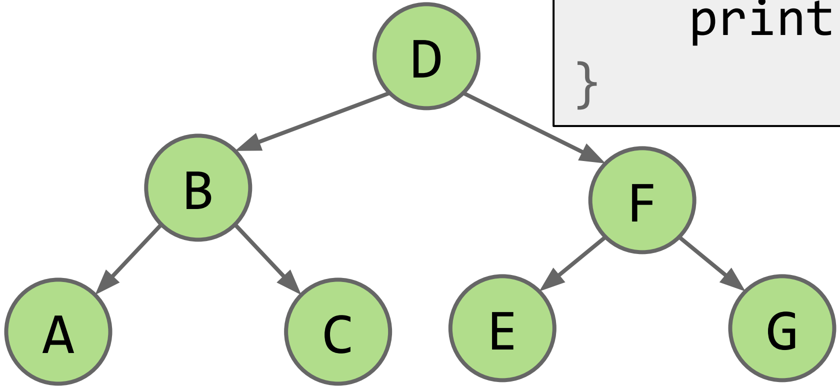
Preorder traversal: “Visit” a node, then traverse its children: DBACFEG

Inorder traversal: Traverse left child, visit, traverse right child: ABCDEFG

Postorder traversal: Traverse left, traverse right, then visit: ACBEGFD

1. DBACEFG
2. GFEDCBA
3. GEFCABD
4. **ACBEGFD**
5. ACBFEGD
6. Other

```
postOrder(BSTNode x) {
    if (x == null) return;
    postOrder(x.left)
    postOrder(x.right)
    print(x.key)
}
```

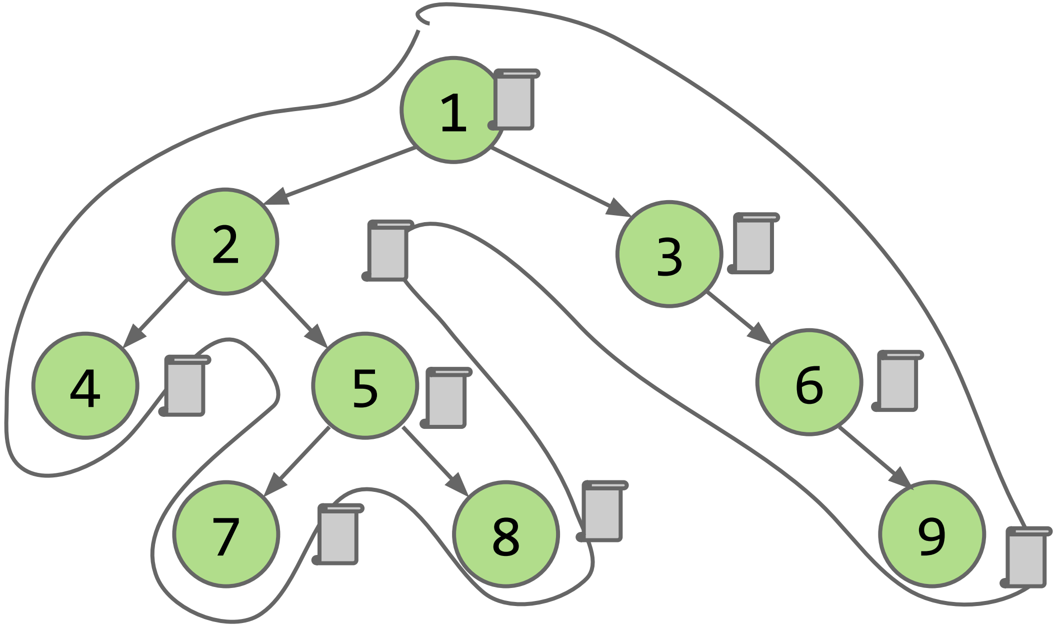


A Weird Trick

- Preorder traversal: We walk the graph, from top going counter-clockwise. Shout every time we pass the LEFT of a node.
- Inorder traversal: Shout when you cross the bottom.
- Postorder traversal: Shout when you cross the right.

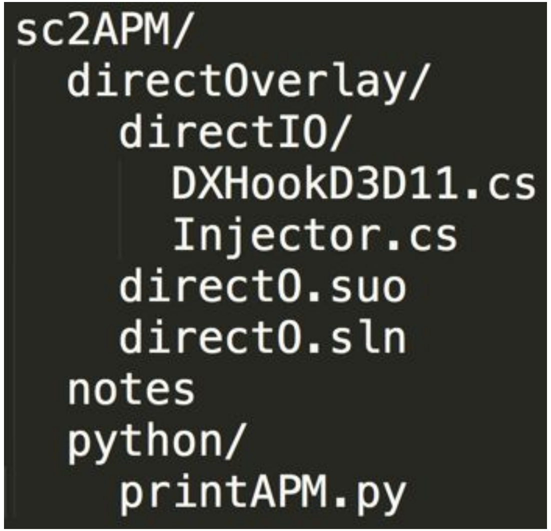
Example: Post-Order Traversal

- 4 7 8 5 2 9 6 3 1

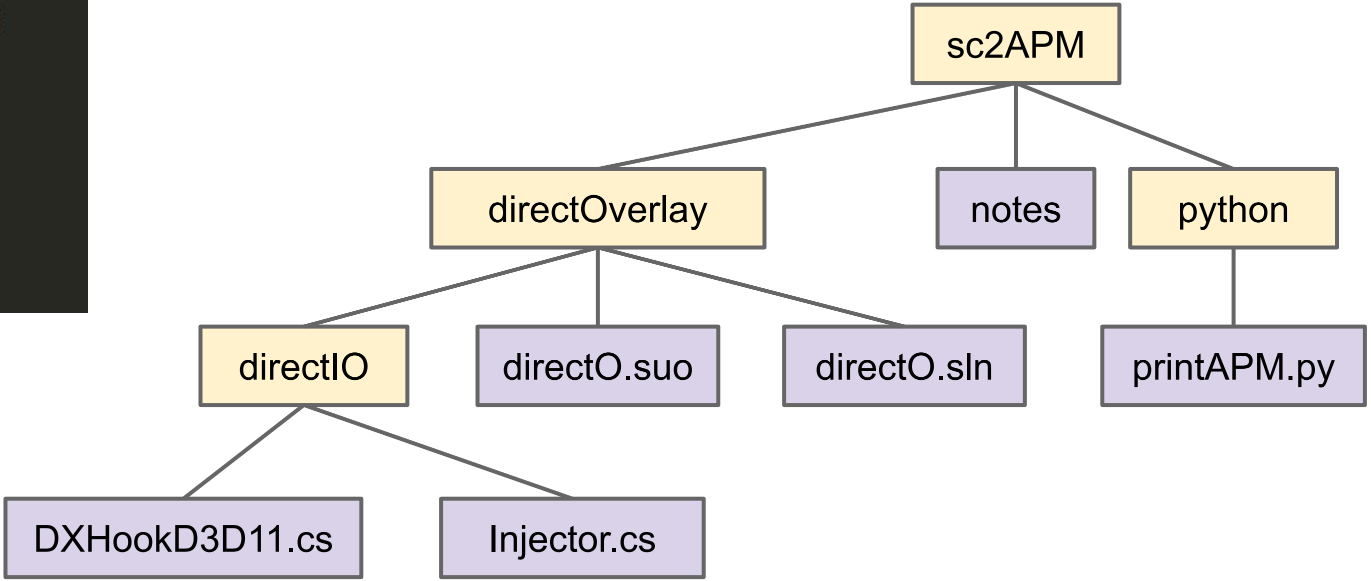


What Good Are All These Traversals?

