

$X=A$, $n=3$: Operators and Kinematic Factors

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January 3, 2025

(4, 1)

(Block 1) Trace = 0, Antisymmetric, C = -1

$$\begin{aligned} O_1^{A(4,1),1} &= O_{2,3,4} - O_{2,4,3} - O_{3,2,4} + O_{3,4,2} + O_{4,2,3} - O_{4,3,2} \\ K_1^{A(4,1),1} &= 0 \end{aligned}$$

$$\begin{aligned} O_2^{A(4,1),1} &= O_{1,3,4} - O_{1,4,3} - O_{3,1,4} + O_{3,4,1} + O_{4,1,3} - O_{4,3,1} \\ K_2^{A(4,1),1} &= 0 \end{aligned}$$

$$\begin{aligned} O_3^{A(4,1),1} &= O_{1,2,4} - O_{1,4,2} - O_{2,1,4} + O_{2,4,1} + O_{4,1,2} - O_{4,2,1} \\ K_3^{A(4,1),1} &= 0 \end{aligned}$$

$$\begin{aligned} O_4^{A(4,1),1} &= O_{1,2,3} - O_{1,3,2} - O_{2,1,3} + O_{2,3,1} + O_{3,1,2} - O_{3,2,1} \\ K_4^{A(4,1),1} &= 0 \end{aligned}$$

(4, 3)

(Block 1) Trace = 0, Symmetric, C = 1

$$O_1^{A(4,3),1} = O_{2,3,4} + O_{2,4,3} + O_{3,2,4} + O_{3,4,2} + O_{4,2,3} + O_{4,3,2}$$

$$K_1^{A(4,3),1} = \frac{p_2(-E(p)^3 - 2E(p)^2 m_N - E(p)m_N^2 + E(p)p_1^2 + E(p)p_2^2 - 5E(p)p_3^2 - 2m_N p_3^2)}{(E(p)(E(p) + m_N))}$$

$$O_2^{A(4,3),1} = O_{1,3,4} + O_{1,4,3} + O_{3,1,4} + O_{3,4,1} + O_{4,1,3} + O_{4,3,1}$$

$$K_2^{A(4,3),1} = \frac{p_1(-E(p)^3 - 2E(p)^2 m_N - E(p)m_N^2 + E(p)p_1^2 + E(p)p_2^2 - 5E(p)p_3^2 - 2m_N p_3^2)}{(E(p)(E(p) + m_N))}$$

$$O_3^{A(4,3),1} = O_{1,2,4} + O_{1,4,2} + O_{2,1,4} + O_{2,4,1} + O_{4,1,2} + O_{4,2,1}$$

$$K_3^{A(4,3),1} = \frac{-2p_1 p_2 p_3 (3E(p) + m_N)}{(E(p)(E(p) + m_N))}$$

$$O_4^{A(4,3),1} = O_{1,2,3} + O_{1,3,2} + O_{2,1,3} + O_{2,3,1} + O_{3,1,2} + O_{3,2,1}$$

$$K_4^{A(4,3),1} = \frac{ip_1 p_2 (E(p)^2 + 2E(p)m_N + m_N^2 - p_1^2 - p_2^2 + 5p_3^2)}{(E(p)(E(p) + m_N))}$$

(4, 4)

(Block 1) Trace = 0, Mixed Symmetry, C = mixed

$$O_1^{A(4,4),1} = O_{2,2,1} + O_{3,3,1} + O_{4,4,1}$$
$$K_1^{A(4,4),1} = \frac{ip_1 p_3 (-E(p)^2 + m_N^2 - p_1^2 + p_2^2 + p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_2^{A(4,4),1} = O_{1,1,2} + O_{3,3,2} + O_{4,4,2}$$
$$K_2^{A(4,4),1} = \frac{ip_2 p_3 (-E(p)^2 + m_N^2 + p_1^2 - p_2^2 + p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_3^{A(4,4),1} = O_{1,1,3} + O_{2,2,3} + O_{4,4,3}$$
$$K_3^{A(4,4),1} = \frac{ip_3^2 (-E(p)^2 - E(p)m_N + p_1^2 + p_2^2)}{(E(p)(E(p) + m_N))}$$

$$O_4^{A(4,4),1} = O_{1,1,4} + O_{2,2,4} + O_{3,3,4}$$
$$K_4^{A(4,4),1} = \frac{-p_3(E(p)^2 + 2E(p)m_N + m_N^2 + p_1^2 + p_2^2 + p_3^2)}{(2E(p) + 2m_N)}$$

