UNIVERSITY OF ZIMBABWE

ASSET ALLOCATION FOR ZIMBABWE DEFINED CONTRIBUTION PENSION SCHEMES:AN APPLICATION OF THE MODERN PORTFOLIO THEORY

A PROJECT SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
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Chapter 1

Introduction

1.1 Introduction

Pension funds are a major concern for any country as they allow people to save for retirement or when they are physically or mentally incapacitated to work , and they also financially assist the pension fund members‘ dependents when he or she passes away. In most countries, the average population age continues to rise at a time when healthcare is improving, with today's employees expecting more retirement years. Thus, pension funds must develop successful investment strategies that provide optimal long-term returns to enable retirees to maintain a good standard of life after retirement.

In 2008, Zimbabwe witnessed a record breaking hyperinflation in the history which resulted in the erosion of pension's monetary values and many pensioners incurred losses. A year later, under the Government of National Unity (GNU), the Zimbabweans economy was dollarized through setting up a multi-currency system in order to ease inflation as the Zimbabwean dollar became valueless. According to the Commission of Inquiry on the conversion of insurance and pension values from the Zimbabwean Dollar (ZWL) to United States Dollar(USD) report issued on March 2017,

many pension fund administrators could not issue the asset values for the pension funds they administered and due to this many pension fund assets were undervalued leading to fewer funds being earned by pensioners.

The Statutory Instrument (SI) 142 was passed on June 24, 2019, making the ZWL a legal tender and initiating de-dollarization. This resulted in yet another repetition of the 2008 hyperinflationary environment, though this time it was not record breaking. However, this had a negative effect on pension values.

Due to this economic turmoil and repeated cycles of hyperinflation, confidence in the pension systems is at its lowest

1.2 Background of the Problem

Fears of a repetition of the events that happened in 2008 after the reintroduction of Zimbabwean dollar has made it difficult for the finance sector in Zimbabwe to grow to its full potential as consumers lack confidence to invest their funds into the finance sector. Savings and investments are essential components of every country's growth. In most countries these investments come from pension funds and the insurance sector.

According to the Insurance and Pensions Commission (IPEC) 2021 Fourth Quarter Pension Report, the average asset share per member (including beneficiaries) of ZWL337,824.58 which equates to US$3099,31 at the of the RBZ auction rate of ZWL109 as at 31 December 2021. This amount is not adequate for one to sustain a decent living after retirement.

The Corona Virus Disease of 2019 (COVID-19) pandemic that hit Zimbabwe in March 2020 resulted in a great depression in the pension industry as many employees were

either laid off work (particularly in the tourism sector) and salaries were reduced as company revenues were reduced due to the lock-down restrictions. In turn, funds channelled towards pension schemes lowered significantly thus fewer investments could occur.

In Zimbabwe, poverty after retirement is high with most retirees depending on real estate or children for income to supplement their inadequate pensions. Those that do not have this privilege, move back to their rural homesteads where they tend to depend on peasant farming to sustain their livelihoods.

All the above reasons proved beyond reasonable doubt that there is need to rapidly put in place and implement investment techniques which are beneficial to pension fund members and improve the nation‘s Gross Domestic Product (GDP).

Portfolio optimization, defined as the process of combining various asset types in order to provide favourable returns with the lowest possible risk. Long-term horizons must be used to determine the asset structure of any long-term fund, such as a pension fund. Strategic Asset Allocation framework, the name given to the fund's long-term goal, should take into account the fund's liabilities, economic conditions, and actuarial expertise while developing the Strategic Asset Allocation framework.

1.3 Statement of the Problem

Pension schemes have limited investment asset options. Currently, available investment assets include property, money market instruments, equities, cash and prescribed assets. Other investment vehicles such as corporate bonds are unavailable on the Zimbabwe Stock Exchange. This has resulted in receiving negative investment which lower the growth of the pensions industry.

Informed by the current situation, this study intends to put into practice asset allocation strategies that reduce the adverse economic events that have stunted the growth of the pensions sector. Many pension funds are currently under dissolution in the country as they fail to pay pensioners and beneficiary's reasonable monthly pensions. This research looks into asset allocation with some optimization techniques that create a good balance between the risk and returns for pension schemes.

1.4 Aims and Objectives

1.4.1 Aim

The aim of this research project is to construct investment portfolios with favourable long-term investment returns and acceptable risk for a pension scheme.

1.4.2 Objectives

The objectives of this research are to:

1. Determine the available investment vehicles for the pensions industry.
2. Calculate the risk and investment return using Modern portfolio theory.
3. Construct efficient frontiers with the calculated risk and investment returns.
4. Find the tangency portfolio using the Sharpe ratio.
5. Obtain the investment vehicles with acceptable risks and favourable long-term investment returns.

1.5 Research Questions

1. Are there viable investment vehicles for Zimbabwe‘s pensions industry?

2. What are the current risk and investment returns on the available vehicles using the Modern Portfolio Theory?

3. What are the existing efficient frontiers using the results from the risk and investment returns on available assets?

4. What is the tangent portfolio using Sharpe ratio?

1.6 Significance of the study

A basic investment concept is that sufficient money should be accessible to meet liabilities as they arise, while also attempting to maximize accumulation within this constraint. Pension funds present a problem in determining the form of liabilities and the asset mix that will best enable them to be met. As a result, it's not unexpected that both individuals and corporations have limited knowledge of how to invest such funds. Mostly qualitative techniques are being used by asset managers and pension fund trustees which tend to be return chasing. There is need for a diversified asset mix which helps to distribute risk over a large number of assets.

It also corrects asset-liability mismatches thereby, reducing poverty after retirement as people are currently finding it difficult to maintain a basic standard of living. There has been heavy reliance on the working population to take care of the retired individuals and also the younger population and this strains the working class. This study serves in the construction of asset mix that distributes risk evenly providing comfort to pension schemes. The solvency of pension schemes can only be done through proper asset allocation.

Traditional investment behaviour tends to be qualitative and judgemental and each asset manager's judgement is subjective. Investment decisions are produced using directives and regulations from other countries are commonly used by asset managers. These directives being used include the Solvency II from Europe and the Solvency

Assessment Management from South Africa. However, there is need for Zimbabwe to develop our own asset allocation methods which are in line with our capital markets. Use of statistical methods which quantitative such as the Modern Portfolio Theory to make investment decisions can be more reliable as compared to using insurance investment policies from other countries.

1.7 Research Limitations

1. There are fewer investment options in Zimbabwe
2. Bonds which are of lower risk are not available in the investment market. Due to this the investment vehicles available for pension schemes are then limited hence reducing diversification.
3. Access to more sensitive information was limited as pension fund companies were not willing to disclose data on their investment portfolios
4. The research will only be looking into asset allocation of equities that are listed on the Zimbabwe Stock Exchange (ZSE) thus limiting the scope of the study and the accuracy of the research as it looks into one asset class.

1.8 Research Delimitations

This research mainly focuses on investment strategies for defined contribution schemes ignoring defined benefit schemes. This is due to defined contributions schemes mainly relying on investments as compared to defined benefit schemes. Furthermore, defined benefit schemes have become less popular in this current century due to their lack of flexibility as employees are constantly changing jobs and fewer employers are willing to take the risk of covering pension deficits that may arise if the member's predetermined pension liability fails to be met.

1.9 Project Layout

This research study is made up of five chapters. Chapter 1 defines the background of the research, the problem statement, research objectives, and research questions, significance of the study, the scope, the limitations and delimitations of the study. Chapter 2 comprises of the literature review for the research. In this chapter, existing work by other researchers on pensions‘ assets is reviewed paying closer attention to the Zimbabwean context. Here, the theoretical and conceptual frameworks are outlined. Chapter 3 focuses mainly on the research methods employed in data collection and description of the study. Furthermore, the chapter outlines sampling procedure, data collection procedure used and how data was analysed. Chapter 4 This focuses on the data analysis and the results and lastly Chapter 5 focuses on presenting the various discussion on the summary of results, conclusions, recommendations, policy implications of the study and areas which needs further studies.

