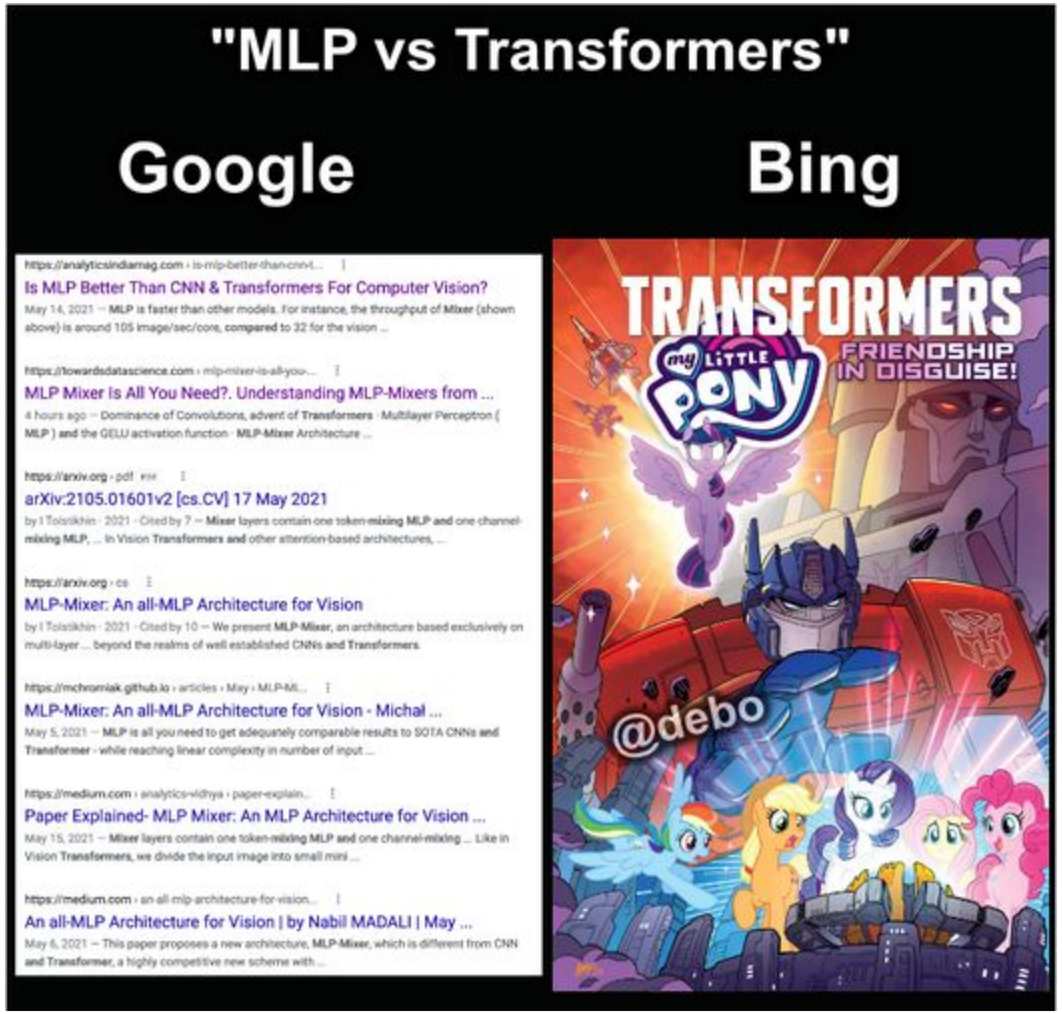
CS5841/EE5841 Machine Learning

Lecture 9: Review in preparation for neural networks

Evan Lucas

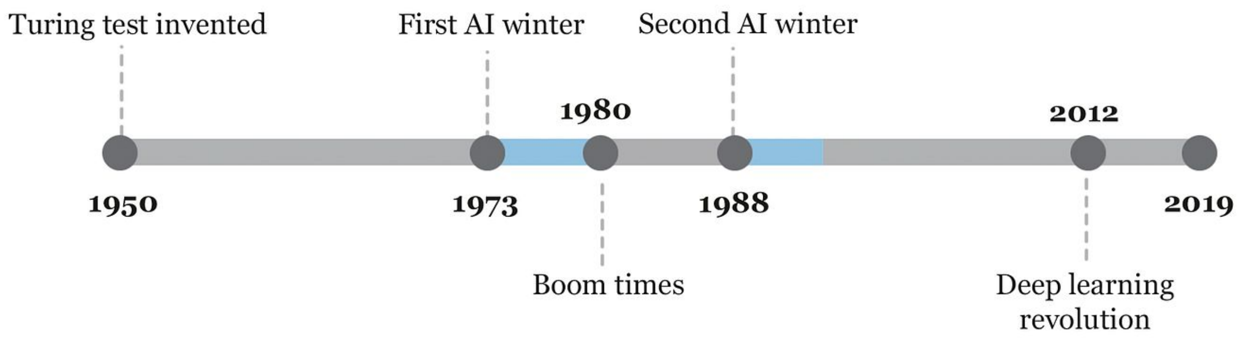
Common abbreviations

- Artificial neural network - ANN
- Multilayer perceptron - MLP
  - MLP used interchangeably with fully connected or dense NN
- Neural networks - NN
  - Used interchangeably with ANN
