Pre-Announcements

Justin: Alpha Kappa Psi Blockchain Event, next Wednesday in Chevron Auditorium

- Companies like Oracle, Ripple, Coinbase, etc.
- An overview of blockchains and its implications.
- 7 PM next Wednesday in iHouse.
    - Pizza, ice cream, and a raffle

Fliers up front if you want to do this thing.

Announcements

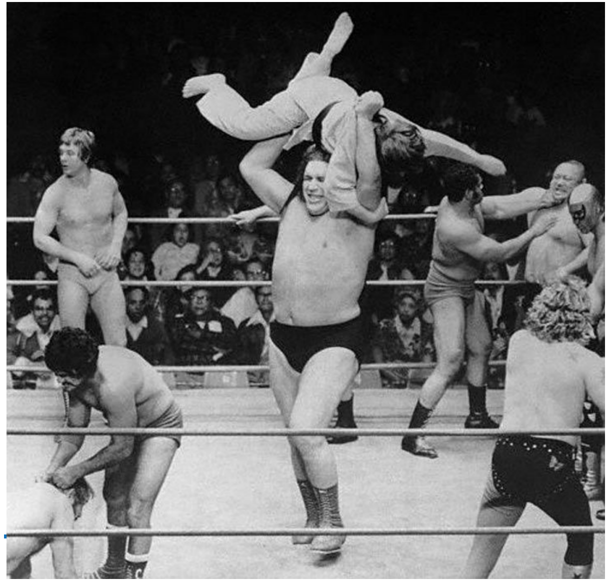
Midterm 2:

- 3/20, 8-10 PM in various rooms.
- Covers material through 3/16 (this Friday).
- Study using study guides.
    - THE KEY IS METACOGNITION: Reflect on your problem solving strategies and those of your fellow students.
    - Understanding a handful of solutions to old midterm problems is less helpful than you might think -- look at answers as late as possible.
- There is an alternate 61C midterm from 6 - 8 in 1 LeConte.

CS61B

Lecture 24: Priority Queues and Heaps

- Priority Queues
- Heaps
- Tree Representations
- Data Structures Summary



The Priority Queue Interface

```java
/** (Min) Priority Queue: Allowing tracking and removal of the
  * smallest item in a priority queue. */
public interface MinPQ<Item> {
    /** Adds the item to the priority queue. */
    public void add(Item x);
    /** Returns the smallest item in the priority queue. */
    public Item getSmallest();
    /** Removes the smallest item from the priority queue. */
    public Item removeSmallest();
    /** Returns the size of the priority queue. */
    public int size();
}
```

Useful if you want to keep track of the “smallest”, “largest”, “best” etc. seen so far.

Usage Example: Unharmonious Texts

Imagine that you're part of Grand Leader's Information Compliance and Happiness Enhancement (`GLICHE`) team.

- Your job: Monitor the text messages of the citizens to make sure that they are not having any unharmonious conversations.
- Each day, you prepare a report of the M messages that seem most unharmonious using the `HarmoniousnessComparator`.

Naive approach: Create a list of all messages sent for the entire day. Sort it using your comparator. Return the M messages that are largest.

Naive Implementation: Store and Sort

```
public List<String> unharmoniousTexts(Sniffer sniffer, int M) {
    ArrayList<String> allMessages = new ArrayList<String>();

    for (Timer timer = new Timer(); timer.hours() < 24; ) {
        allMessages.add(sniffer.getNextMessage());
    }

    Comparator<String> cmptr = new HarmoniousnessComparator();
    Collections.sort(allMessages, cmptr, Collections.reverseOrder());

    return allMessages.sublist(0, M);
}
```

Potentially uses a huge amount of memory Θ(N), where N is number of texts.

- Goal: Do this in Θ(M) memory using a MinPQ.

```
MinPQ<String> unharmoniousTexts = new HeapMinPQ<Transaction>(cmptr);
```

Better Implementation: Track the M Best

```
public List<String> unharmoniousTexts(Sniffer sniffer, int M) {
    Comparator<String> cmptr = new HarmoniousnessComparator();
    MinPQ<String> unharmoniousTexts = new HeapMinPQ<Transaction>(cmptr);
    for (Timer timer = new Timer(); timer.hours() < 24; ) {
        unharmoniousTexts.add(sniffer.getNextMessage());
        if (unharmoniousTexts.size() > M)
           { unharmoniousTexts.removeSmallest(); }
    }
    ArrayList<String> textlist = new ArrayList<String>();
    while (unharmoniousTexts.size() > 0) {
           textlist.add(unharmoniousTexts.removeSmallest());
    }
    return textlist;
}
```

Can track top M transactions using only M memory. API for MinPQ also makes code very simple (don’t need to do explicit comparisons).

How Would We Implement a MinPQ?

Some possibilities:

- Ordered Array
- Bushy BST: Maintaining bushiness is annoying. **Handling duplicate priorities is awkward.**
- HashTable: No good! Items go into random places.

	Ordered Array	Bushy BST	Hash Table	Heap
add	$\Theta(N)$	$\Theta(\log N)$	$\Theta(1)$	
getSmallest	$\Theta(1)$	$\Theta(\log N)$	$\Theta(N)$	
removeSmallest	$\Theta(N)$	$\Theta(\log N)$	$\Theta(N)$	
Caveats		Dups tough		

Worst Case $\Theta(\cdot)$ Runtimes

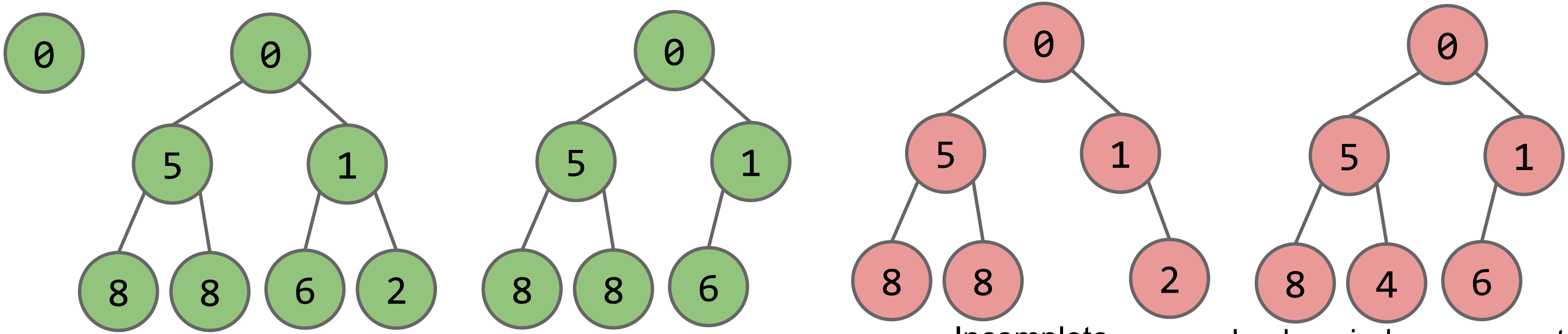
Heaps

Introducing the Heap

BSTs would work, but need to be kept bushy and duplicates are awkward.

