

# $X=V$ , $n=3$ : Operators and Kinematic Factors

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**(4, 1)**

**(Block 1) Trace = 0, Mixed Symmetry, C = mixed**

$$O_1^{V(4,1),1} = O_{2,1,2} + O_{3,1,3} + O_{4,1,4}$$

$$K_1^{V(4,1),1} = \frac{ip_1(E(p)^3 + 2E(p)^2m_N + E(p)m_N^2 + E(p)p_1^2 - E(p)p_2^2 - E(p)p_3^2 - 2m_Np_2^2 - 2m_Np_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_2^{V(4,1),1} = O_{1,2,1} + O_{3,2,3} + O_{4,2,4}$$

$$K_2^{V(4,1),1} = \frac{ip_2(E(p)^3 + 2E(p)^2m_N + E(p)m_N^2 - E(p)p_1^2 + E(p)p_2^2 - E(p)p_3^2 - 2m_Np_1^2 - 2m_Np_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_3^{V(4,1),1} = O_{1,3,1} + O_{2,3,2} + O_{4,3,4}$$

$$K_3^{V(4,1),1} = \frac{ip_3(E(p)^3 + 2E(p)^2m_N + E(p)m_N^2 - E(p)p_1^2 - E(p)p_2^2 + E(p)p_3^2 - 2m_Np_1^2 - 2m_Np_2^2)}{(2E(p)(E(p) + m_N))}$$

$$O_4^{V(4,1),1} = O_{1,4,1} + O_{2,4,2} + O_{3,4,3}$$

$$K_4^{V(4,1),1} = p_1^2 + p_2^2 + p_3^2$$

**(Block 2) Trace = 0, Mixed Symmetry, C = mixed**

$$O_1^{V(4,1),2} = O_{2,2,1} + O_{3,3,1} + O_{4,4,1}$$

$$K_1^{V(4,1),2} = \frac{ip_1(E(p)^3 + 2E(p)^2m_N + E(p)m_N^2 + E(p)p_1^2 - E(p)p_2^2 - E(p)p_3^2 - 2m_Np_2^2 - 2m_Np_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_2^{V(4,1),2} = O_{1,1,2} + O_{3,3,2} + O_{4,4,2}$$

$$K_2^{V(4,1),2} = \frac{ip_2(E(p)^3 + 2E(p)^2m_N + E(p)m_N^2 - E(p)p_1^2 + E(p)p_2^2 - E(p)p_3^2 - 2m_Np_1^2 - 2m_Np_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_3^{V(4,1),2} = O_{1,1,3} + O_{2,2,3} + O_{4,4,3}$$

$$K_3^{V(4,1),2} = \frac{ip_3(E(p)^3 + 2E(p)^2m_N + E(p)m_N^2 - E(p)p_1^2 - E(p)p_2^2 + E(p)p_3^2 - 2m_Np_1^2 - 2m_Np_2^2)}{(2E(p)(E(p) + m_N))}$$

$$O_4^{V(4,1),2} = O_{1,1,4} + O_{2,2,4} + O_{3,3,4}$$

$$K_4^{V(4,1),2} = p_1^2 + p_2^2 + p_3^2$$

**(Block 3) Trace = 0, Symmetric, C = -1**

$$O_1^{V(4,1),3} = O_{1,1,1}$$

$$K_1^{V(4,1),3} = \frac{-ip_1^3}{E(p)}$$

$$O_2^{V(4,1),3} = O_{2,2,2}$$

$$K_2^{V(4,1),3} = \frac{-ip_2^3}{E(p)}$$

$$O_3^{V(4,1),3} = O_{3,3,3}$$

$$K_3^{V(4,1),3} = \frac{-ip_3^3}{E(p)}$$

$$O_4^{V(4,1),3} = O_{4,4,4}$$

$$K_4^{V(4,1),3} = \frac{-E(p)(E(p)^2 + 2E(p)m_N + m_N^2 + p_1^2 + p_2^2 + p_3^2)}{(2E(p) + 2m_N)}$$