- Convolutional neural network - CNN



History moment

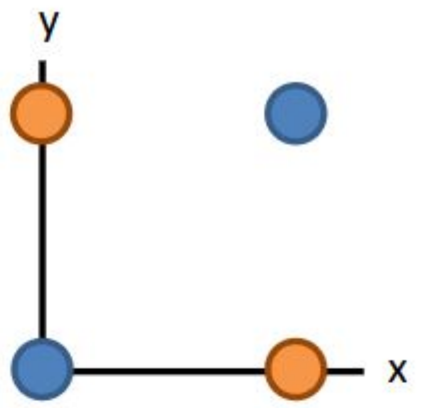
- First AI Winter - criticisms of ANN by Minksy and others
  - They didn't know how to train them - no backpropagation for ANN's until 1982

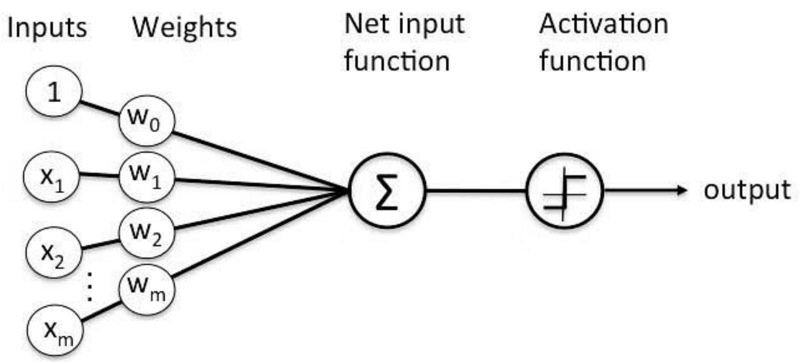


Minsky and Perceptrons

- Published major work on limitations of perceptrons
  - namely that they can’t solve XOR