Binary min-heap: Binary tree that is ***complete*** and obeys ***min-heap property***.

- Min-heap: Every node is less than or equal to both of its children.
- Complete: Missing items only at the bottom level (if any), all nodes are as far left as possible.



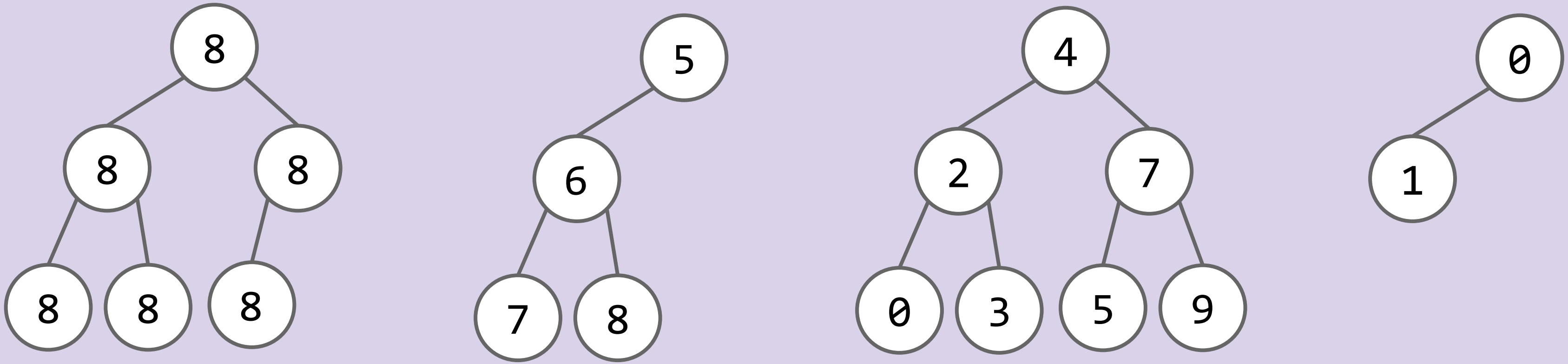
Incomplete

Lacks min-heap property

Heap Comprehension Test: http://yellkey.com/baby

How many of these are min heaps?

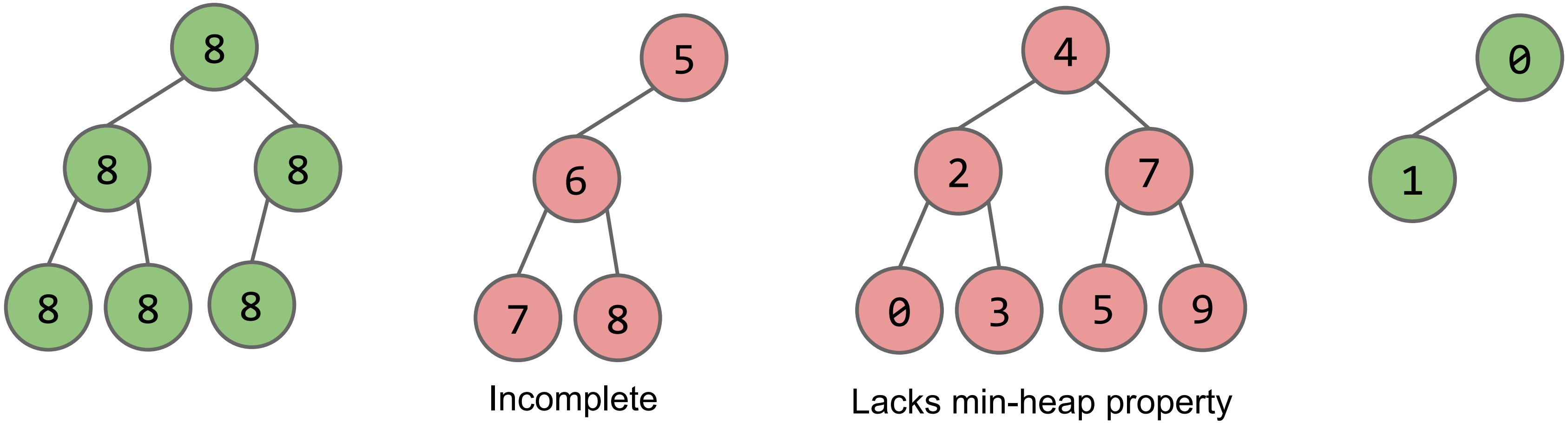
A. 0
B. 1
C. 2
D. 3
E. 4



Heap Comprehension Test: http://yellkey.com/present

How many of these are min heaps?

A. 0
B. 1
C. 2
D. 3
E. 4

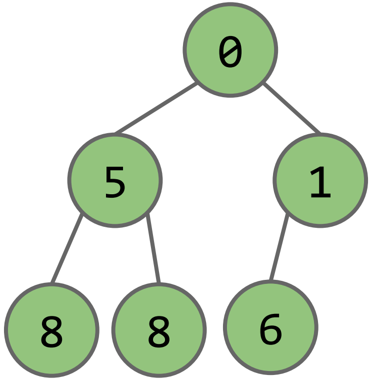


What Good Are Heaps?

Heaps lend themselves very naturally to implementation of a priority queue.

Hopefully easy question:

- How would you support `getSmallest()`?