Chapter 2

Literature Review

2.1 Introduction

Literature review can be regarded to be of utmost importance in informing a meaningful researches as it presents the existing knowledge which aids the researcher in identifying research gaps which increases the understanding of the topic (Winchester and Saliji, 2016). It is with this background that Literature Review chapter aims at providing a thorough review of existing literature on the subject of pensions and asset management of life schemes funds in order to strategically position this research among the existing body of knowledge. This chapter presents the frameworks that inform the asset allocation for pension funds. Furthermore, this chapter focuses on pension's asset allocation hypotheses as well as factual studies and the Zimbabwean insurance market focusing on the regulatory instruments.

2.2 Glossary of terms

2.2.1 Financial Asset

This is a liquid asset (can easily be converted into cash) whose value is derived from a contractual right or ownership claim. Examples include stocks, cash, bonds, mutual funds, and bank deposits. As opposed to tangible assets such as land, property or

commodities, financial assets do not necessarily poses inherent physical worth or even a physical form. Their value reflects factors of supply and demand in the marketplace in which they trade, as well as the degree of risk they carry.

2.2.2 Asset Allocation

Asset allocation is defined as an investment strategy where investors divide their portfolios between different dissimilar asset classes in order to minimize investment risks. The investment asset classes can be categorized into fixed-income, cash and equivalents as well as stocks.

2.2.3 Defined Contribution (DC)

Defined Contribution (DC) schemes are defined as occupational pension schemes where employee‘s contributions and the employer's contributions are both invested and the proceeds are then used to buy a pension and/or other benefits at retirement. The total value is the pension benefit payable to the employee under DC scheme depends on the amount of contributions paid during the working years the investment return achieved less any fees and charges, and the cost of buying the benefits

2.2.4 Pension Scheme

Is the pool of assets that form an independent legal entity that are bought with the contributions to a pension plan for the exclusive purpose of financing pension plan benefits. The plan/fund members have a legal or beneficial right or some other contractual claim against the assets of the pension fund. Pension funds take the form of either a special purpose entity with legal personality (such as a trust, foundation, or corporate entity) or a legally separated fund without legal personality managed by a dedicated provider (Pension Fund Management Company) or other financial institution on behalf of the plan/fund members.

2.2.5 Investment vehicle

Is defined as investment tools possessed by investment managers. It furthermore denotes a collection of chosen investment assets.

2.2.6 Contribution

Is defined as the payment made to a pension plan by a plan sponsor or member.

2.2.7 Fund member

A fund member is an individual who is either active (working or contributing, and hence actively accumulating assets) or passive (retired, and hence receiving benefits), or deferred (holding deferred benefits) participant in a pension plan.

2.2.8 Asset Mix

Refers to the breakdown of assets in a given portfolio which is usually created from equities (stocks) tend to offer the greatest long-term growth potential and can help you beat inflation; they also carry the most risk. Therefore, an asset mix can be regarded to as an asset portfolio as it is collection of different types of assets.

2.2.9 Investment Risk

This is regarded as the risk associated with investing due to fluctuations in the value of investments. This risk can be broken down into three broad categories namely, Market risk, credit risk and liquidity risk.

2.2.10 Financial Deficit

This is when a company‘s expenditures exceed its income, or liabilities exceed assets. This is synonymous with a shortfall or loss and is the exact opposite of a financial

surplus. This results in a situation where available funds are not enough to cover the company's costs.

2.2.11 Solvency

Is regarded as the company‘s ability to pay long-term liabilities. This is a useful measure of a company‘s financial health as it resembles a company's ability to continue its operations as a going concern.

2.2.12 Asset-Liability Match

This is a measure of an insurance company's liquidity that matches a company's assets to its liabilities. It shows the company‘s ability to cover future claims.

2.2.13 Investment Management

This refers to the decisions over the investments so that they are done well. It involves the managerial aspects that include organizing, controlling and planning the investments of corporates and individuals.

2.2.14 Pensions

According to the Oxford Dictionary, a pension is a regular payment made during a person's retirement from an investment fund to which that person or their employer has contributed during their working life.

2.3 Pensions Products

In accordance to the Insurance and Pensions Commission (IPEC), a pension is a stream of income that is paid usually when one attains retirement age, but it can also be paid to at dependent be it a child widow/widower upon the death of a member of the pension fund. IPEC further regards. Pension funds are intended to carter for

the welfare of an individual on his retirement and at other times of transition during his working life-time. Bodie (1989) described pension funds as a type of retirement insurance. There are basically two main types of pension products which are as follows:

2.3.1 Defined Benefit (DB) Fund

This is an arrangement whereby the value of a member‘s benefits is known in advance with fund‘s sponsor bearing the cost of ensuring that the member receives the full benefit. This benefit focuses the benefits flow which the member will receive in the future. A typical DB plan determines the employee's benefit as a function of both years of service and wage history.

2.3.2 Defined Contribution (DC) Fund

This a pension product whereby the contribution made by the employer and the member is determined in advance and the member receives the accumulated value of these contributions on retirement. The benefit therefore dependent on many factors, amongst them the investment return earned on the fund assets. The DC arrangement is regarded to be simpler compared to DB. The employer and the employee at times, make regular contributions into the employee's retirement account. These contributions are often a predetermined percentage of the employee‘s which tends to vary during the course of a career. All contributions made are tax-deductible, and investment income accrues tax-free.

The employee is usually entitled to determine how his account is to be invested. In principle, contributions may be invested in any security though most plans limit investment options to specified financial assets. Upon retirement, the member either receives a lump sum or an annuity whose size depends on the accumulated value of

the funds in the retirement account. Therefore, the employee endures investment risk as the retirement account is fully funded with the employer‘s only obliged to making its periodic contribution. DC plan valuation is measured as the market value of the assets held in the retirement account. However, as a guide for personal financial planning, the DC plan sponsor often provides workers with the indicated size of a life annuity starting at retirement age that could be purchased now with the accumulation in their account under different scenarios. The actual size of the retirement annuity will depend on the realized investment performance of the retirement fund, the interest rate at retirement, and the ultimate wage path of the employee.

2.3.3 Hybrid Plan

- These plans combine both DB and DC in order to reduce the risk associated with each of the above plans. Of late, there has been a transition to DC funds from DB funds. This transition shifts investment risk from the corporate sector to households thus they are becoming increasingly exposed to financial markets, and retirement income may be subject to greater variability than before. T. DC plans now occupy for the majority of invested assets in private sector occupational pension plans. This was a deliberate effort to respond to inequalities brought about by DB funds as they provided unfair withdrawal values when an employee changed jobs (Andrew, 2004). DC contributions can be fixed as a predictable share of payroll, migrating to a DC plan helps employers reduce balance sheet and earnings volatility in the long run.

The shift towards DC pensions does have advantages to both employees and employers. These include supports labour market mobility as accrual risk is reduced as benefits in DB plans tend to be back-loaded, so that workers who change employers can lose a great portion of expected benefits if these are not transferable from one employer to another. However, such a shift also reallocates investment risk within

the financial system from the corporate to the household sector, which may have implications for financial stability.

2.4 Zimbabwean Pension Industry

The Zimbabwean Pensions Industry is guided by the Pension and Provident Funds Act Chapter 24:09 and is regulated by IPEC. The pensions industry may be categorized into four main classes as follows:

2.4.1 Occupational Pensions Schemes

These are funds set up by employers to benefit their staff and are usually DC schemes. Occupation pensions fund can either be self-administered or insured funds. Self-administered funds allow fund assets to be registered in the name of the particular fund while in insured fund, assets are registered in the name of the insurer who makes certain promises to members of the fund. Trustees of the self-administered funds are fully responsible for investment decisions and performance of the fund. Self-administered funds can either own as well as control their respective administrative structures; or outsource administration services from life insurers or professional pension fund administrators. Insured funds are usually small funds whose assets are pooled together and invested in the name of the life insurance company that administer them. Investment decision are done on behalf of the funds by the insurance company.