Example: Preorder Traversal for printing directory listing:



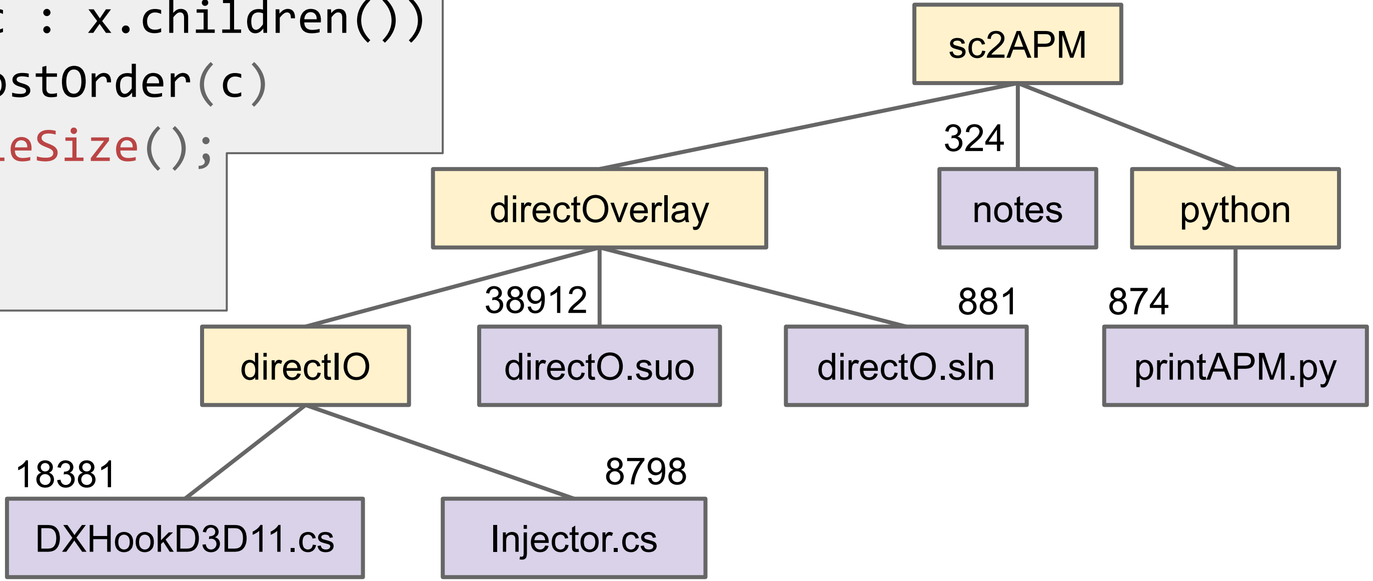
```
sc2APM/
  directOverlay/
    directIO/
      DXHookD3D11.cs
      Injector.cs
    directO.suo
    directO.sln
  notes
  python/
    printAPM.py
```



What Good Are All These Traversals?

Example: Postorder Traversal for gathering file sizes.

```
postOrder(BSTNode x) {
    if (x == null) return 0;
    int total = 0;
    for (BSTNode c : x.children())
           total += postOrder(c)
    total += x.fileSize();
    return total;
}
```



What Good Are All These Traversals?

Example: Postorder Traversal for gathering file sizes.

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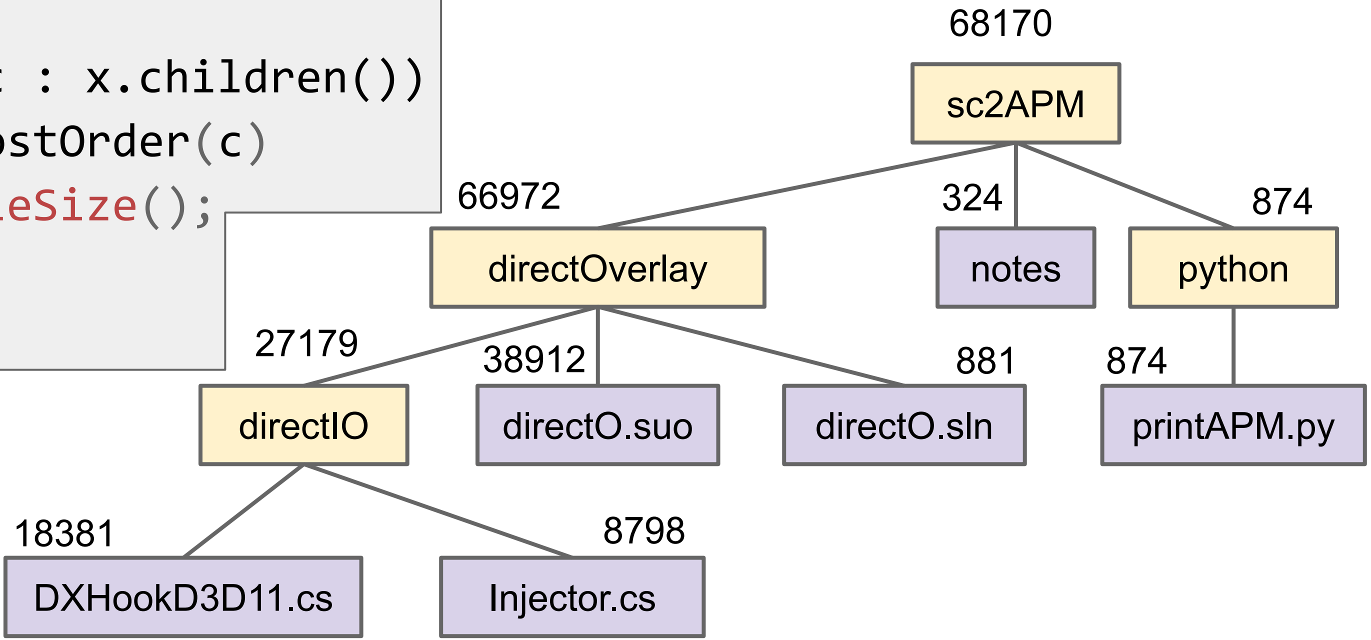
postOrder(BSTNode x) {
    if (x == null) return 0;
    int total = 0;
    for (BSTNode c : x.children())
       total += postOrder(c)
    total += x.fileSize();
    return total;
}
```



Visitor Pattern (Patterns)

When writing general tree traversal code. Avoid rewriting traversal for every task of interest (print, sum filesizes, etc.) by using the Visitor pattern.

```
void preorderTraverse(Tree<Label> T, Action<Label> whatToDo) {
   if (T == null) { return; }
   whatToDo.visit(T); /* before we hard coded a print */
   preorderTraverse(T.left, whatToDo);
   preorderTraverse(T.right, whatToDo);
}
```

```
interface Action<Label> {
   void visit(Tree<Label> T);
}
```

```
class FindPig implements Action<String> {
   boolean found = false;
   @Override
   void visit(Tree<String> T) {
      if ("pig".equals(T.label))
         { found = true; }
   }
```

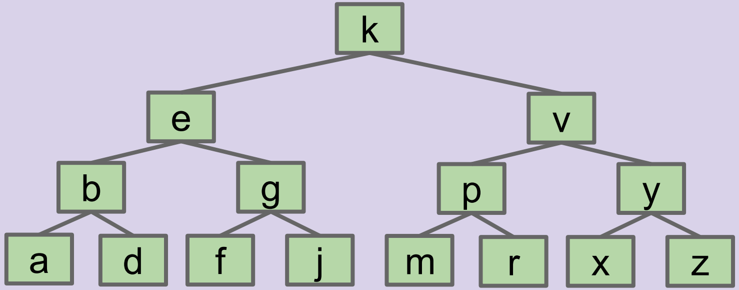
```
preorderTraverse(someTree, new FindPig());
```

The real visitor pattern is more complex.

Preorder Traversal Runtime: http://yellkey.com/most

What is the runtime of a preorder traversal in terms of N, the number of nodes? (in code below, assume the visit action takes constant time)

1. Θ(1)
2. Θ(log N)
3. Θ(N)
4. Θ(N log N)
5. $\Theta(2^N)$