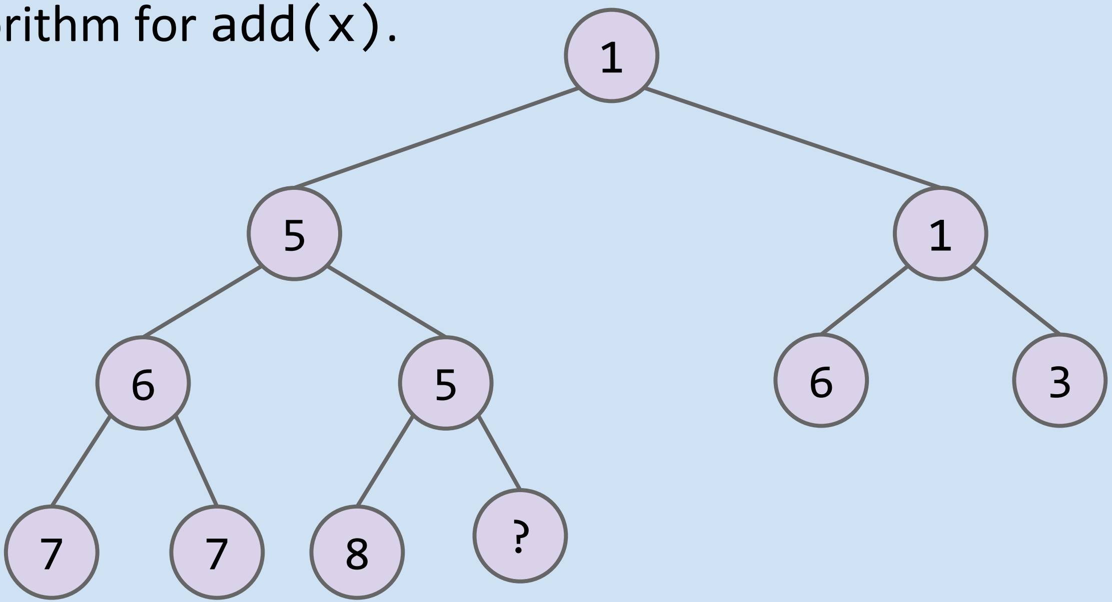


How Do We Add To A Heap?

Challenge: Come up with an algorithm for `add(x)`.

- How would we insert 3?

Runtime must be logarithmic.



Bonus: Come up with an algorithm for `removeSmallest()`.

Solution: See https://goo.gl/wBKdFQ for an animated demo.

Heap Operations Summary

Given a heap, how do we implement PQ operations?

- `getSmallest()` - return the item in the root node.
- `add(x)` - place the new employee in the last position, and promote as high as possible.
- `removeSmallest()` - assassinate the president (of the company), promote the rightmost person in the company to president. Then demote repeatedly, always taking the ‘better’ successor.

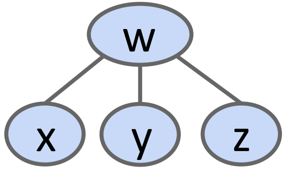
See https://goo.gl/wBKdFQ for an animated demo.

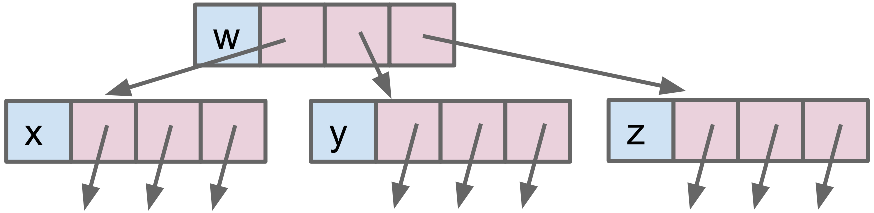
Remaining question: How would we do all this in Java?

Tree Representations

How do we Represent a Tree in Java?

Approach 1a, 1b and 1c: Create mapping from node to children.





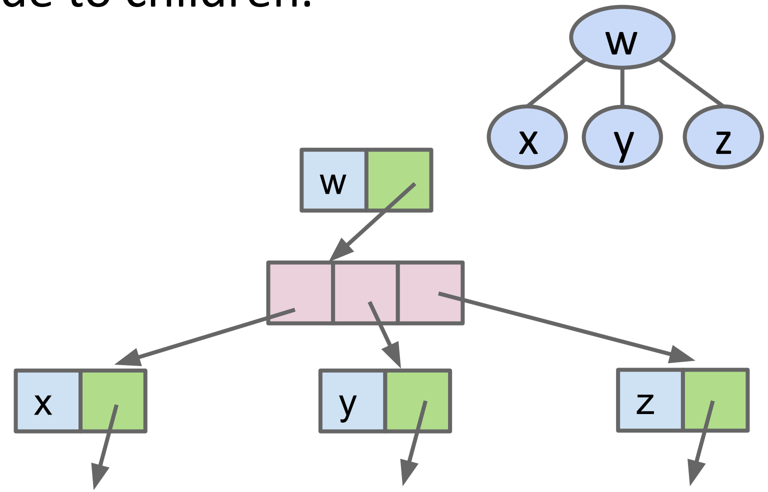
1a: Fixed-Width Nodes (BSTMap used this approach)

```
public class Tree1A<Key> {
   Key k; // e.g. 0
   Tree1A left;
   Tree1A middle;
   Tree1A right;
   ...
```

How do we Represent a Tree in Java?

Approach 1a, 1b and 1c: Create mapping from node to children.

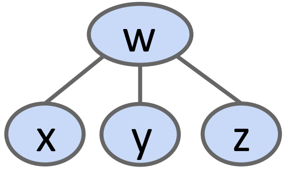
```
public class Tree1B<Key> {
   Key k; // e.g. 0
   Tree1B[] children;
   ...
```



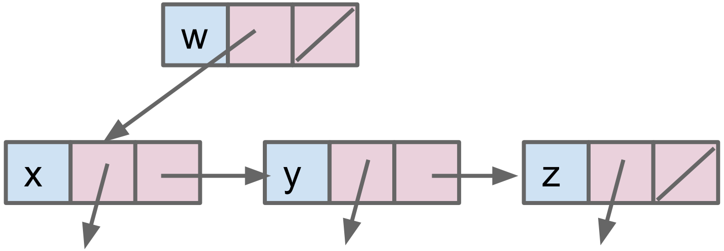
1b: Variable-Width Nodes

How do we Represent a Tree in Java?

Approach 1a, 1b and 1c: Create mapping from node to children.



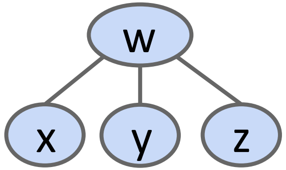

```
public class Tree1C<Key> {
   Key k; // e.g. 0
   Tree1C favoredChild;
   Tree1C sibling;
   ...
```

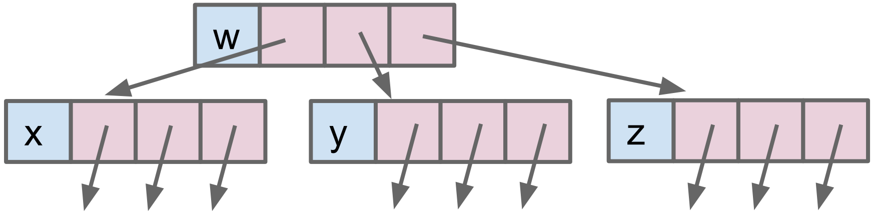


1c: Sibling Tree

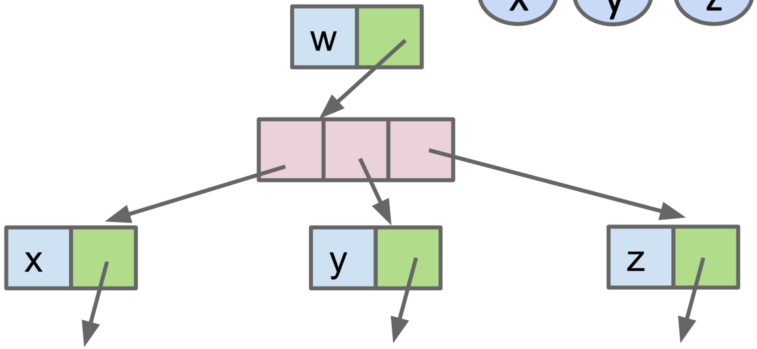
How do we Represent a Tree in Java?