2.4.2 Personal Pension Plans (Individual Schemes)

These are pension plans set up by life insurers targeting individual members without focusing on their employment status and are ideal for those in the informal sector.

2.4.3 Public Service Pension Scheme

This is a pay-as-you–go defined scheme catering for civil servants. The scheme was established by an Act Parliament and is not regulated by IPEC.

2.4.4 National Pension Scheme

This is a compulsory pension scheme for all employees administered by the National Social Security Authority (NSSA) and is commonly known as "NSSA Pension". Just like the public service pension scheme, NSSA is currently not under the supervision of IPEC.

2.5 Asset Management in Zimbabwe

Asset management is regarded to be continuous process since risks are usually time sensitive usually due to the instabilities in any given economy. The Zimbabwean economy being a good example of such fluctuations as evidenced by the two currency changes in the past 12 years. This therefore possess the need constant revival of investment techniques. To evade insolvency, pension fund managers must monitor asset allocation techniques as they are critical in corporate growth as well as profitability

2.6 Modern Portfolio Theory

In his seminal paper Markowitz's (1952) entitled Portfolio Selection, he presented a methodology to portfolio allocation trading off risk and return. He suggested that portfolio allocation involves analysing available data in creating investable insight and converting these insights into real portfolios. Furthermore, he put forward the suggestion that in portfolio allocation, returns maximization is not a sufficient investor's objective as it fails to ensure satisfactorily portfolio diversification.

Markowitz's approach favours portfolios that allocate resources to a variety of capital assets considering the correlation between the constituents and diversify the effective bets investors make on factors common to subsets of their opportunity set. Other scholars such as Roy (1952) of the Safety First Criterion, Perold, (2004) Tobin (1958) played a pivotal role in shaping Modern Portfolio Theory (MPT) as the provided a framework of utilising investment insights in forming efficient portfolios considering the investor's levels of risk averseness

2.6.1 Model Development

In order to successfully apply the Modern Portfolio Theory framework in the crafting of pension portfolios, assumptions are essential. Individual investment vehicle returns are statistically distributed using a symmetric probability distribution. In this research, the Mean Variance approach will be employed as well as Lagrangian procedures in modelling investments for pension schemes.

2.6.2 Model Assumption

The MPT is based on the following important assumptions:

- The investor is assumed to prefer higher levels of return compared to lower asset returns thus for a given two portfolios with the same risk the investor will take the one with the higher expected return. This is known as the assumption of non-satiation.
- Investor's decisions are based on return and risk.
- Investors are risk averse that is they prefer the portfolio with lower risk. This is known as the assumption of risk aversion.
- All expected returns, variances and co-variances of pairs of assets are known.

- There is a fixed single-step time horizon.
- Taxes and transactions costs are not present.
- Assets can be invested in any amounts.

2.6.3 Expected Return

The Expected Return for an individual asset is calculated using the following formula :

$$E(R_i) = \sum_{i=1}^{n} P_i(R_i)\ Single\ Asset \tag{2.6.1}$$

For a portfolio of assets, the Expected Return is computed using weighted average rates of returns of individual assets in the portfolio. These weights are proportions of the value of investment. The mathematical representation of the formula is as follows:

$$E(R_p) = \sum_{i=1}^{n} \omega_i E(R_i) \tag{2.6.2}$$

where ω_i is the percentage of the portfolio invested in asset i and $E(R_i)$ is the Expected Return for asset i. In accordance to Markowitz (1959) the rate of return of an asset is computed as follows:

$$R_n = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{2.6.3}$$

This equation takes into account assets value changes in the market in the interval time t and $t-1$ as asset return are random variable, thus return on a portfolio can be calculated using probabilities using the formula:

$$R_n = p_1 x_1 R_1 + p_2 x_2 R_2 + ... + p_n x_n R_n \tag{2.6.4}$$

2.6.4 Variance and standard deviation of returns

Variance can be regarded as the measure of the variation of possible returns from the Expected Returns which is the risk in the investment and is calculated as follows:

$$\sigma_i^2 = \sum_{i=1}^{n} P_i[R_i - E(R_i)]^2 \tag{2.6.5}$$

where P_i is the probability of R_i. Therefore the standard deviation is calculated as the square root of the variance

2.6.5 Covariance of Returns

Covariance is the measure of the degree to which two investment assets move together relative to their individual mean values over time. In portfolio analysis it's important to pay particular attention to covariance of expected returns instead of prices or any other variable.

- A positive covariance indicates that two assets move in the same direction relative to their individual expected returns.
- A negative covariance indicates that expected returns move in opposite directions relative to their expected returns.

Given two assets, i and j, the covariance is calculated as follows:

$$Cov_{i,j} = E\{[R_i - E(R_i)][R_j - E(R_j)]\} \tag{2.6.6}$$

2.6.6 Risk

(Bell, 1995) put forward that variability of returns associated with any investment is independent of the original wealth available and only dependent on investor's decision on wealth distribution. Therefore, risk can be represented in mathematical terms as

follows:

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_{ij}x_i x_j \tag{2.6.7}$$

This can be represented by a matrix for all pairs as;

$$\Omega = \begin{bmatrix} \sigma_{11} & \sigma_{12} & .... & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & .... & \sigma_{2n} \\ . & . & .... & . \\ . & . & .... & . \\ \sigma_{i1} & \sigma_{i2} & .... & \sigma_{ij} \end{bmatrix} \tag{2.6.8}$$

With the above variance-covariance matrix, the risk can be calculated as follows:

$$Var(R_p) \;=\; \sigma_p^2 \;=\; x^T\Omega x \tag{2.6.9}$$

2.7 Langrangian Techniques

Letting $x = (x_1, x_2, \ldots, x_n)$ to be the portfolio assets where x_{is} are individual assets in a given portfolio then the model can be expressed as follows:

$$f(x_1, x_2, \ldots, x_n) \;=\; f(X) \tag{2.7.10}$$

This is subject to :

$$g_j(x_1, x_2, \ldots, x_n) \;=\; b_j \tag{2.7.11}$$

$f(X)$ and $g(X)$ result in the Lagrangian function:

$$L(x_1, x_2, \ldots, x_n, \lambda) = [f(X) - \lambda g(X)] \tag{2.7.12}$$

To optimize $L(X, \lambda)$ we find the partial derivatives of x_{is} and $\lambda' s$

$$\begin{aligned} \frac{\partial L}{\partial x_i} &= \frac{\partial f}{\partial x_i} - \lambda\frac{\partial g}{\partial x_i} = 0 \\ \frac{\partial L}{\partial \lambda_i} &= -(g(X)) \end{aligned} \tag{2.7.13}$$

Deducing from our optimisation model, it becomes;

$$L = x^T \Omega x - \lambda_1[E(R_p) - E_r] - \lambda_2[\sum_{i=1}^{n} -1] \tag{2.7.14}$$

Partial derivatives from equation 2.7.14 becomes:

$$\frac{\partial L}{\partial x_i} = 2\sum_{i=1}^{n} \sigma_{ij} - \lambda_i E_i - \lambda_2 = 0 \tag{2.7.15}$$

$$\frac{\partial L}{\partial x_i} = -(\sum_{i=1}^{n} x_i E_i - E(R_p)) = 0 \tag{2.7.16}$$

$$\frac{\partial L}{\partial x_i} = -(\sum_{i=1}^{n} x_i - 1) = 0 \tag{2.7.17}$$

Equations above need to be expressed in matrix form $Ax = b$ where,

$$A = \begin{bmatrix} 2\Omega & -E & -1 \\ E^T & 0 & 0 \\ 1^T & 0 & 0 \end{bmatrix},\ b = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ E^T \\ 1 \end{bmatrix},\ E = \begin{bmatrix} E_1 \\ E_2 \\ . \\ . \\ E_n \\ \end{bmatrix},\ X = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix},\ 1 = \begin{bmatrix} 1 \\ 1 \\ . \\ . \\ 1 \end{bmatrix} \tag{2.7.18}$$