```
void preorderTraverse(Tree<Label> T, Action<Label> whatToDo) {
   if (T == null) { return; }
   whatToDo.visit(T);
   preorderTraverse(T.left, whatToDo);
   preorderTraverse(T.right, whatToDo);
}
```

Preorder Traversal Runtime

What is the runtime of a preorder traversal in terms of N, the number of nodes? (in code below, assume the visit action takes constant time)

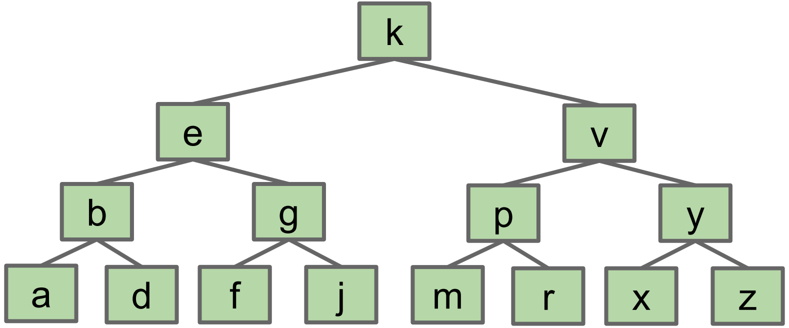
3. Θ(N) : Every node visited exactly once. Constant work per visit.

Runtime is exponential in height of the tree, not number of items.

- $\Theta(2^H)$, but $H = \Theta(\log N)$
- This is not a proof of runtime, but rather a response to a possible objection.

N = 15

H = 3



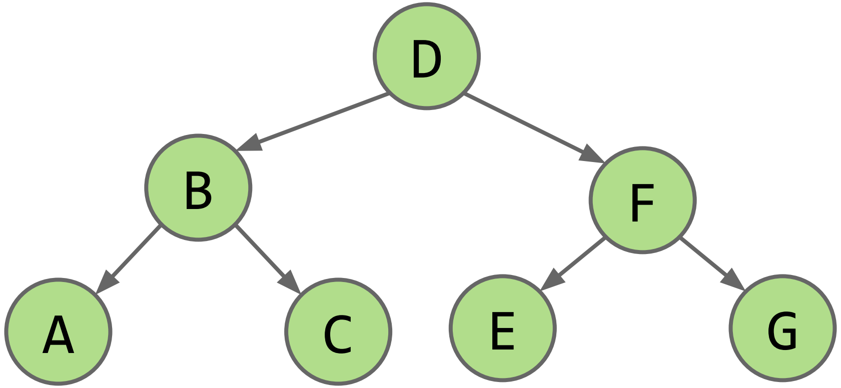
Level Order Traversal

Tree Traversal: Level Order Traversal

The Level Order Traversal is the result of reading the nodes “like a book”, one level at a time.

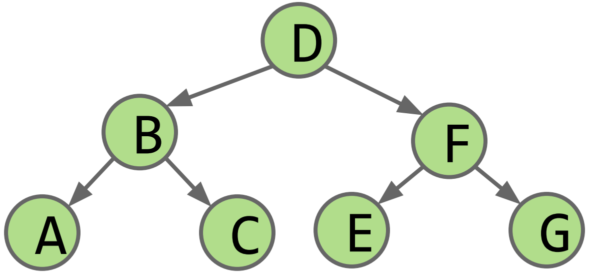
How would we implement a level order traversal?

- Level order: D B F A C E G
- Goal: Visit nodes on 0th level, then 1st level, then 2nd level, etc.



Level-Order Traversal through Iterative Deepening

```
public void levelOrder(Tree T, Action toDo) {
   for (int i = 0; i < T.height(); i += 1) {
      visitLevel(T, i, toDo);
   }
}
```



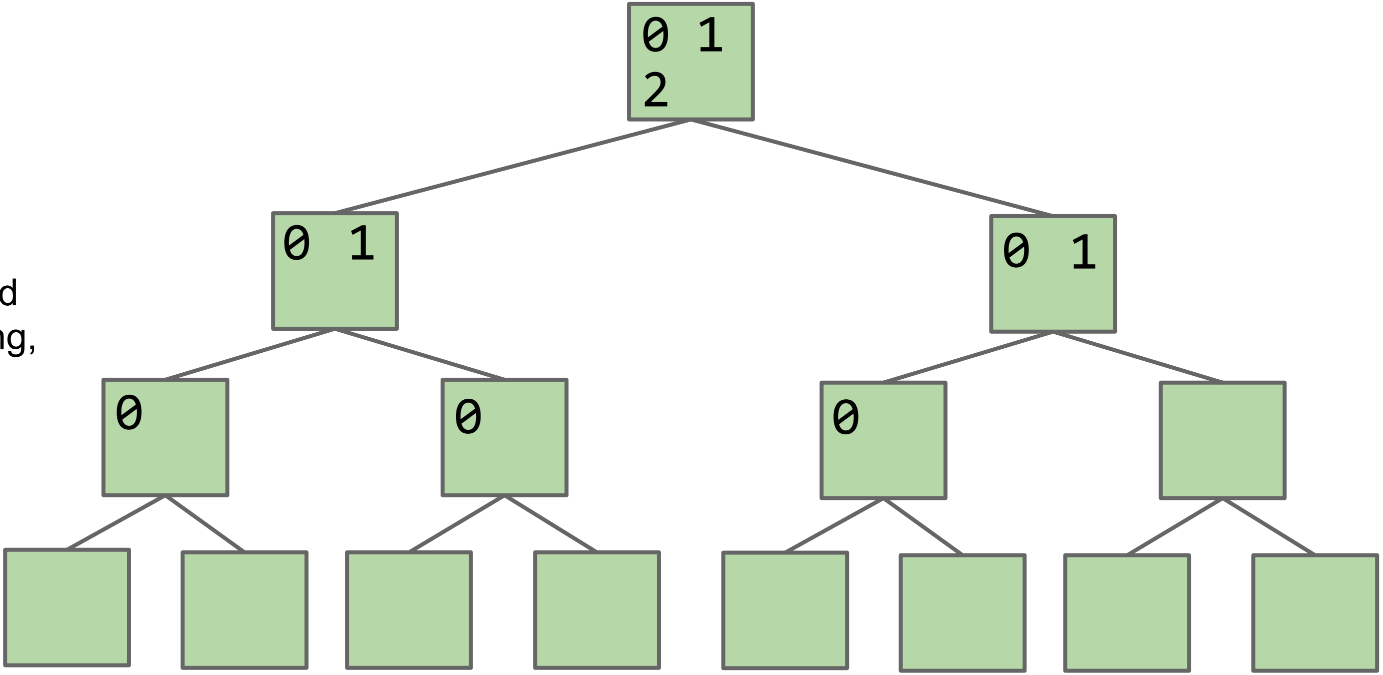
Run visitLevel H times, one for each level.