**(Block 4) Trace = 0, Mixed Symmetry, C = -1**

$$O_1^{V(4,1),4} = O_{1,2,2} + O_{1,3,3} + O_{1,4,4}$$

$$K_1^{V(4,1),4} = \frac{ip_1(E(p)^2 - p_2^2 - p_3^2)}{E(p)}$$

$$O_2^{V(4,1),4} = O_{2,1,1} + O_{2,3,3} + O_{2,4,4}$$

$$K_2^{V(4,1),4} = \frac{ip_2(E(p)^2 - p_1^2 - p_3^2)}{E(p)}$$

$$O_3^{V(4,1),4} = O_{3,1,1} + O_{3,2,2} + O_{3,4,4}$$

$$K_3^{V(4,1),4} = \frac{ip_3(E(p)^2 - p_1^2 - p_2^2)}{E(p)}$$

$$O_4^{V(4,1),4} = O_{4,1,1} + O_{4,2,2} + O_{4,3,3}$$

$$K_4^{V(4,1),4} = \frac{(p_1^2 + p_2^2 + p_3^2)(m_N(E(p) + m_N - p_3) + m_N(E(p) + m_N + p_3) + (E(p) - p_3)(E(p) + m_N - p_3) + (E(p) + p_3)(E(p) + m_N - p_3))}{(4E(p)(E(p) + m_N))}$$

**(4, 2)**

**(Block 1) Trace = 0, Symmetric, C = -1**

$$O_1^{V(4,2),1} = O_{2,3,4} + O_{2,4,3} + O_{3,2,4} + O_{3,4,2} + O_{4,2,3} + O_{4,3,2}$$

$$K_1^{V(4,2),1} = \frac{p_2 p_3 (5E(p)^2 + 6E(p)m_N + m_N^2 + p_1^2 + p_2^2 + p_3^2)}{(E(p)(E(p) + m_N))}$$

$$O_2^{V(4,2),1} = O_{1,3,4} + O_{1,4,3} + O_{3,1,4} + O_{3,4,1} + O_{4,1,3} + O_{4,3,1}$$

$$K_2^{V(4,2),1} = \frac{p_1 p_3 (5E(p)^2 + 6E(p)m_N + m_N^2 + p_1^2 + p_2^2 + p_3^2)}{(E(p)(E(p) + m_N))}$$

$$O_3^{V(4,2),1} = O_{1,2,4} + O_{1,4,2} + O_{2,1,4} + O_{2,4,1} + O_{4,1,2} + O_{4,2,1}$$

$$K_3^{V(4,2),1} = \frac{p_1 p_2 (5E(p)^2 + 6E(p)m_N + m_N^2 + p_1^2 + p_2^2 + p_3^2)}{(E(p)(E(p) + m_N))}$$

$$O_4^{V(4,2),1} = O_{1,2,3} + O_{1,3,2} + O_{2,1,3} + O_{2,3,1} + O_{3,1,2} + O_{3,2,1}$$

$$K_4^{V(4,2),1} = \frac{-6ip_1 p_2 p_3}{E(p)}$$

**(4, 4)**

**(Block 1) Trace = 0, Antisymmetric, C = 1**

$$\begin{aligned} O_1^{V(4,4),1} &= O_{2,3,4} - O_{2,4,3} - O_{3,2,4} + O_{3,4,2} + O_{4,2,3} - O_{4,3,2} \\ K_1^{V(4,4),1} &= 0 \end{aligned}$$

$$\begin{aligned} O_2^{V(4,4),1} &= O_{1,3,4} - O_{1,4,3} - O_{3,1,4} + O_{3,4,1} + O_{4,1,3} - O_{4,3,1} \\ K_2^{V(4,4),1} &= 0 \end{aligned}$$

$$\begin{aligned} O_3^{V(4,4),1} &= O_{1,2,4} - O_{1,4,2} - O_{2,1,4} + O_{2,4,1} + O_{4,1,2} - O_{4,2,1} \\ K_3^{V(4,4),1} &= 0 \end{aligned}$$

$$\begin{aligned} O_4^{V(4,4),1} &= O_{1,2,3} - O_{1,3,2} - O_{2,1,3} + O_{2,3,1} + O_{3,1,2} - O_{3,2,1} \\ K_4^{V(4,4),1} &= 0 \end{aligned}$$