(Block 2) Trace = 0, Mixed Symmetry, C = 1

$$O_1^{A(4,4),2} = O_{1,2,2} + O_{1,3,3} + O_{1,4,4}$$

$$K_1^{A(4,4),2} = \frac{ip_1p_3(-E(p)^2 + p_2^2 + p_3^2)}{(E(p)(E(p) + m_N))}$$

$$O_2^{A(4,4),2} = O_{2,1,1} + O_{2,3,3} + O_{2,4,4}$$

$$K_2^{A(4,4),2} = \frac{ip_2p_3(-E(p)^2 + p_1^2 + p_3^2)}{(E(p)(E(p) + m_N))}$$

$$O_3^{A(4,4),2} = O_{3,1,1} + O_{3,2,2} + O_{3,4,4}$$

$$K_3^{A(4,4),2} = \frac{i(-E(p)^2 + p_1^2 + p_2^2)(m_N(E(p) + m_N - p_3) + m_N(E(p) + m_N + p_3) + (E(p) - p_3)(E(p) + m_N - p_3) + (E(p) + p_3)(E(p) + m_N - p_3))}{(4E(p)(E(p) + m_N))}$$

$$O_4^{A(4,4),2} = O_{4,1,1} + O_{4,2,2} + O_{4,3,3}$$

$$K_4^{A(4,4),2} = \frac{p_3(-p_1^2 - p_2^2 - p_3^2)}{E(p)}$$

(Block 3) Trace = 0, Symmetric, C = 1

$$O_1^{A(4,4),3} = O_{1,1,1}$$

$$K_1^{A(4,4),3} = \frac{ip_1^3 p_3}{(E(p)(E(p) + m_N))}$$

$$O_2^{A(4,4),3} = O_{2,2,2}$$

$$K_2^{A(4,4),3} = \frac{ip_2^3 p_3}{(E(p)(E(p) + m_N))}$$

$$O_3^{A(4,4),3} = O_{3,3,3}$$

$$K_3^{A(4,4),3} = \frac{ip_3^2(E(p)^2 + 2E(p)m_N + m_N^2 - p_1^2 - p_2^2 + p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_4^{A(4,4),3} = O_{4,4,4}$$

$$K_4^{A(4,4),3} = E(p)p_3$$

(Block 4) Trace = 0, Mixed Symmetry, C = mixed

$$O_1^{A(4,4),4} = O_{2,1,2} + O_{3,1,3} + O_{4,1,4}$$

$$K_1^{A(4,4),4} = \frac{ip_1 p_3 (-E(p)^2 + m_N^2 - p_1^2 + p_2^2 + p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_2^{A(4,4),4} = O_{1,2,1} + O_{3,2,3} + O_{4,2,4}$$

$$K_2^{A(4,4),4} = \frac{ip_2 p_3 (-E(p)^2 + m_N^2 + p_1^2 - p_2^2 + p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_3^{A(4,4),4} = O_{1,3,1} + O_{2,3,2} + O_{4,3,4}$$

$$K_3^{A(4,4),4} = \frac{ip_3^2 (-E(p)^2 - E(p)m_N + p_1^2 + p_2^2)}{(E(p)(E(p) + m_N))}$$

$$O_4^{A(4,4),4} = O_{1,4,1} + O_{2,4,2} + O_{3,4,3}$$

$$K_4^{A(4,4),4} = \frac{-p_3(E(p)^2 + 2E(p)m_N + m_N^2 + p_1^2 + p_2^2 + p_3^2)}{(2E(p) + 2m_N)}$$

(8, 1)