```
public void visitLevel(Tree T, int level, Action toDo) {
   if (T == null)
      { return; }
   if (lev == 0)
      { toDo.visit(T.key); }
   else {
      visitLevel(T.left(), lev - 1, toDo);
      visitLevel(T.right(), lev - 1, toDo);
   }
}
```

The strategy described on this slide is called “Iterative Deepening”.

Level-Order Traversal through Iterative Deepening

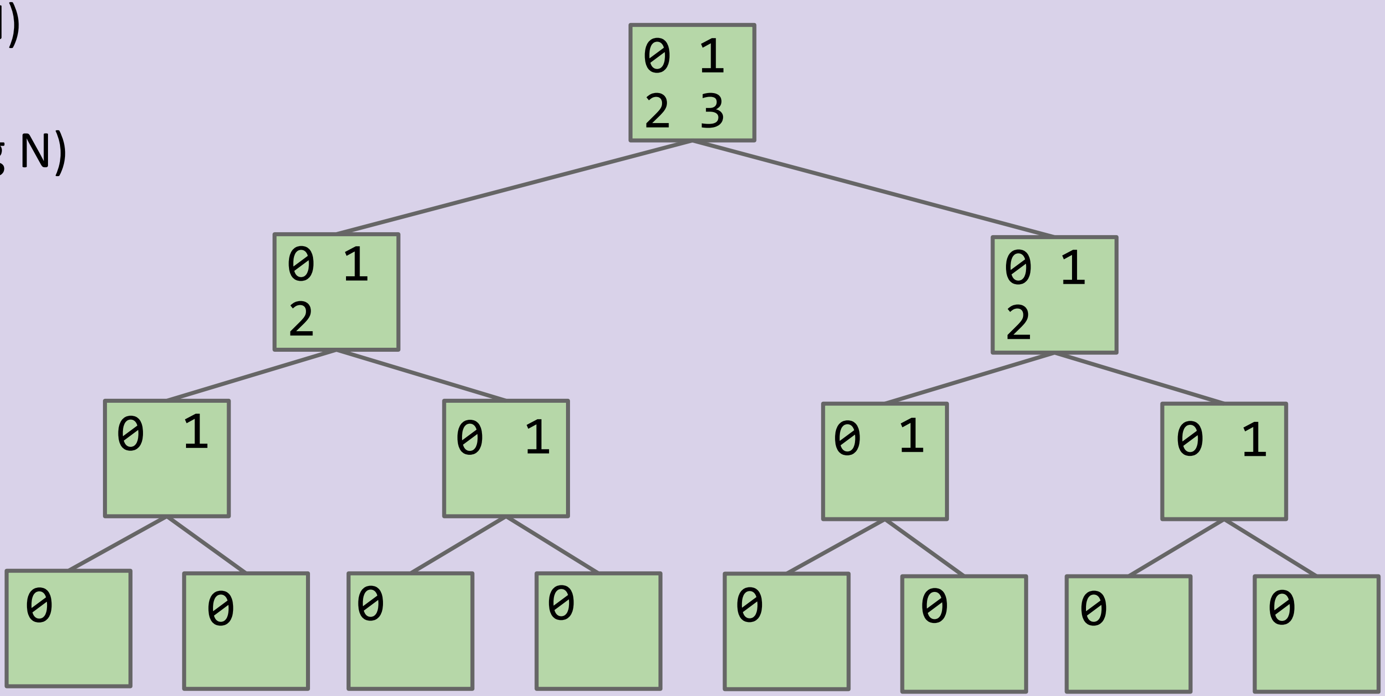
Partially completed Iterative Deepening,



Iterative Deepening Runtime: http://yellkey.com/nor

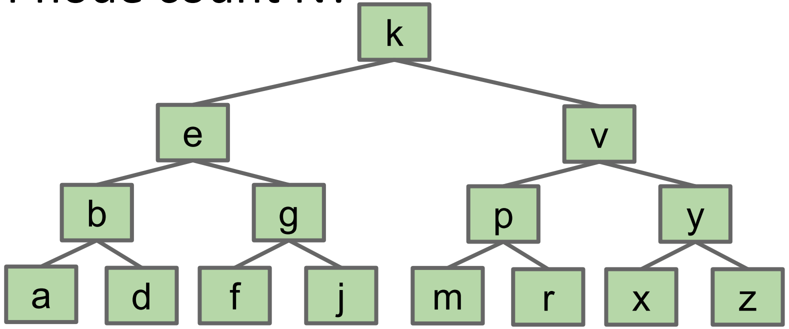
What is the runtime to complete iterative deepening on a **complete** tree (as shown below) as a function of node count N?

1. Θ(log N)
2. Θ(N)
3. Θ(N log N)
4. $\Theta(N^2)$
5. $\Theta(2^N)$



Preorder Traversal and Prefix Expressions

What is the runtime to complete iterative deepening on a **complete** tree (as shown below) as a function of node count N?



1. **Θ(N)**

Top level considered: 1

Then top two levels considered: 1 + 2 = 3

Then top three levels considered: 1 + 2 + 4 = 7

Then top four: 1 + 2 + 4 + 8 = 15

Top H levels: $2^1+2^2+2^3+\ldots+2^H$ - H = **Θ(N)**

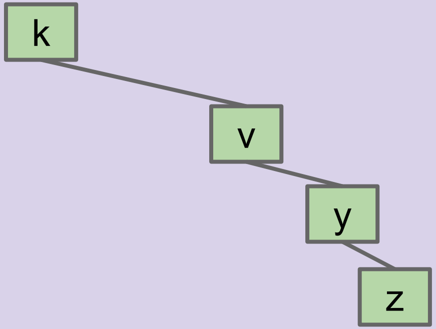
Note: Exact sum doesn't matter, the order of growth (and hence the pattern) is what is important.

Interesting aside: Much harder to solve as "4 visits at level 0" then "6 visits at level 1", etc.

Iterative Deepening Runtime: http://yellkey.com/work

What is the runtime for iterative deepening on a “spindly” tree?

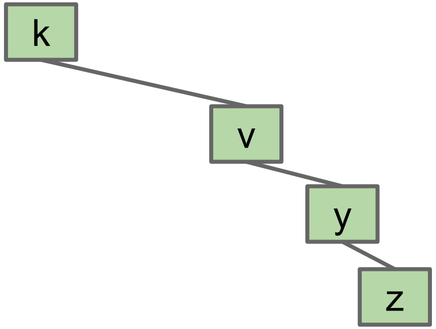
1. $\Theta(\log N)$
2. $\Theta(N)$
3. $\Theta(N \log N)$
4. $\Theta(N^2)$
5. $\Theta(2^N)$



Iterative Deepening Runtime

What is the runtime for iterative deepening on a “spindly” tree?

4. **Θ(N^2)**



Top level considered: 1

Then top two levels: 1 + 1 = 2

Then top three levels: 1 + 1 + 1 = 3

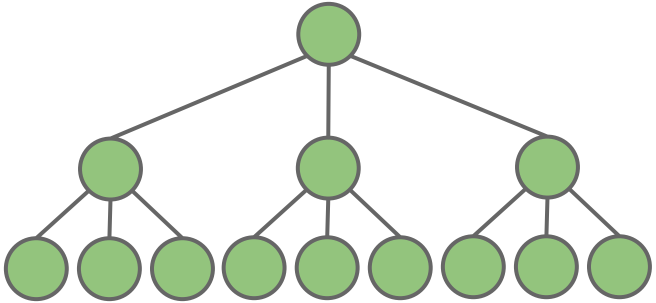
Top H levels: H

Total: 1 + 2 + 3 + … + H = H^2

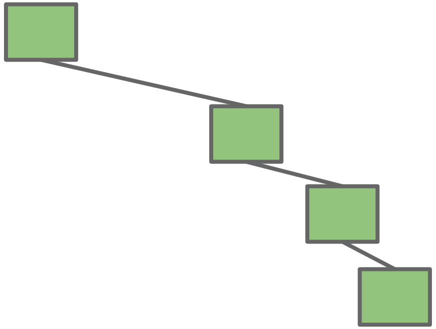
H = N - 1, so **Θ(N^2)**

Tree Height

For algorithms whose runtime depends on height, difference between bushy tree and spindly tree can be huge!



H = Θ(log(N))



H = Θ(N)

Iterative deepening runtimes: Θ(N) vs. Θ(N^2)

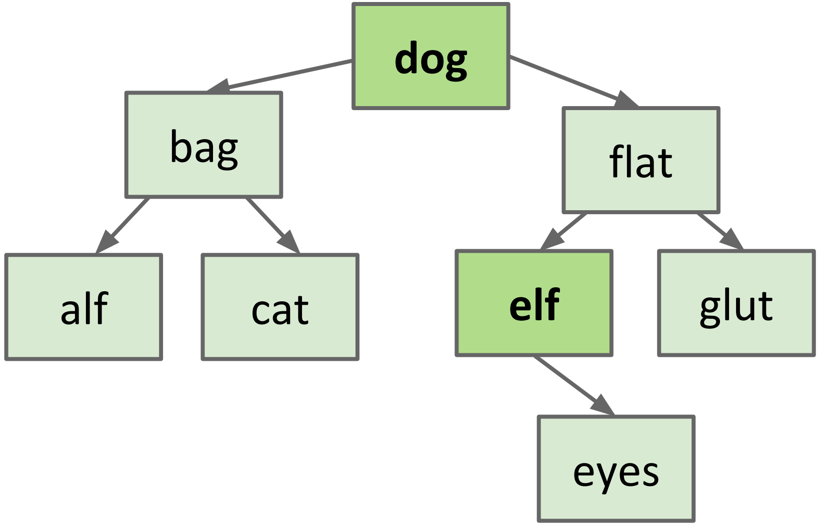
- Note: No simple mapping from height to runtime.
- Extra for experts: Write a better level order Traversal algorithm.

Range Finding

Geometric Search

Suppose we want an operation that returns all items in a range:

- `public Set<Label> findInRange(Tree T, Label min, Label max)`



Example:

- `findInRange(T, “dog”, “elves”)`
- Should return:
  - `{“dog”, “elf”}`

Geometric Search

Easy approach, just do a traversal of the whole tree, and use visitor pattern to collect matching items.

```
class rangeFind implements Action<String> {
   private Label min, max;
   public Set<Label> inRange;
   public rangeFind(Label min, Label max) {
      this.min = min; this.max = max;
      inRange = new HashSet<Label>();
   }