X	Y	F(x,y)
0	0	0
0	1	1
1	0	1
1	1	0



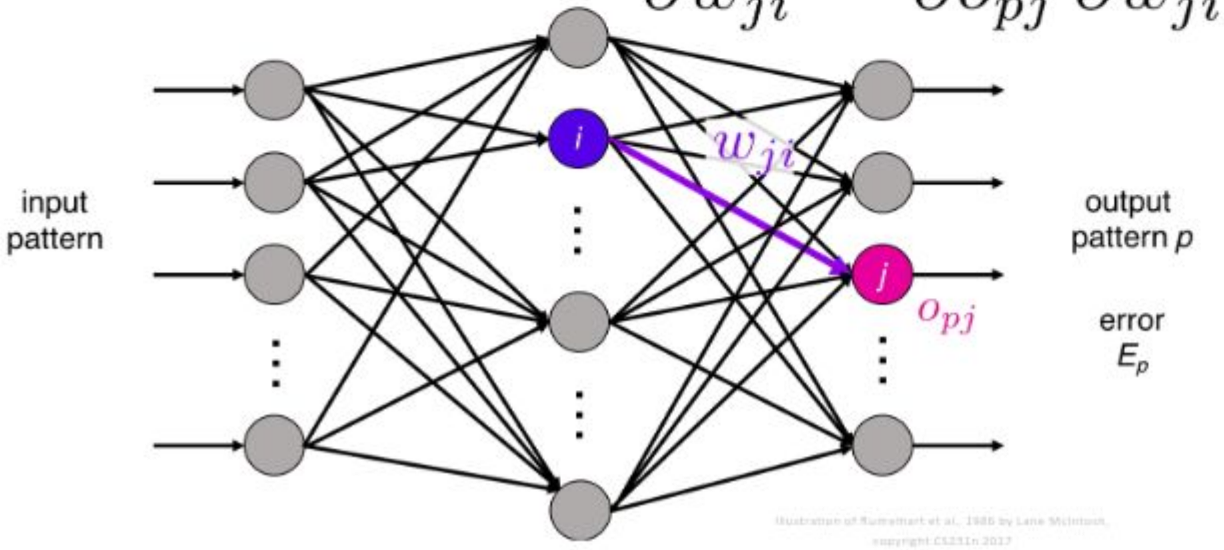


Training perceptrons - Backpropagation Rumelhart, Hinton, and Williams, 1986

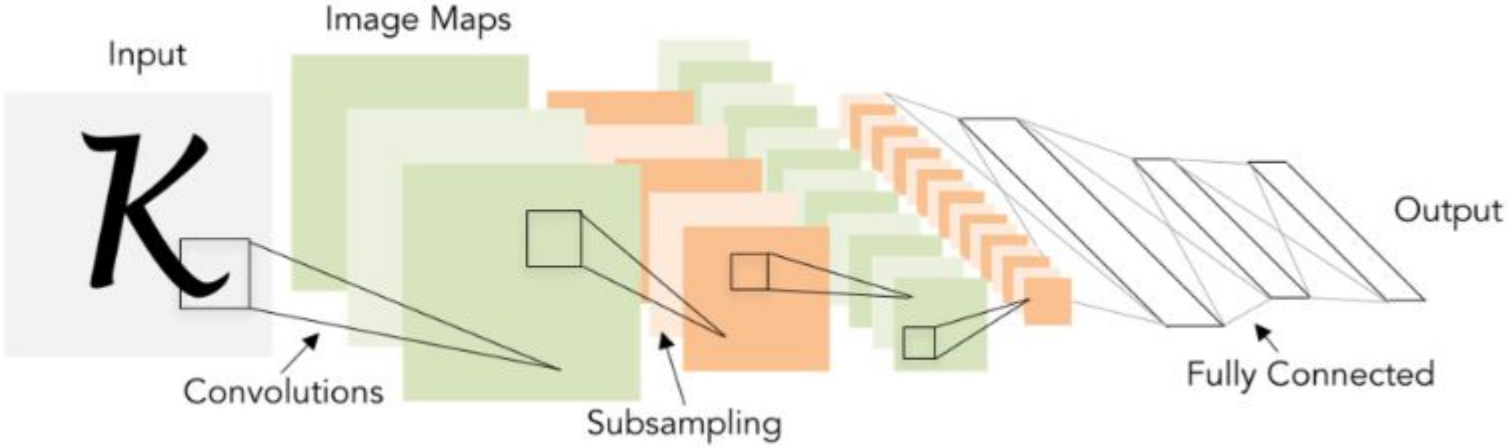
- Allowed easy training of multiple layer perceptrons

$$\frac{\partial E_p}{\partial w_{ji}} = \frac{\partial E_p}{\partial o_{pj}} \frac{\partial o_{pj}}{\partial w_{ji}}$$