**(8, 1)**

**(Block 1) Trace = 0, Mixed Symmetry, C = mixed**

$$O_1^{V(8,1),1} = O_{2,1,2} - O_{3,1,3}/2 - O_{4,1,4}/2$$

$$K_1^{V(8,1),1} = \frac{ip_1(-E(p)^3 - 2E(p)^2m_N - E(p)m_N^2 - E(p)p_1^2 - 5E(p)p_2^2 + E(p)p_3^2 - 4m_Np_2^2 + 2m_Np_3^2)}{(4E(p)(E(p) + m_N))}$$

$$O_2^{V(8,1),1} = O_{1,2,1} - O_{3,2,3}/2 - O_{4,2,4}/2$$

$$K_2^{V(8,1),1} = \frac{ip_2(-E(p)^3 - 2E(p)^2m_N - E(p)m_N^2 - 5E(p)p_1^2 - E(p)p_2^2 + E(p)p_3^2 - 4m_Np_1^2 + 2m_Np_3^2)}{(4E(p)(E(p) + m_N))}$$

$$O_3^{V(8,1),1} = O_{1,3,1} + O_{2,3,2} - 2O_{4,3,4}$$

$$K_3^{V(8,1),1} = \frac{-ip_3(E(p)^3 + 2E(p)^2m_N + E(p)m_N^2 + 2E(p)p_1^2 + 2E(p)p_2^2 + E(p)p_3^2 + m_Np_1^2 + m_Np_2^2)}{(E(p)(E(p) + m_N))}$$

$$O_4^{V(8,1),1} = O_{1,4,1} + O_{2,4,2} - 2O_{3,4,3}$$

$$K_4^{V(8,1),1} = p_1^2 + p_2^2 - 2p_3^2$$

$$O_5^{V(8,1),1} = O_{3,1,3} - O_{4,1,4}$$

$$K_5^{V(8,1),1} = \frac{-ip_1(E(p)^3 + 2E(p)^2m_N + E(p)m_N^2 + E(p)p_1^2 + E(p)p_2^2 + 3E(p)p_3^2 + 2m_Np_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_6^{V(8,1),1} = O_{3,2,3} - O_{4,2,4}$$

$$K_6^{V(8,1),1} = \frac{-ip_2(E(p)^3 + 2E(p)^2m_N + E(p)m_N^2 + E(p)p_1^2 + E(p)p_2^2 + 3E(p)p_3^2 + 2m_Np_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_7^{V(8,1),1} = O_{1,3,1} - O_{2,3,2}$$

$$K_7^{V(8,1),1} = \frac{ip_3(-p_1^2 + p_2^2)}{E(p)}$$

$$O_8^{V(8,1),1} = O_{1,4,1} - O_{2,4,2}$$

$$K_8^{V(8,1),1} = p_1^2 - p_2^2$$



(Block 2) Trace = 0, Mixed Symmetry, C = mixed

$$O_1^{V(8,1),2} = O_{2,2,1} - O_{3,3,1}/2 - O_{4,4,1}/2$$

$$K_1^{V(8,1),2} = \frac{ip_1(-E(p)^3 - 2E(p)^2m_N - E(p)m_N^2 - E(p)p_1^2 - 5E(p)p_2^2 + E(p)p_3^2 - 4m_Np_2^2 + 2m_Np_3^2)}{(4E(p)(E(p) + m_N))}$$

$$O_2^{V(8,1),2} = O_{1,1,2} - O_{3,3,2}/2 - O_{4,4,2}/2$$

$$K_2^{V(8,1),2} = \frac{ip_2(-E(p)^3 - 2E(p)^2m_N - E(p)m_N^2 - 5E(p)p_1^2 - E(p)p_2^2 + E(p)p_3^2 - 4m_Np_1^2 + 2m_Np_3^2)}{(4E(p)(E(p) + m_N))}$$

$$O_3^{V(8,1),2} = O_{1,1,3} + O_{2,2,3} - 2O_{4,4,3}$$

$$K_3^{V(8,1),2} = \frac{-ip_3(E(p)^3 + 2E(p)^2m_N + E(p)m_N^2 + 2E(p)p_1^2 + 2E(p)p_2^2 + E(p)p_3^2 + m_Np_1^2 + m_Np_2^2)}{(E(p)(E(p) + m_N))}$$