   void action(Tree<Label> T) {
      if (T.label ⩽ max && T.label ⩾min) {
        inRange.add(T.label);
      }
   }
}
```

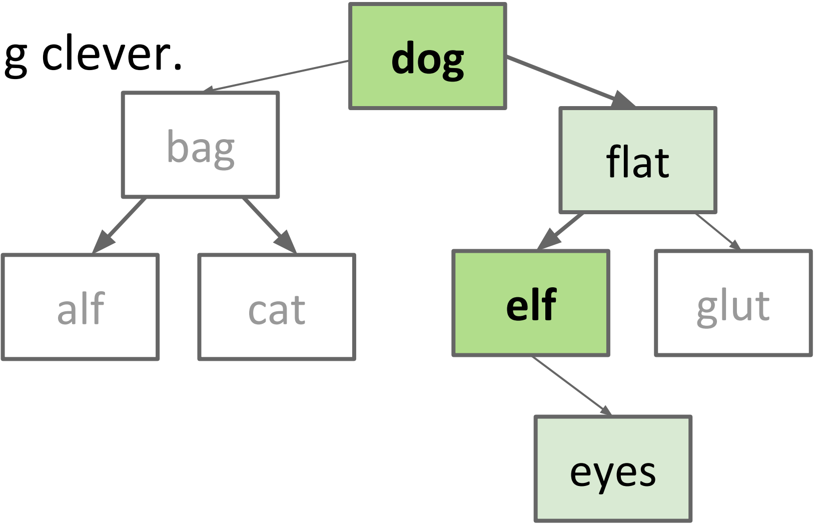
Runtime is Θ(N)

Geometric Search

Suppose we want an operation that returns all items in a range:

- `public Set<Label> findInRange(Tree T, Label min, Label max)`

Can avoid need to traverse entire tree by being clever.



Example:

- `findInRange(T, “dog”, “elves”)`
- No need to look:
  - Left of dog.
  - Right of flat.

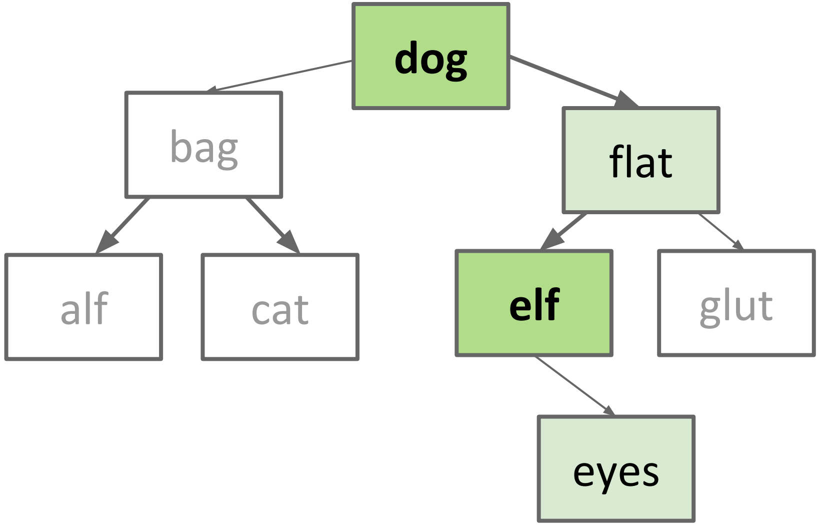
Nodes inspected: dog, flat, elf, eyes
Nodes matching: dog, elf

Pruning and findInRange Runtime

Suppose we want an operation that returns all items in a range:

- `public Set<Label> findInRange(Tree T, Label min, Label max)`

Pruning: Restricting our search to only nodes that might contain the answers we seek.



Nodes inspected: dog, flat, elf, eyes
Nodes matching: dog, elf

Pruning and findInRange Runtime

Suppose we want an operation that returns all items in a range:

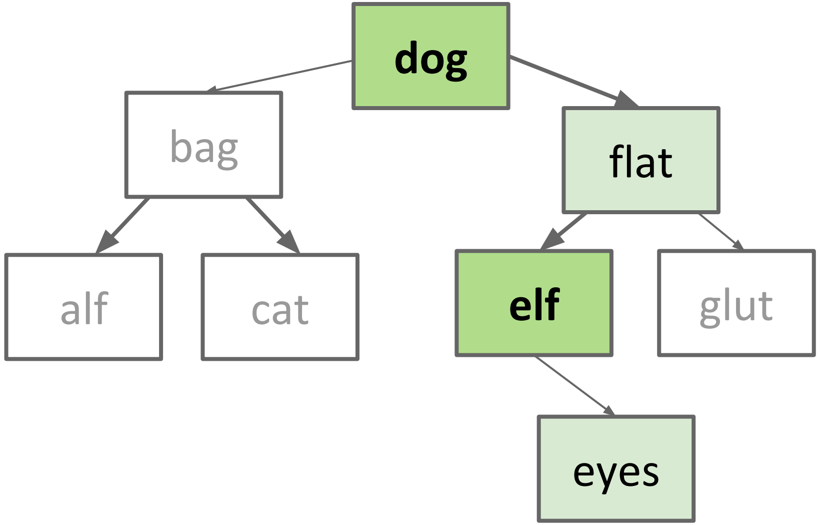
- `public Set<Label> findInRange(Tree T, Label min, Label max)`

Pruning: Restricting our search to only nodes that might contain the answers we seek.

Runtime for our search: Θ(log N + R)

- N: Total number of items in tree.
- R: Number of matches.

See study guide A-level problems for proof.



Nodes inspected: dog, flat, elf, eyes
Nodes matching: dog, elf

Spatial Trees

2D Range Finding

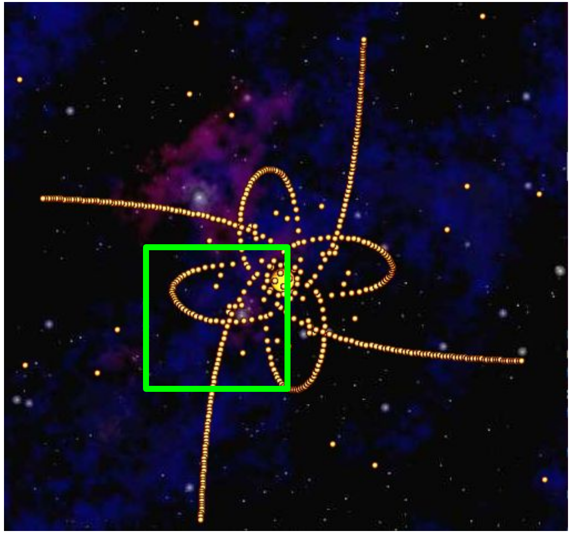
Suppose we want to do range finding on Planets in space.

- Query: How many objects are in the highlighted rectangle?

Could iterate through all objects in Θ(N) time.

- But could we do some sort of tree + pruning?

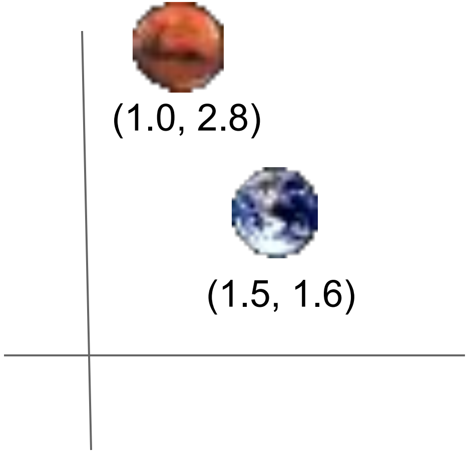
Pruning implies we need some kind of tree, but ...