recognizable math



Convolutional Neural Networks LeCun et al, 1998

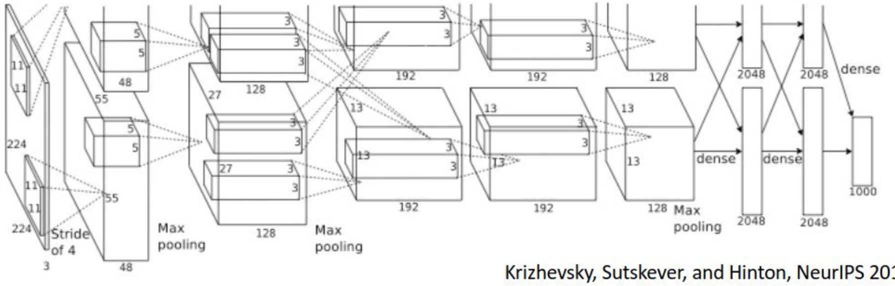


Applied backprop to an architecture involving convolutional steps

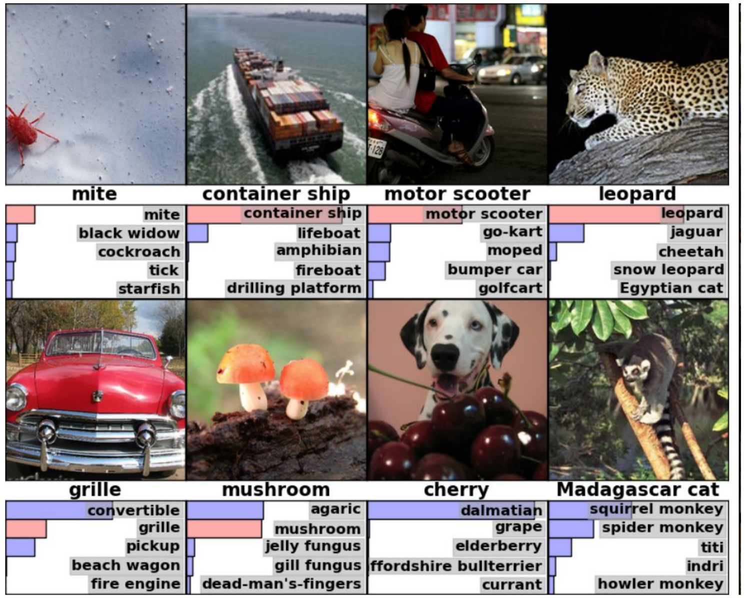
Very similar to modern CNNs

Deep NNs

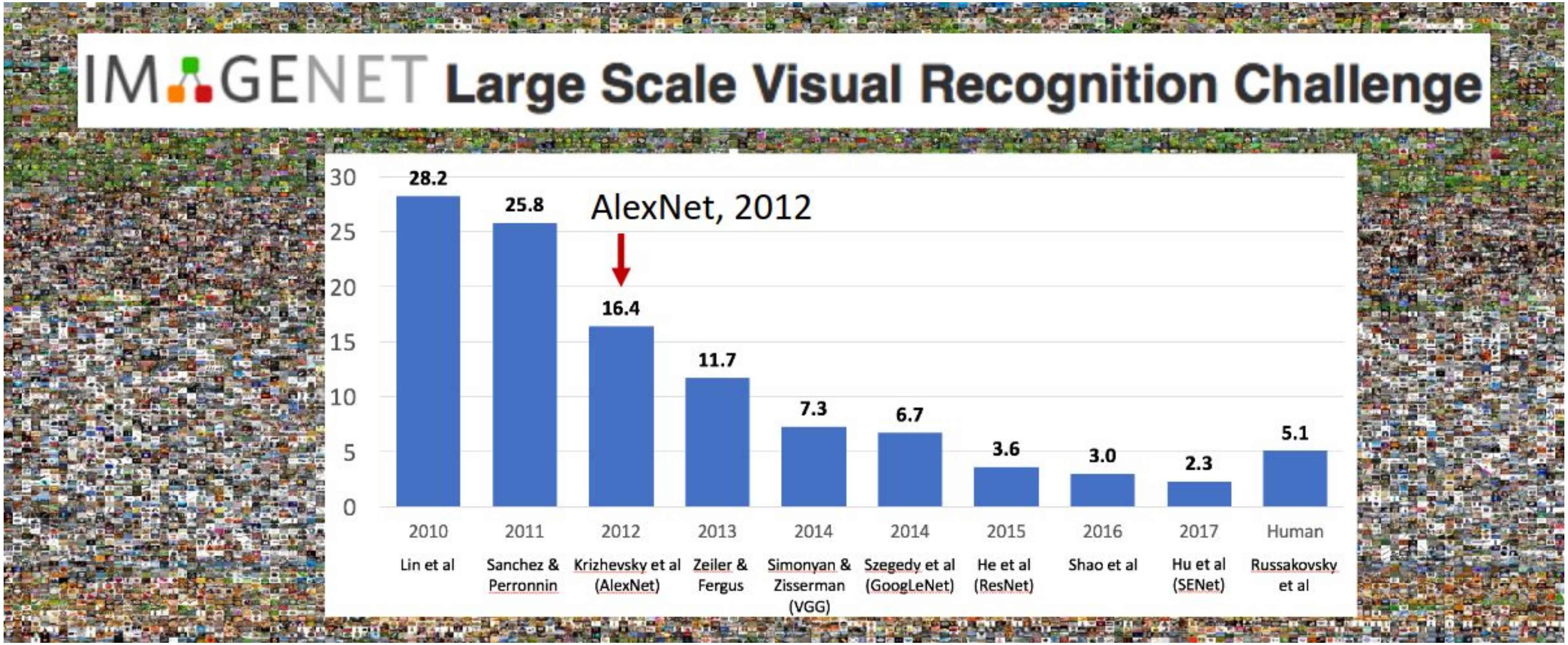