In matrix $A, 2\Omega$ is a result of partial derivative of $\sum_{i=1}^{N}\sum_{j=1}^{N} \sigma_{ij} x_i x_j$. Solving for X in $AX = b$ it becomes $b^T A_{-1} = X$ which gives the optimal portfolio $X^* = (x_1, x_2, ..., x_n$

2.8 Efficient Market Instrument

2.9 Efficient Frontier

An Efficient Frontier (EF) is a hyperbolic diagram plotted by matching expected return and standard deviation (σ, R). This diagram represents the set of optimal portfolios offering the highest expected return at a given level of risk or the lowest

risk for a given level of expected return. Figure 2.1 below shows a typical efficient frontier: Portfolios that lie below the efficient frontier are sub-optimal as they fail to

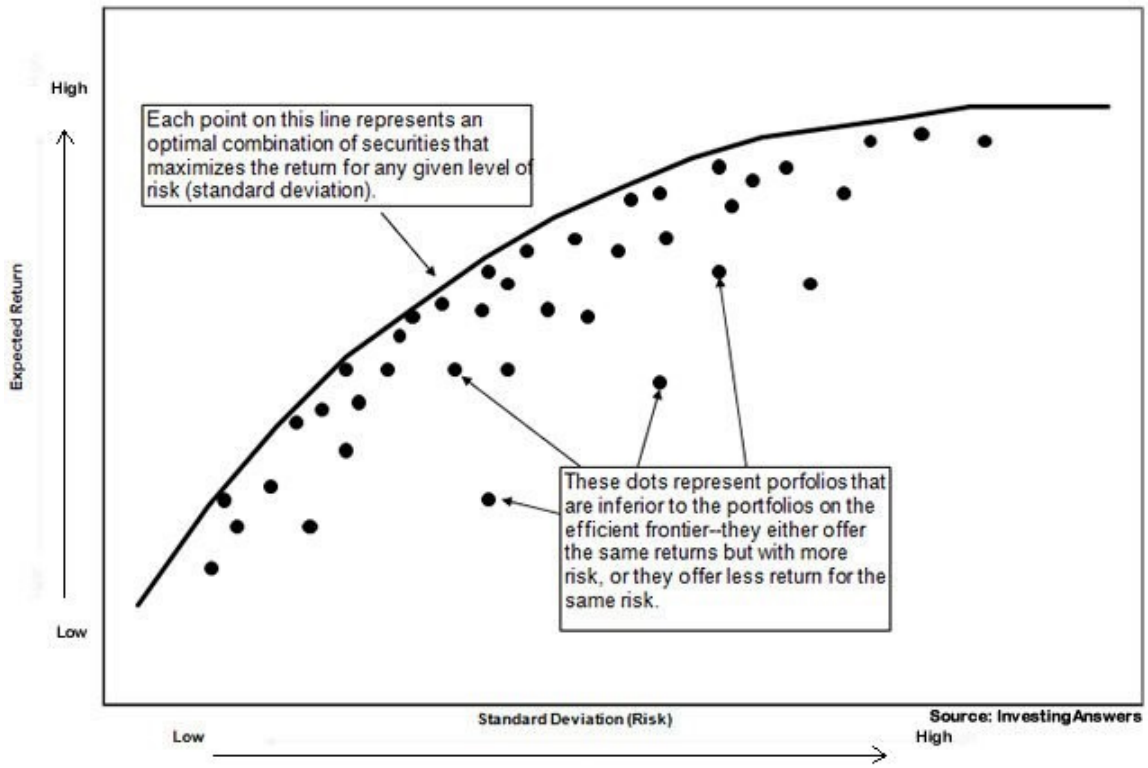

Figure 2.1: The Efficient Frontier

provide enough return for the given level of risk. Portfolios clustering right of the efficient frontier are sub-optimal since poses a higher risk for the given rate of return.

Chapter 3

Methodology

3.1 Introduction

In order to achieve the research's goals, existing models were utilized. The study did not use any newly developed asset allocation models. The mean-variance technique was paired with several Lagrangian approaches to analyse investments for insurers, as detailed in the prior chapter. Some models will be developed and implemented as needed to support the mean-variance model.

3.2 Data Sources and Quality

The data on equity investment for the Pension Fund currently under study was obtained asset managers report on a weekly basis. Since the data was from the investment managers of the Fund it can be said to be reliable and valid which makes it well suited for this research.

3.3 Data Pre-processing

Data required was for respective equity market values in the company's investment portfolio. It was collected on a weekly basis for the year 2021.

3.4 Structure of the Data

The final dataset consists of 10 equities. Below is a description of each corporation being analysed in this research:

3.4.1 Axia Corporation

Axia Corporation Limited is a company that specializes in speciality retail, distribution and logistics. Distribution Group Africa (DGA), TV Sales and Home (TVSH), and Transerv are its three operating business segments. Inbound clearing and bonded warehousing, ambient and chilled/frozen warehousing, logistics, marketing, sales, and merchandising services are among DGA's core competencies. TVSH is a national furniture and electrical appliance retailer with locations across the United States. Transerv sells auto-mobile parts through a variety of channels to meet the demands of its clients around the country.

3.4.2 British American Tobacco

British American Tobacco Zimbabwe Holdings Ltd is a tobacco industry holding company established in Zimbabwe. It specializes in the production and distribution of cigarettes and pipe tobaccos. The company has two primary production lines, one for cigarettes and the other for chopped rags. Dunhill, Newbury, Madison, Everest, Kings-gate, and Berkeley are among the brands in its portfolio. British American Tobacco Zimbabwe Holdings Ltd manufactures and distributes tobacco for both domestic and international markets. British American Tobacco International Holdings (UK) Limited was the Company's largest shareholder as of December 31, 2009, with 56.95% of the share capital.

3.4.3 Cassava Smartech

Cassava SmarTech Zimbabwe Limited (currently known as EcoCash Holdings) is a diversified "SmarTech" group that has been de-merged from Econet Wireless Zimbabwe Limited and operates separately and independently from Econet, allowing the two separate Groups of Companies to exploit their synergies. The organization uses digital technologies to help Zimbabweans achieve socio-economic development and improve their overall quality of life.

3.4.4 Delta Corporation

Its primary businesses are cold beverages and agriculture. Lager beer, Chibuku, carbonated soft drinks, non-carbonated beverages, distribution and services, and maltings are among the Group's products.

3.4.5 Hippo Valley Estates Limited

Hippo Valley Estates Limited is a farming company that grows and mills sugar-cane as well as other crops. The Company's subsidiaries are in the business of developing and selling township lots. Mkwasine Estate, which grows sugar-cane and conducts other agricultural operations, and Zimbabwe Sugar Sales (Private) Limited, which works as a broker for sugar millers, are both owned by the Company. The Tokwane Consortium owns 32.56% of the company. The Company also owns a 33.3% stake in Sugar Industries (Proprietary) Limited, a Botswana-based packer and distributor of refined sugar, and a 49 percent stake in NCP Distillers (Zimbabwe) (Private) Limited, which converts molasses to alcohol.

3.4.6 Innscor Africa

The Group's primary activities include providing fast food services, manufacturing and selling biological assets, and manufacturing and selling domestic goods. The

following are the divisions: Fast food restaurants (such as Bakers Inn, Chicken Inn, Creamy Inn, Nando's, and Pizza Inn) Retail and distribution (Distribution Group Africa, Innscor Credit Retail, Kodak Photo, and the Spar franchised outlets), agro-processing (Niloticus and Colcom Holdings), and manufacturing (Distribution Group Africa, Innscor Credit Retail, Kodak Photo, and the Spar franchised outlets) (Bakeries, National Foods and appliance manufacturing).