Building Trees of Two Dimensional Data

So far, we've only considered one dimensional data.

- There exists a total order!
  - 5 < 10
  - "alf" < "elf"

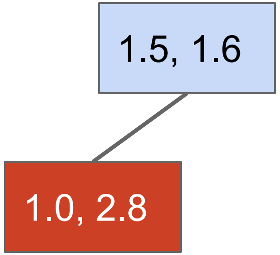


Some data is two dimensional, e.g. the location of Planets.

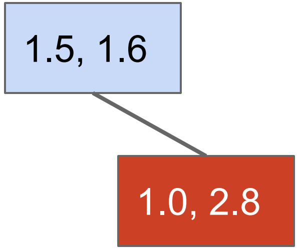
- earth.xPos = 1.5, earth.yPos = 1.6
- mars.xPos = 1.0, mars.yPos = 2.8

If we're comparing by location:

- In xPos, Mars < Earth
- In yPos, Mars > Earth



X-Based Tree



Y-Based Tree

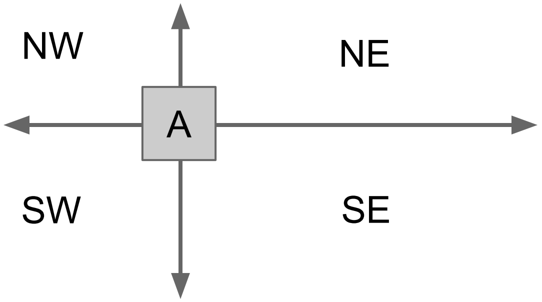
Handling Multidimensional Data: Quadtrees

Quadtrees:

- Divide and conquer by splitting 2D space into four quadrants.
    - Store items into appropriate quadrant.
    - Repeat recursively if more than one item in a quadrant.

Definition, quadtree is either:

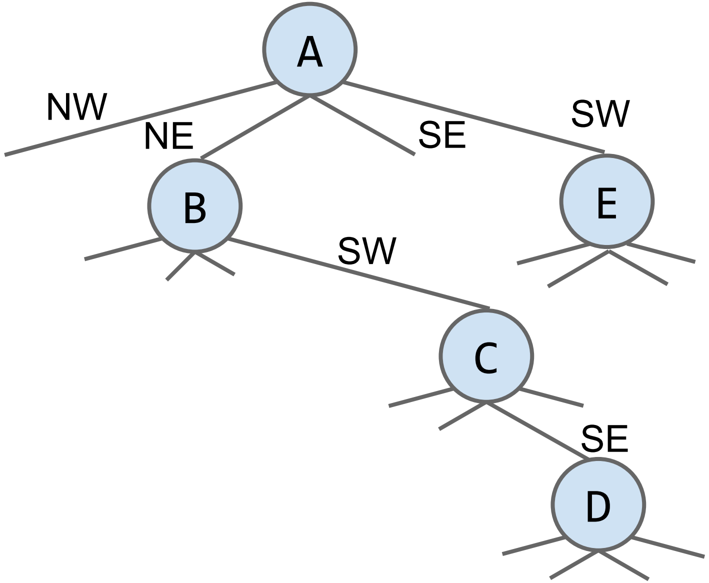
- Empty
- A ‘root’ item at some position (x, y) AND four quadtrees that are northwest, northeast, southwest, southeast of (x, y)
- Use TWO compares to decide which direction to go.

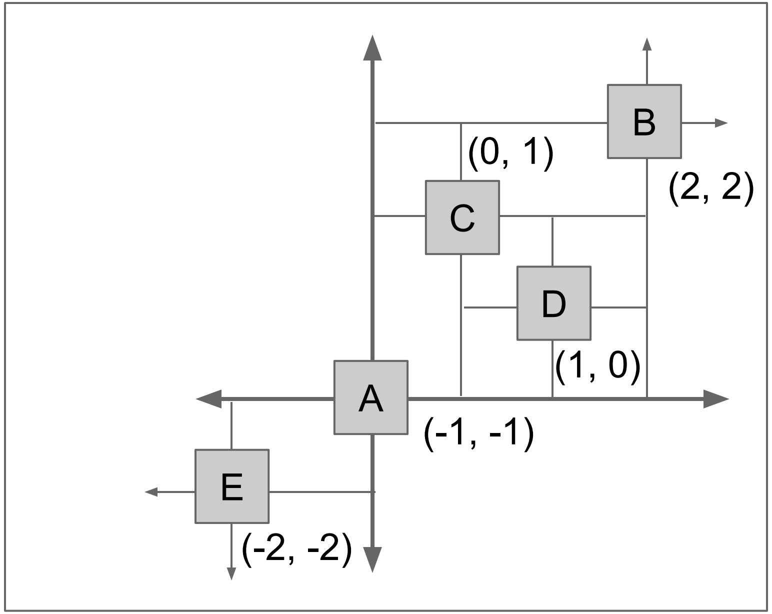


Quadtree Demo

Below: Quadtree Representation of 5 objects in 2D space.

- Demo: Link

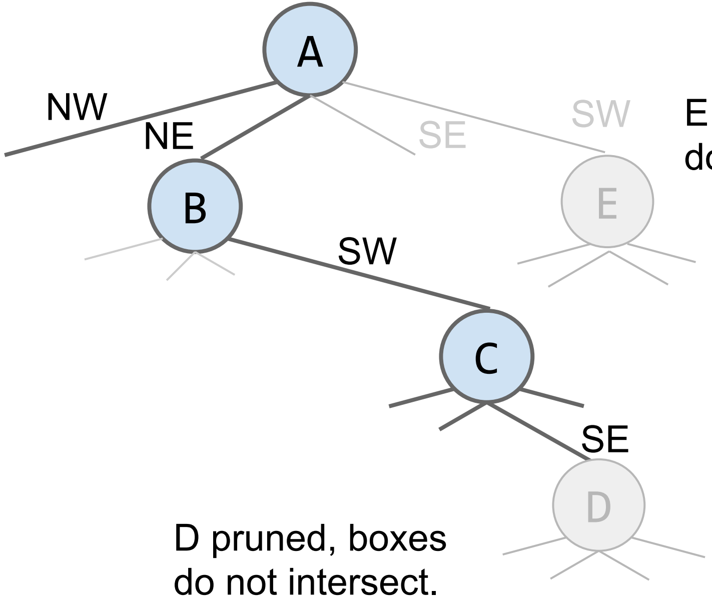




Quadtree Demo

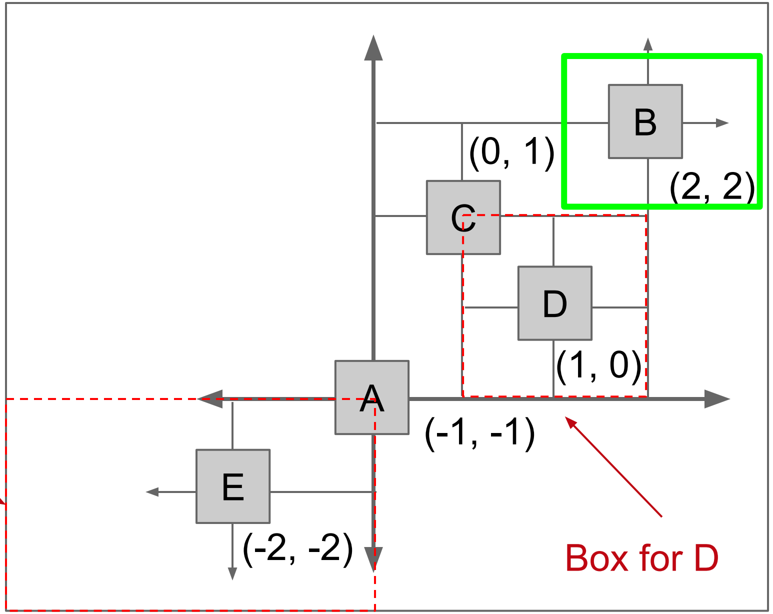
Quadtrees allow us to prune when performing a rectangle search.

- Basic rule: Prune a branch if the search rectangle doesn’t overlap a quadrant of potential interest.



E pruned, boxes do not intersect.

D pruned, boxes do not intersect.



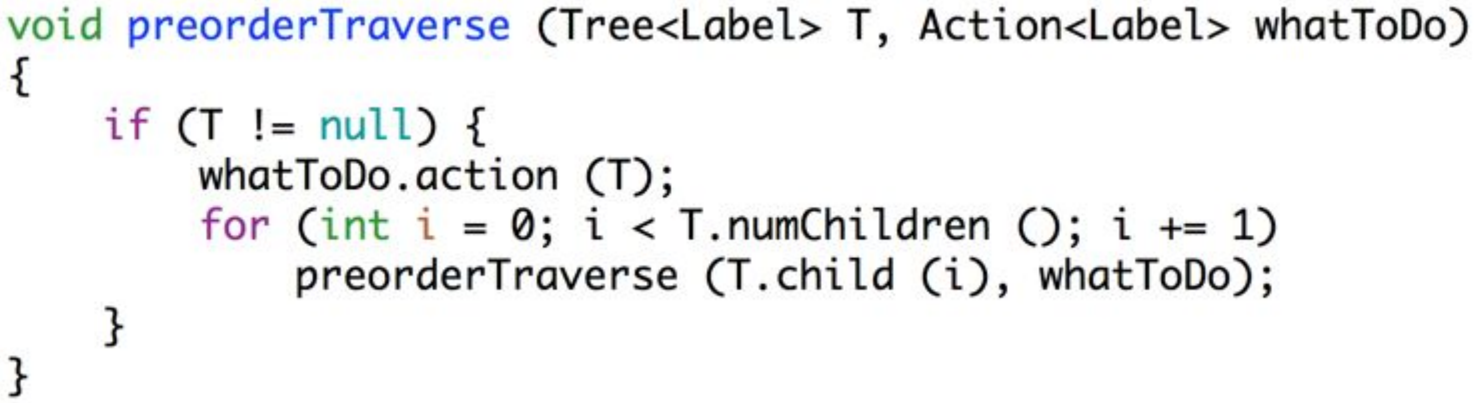
Only item that intersects box is B.

Optional: Tree Iterators

Iterators

Suppose we want to iterate through a tree using the : operator.

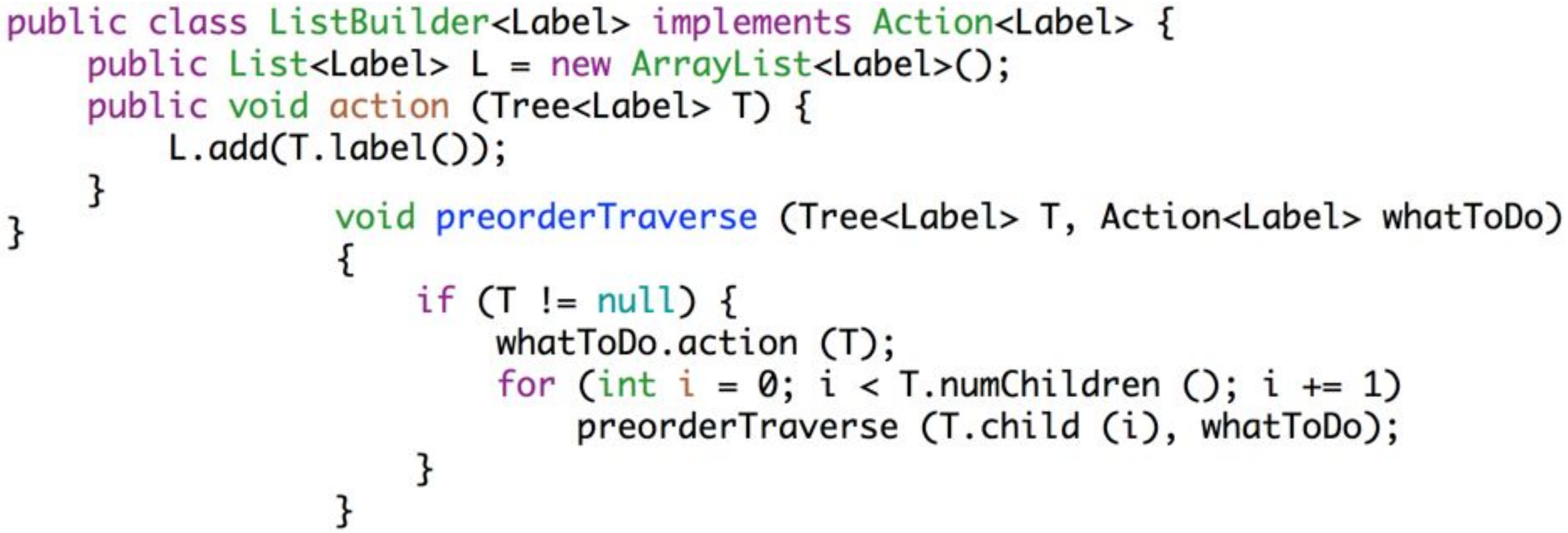
How can we adapt our traversal code to implement next() and hasNext()?