Approach 1a, 1b and 1c: Create mapping from node to children.

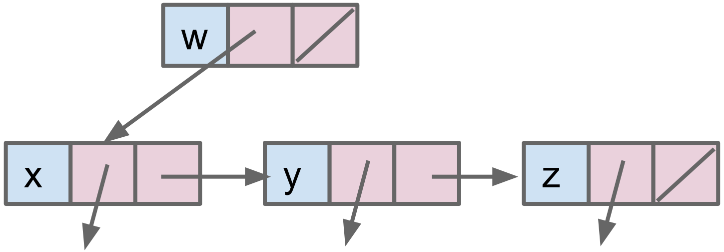




1a: Fixed-Width Nodes



1b: Variable-Width Nodes



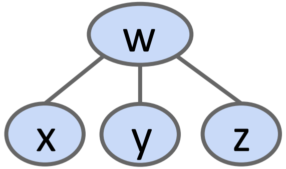
1c: Sibling Tree

How do we Represent a Tree in Java?

Approach 2: Store keys in an array. Store parentIDs in an array.

- Similar to what we did with disjointSets.

```java
public class Tree2<Key> {
   Key[] keys;
   int[] parents;
   ...
```

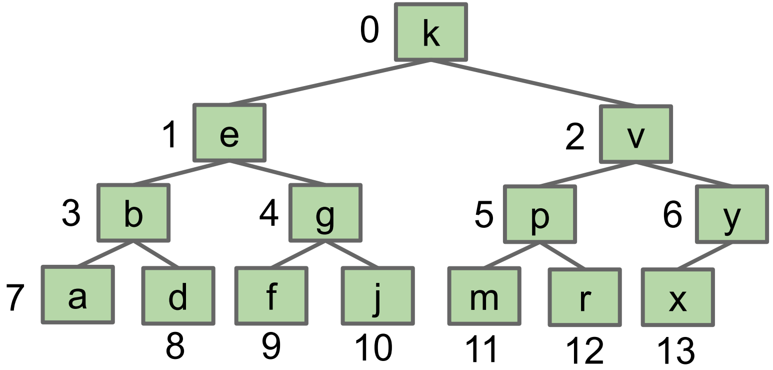


Key[] keys

w	x	y	z

int[] parents

0	0	0	0
0	1	2	3



Key[] keys

k	e	v	b	g	p	y	a	d	f	j	m	r	x
0	1	2	3	4	5	6	7	8	9	10	11	12	13

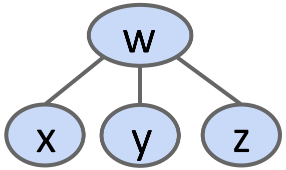
int[] parents

0	0	0	1	1	2	2	3	3	4	4	5	5	6
0	1	2	3	4	5	6	7	8	9	10	11	12	13

How do we Represent a Tree in Java?

Approach 3: Store keys in an array. Don't store structure anywhere.

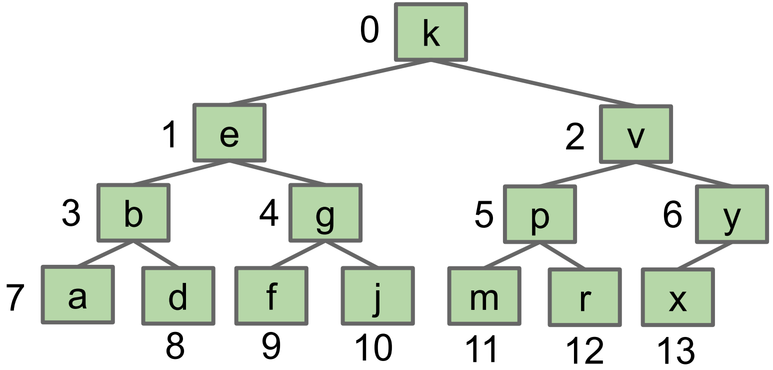
- To interpret array: Simply assume tree is complete.
- Obviously only works for "complete" trees.




```java
public class Tree3<Key> {
   Key[] keys;
   ...
```

`Key[] keys`

w	x	y	z
0	1	2	3



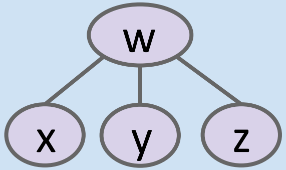
`Key[] keys`

k	e	v	b	g	p	y	a	d	f	j	m	r	x
0	1	2	3	4	5	6	7	8	9	10	11	12	13

A Deep Look at Approach 3

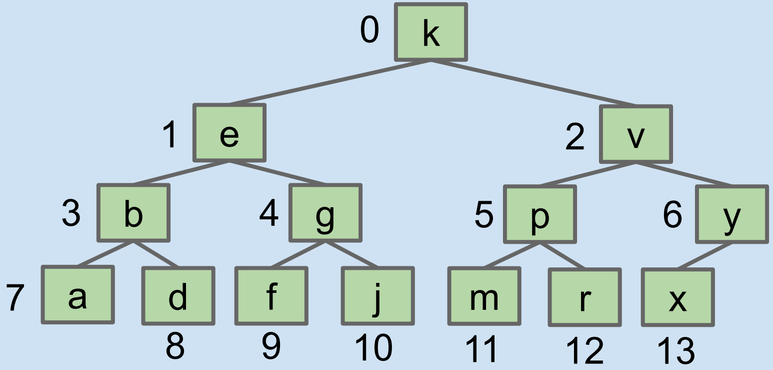
Challenge: Write the `parent(k)` method for approach 3.

```
public void swim(int k) {
    if (keys[parent(k)] > keys[k]) {
        swap(k, parent(k));
        swim(parent(k));
    }
}
```