- Really big NNs
- Deep - many layers
- First really successful one was AlexNet (2012)



Krizhevsky, Sutskever, and Hinton, NeurIPS 2012



ImageNet dataset



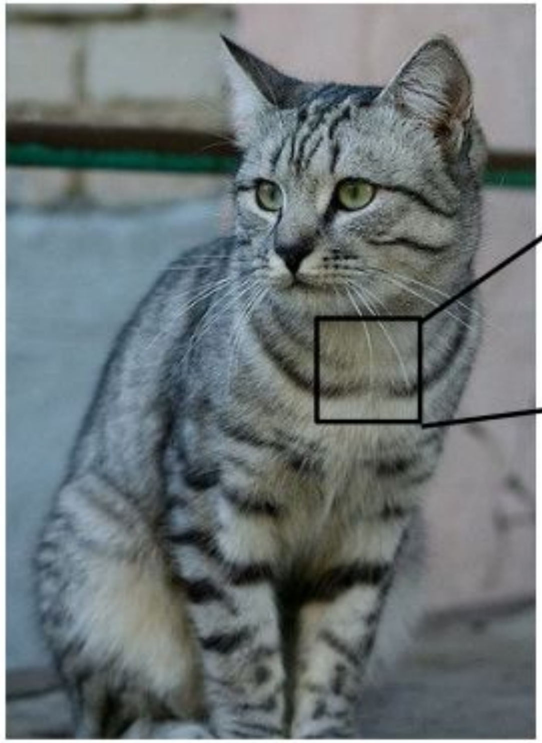
https://proceedings.neurips.cc/paper/2012/file/c399862d3b9d6b76c8436e924a68c45b-Paper.pdf