```
void preorderTraverse (Tree<Label> T, Action<Label> whatToDo)
{
    if (T != null) {
        whatToDo.action (T);
        for (int i = 0; i < T.numChildren (); i += 1)
            preorderTraverse (T.child (i), whatToDo);
    }
}
```

Iteration: The Obvious Way

One approach: Create an action class that puts visited item in a list.



```
public class ListBuilder<Label> implements Action<Label> {
    public List<Label> L = new ArrayList<Label>();
    public void action (Tree<Label> T) {
        L.add(T.label());
    }
}
```

```
void preorderTraverse (Tree<Label> T, Action<Label> whatToDo)
{
    if (T != null) {
        whatToDo.action (T);
        for (int i = 0; i < T.numChildren (); i += 1)
            preorderTraverse (T.child (i), whatToDo);
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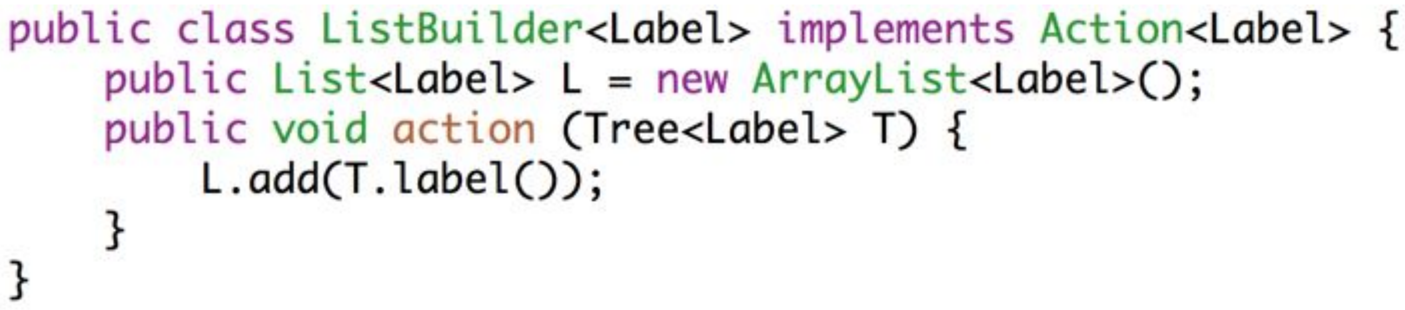
}
```

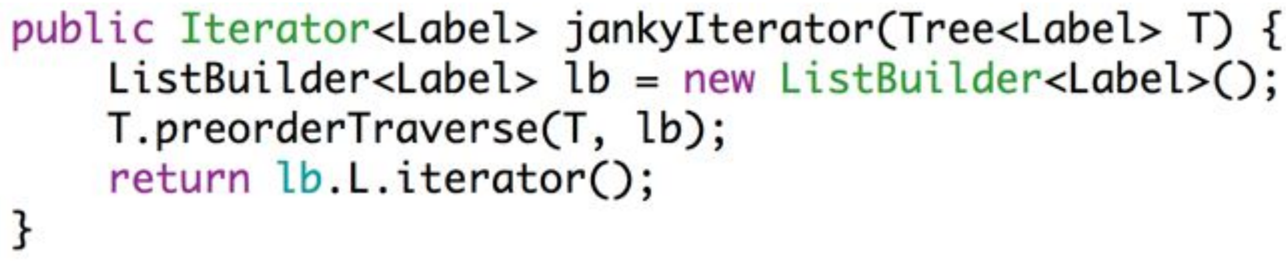
Iteration: The Obvious Way

One approach: Create an action class that puts visited item in a list.

- iterator method creates such a list and returns an iterator to it.
- What’s the downside of this solution?



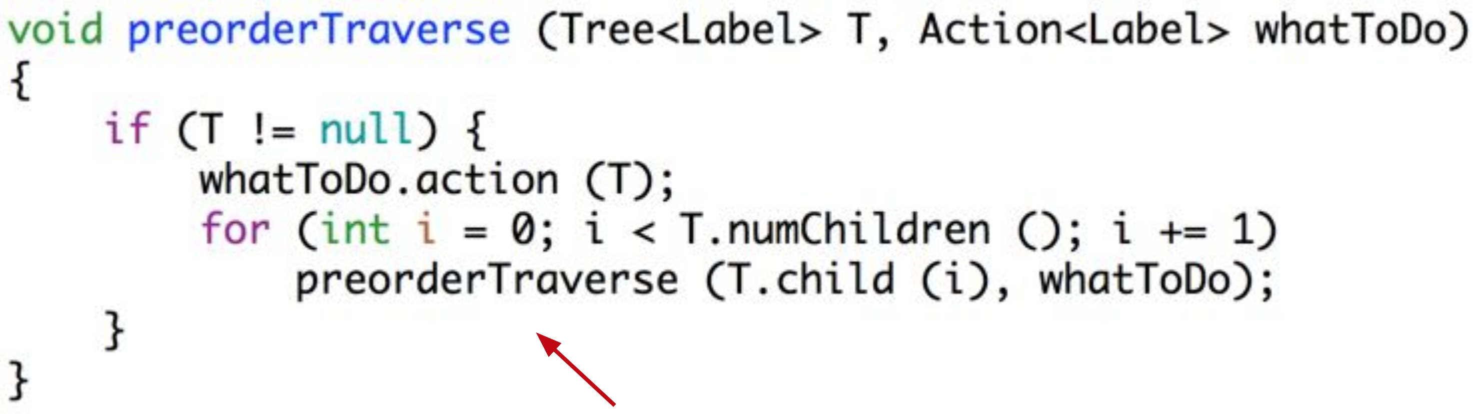
```
public class ListBuilder<Label> implements Action<Label> {
    public List<Label> L = new ArrayList<Label>();
    public void action (Tree<Label> T) {
        L.add(T.label());
    }
}
```



```
public Iterator<Label> jankyIterator(Tree<Label> T) {
    ListBuilder<Label> lb = new ListBuilder<Label>();
    T.preorderTraverse(T, lb);
    return lb.L.iterator();
}
```

Iteration: Space-saving Approach

Tricky question: How could convert our recursive traversal into iterative code using a stack?