(Block 1) Trace = 0, Mixed Symmetry, C = mixed

$$O_1^{A(8,1),1} = O_{2,3,4} - O_{2,4,3} - O_{3,4,2} + O_{4,3,2}$$

$$K_1^{A(8,1),1} = \frac{p_2(E(p)^3 + 2E(p)^2 m_N + E(p)m_N^2 - E(p)p_1^2 - E(p)p_2^2 - E(p)p_3^2 - 2m_N p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_2^{A(8,1),1} = O_{1,3,4} - O_{1,4,3} - O_{3,4,1} + O_{4,3,1}$$

$$K_2^{A(8,1),1} = \frac{p_1(E(p)^3 + 2E(p)^2 m_N + E(p)m_N^2 - E(p)p_1^2 - E(p)p_2^2 - E(p)p_3^2 - 2m_N p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_3^{A(8,1),1} = O_{1,2,4} - O_{2,1,4} - O_{4,1,2} + O_{4,2,1}$$

$$K_3^{A(8,1),1} = 0$$

$$O_4^{A(8,1),1} = O_{1,2,3} - O_{2,1,3} - O_{3,1,2} + O_{3,2,1}$$

$$K_4^{A(8,1),1} = 0$$

$$O_5^{A(8,1),1} = O_{2,3,4} + O_{2,4,3} - 2O_{3,2,4} + O_{3,4,2} - 2O_{4,2,3} + O_{4,3,2}$$

$$K_5^{A(8,1),1} = \frac{p_2(E(p)^3 + 2E(p)^2 m_N + E(p)m_N^2 - E(p)p_1^2 - E(p)p_2^2 - E(p)p_3^2 + 2m_N p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_6^{A(8,1),1} = O_{1,3,4} + O_{1,4,3} - 2O_{3,1,4} + O_{3,4,1} - 2O_{4,1,3} + O_{4,3,1}$$

$$K_6^{A(8,1),1} = \frac{p_1(E(p)^3 + 2E(p)^2 m_N + E(p)m_N^2 - E(p)p_1^2 - E(p)p_2^2 - E(p)p_3^2 + 2m_N p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_7^{A(8,1),1} = O_{1,2,4} - 2O_{1,4,2} + O_{2,1,4} - 2O_{2,4,1} + O_{4,1,2} + O_{4,2,1}$$

$$K_7^{A(8,1),1} = \frac{-2m_N p_1 p_2 p_3}{(E(p)(E(p) + m_N))}$$

$$O_8^{A(8,1),1} = O_{1,2,3} - 2O_{1,3,2} + O_{2,1,3} - 2O_{2,3,1} + O_{3,1,2} + O_{3,2,1}$$

$$K_8^{A(8,1),1} = \frac{ip_1 p_2 (E(p)^2 + 2E(p)m_N + m_N^2 - p_1^2 - p_2^2 - p_3^2)}{(E(p)(E(p) + m_N))}$$

(Block 2) Trace = 0, Mixed Symmetry, C = mixed

$$O_1^{A(8,1),2} = O_{2,3,4} - O_{2,4,3} + O_{3,2,4} - O_{4,2,3}$$

$$K_1^{A(8,1),2} = \frac{p_2(-E(p)^3 - 2E(p)^2 m_N - E(p)m_N^2 + E(p)p_1^2 + E(p)p_2^2 + E(p)p_3^2 + 2m_N p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_2^{A(8,1),2} = O_{1,3,4} - O_{1,4,3} + O_{3,1,4} - O_{4,1,3}$$

$$K_2^{A(8,1),2} = \frac{p_1(-E(p)^3 - 2E(p)^2 m_N - E(p)m_N^2 + E(p)p_1^2 + E(p)p_2^2 + E(p)p_3^2 + 2m_N p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_3^{A(8,1),2} = O_{1,4,2} - O_{2,4,1} + O_{4,1,2} - O_{4,2,1}$$

$$K_3^{A(8,1),2} = 0$$

$$O_4^{A(8,1),2} = O_{1,3,2} - O_{2,3,1} + O_{3,1,2} - O_{3,2,1}$$

$$K_4^{A(8,1),2} = 0$$

$$O_5^{A(8,1),2} = O_{2,3,4} + O_{2,4,3} + O_{3,2,4} - 2O_{3,4,2} + O_{4,2,3} - 2O_{4,3,2}$$

$$K_5^{A(8,1),2} = \frac{p_2(E(p)^3 + 2E(p)^2 m_N + E(p)m_N^2 - E(p)p_1^2 - E(p)p_2^2 - E(p)p_3^2 + 2m_N p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_6^{A(8,1),2} = O_{1,3,4} + O_{1,4,3} + O_{3,1,4} - 2O_{3,4,1} + O_{4,1,3} - 2O_{4,3,1}$$