Key[] keys

0	1	2	3
w	x	y	z



Key[] keys

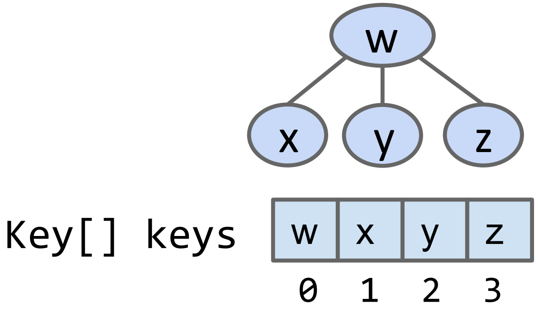
0	1	2	3	4	5	6	7	8	9	10	11	12	13
k	e	v	b	g	p	y	a	d	f	j	m	r	x

```
public class Tree3<Key> {
    Key[] keys;
    ...
```

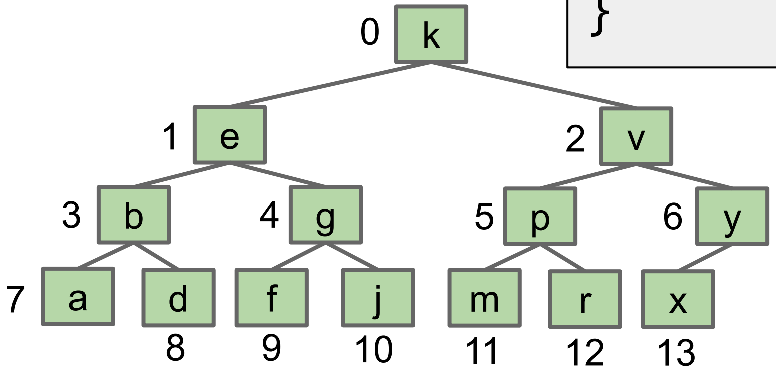
A Deep Look at Approach 3

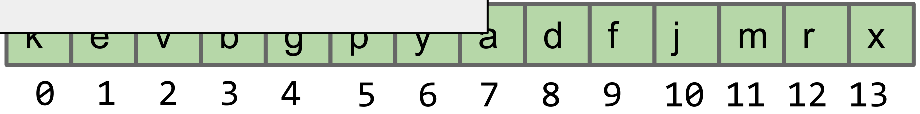
Challenge: Write the `parent(k)` method for approach 3.

```java
public void swim(int k) {
    if (keys[parent(k)] ≻ keys[k]) {
        swap(k, parent(k));
        swim(parent(k));
    }
}
```



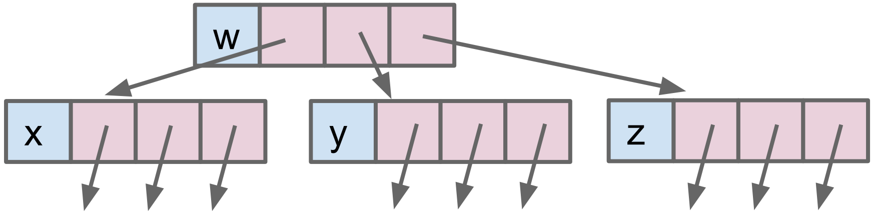

```java
public int parent(int k) {
    return (k - 1) / 2;
}
```



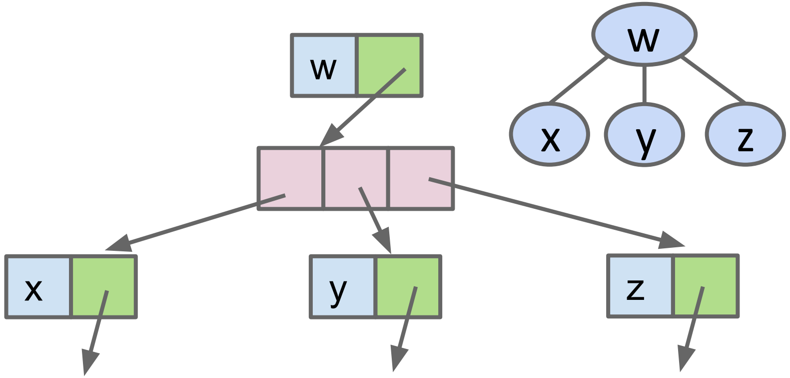



```java
public class Tree3<Key> {
    Key[] keys;
    ...
```

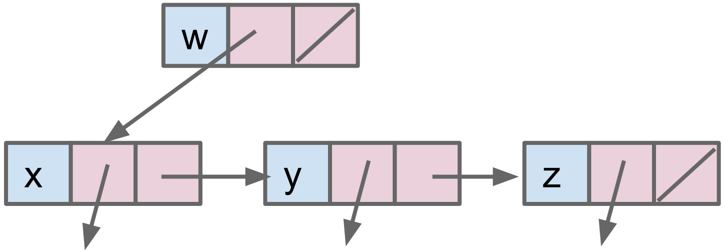
Tree Representations (Summary)



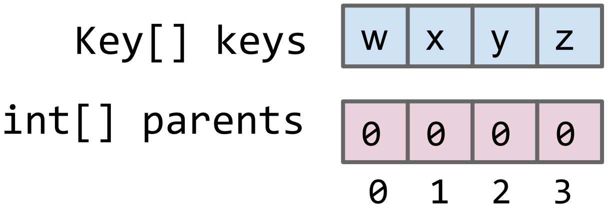
1a: Fixed Number of Links (One Per Child)



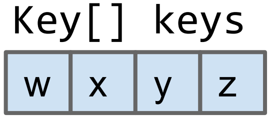
1b: Array of Child Links



1c: FirstBorn/Sibling Links



2: Array of Keys, Array of Structure

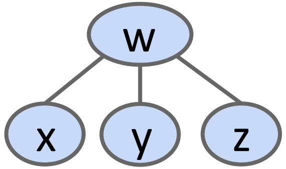


3: Array of Keys

Approach 3B (book implementation): Leaving One Empty Spot

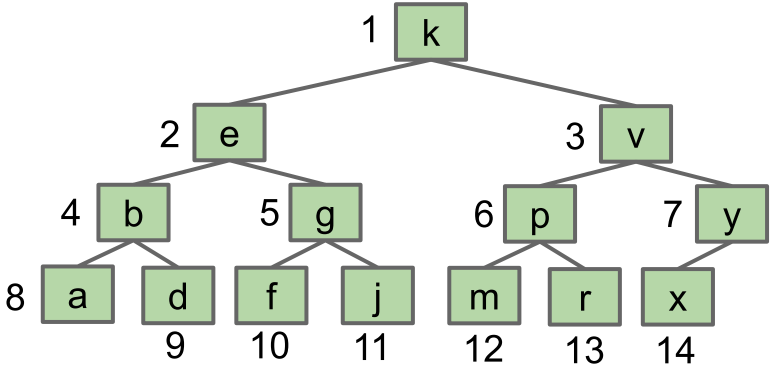
Approach 3b: Store keys in an array. Offset everything by 1 spot.