An illustrated example of neural networks

(assume given a set of possible labels)
{dog, cat, truck, plane, ...}

$\longrightarrow$ cat

The Problem: Semantic Gap



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 [ 91  98 102 106 104  79  98 103  99 105 123 136 110 105  94  85]
 [ 76  85  90 105 128 105  87  96  95  99 115 112 106 103  99  85]
 [ 99  81  81  93 120 131 127 100  95  98 102  99  96  93 101  94]
 [106  91  61  64  69  91  88  85 101 107 109  98  75  84  96  95]
 [114 108  85  55  55  69  64  54  64  87 112 129  98  74  84  91]
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 [122 121 102  80  82  86  94 117 145 148 153 102  58  78  92 107]
 [122 164 148 103  71  56  78  83  93 103 119 139 102  61  69  84]]
```

What the computer sees

An image is a tensor of integers between [0, 255]:

e.g. 800 x 600 x 3
(3 channels RGB)

First classifier: **Nearest Neighbor**

deer bird plane cat car

Training data with labels

?

query data

Distance Metric |

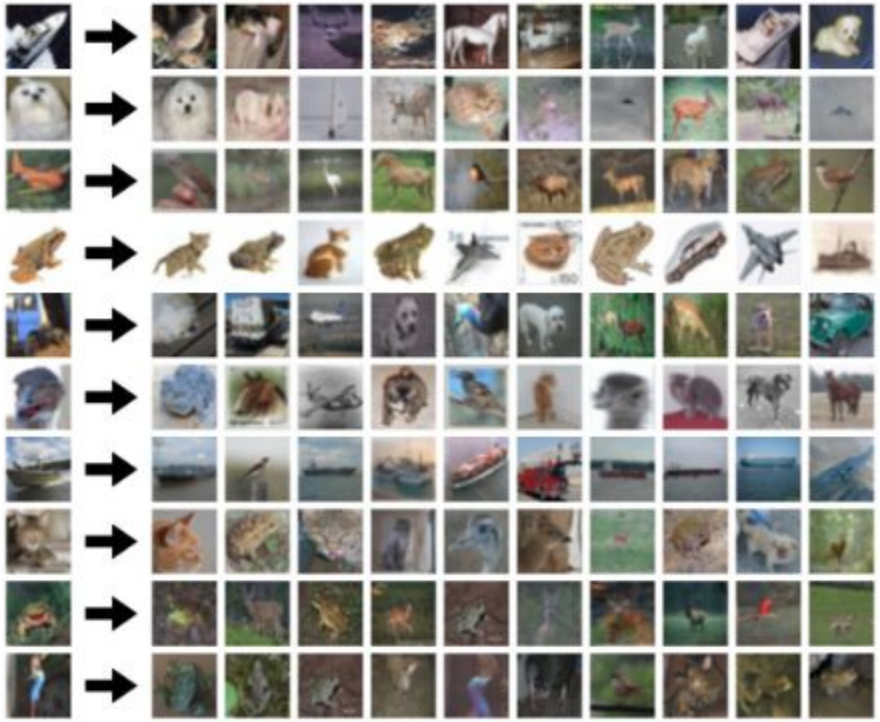
,

| $\to \mathbb{R}$

Example Dataset: **CIFAR10**

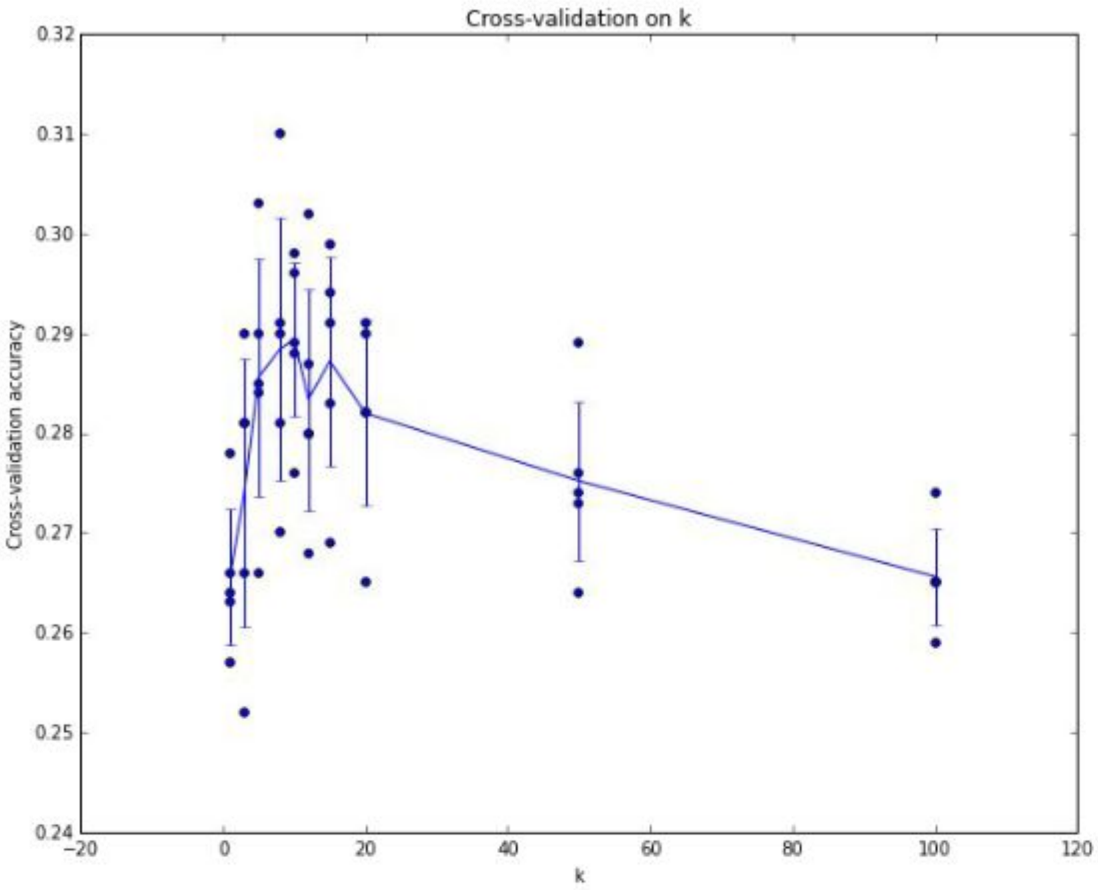
10 classes
50,000 training images
10,000 testing images

Test images and nearest neighbors



Alex Krizhevsky, "Learning Multiple Layers of Features from Tiny Images", Technical Report, 2009.