$$O_4^{V(8,1),2} = O_{1,1,4} + O_{2,2,4} - 2O_{3,3,4}$$

$$K_4^{V(8,1),2} = p_1^2 + p_2^2 - 2p_3^2$$

$$O_5^{V(8,1),2} = O_{3,3,1} - O_{4,4,1}$$

$$K_5^{V(8,1),2} = \frac{-ip_1(E(p)^3 + 2E(p)^2m_N + E(p)m_N^2 + E(p)p_1^2 + E(p)p_2^2 + 3E(p)p_3^2 + 2m_Np_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_6^{V(8,1),2} = O_{3,3,2} - O_{4,4,2}$$

$$K_6^{V(8,1),2} = \frac{-ip_2(E(p)^3 + 2E(p)^2m_N + E(p)m_N^2 + E(p)p_1^2 + E(p)p_2^2 + 3E(p)p_3^2 + 2m_Np_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_7^{V(8,1),2} = O_{1,1,3} - O_{2,2,3}$$

$$K_7^{V(8,1),2} = \frac{ip_3(-p_1^2 + p_2^2)}{E(p)}$$

$$O_8^{V(8,1),2} = O_{1,1,4} - O_{2,2,4}$$

$$K_8^{V(8,1),2} = p_1^2 - p_2^2$$

**(Block 3) Trace = 0, Mixed Symmetry, C = -1**

$$O_1^{V(8,1),3} = O_{1,2,2} - O_{1,3,3}/2 - O_{1,4,4}/2$$

$$K_1^{V(8,1),3} = \frac{ip_1(-E(p)^2 - 2p_2^2 + p_3^2)}{(2E(p))}$$

$$O_2^{V(8,1),3} = O_{2,1,1} - O_{2,3,3}/2 - O_{2,4,4}/2$$

$$K_2^{V(8,1),3} = \frac{ip_2(-E(p)^2 - 2p_1^2 + p_3^2)}{(2E(p))}$$

$$O_3^{V(8,1),3} = O_{3,1,1} + O_{3,2,2} - 2O_{3,4,4}$$

$$K_3^{V(8,1),3} = \frac{ip_3(-2E(p)^2 - p_1^2 - p_2^2)}{E(p)}$$

$$O_4^{V(8,1),3} = O_{4,1,1} + O_{4,2,2} - 2O_{4,3,3}$$

$$K_4^{V(8,1),3} = \frac{(p_1^2 + p_2^2 - 2p_3^2)(m_N(E(p) + m_N - p_3) + m_N(E(p) + m_N + p_3) + (E(p) - p_3)(E(p) + m_N - p_3) + (E(p) + p_3)(E(p) + m_N - p_3))}{(4E(p)(E(p) + m_N))}$$

$$O_5^{V(8,1),3} = O_{1,3,3} - O_{1,4,4}$$

$$K_5^{V(8,1),3} = \frac{ip_1(-E(p)^2 - p_3^2)}{E(p)}$$

$$O_6^{V(8,1),3} = O_{2,3,3} - O_{2,4,4}$$

$$K_6^{V(8,1),3} = \frac{ip_2(-E(p)^2 - p_3^2)}{E(p)}$$

$$O_7^{V(8,1),3} = O_{3,1,1} - O_{3,2,2}$$

$$K_7^{V(8,1),3} = \frac{ip_3(-p_1^2 + p_2^2)}{E(p)}$$

$$O_8^{V(8,1),3} = O_{4,1,1} - O_{4,2,2}$$

$$K_8^{V(8,1),3} = \frac{(p_1^2 - p_2^2)(m_N(E(p) + m_N - p_3) + m_N(E(p) + m_N + p_3) + (E(p) - p_3)(E(p) + m_N - p_3) + (E(p) + p_3)(E(p) + m_N - p_3))}{(4E(p)(E(p) + m_N))}$$

**(8, 2)**

**(Block 1) Trace = 0, Mixed Symmetry, C = mixed**

$$O_1^{V(8,2),1} = O_{2,3,4} + O_{2,4,3} - O_{3,2,4} - O_{4,2,3}$$

$$K_1^{V(8,2),1} = \frac{p_2 p_3 (E(p)^2 - m_N^2 - p_1^2 - p_2^2 - p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_2^{V(8,2),1} = O_{1,3,4} + O_{1,4,3} - O_{3,1,4} - O_{4,1,3}$$

$$K_2^{V(8,2),1} = \frac{p_1 p_3 (E(p)^2 - m_N^2 - p_1^2 - p_2^2 - p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_3^{V(8,2),1} = O_{1,4,2} + O_{2,4,1} - O_{4,1,2} - O_{4,2,1}$$