3.4.7 Seed Co

The majority of Seed Co's proprietary hybrid and non-hybrid grain and oil crop seed varieties were produced and bred at its research stations through market-driven research and breeding programs. The seed is generated under contract by an established producer network using Seed Co's own parent seed.

3.4.8 Simbisa Brands Limited

Simbisa Brands Limited is a public company that owns, runs, and franchises a collection of Quick Service Restaurant (QSR) brands. Simbisa is unique in that it not only controls the intellectual property rights to the brands in its portfolio, but it also owns and operates the bulk of the QSR stores. Simbisa is a Pan-African quick service restaurant (QSR) operator with locations in 11 African nations with plans to expand further in the future.

3.4.9 Old Mutual Limited

Old Mutual Limited is an African financial services conglomerate that provides a wide range of financial services to retail and corporate customers in 17 countries. In Africa, the company offers solutions for investing, savings, life insurance, asset management, banking, and property and personal insurance.

3.5 Portfolio Simulation

Microsoft Excel will be used to model the respective portfolios from the dataset. Using the calculations in Chapter Two, the expected return and standard deviation or risk will be determined using this software.

3.6 Efficient Frontier

After calculating the projected return and risk, the results will be shown on a graph using the R programming language. It will be possible to create a collection of efficient investment portfolios.

3.7 Portfolio Optimization

The Solver TOOLPAK in MS Excel will be used to calculate the best portfolio weights. This is a type of linear programming that mimics asset weights for the best portfolio while keeping specific limits in mind. These weights are calculated using the values from the portfolio with the highest return at a reasonable risk.

3.8 Software packages

In this study, two statistical software tools will be used: Microsoft Excel and R programming.

Table 3.1: Portfolio Proportions

PF	**Return**	**Risk**	**BAT**	**CSZL**	**DLTA**	**ECO**	**HIPO**	**INN**	**SEED**	**SIM**	**OMZIL**
1	0.1170	0.207	-	0.0904	-	0.9096	-	-	-	-	-
2	0.0840	0.099	-	0.2040	-	0.1435	-	-	0.1250	0.0857	0.4418
3	0.0650	0.093	-	0.0340	-	-	0.2956	0.0868	0.0749	-	0.5086
4	0.0590	0.183	-	-	-	-	1.0000	-	-	-	-
5	0.0790	0.093	-	0.1935	-	0.0932	0.0580	-	0.1444	0.0485	0.4624
6	0.1190	0.218	-	-	-	1.0000	-	-	-	-	-
7	0.0870	0.105	-	0.2018	-	0.2040	-	-	0.0973	0.0876	0.4092
8	0.0890	0.109	-	0.2004	-	0.2443	-	-	0.0789	0.0889	0.3875
9	0.1080	0.169	-	0.1751	-	0.6568	-	-	-	0.0462	0.1219
10	0.1120	0.186	-	0.1677	-	0.7487	-	-	-	0.0278	0.0558
11	0.1100	0.178	-	0.1714	-	0.7027	-	-	-	0.0370	0.0888
12	0.0600	0.139	-	-	-	-	0.7734	-	-	-	0.2266
13	0.0750	0.089	0.0300	0.1808	-	0.0652	0.1132	-	0.1487	0.0029	0.4591
14	0.1030	0.15133	-	0.1843	-	0.5418	-	-	-	0.0693	0.2045
15	0.0730	0.089	0.0457	0.1752	-	0.0328	0.1357	-	0.1489	-	0.4617
16	0.0910	0.114	-	0.1990	-	0.2846	-	-	0.0605	0.0902	0.3658
17	0.0800	0.09428	-	0.1964	-	0.1002	0.0430	-	0.1421	0.0579	0.4603
18	0.1060	0.162	-	0.1788	-	0.6108	-	-	-	0.0555	0.1549
19	0.0890	0.109	-	0.2004	-	0.2443	-	-	0.0789	0.0889	0.3875
20	0.0940	0.122	-	0.1969	-	0.3451	-	-	0.0328	0.0920	0.3332

Chapter 4

Data Analysis and Results

4.1 Introduction

This chapter shows the research findings and simulations that were carried out in accordance with the study's objectives. The study was carried out according to the technique outlined in Chapter 3, and the required parameters were determined using formulas outlined in Chapter 2. The study topic presents a quantitative method for allocating assets to portfolios that are efficient. It solves the problem of determining how much a company should invest in each asset in order to reduce risk and maximize benefits. Pension funds face challenges in deciding which asset classes to invest in in order to improve their financial health. The portfolios were simulated in Microsoft Excel, and the Efficient Frontier curves were created in R.

4.2 Source of Data

The data was from a Defined Contribution pension fund in Zimbabwe and we were provided with asset managers reports with all the listed equities on Zimbabwe Stock Exchange they have investments in. The researcher chose ten of the highest performing shares from their listed equities portfolio in our asset allocation investigation. Market values for each equity investment were used to calculate returns and monitor

their returns.

4.3 Investment Portfolio

In this study ten high performing equities were used that the pension fund can invest into to protect the pension fund members' contributions. The equities included Axia, British American Tobacco, Cassava Technologies, Delta Beverages, Econet, Hippo Valley, Innscor, SeedCo, and Old Mutual Zimbabwe. These ten equities form a diverse portfolio as they are from companies that specialize in various industries in Zimbabwe such as agricultural and financial services sector. Portfolios were simulated using these ten equities and the following results were produced shown in Figure 4.1

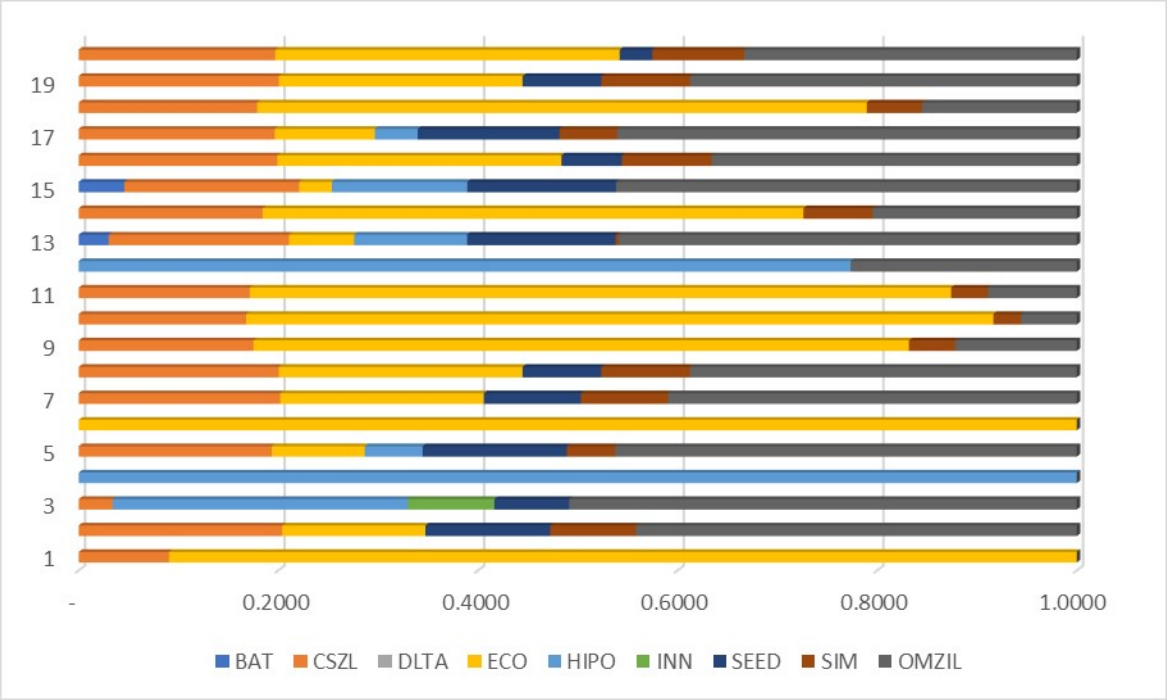

Figure 4.1: Simulated Portfolios

The portfolios simulated consist of their calculated returns, risk and weights for each equity available for investment. The portfolio proportions are represented in Table 3.1 for each equity. If an equity has a weight of 0 this means nothing should be invested in that equity. For example, for portfolio 4, the investments in Hippo Valley shares is equal to 1 which indicates that all of the pensioner's wealth should be invested in Hippo Valley equities to earn a return of 5.9%. However, investing in such a portfolio is of high-risk margin of 0.18267. This investment risk level is quite high compared to other portfolios. This supports numerous financial optimization research and theories. This model focuses on wealth in one asset when the minimum and maximum returns for each equity are used as proxies. As shown in Table 3.1 investments in more assets leads to lower investment risks for example portfolio 13 has the lowest investment risk of 0.08985 and holds investments in 7 equities. More investments in different types of equities allows for spreading out of investment risk. This supports the Markowitz theory on diversification.