$$K_6^{A(8,1),2} = \frac{p_1(E(p)^3 + 2E(p)^2 m_N + E(p)m_N^2 - E(p)p_1^2 - E(p)p_2^2 - E(p)p_3^2 + 2m_N p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_7^{A(8,1),2} = O_{1,2,4} - O_{1,4,2}/2 + O_{2,1,4} - O_{2,4,1}/2 - O_{4,1,2}/2 - O_{4,2,1}/2$$

$$K_7^{A(8,1),2} = \frac{m_N p_1 p_2 p_3}{(E(p)(E(p) + m_N))}$$

$$O_8^{A(8,1),2} = O_{1,2,3} - O_{1,3,2}/2 + O_{2,1,3} - O_{2,3,1}/2 - O_{3,1,2}/2 - O_{3,2,1}/2$$

$$K_8^{A(8,1),2} = \frac{ip_1 p_2 (-E(p)^2 - 2E(p)m_N - m_N^2 + p_1^2 + p_2^2 + p_3^2)}{(2E(p)(E(p) + m_N))}$$

(8, 2)

(Block 1) Trace = 0, Mixed Symmetry, C = 1

$$O_1^{A(8,2),1} = O_{1,3,3} - O_{1,4,4}$$

$$K_1^{A(8,2),1} = \frac{ip_1 p_3 (E(p)^2 + p_3^2)}{(E(p)(E(p) + m_N))}$$

$$O_2^{A(8,2),1} = O_{2,3,3} - O_{2,4,4}$$

$$K_2^{A(8,2),1} = \frac{ip_2 p_3 (E(p)^2 + p_3^2)}{(E(p)(E(p) + m_N))}$$

$$O_3^{A(8,2),1} = O_{3,1,1} - O_{3,2,2}$$

$$K_3^{A(8,2),1} = \frac{i(p_1^2 - p_2^2)(m_N(E(p) + m_N - p_3) + m_N(E(p) + m_N + p_3) + (E(p) - p_3)(E(p) + m_N - p_3) + (E(p) + p_3)(E(p) + m_N + p_3))}{(4E(p)(E(p) + m_N))}$$

$$O_4^{A(8,2),1} = O_{4,1,1} - O_{4,2,2}$$

$$K_4^{A(8,2),1} = \frac{p_3(-p_1^2 + p_2^2)}{E(p)}$$

$$O_5^{A(8,2),1} = O_{1,2,2} - O_{1,3,3}/2 - O_{1,4,4}/2$$

$$K_5^{A(8,2),1} = \frac{ip_1 p_3 (E(p)^2 + 2p_2^2 - p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_6^{A(8,2),1} = O_{2,1,1} - O_{2,3,3}/2 - O_{2,4,4}/2$$

$$K_6^{A(8,2),1} = \frac{ip_2 p_3 (E(p)^2 + 2p_1^2 - p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_7^{A(8,2),1} = O_{3,1,1} + O_{3,2,2} - 2O_{3,4,4}$$

$$K_7^{A(8,2),1} = \frac{i(2E(p)^2 + p_1^2 + p_2^2)(m_N(E(p) + m_N - p_3) + m_N(E(p) + m_N + p_3) + (E(p) - p_3)(E(p) + m_N - p_3) + (E(p) + p_3)(E(p) + m_N + p_3))}{(4E(p)(E(p) + m_N))}$$

$$O_8^{A(8,2),1} = O_{4,1,1} + O_{4,2,2} - 2O_{4,3,3}$$

$$K_8^{A(8,2),1} = \frac{p_3(-p_1^2 - p_2^2 + 2p_3^2)}{E(p)}$$

(Block 2) Trace = 0, Mixed Symmetry, C = mixed

$$O_1^{A(8,2),2} = O_{3,3,1} - O_{4,4,1}$$

$$K_1^{A(8,2),2} = \frac{ip_1 p_3 (3E(p)^2 + 4E(p)m_N + m_N^2 - p_1^2 - p_2^2 + p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_2^{A(8,2),2} = O_{3,3,2} - O_{4,4,2}$$

$$K_2^{A(8,2),2} = \frac{ip_2 p_3 (3E(p)^2 + 4E(p)m_N + m_N^2 - p_1^2 - p_2^2 + p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_3^{A(8,2),2} = O_{1,1,3} - O_{2,2,3}$$