$$K_3^{V(8,2),1} = \frac{p_1 p_2 (E(p)^2 - m_N^2 - p_1^2 - p_2^2 - p_3^2)}{(E(p)(E(p) + m_N))}$$

$$O_4^{V(8,2),1} = O_{1,3,2} + O_{2,3,1} - O_{3,1,2} - O_{3,2,1}$$

$$K_4^{V(8,2),1} = 0$$

$$O_5^{V(8,2),1} = O_{2,3,4} - O_{2,4,3} - O_{3,2,4} - 2O_{3,4,2} + O_{4,2,3} + 2O_{4,3,2}$$

$$K_5^{V(8,2),1} = \frac{3p_2 p_3 (-E(p)^2 + m_N^2 + p_1^2 + p_2^2 + p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_6^{V(8,2),1} = O_{1,3,4} - O_{1,4,3} - O_{3,1,4} - 2O_{3,4,1} + O_{4,1,3} + 2O_{4,3,1}$$

$$K_6^{V(8,2),1} = \frac{3p_1 p_3 (-E(p)^2 + m_N^2 + p_1^2 + p_2^2 + p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_7^{V(8,2),1} = O_{1,2,4} + O_{1,4,2}/2 - O_{2,1,4} - O_{2,4,1}/2 - O_{4,1,2}/2 + O_{4,2,1}/2$$

$$K_7^{V(8,2),1} = 0$$

$$O_8^{V(8,2),1} = O_{1,2,3} + O_{1,3,2}/2 - O_{2,1,3} - O_{2,3,1}/2 - O_{3,1,2}/2 + O_{3,2,1}/2$$

$$K_8^{V(8,2),1} = 0$$

(Block 2) Trace = 0, Mixed Symmetry, C = mixed

$$O_1^{V(8,2),2} = O_{2,3,4} + O_{2,4,3} - O_{3,4,2} - O_{4,3,2}$$

$$K_1^{V(8,2),2} = \frac{p_2 p_3 (E(p)^2 - m_N^2 - p_1^2 - p_2^2 - p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_2^{V(8,2),2} = O_{1,3,4} + O_{1,4,3} - O_{3,4,1} - O_{4,3,1}$$

$$K_2^{V(8,2),2} = \frac{p_1 p_3 (E(p)^2 - m_N^2 - p_1^2 - p_2^2 - p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_3^{V(8,2),2} = O_{1,2,4} + O_{2,1,4} - O_{4,1,2} - O_{4,2,1}$$

$$K_3^{V(8,2),2} = \frac{p_1 p_2 (E(p)^2 - m_N^2 - p_1^2 - p_2^2 - p_3^2)}{(E(p)(E(p) + m_N))}$$

$$O_4^{V(8,2),2} = O_{1,2,3} + O_{2,1,3} - O_{3,1,2} - O_{3,2,1}$$

$$K_4^{V(8,2),2} = 0$$

$$O_5^{V(8,2),2} = O_{2,3,4} - O_{2,4,3} + 2O_{3,2,4} + O_{3,4,2} - 2O_{4,2,3} - O_{4,3,2}$$

$$K_5^{V(8,2),2} = \frac{3p_2 p_3 (E(p)^2 - m_N^2 - p_1^2 - p_2^2 - p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_6^{V(8,2),2} = O_{1,3,4} - O_{1,4,3} + 2O_{3,1,4} + O_{3,4,1} - 2O_{4,1,3} - O_{4,3,1}$$

$$K_6^{V(8,2),2} = \frac{3p_1 p_3 (E(p)^2 - m_N^2 - p_1^2 - p_2^2 - p_3^2)}{(2E(p)(E(p) + m_N))}$$

$$O_7^{V(8,2),2} = O_{1,2,4} + 2O_{1,4,2} - O_{2,1,4} - 2O_{2,4,1} + O_{4,1,2} - O_{4,2,1}$$

$$K_7^{V(8,2),2} = 0$$

$$O_8^{V(8,2),2} = O_{1,2,3} + 2O_{1,3,2} - O_{2,1,3} - 2O_{2,3,1} + O_{3,1,2} - O_{3,2,1}$$

$$K_8^{V(8,2),2} = 0$$