Investment portfolios with the highest return tend to yield the highest investment risk for example portfolio 6 has the highest risk of 0.21754 and has the return of 11.9%. All of the investment wealth is allocated into one equity which is the Econet equity. The Markowitz approach outlines that portfolios with highly uncorrelated individual asset returns also minimize risk. The investment assets with the computed results are perfectly uncorrelated according to the calculated variance covariance matrix Ω. The efficient frontier now has to be constructed to show the optimal investment portfolio. The figure in 4.1 illustrates the asset allocation for the equities for each simulated portfolio.

4.3.1 Equity Weights

The simulated portfolios have a minimum and maximum return of 4% to 12%. This is because, according to the statistics presented, the weekly returns for all equities are in this range. The mean asset returns are obtained by using the $formula = AVERAGE()$ in MS Excel and the results are as shown in Table 4.1.

These mean average returns were also utilized as a guideline for the projected returns in which all portfolios are populated between 5% and 12%. The majority of financial testing includes utilizing realized returns as a benchmark for expected returns. Because realized returns are unbiased estimates of predicted portfolio returns E(Rp), they are utilized as a proxy for anticipated returns. The returns shown in Table 3.1 are investment yields obtained using the company's current strategy. The pension fund's investment plan is a professional judgement strategy, such as the application of Investment Policy Strategy (IPS).

However, these qualitative measures do not allow the pension funds to achieve an optimal investment portfolio and better asset allocation strategies. It is recommended that the fund switch from this strategy to Markowitz. The Markowitz approach is a quantitative strategy which uses statistical analysis approach in order to monitor investment returns and produce the best trade offs for assets which minimize risk and maximise returns. The expected return of the portfolio is calculated by Excel SOLVER. The model changes the assumed return and weights for each asset class that is $XE(Rp) \geq Er$ then the statistical software goal seeks both equity weights and $E(Rp)$ until an optimal investment is found.

Table 4.1: Equity Weights

Equity	**Mean Return**
Axia	7.53%
BAT	7.81%
CSZL	9.70%
DLTA	9.05%
ECO	11.90%
HIPO	5.93%
INN	7.56%
SEED	7.86%
SIM	11.40%
OMZIL	6.24%

4.4 Investment Risk

In the Markowitz Approach the standard deviation of asset returns is the investment risk. It is computed using MS Excel with the formula below

$$Investment\ Risk = MMULT(MMULT(T\ RANSP\ OSE(X) : \Omega) : X) \quad (4.4.1)$$

where X is a column vector of all 6 available assets and Ω is the variance-covariance matrix for the returns. Calculations were done and the following results were obtained.

From Table 4.3 it can be noted that OMZIL, INN, SEED, BAT and HIPO have the lower risks. The possible optimal portfolio that was constructed gives allocations

to these asset classes. The Markowitz approach gives greater allocation to the less risky assets in order to minimize and gives balance between these risks and returns. Since the goal achieve higher rewards from investment greater allocations are given to the risky assets. These riskier equities include SIM, AXIA and CSZL. This is in line with the many theories and studies in financial economics that support high risk to high return concept.

4.5 The Efficient Frontier

An efficient frontier curve was plotted using the 20 portfolios simulated in MS Excel shown in Table 3.1. In the portfolios that were plotted a minimum variance portfolio is said to be the Optimal Portfolio of Risky Assets (OPRA). This portfolio contains the minimum possible risk and acceptable investment risk. The turning point of this frontier curve is the one with the optimal portfolio since the Markowitz approach assumes that investors are risk averse. The constructed portfolios have the floor and ceiling values of 5% and 12% respectively. Figure 4.2 below gives the best description of our portfolios.

The efficient frontier was plotted using risks and returns. The portfolio with return of 0.075 and risk of 0.08985 is said to be our minimum variance portfolio since all other portfolios below this portfolio can be obtained on the higher points of the frontier with higher returns and same risk. This is backed up by financial and economic theories which state that economic agents are non-satiated which means they prefer more to less. Investors will not consider a larger risk portfolio unless they expect a higher return, according to Dimson E. et al. (2002). As a result, when risk averse investors have to choose between two portfolios with the same amount of return, they select the one with the lowest investment risk. As a result, an investor will only take on a riskier portfolio in exchange for a higher return. Therefore, all portfolios with

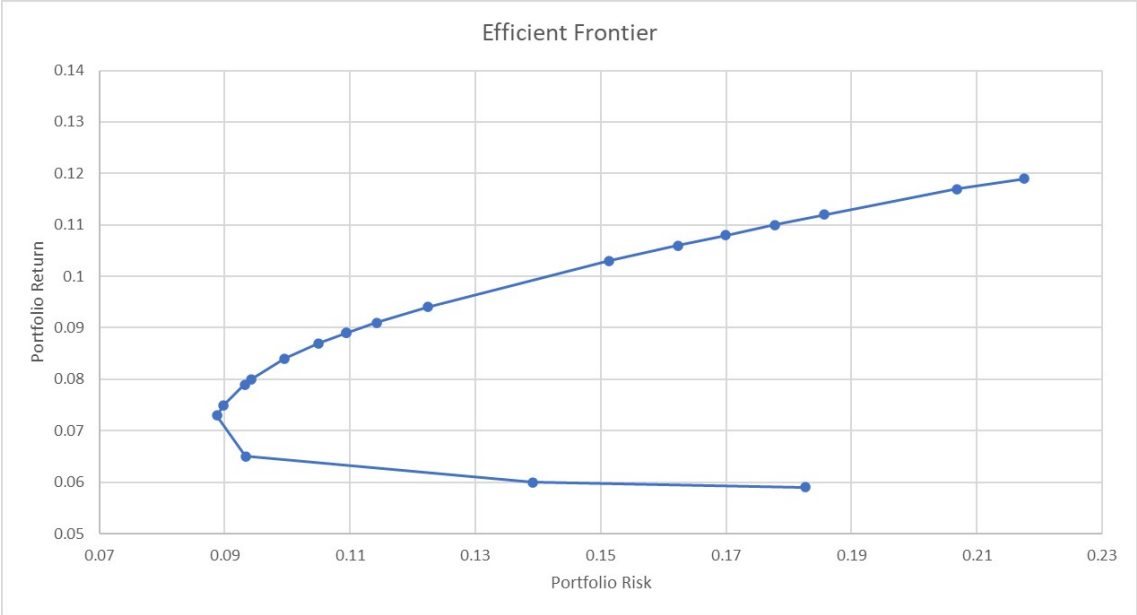

Figure 4.2: Efficient Frontier

return below 0.075 are inefficient. Efficient portfolios are the one with returns above 0.075. An explanation is given by the figure 4.3 given below.