- Same as 3, but leave spot 0 empty.
- Makes computation of children/parents “nicer”.
  - `leftChild(k) = k*2`
  - `rightChild(k) = k*2 + 1`
  - `parent(k) = k/2`



`Key[] keys`

-	w	x	y	z
0	1	2	3	4



`Key[] keys`

-	k	e	v	b	g	p	y	a	d	f	j	m	r	x
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

In next week’s lab, you’ll implement a MinPQ.

Heap Implementation of a Priority Queue

	Ordered Array	Bushy BST	Hash Table	Heap
add	Θ(N)	Θ(log N)	Θ(1)	Θ(log N)
getSmallest	Θ(1)	Θ(log N)	Θ(N)	Θ(1)
removeSmallest	Θ(N)	Θ(log N)	Θ(N)	Θ(log N)

Items with same priority hard to handle.

Notes:

- Why “priority queue”? Can think of position in tree as its “priority.”
- Heap is log N time AMORTIZED (some resizes, but no big deal).
- BST can have constant getSmallest if you keep a pointer to smallest.
- Heaps handle duplicate priorities much more naturally than BSTs.
- Array based heaps take less memory (very roughly about 1/3rd the memory of representing a tree with approach 1a).

Some Implementation Questions

1. How does a PQ know how to determine which item in a PQ is larger?
   a. What could we change so that there is a default comparison?
2. What constructors are needed to allow for different orderings?

```
/** (Min) Priority Queue: Allowing tracking and removal of the
  * smallest item in a priority queue. */
public interface MinPQ<Item> {
    /** Adds the item to the priority queue. */
    public void add(Item x);
    /** Returns the smallest item in the priority queue. */
    public Item getSmallest();
    /** Removes the smallest item from the priority queue. */
    public Item removeSmallest();
    /** Returns the size of the priority queue. */
    public int size();
}
```

Data Structures Summary

The Search Problem

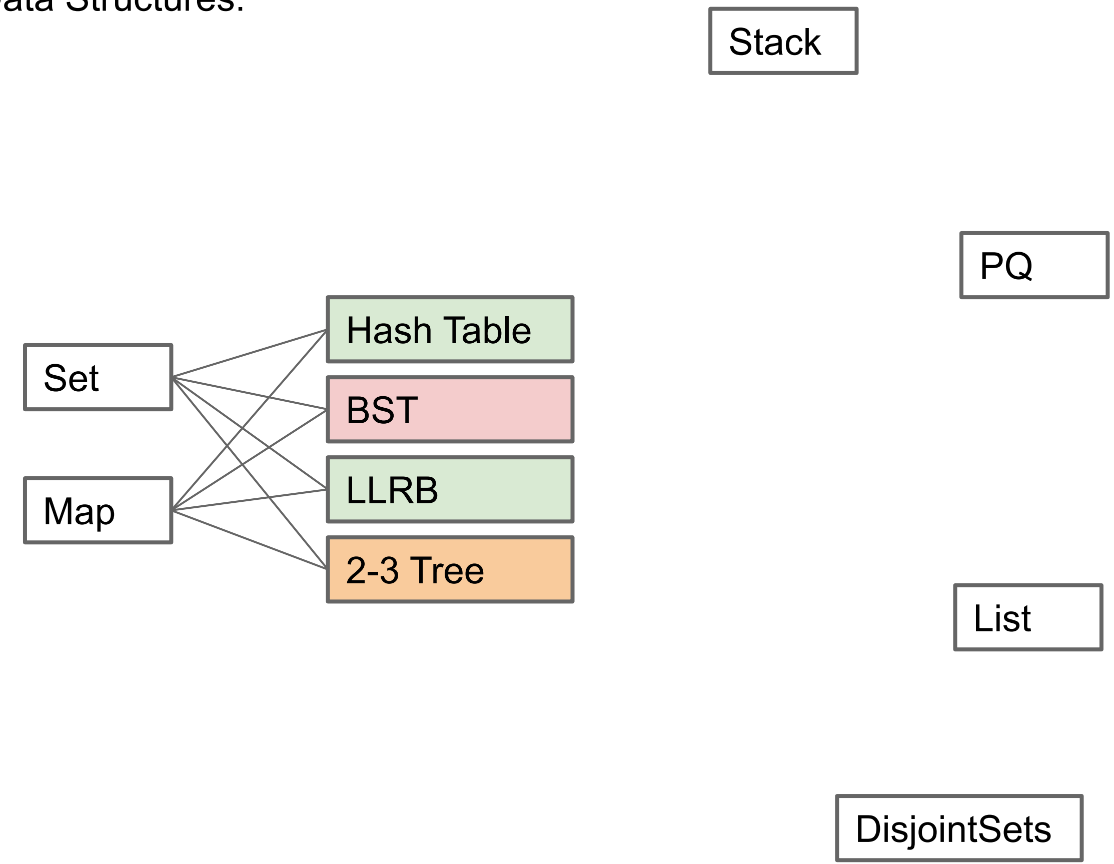
Given a stream of data, retrieve information of interest.

- Examples:
  - Website users post to personal page. Serve content only to friends.
  - Given logs for thousands of weather stations, display weather map for specified date and time.