$$K_3^{A(8,2),2} = \frac{ip_3^2(p_1^2 - p_2^2)}{(E(p)(E(p) + m_N))}$$

$$O_4^{A(8,2),2} = O_{1,1,4} - O_{2,2,4}$$

$$K_4^{A(8,2),2} = \frac{p_3(-p_1^2 + p_2^2)}{(E(p) + m_N)}$$

$$O_5^{A(8,2),2} = O_{2,2,1} - O_{3,3,1}/2 - O_{4,4,1}/2$$

$$K_5^{A(8,2),2} = \frac{ip_1 p_3 (E(p)^2 - m_N^2 + p_1^2 + 5p_2^2 - p_3^2)}{(4E(p)(E(p) + m_N))}$$

$$O_6^{A(8,2),2} = O_{1,1,2} - O_{3,3,2}/2 - O_{4,4,2}/2$$

$$K_6^{A(8,2),2} = \frac{ip_2 p_3 (E(p)^2 - m_N^2 + 5p_1^2 + p_2^2 - p_3^2)}{(4E(p)(E(p) + m_N))}$$

$$O_7^{A(8,2),2} = O_{1,1,3} + O_{2,2,3} - 2O_{4,4,3}$$

$$K_7^{A(8,2),2} = \frac{ip_3^2(2E(p)^2 + 2E(p)m_N + p_1^2 + p_2^2)}{(E(p)(E(p) + m_N))}$$

$$O_8^{A(8,2),2} = O_{1,1,4} + O_{2,2,4} - 2O_{3,3,4}$$

$$K_8^{A(8,2),2} = \frac{p_3(E(p)^2 + 2E(p)m_N + m_N^2 - 2p_1^2 - 2p_2^2 + p_3^2)}{(E(p) + m_N)}$$

(Block 3) Trace = 0, Mixed Symmetry, C = mixed

$$O_1^{A(8,2),3} = O_{3,1,3} - O_{4,1,4}$$

$$K_1^{A(8,2),3} = \frac{ip_1 p_3 (3E(p)^2 + 4E(p)m_N + m_N^2 - p_1^2 - p_2^2 + p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_2^{A(8,2),3} = O_{3,2,3} - O_{4,2,4}$$

$$K_2^{A(8,2),3} = \frac{ip_2 p_3 (3E(p)^2 + 4E(p)m_N + m_N^2 - p_1^2 - p_2^2 + p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_3^{A(8,2),3} = O_{1,3,1} - O_{2,3,2}$$

$$K_3^{A(8,2),3} = \frac{ip_3^2(p_1^2 - p_2^2)}{(E(p)(E(p) + m_N))}$$

$$O_4^{A(8,2),3} = O_{1,4,1} - O_{2,4,2}$$

$$K_4^{A(8,2),3} = \frac{p_3(-p_1^2 + p_2^2)}{(E(p) + m_N)}$$

$$O_5^{A(8,2),3} = O_{2,1,2} - O_{3,1,3}/2 - O_{4,1,4}/2$$

$$K_5^{A(8,2),3} = \frac{ip_1 p_3 (E(p)^2 - m_N^2 + p_1^2 + 5p_2^2 - p_3^2)}{(4E(p)(E(p) + m_N))}$$

$$O_6^{A(8,2),3} = O_{1,2,1} - O_{3,2,3}/2 - O_{4,2,4}/2$$

$$K_6^{A(8,2),3} = \frac{ip_2 p_3 (E(p)^2 - m_N^2 + 5p_1^2 + p_2^2 - p_3^2)}{(4E(p)(E(p) + m_N))}$$

$$O_7^{A(8,2),3} = O_{1,3,1} + O_{2,3,2} - 2O_{4,3,4}$$

$$K_7^{A(8,2),3} = \frac{ip_3^2(2E(p)^2 + 2E(p)m_N + p_1^2 + p_2^2)}{(E(p)(E(p) + m_N))}$$

$$O_8^{A(8,2),3} = O_{1,4,1} + O_{2,4,2} - 2O_{3,4,3}$$

$$K_8^{A(8,2),3} = \frac{p_3(E(p)^2 + 2E(p)m_N + m_N^2 - 2p_1^2 - 2p_2^2 + p_3^2)}{(E(p) + m_N)}$$