The portfolios shown in the Figure 4.3 are all efficient. This is because, despite changing risks, they have higher investment returns. Feasible portfolios can be found in the area beneath the frontier curve. This is because the portfolios have returns ranging from 7.5% to 12%. Because the research assumes that pension funds are risk conservative, we consider the least variance portfolio to be our best option.

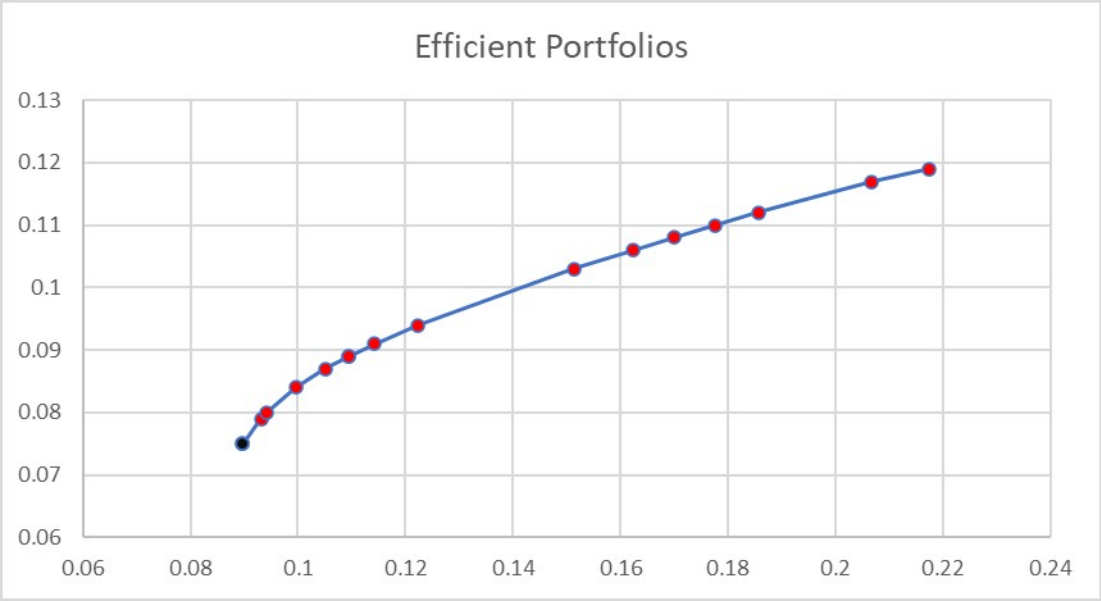

Figure 4.3: Efficient Portfolio

4.6 Parameter Calculations

All parameter computations were done in MS Excel. The predicted return of each portfolio was calculated using the formula below:

$$Predicted\ Return = SUMPRODUCT()\ of\ weights\ and\ individual\ asset\ returns\ (R_{is}). \tag{4.6.2}$$

SOLVER aim targeting the portfolio expected return E was used to calculate the asset allocations (Rp). In addition, the variance covariance matrix was calculated using the Analysis Toolpak in MS Excel. The asset-mix trade-offs were calculated using the formulas from Chapter 2. The returns, risks, and weights of investment vehicles were calculated using Microsoft Excel.

4.7 Optimization Model

The return, risk, and weights were optimized using Excel SOLVER. Given a return option, this non-linear programming model was utilized to calculate the risk, return, and weight of each portfolio.

4.8 Equity proportions for the Optimal Portfolio

Each portfolio has a proportion of total money invested in each equity. All of the proportions for all equities are non-negative because it is assumed that pension funds do not engage in short selling in their assets. The portfolio with the best balance, shown by the black point on the efficient frontier, has the following asset class weights

The optimal portfolio weights shown in Figure 4.4 below encourage investments in OMZIL, SIM and SEEDCO. Such an investment portfolio allows the pension fund to balance its tradeoffs.

The simulated portfolios in Table 3.1 was used to create the bar chart in Figure 4.4 that depicts the proportions. According to the portfolio weights, the ideal portfolio has seven equities: BAT, CSZL, ECO, HIPO, SEED, OMZIL and SIM. Table 4.4 shows that equities with relatively minor to no variance are included in the portfolio. This suggests that all less risky assets should be invested in order to increase investment returns and is good for risk averse investors.

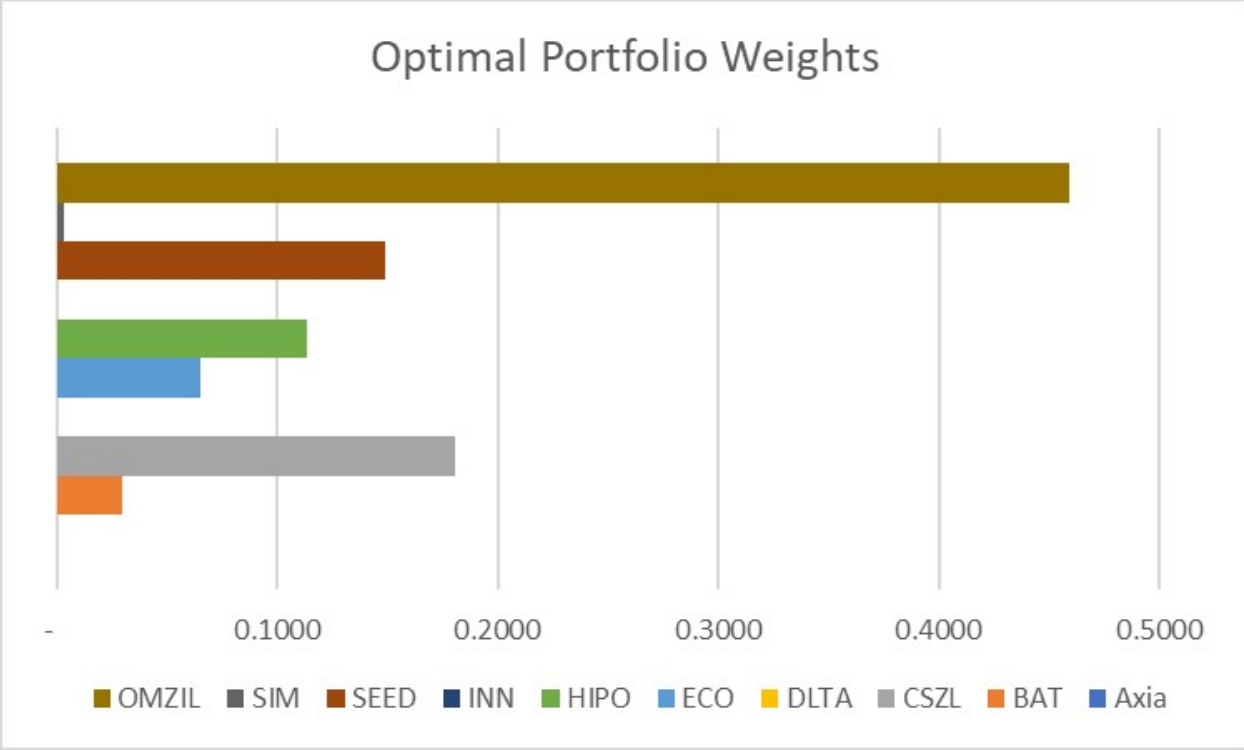

Figure 4.4: Optimum Portfolio Weights

4.9 Optimal Portfolio Using the Risk-Return Ratio

Another approach used in this study to find the optimal portfolio is the return-risk ratio which measures return per unit investment risk. Therefore, the portfolio with the highest ratio is the most feasible. For this research using the approach the optimal portfolio was at portfolio 7 with a ratio of 0.8485. This portfolio yields a return of 8% with an acceptable risk of 0.0943. The optimal portfolio weights are as follows in Figure 4.5.