```
void preorderTraverse (Tree<Label> T, Action<Label> whatToDo)
{
    if (T != null) {
        whatToDo.action (T);
        for (int i = 0; i < T.numChildren (); i += 1)
            preorderTraverse (T.child (i), whatToDo);
    }
}
```

Observation: Each call to preorderTraverse is the equivalent of putting a call on the call stack.

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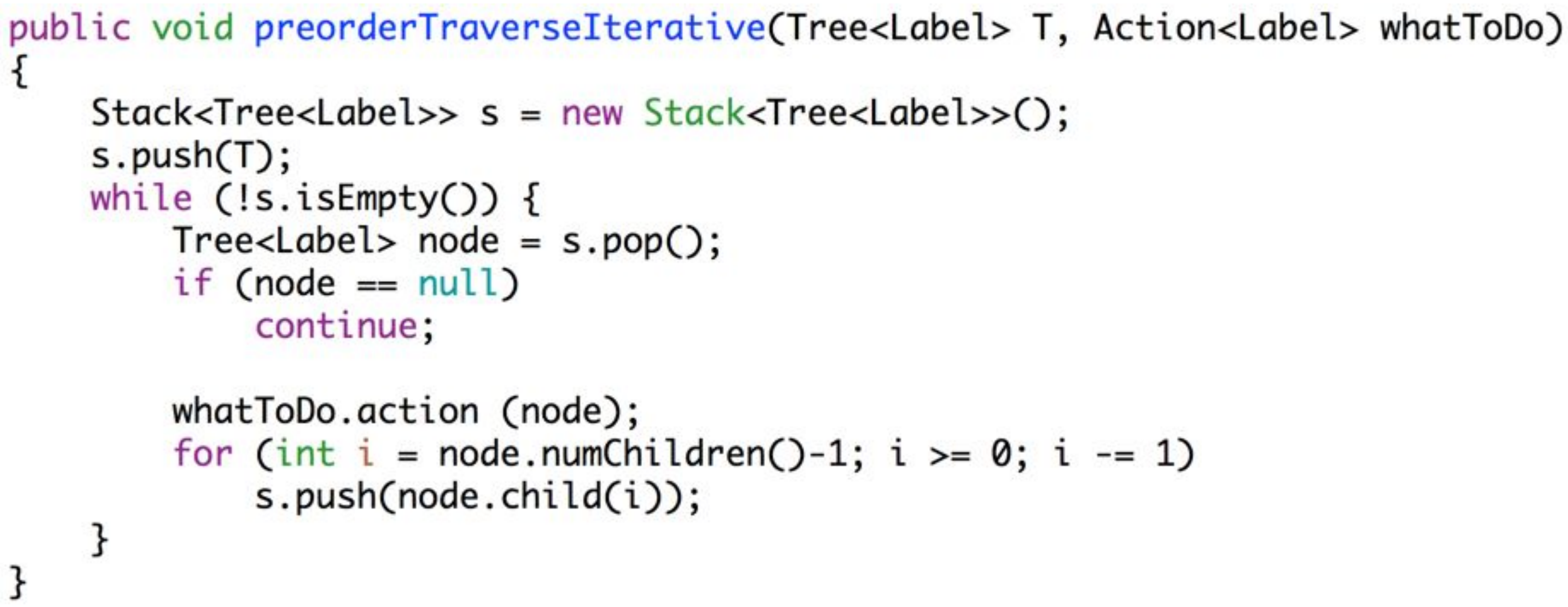
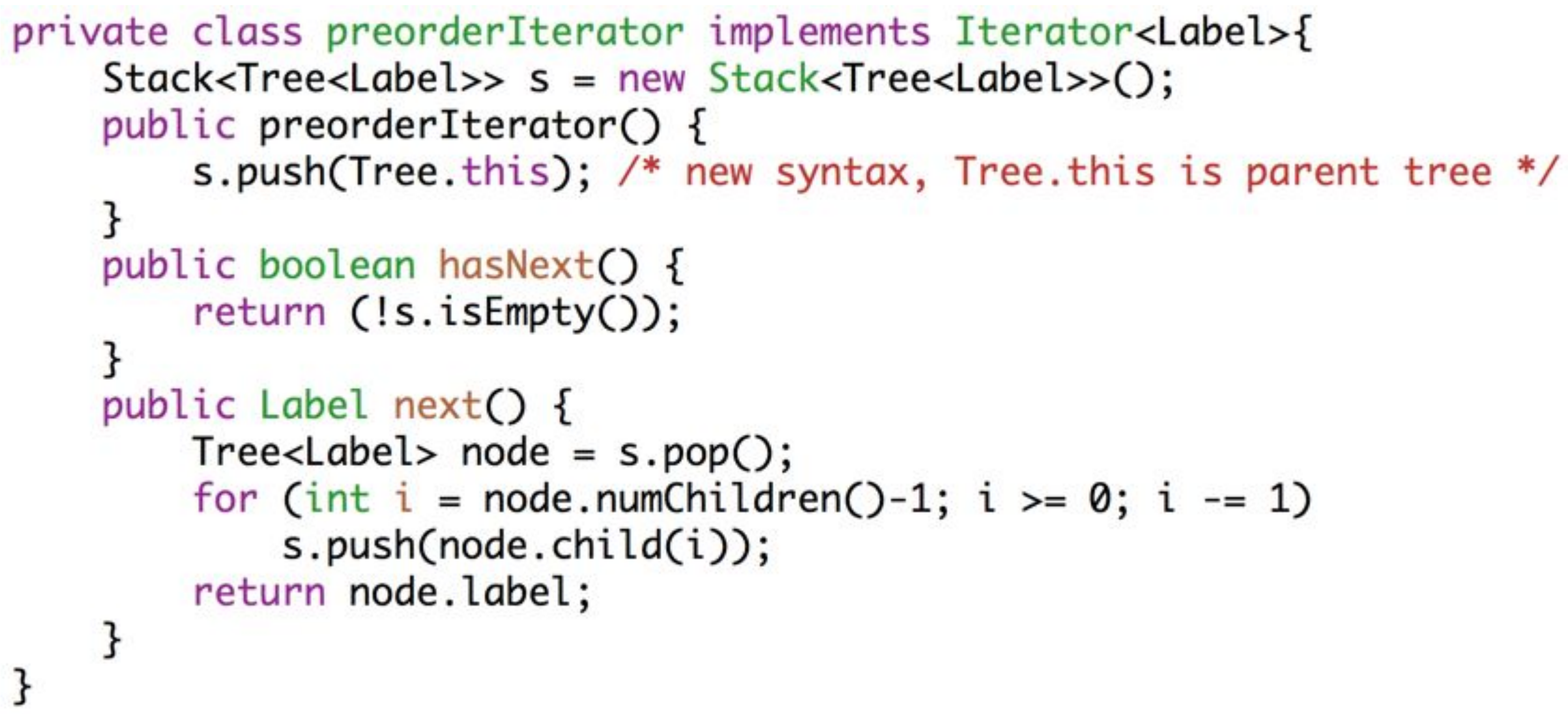


```
public void preorderTraverseIterative(Tree<Label> T, Action<Label> whatToDo)
{
    Stack<Tree<Label>> s = new Stack<Tree<Label>>();
    s.push(T);
    while (!s.isEmpty()) {
        Tree<Label> node = s.pop();
        if (node == null)
            continue;

        whatToDo.action (node);
        for (int i = node.numChildren()-1; i >= 0; i -= 1)
            s.push(node.child(i));
    }
}
```

Iteration: Space-saving Approach

Use our stack-based approach, but use next() instead of looping.



```
private class preorderIterator implements Iterator<Label>{
    Stack<Tree<Label>> s = new Stack<Tree<Label>>();
    public preorderIterator() {
        s.push(Tree.this); /* new syntax, Tree.this is parent tree */
    }
    public boolean hasNext() {
        return (!s.isEmpty());
    }
    public Label next() {
        Tree<Label> node = s.pop();
        for (int i = node.numChildren()-1; i >= 0; i -= 1)
            s.push(node.child(i));
        return node.label;
    }
}
```

Citations

Title figure: A thing I made (one of the first Java programs I wrote during my teaching career)

Pruning image: https://res.cloudinary.com/dc8hy36qb/image/upload/v1435213404/Fruit-Tree-Pruning-Methods_o7ieen_atkmmq.jpg

Jonathan Shewchuk: Nice intuitive use cases for various traversals.