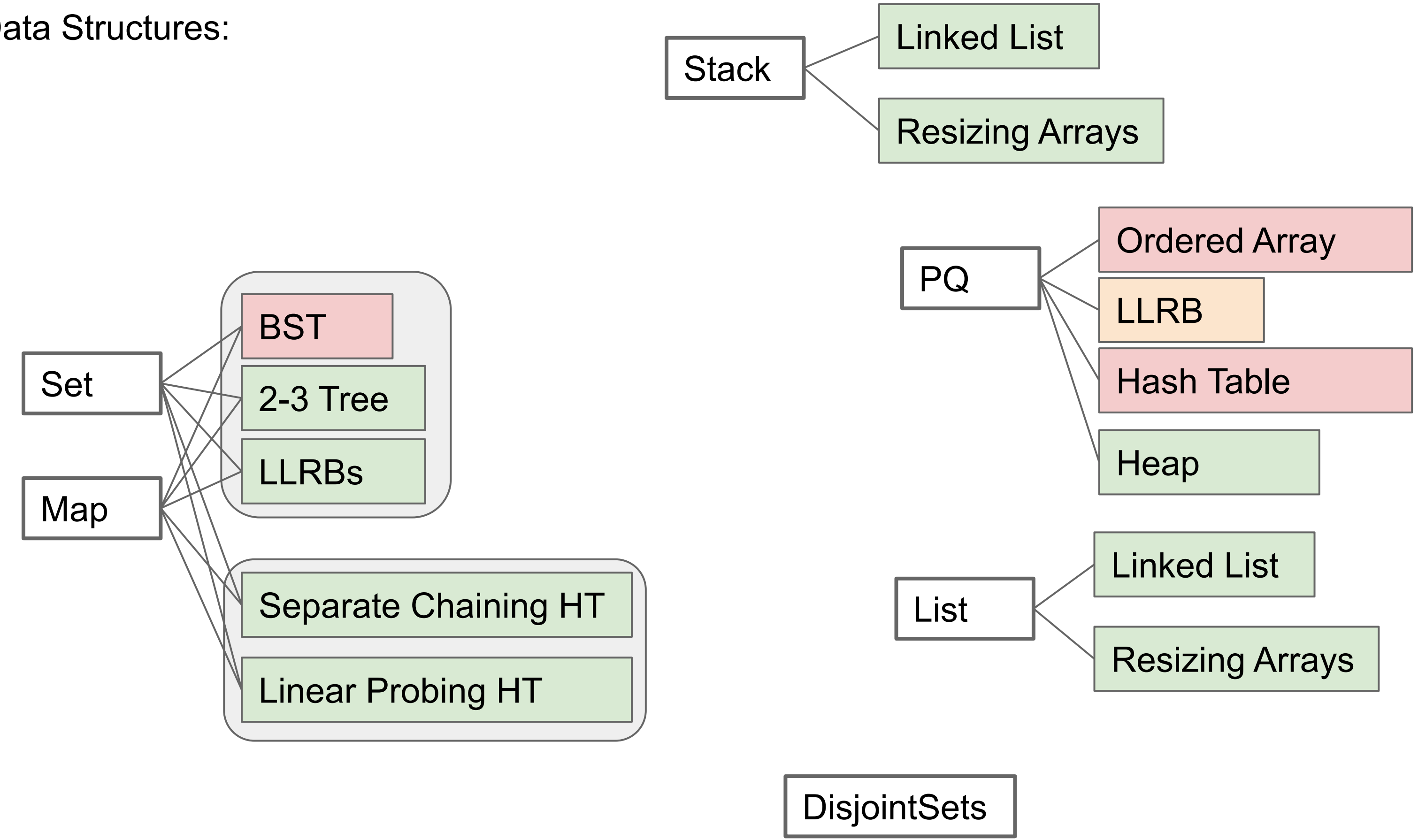
Search Data Structures (The particularly abstract ones)

Name	Storage Operation(s)	Primary Retrieval Operation	Retrieve By:
List	`add(key)` `insert(key, index)`	`get(index)`	index
Map	`put(key, value)`	`get(key)`	key identity
Set	`add(key)`	`containsKey(key)`	key identity
PQ	`add(key)`	`getSmallest()`	key order (a.k.a. key size)
Disjoint Sets	`connect(int1, int2)`	`isConnected(int1, int2)`	two int values

Searching Data Structures:

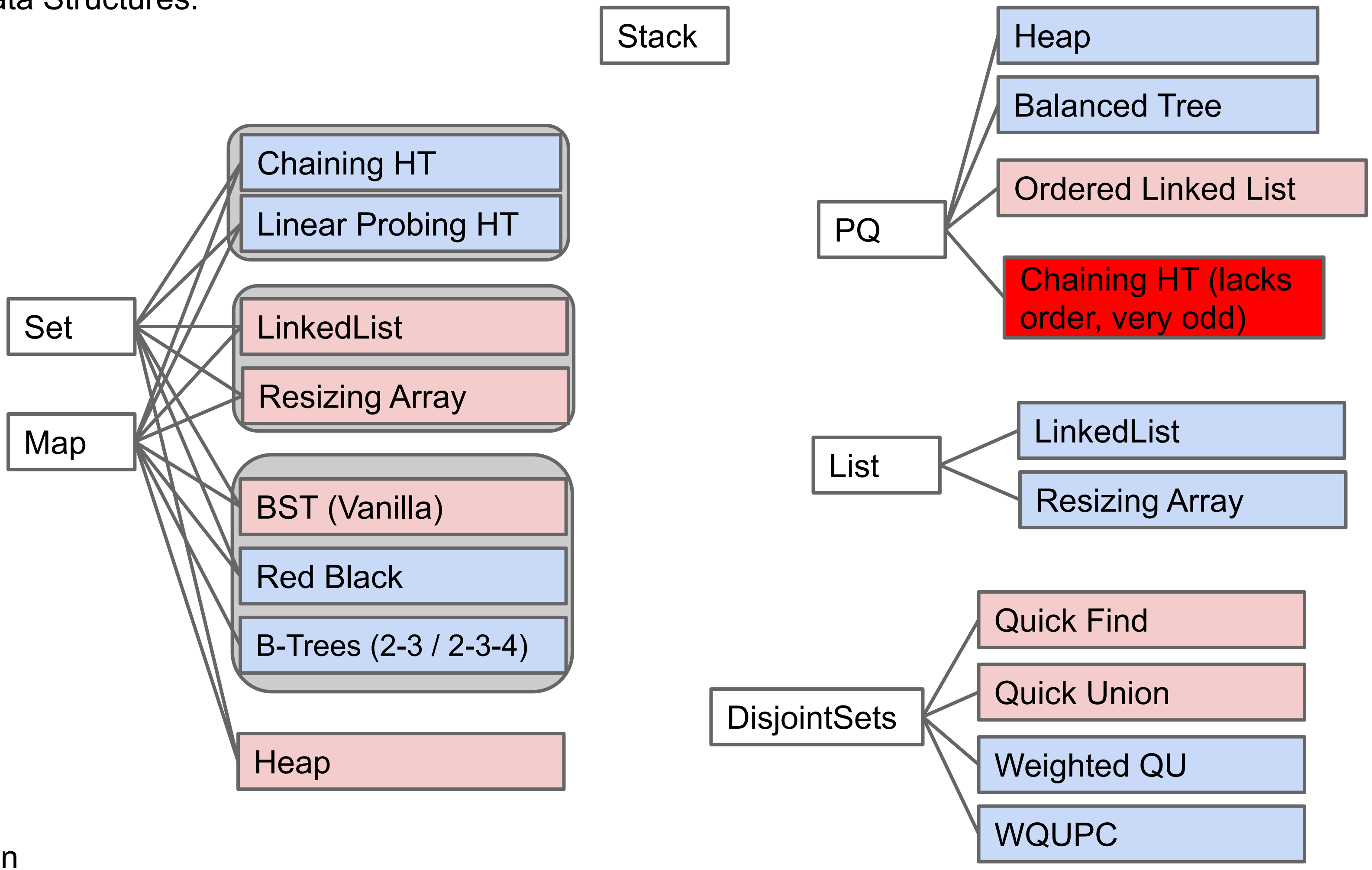


Searching Data Structures:



