

▣ Multiplicative inverse of  $-7$  modulo 20:

⇒ We need to find  $x$  such that  $-7x \equiv 1 \pmod{20}$

→ if  $ax \equiv 1 \pmod{m}$ , then  $x$  is the multiplicative inverse of  $a$  modulo  $m$ .

$$\rightarrow -7 \equiv 13 \pmod{20}$$

We are looking for an integer  $x$  such that  $13x = 20k + 1$  for some integer  $k$ .

We can test values of  $x$ :

if  $x = 1$ ,  $13(1) = 13 \not\equiv 1 \pmod{20}$

if  $x = 2$ ,  $13(2) = 26 \not\equiv 1 \pmod{20}$

if  $x = 3$ ,  $13(3) = 39 \not\equiv 1 \pmod{20}$

if  $x = 4$ ,  $13(4) = 52 \not\equiv 1 \pmod{20}$

if  $x = 5$ ,  $13(5) = 65 \not\equiv 1 \pmod{20}$

if  $x = 6$ ,  $13(6) = 78 \not\equiv 1 \pmod{20}$

if  $x = 7$ ,  $13(7) = 91 \not\equiv 1 \pmod{20}$

if  $x = 8$ ,  $13(8) = 104 \not\equiv 1 \pmod{20}$

if  $x = 9$ ,  $13(9) = 117 \not\equiv 1 \pmod{20}$

if  $x = 10$ ,  $13(10) = 130 \equiv 10 \pmod{20}$

if  $x = 11$ ,  $13(11) = 143 \equiv 3 \pmod{20}$

if  $x = 12$ ,  $13(12) = 156 \equiv 16 \pmod{20}$

if  $x = 13$ ,  $13(13) = 169 \equiv 9 \pmod{20}$



$$\text{if } x=14, \quad 13(14) = 182 \equiv 2 \pmod{20}$$

$$\text{if } x=15, \quad 13(15) = 195 \equiv 15 \pmod{20}$$

$$\text{if } x=16, \quad 13(16) = 208 \equiv 8 \pmod{20}$$

$$\text{if } x=17, \quad 13(17) = 221 \equiv 1 \pmod{20}$$

Thus  $x=17$  is multiplicative inverse of 13 modulo 20.

$\therefore$  The multiplicative inverse of  $-7$  made of 20 is 17.

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$$* \quad -17 \pmod{23} = (-17 + 23) = 6$$

$$23) -77 \overline{) -1}$$

$$1 \neq 23$$

$$2 \neq 46$$

$$3 \neq 69$$

$$4 \neq 92$$

$$5 \neq 115$$

$$6 \neq 138$$

$$7 \neq 161$$

$$8 \neq 184$$

$$\Rightarrow -17 = (-1 \times 23) + 6$$

$$-17 \pmod{23} = 6 \quad (\text{Ans})$$



\* Multiplication Inverse of  $-13 \bmod 23$ :

$\Rightarrow$  The multiplicative inverse of a number  $a$  made  $m$  is a  $x$  such that :  $ax \equiv 1 \bmod m$ .

In our case, we are looking for a number  $x$  such that:

$$-13x \equiv 1 \bmod 23$$

To simplify, we first convert  $-13$  into a positive equivalent modulo 23.

$$-13 \bmod 23 = -13 + 23 = 10$$

so, the equivalent equation become:

$$10x \equiv 1 \bmod 23$$

Now, we find the integer  $x$  such that  $10x \equiv 1 \bmod 23$

$$\text{if } x=1, 10 \times 1 = 10 \not\equiv 1 \bmod 23$$

$$\text{if } x=2, 10 \times 2 = 20 \not\equiv 1 \bmod 23$$

$$\text{if } x=3, 10 \times 3 = 30 \equiv 7 \bmod 23$$

if $x=4$	$10 \times 4 = 40 \equiv 17$	$\text{mod } 23$
if $x=5$	$10 \times 5 = 50 \equiv 4$	$\text{mod } 23$
if $x=6$	$10 \times 6 = 60 \equiv 14$	$\text{mod } 23$
if $x=7$	$10 \times 7 = 70 \equiv 1$	$\text{mod } 23$

We found it :  $10 \cdot 7 = 70 \equiv 1 \pmod{23}$

Since  $-13 \equiv 10 \pmod{23}$  and  $10^{-1} \pmod{23} = 7$

We conclude -

The multiplicative inverse of  $-13 \pmod{23}$  is 7 (Ans).