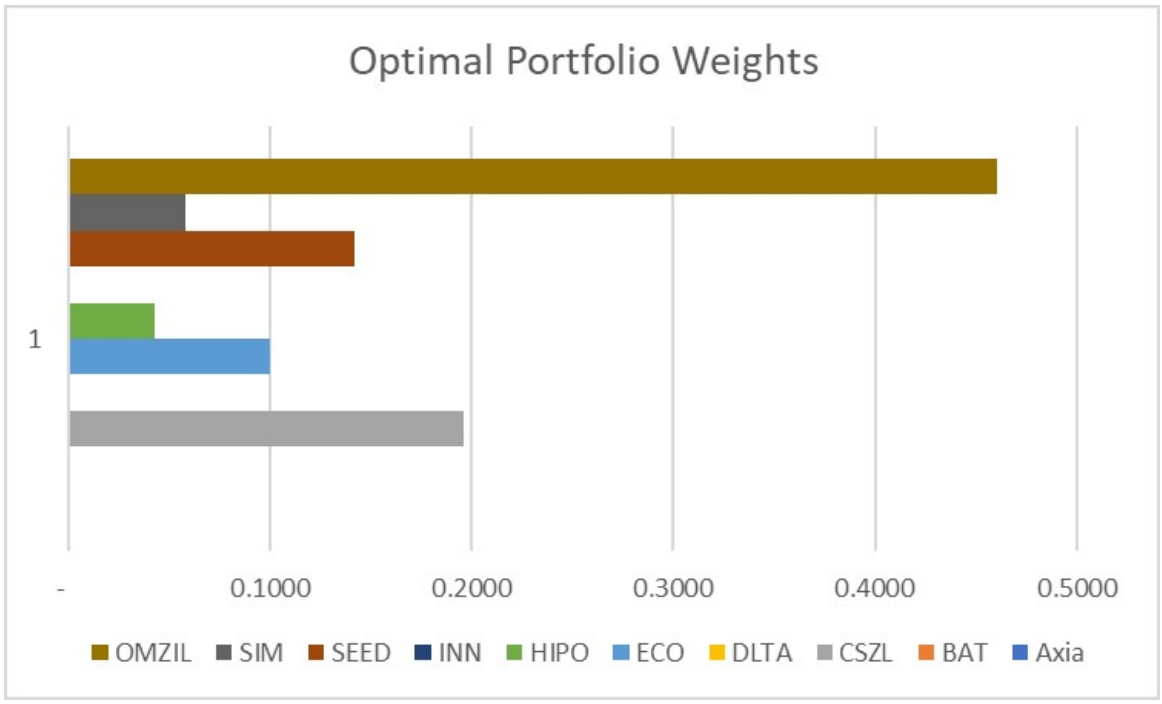

Figure 4.5: Optimum Portfolio Weights

From Figure 4.5, the highest equity portfolio weights are in OMZIL, CSZL, SEED and ECO. This portfolio also allocates wealth to the equities that are less risky but yield favorable just like the efficient frontier approach.

Table 4.2: Investment Portfolio Risk

Portfolio	**Return**	**Risk**
1	0.1170	0.20678
2	0.0840	0.09957
3	0.0650	0.09331
4	0.0590	0.18267
5	0.0790	0.09320
6	0.1190	0.21754
7	0.0870	0.10502
8	0.0890	0.10939
9	0.1080	0.16993
10	0.1120	0.18567
11	0.1100	0.17772
12	0.0600	0.13915
13	0.0750	0.08985
14	0.1030	0.15133
15	0.0730	0.08874
16	0.0910	0.11425
17	0.0800	0.09428
18	0.1060	0.16232
19	0.0890	0.10939
20	0.0940	0.12236

Table 4.3: Individual Asset Risk

Equity	**Risk**
Axia	0.2556
BAT	0.1789
CSZL	0.2332
DLTA	0.1929
ECO	0.2220
HIPO	0.1864
INN	0.1618
SEED	0.2099
SIM	0.2502
OMZIL	0.1524

Table 4.4: Ideal Portfolio Risk

Equity	**Risk**
BAT	0.0300
CSZL	0.1808
ECO	0.0652
HIPO	0.1132
SEED	0.1487
SIM	0.0029
OMZIL	0.4591

Table 4.5: Optimal Portfolio Using the Risk-Return Ratio

Portfolio	**Port. Std**	**Port.Mean**	**Ratio**
1	0.1827	0.0590	0.3230
2	0.1392	0.0600	0.4312
3	0.0933	0.0650	0.6966
4	0.0887	0.0730	0.8226
5	0.0898	0.0750	0.8347
6	0.0932	0.0790	0.8476
7	0.0943	0.0800	0.8485
8	0.0996	0.0840	0.8436
9	0.1050	0.0870	0.8284
10	0.1094	0.0890	0.8136
11	0.1094	0.0890	0.8136
12	0.1143	0.0910	0.7965
13	0.1224	0.0940	0.7682
14	0.1513	0.1030	0.6806
15	0.1623	0.1060	0.6530
16	0.1699	0.1080	0.6356
17	0.1777	0.1100	0.6190
18	0.1857	0.1120	0.6032
19	0.2068	0.1170	0.5658
20	0.2175	0.1190	0.5470

Chapter 5

Conclusions and Recommendations

5.1 Introduction

The research comes to an end in this chapter. The outcomes of the study are all relevant to the present body of literature and established portfolio management principles. This part also summarizes the findings of the investigation and makes recommendations.

5.2 Conclusions

The investment management of pension fund investments is the subject of this study. The study discovered that using Modern Portfolio Theory as an investment analysis tool in portfolio management can be extremely beneficial. It was taken into consideration because of the model's future returns. These investment returns strike a good balance between risk and reward. Various equities have been put together in an equities portfolio and the optimal portfolio with best balance trade-offs (0.08985;0.075) was chosen using the efficient frontier.

Portfolios have a variety of specific investment trade-offs to spread investment risks. The ideal portfolio stated that all risky investment vehicles should be avoided, and

the proportions of each portfolio were also calculated. In this study the pension fund is assumed to be risk averse investor. It's worth noting that organisations with a larger capital base and more wealth aren't always risk averse. The investor becomes more focused with the investment return and less worried with the risk of the investment. The return-risk ratio was used to create an alternative portfolio. Returns and investment risks from the simulated portfolios were used to compute these ratios. The alternative portfolio had the coordinates (0.0943;0.08).

5.3 Limitations

The fewer investment options in Zimbabwe limits the study. Bonds which are of lower risk are not available in the investment market. Due to this the investment vehicles available for pension schemes are then limited hence reducing diversification. The research will only be looking into asset allocation of equities that are listed on the Zimbabwe Stock Exchange. As a result, this limits the scope of the study and the accuracy of the research as it looks into one asset class.

5.4 Recommendations

The researcher will offer comments and advise to the research stakeholders based on the findings. The model used in this study could be beneficial to defined contribution pension funds. As is the case in academia, fellow researchers may be interested in this subject of study. They can use this to improve the model's precision when allocating wealth to securities and portfolios.

Optimal portfolios generated using the Modern portfolio model are responsive to changes in investment performance caused by economic variables. Some theories and more information must be provided to the ideal outcome in order to determine the

optimality. The mean variance technique, to be more exact, does not work on its own. It necessitates the application of additional approaches in order to generate more insightful outcomes. The Mean Variance technique has produced good returns in the past, making it a model that can be utilized in insurance asset management to reduce asset liability mismatches during downturns.

Finally, the study suggests that pension funds use this strategy. To be more specific, because businesses have variable quantities of wealth, risk aversion, and utility, they have varying levels of risk aversion and utility. Risk neutrality is possible for large investors, such as pension funds with solid financial positions. These businesses may be careless about investment risk and only care about the payout.

Because their capital market is limited, the report also urges pension funds to invest offshore. Asset managers are encouraged to participate actively in the rehabilitation of Zimbabwe's bond market because practically all of the assets accessible in Zimbabwe are risky. Because government bonds are risk-free, they can be used to diversify portfolios while reducing risk. It's also worth noting that the model can be utilized to deal with solvency difficulties, particularly liability modeling, which has an impact on pension fund funding levels.

Bibliography