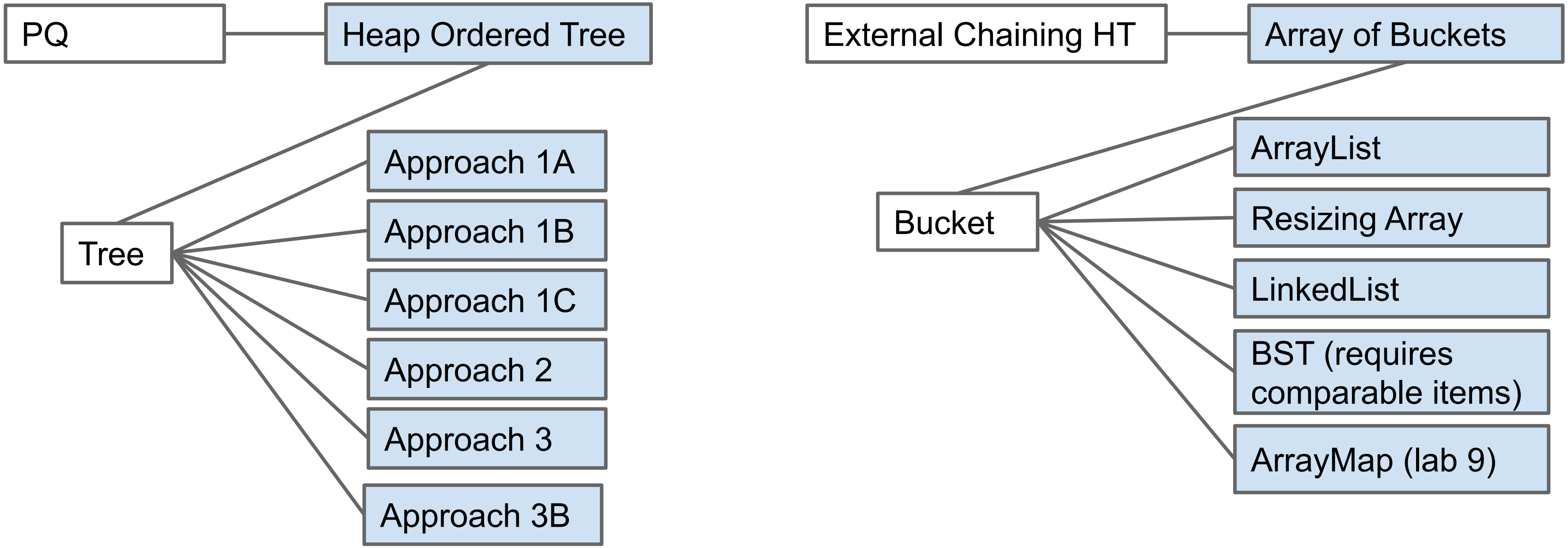
2017 version

Searching Data Structures:

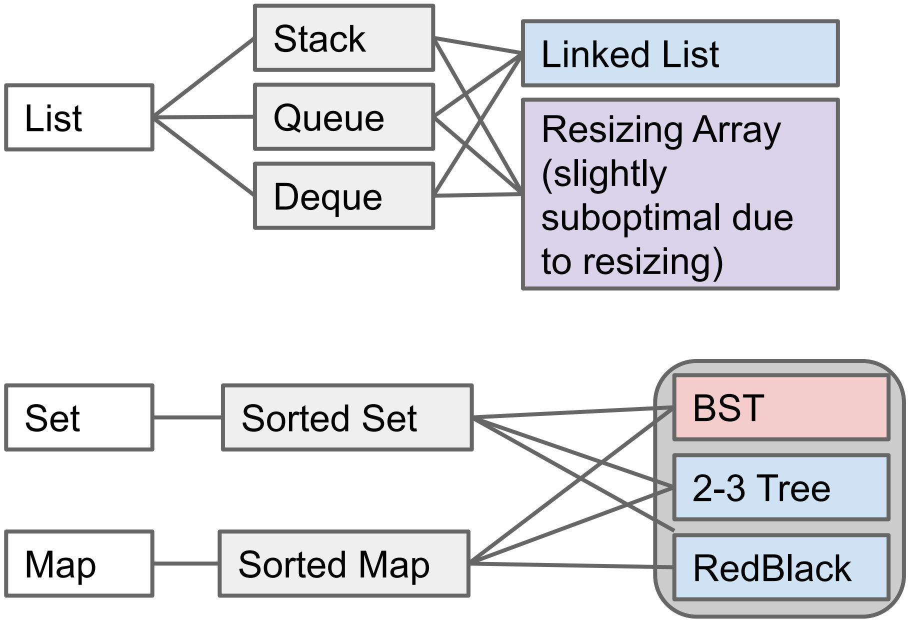


2016 version

Abstraction often happens in layers!



Specialized Searching Data Structures:



In Java:
java.util.SortedSet

java.util.SortedMap

Don’t usually consider MinPQ and MaxPQ to be different data structures, since we can just provide the opposite comparator.

Data Structures

Data Structure: A particular way of organizing data.

- We've covered many of the most fundamental abstract data types, their common implementations, and the tradeoffs thereof.
- We'll do two more in this class:
  - The quadtree (Friday).
  - The graph (starting after exam).

V·T·E	Data structures [hide]
Types	Collection · Container
Abstract	Associative array · Double-ended priority queue · Double-ended queue · List · Map · Multimap · Priority queue · Queue · Set (multiset) · Disjoint Sets · Stack
Arrays	Bit array · Circular buffer · Dynamic array · Hash table · Hashed array tree · Sparse array
Linked	Association list · Linked list · Skip list · Unrolled linked list · XOR linked list
Trees	B-tree · Binary search tree (AA · AVL · red-black · self-balancing · splay) · Heap (binary · binomial · Fibonacci) · R-tree (R* · R+ · Hilbert) · Trie (Hash tree)
Graphs	Binary decision diagram · Directed acyclic graph · Directed acyclic word graph

Citations

Title slide Andre the Giant picture: Unknown source

Friendster screenshot: http://jeremy.zawodny.com/i/friendster_rss.jpg

Weather screenshot: weather.com