Setting Hyperparameters



Example of
5-fold cross-validation
for the value of **k.**

Each point: single
outcome.

The line goes
through the mean, bars
indicated standard
deviation

(Seems that k ~= 7 works best
for this data)

What does this look like?

k-Nearest Neighbor with pixel distance **never used.**

- Distance metrics on pixels are not informative

Original | Occluded | Shifted (1 pixel) | Tinted

(All three images on the right have the same pixel distances to the one on the left)

Parametric Approach: Linear Classifier

$$f(x,W) = Wx$$

Image

Array of **32x32x3** numbers
(3072 numbers total)

$\longrightarrow f(\mathbf{x},\mathbf{W}) \longrightarrow$ **10** numbers giving class scores

$\uparrow$

W

parameters
or weights

Parametric Approach: Linear Classifier

$$\underbrace{\boxed{f(x,W)}}_{\textbf{10x1}} = \underbrace{\boxed{W}}_{\textbf{10x3072}}\overbrace{\boxed{x}}^{\textbf{3072x1}}$$

Image

Array of **32x32x3** numbers
(3072 numbers total)

$\longrightarrow f(\mathbf{x},\mathbf{W}) \longrightarrow$ **10** numbers giving class scores

$\uparrow$

W

parameters
or weights

Parametric Approach: Linear Classifier

3072x1

$$\boxed{f(x,W)} = \boxed{W}\boxed{x} + \boxed{b}$$

10x1

10x1 10x3072

Image

Array of **32x32x3** numbers
(3072 numbers total)

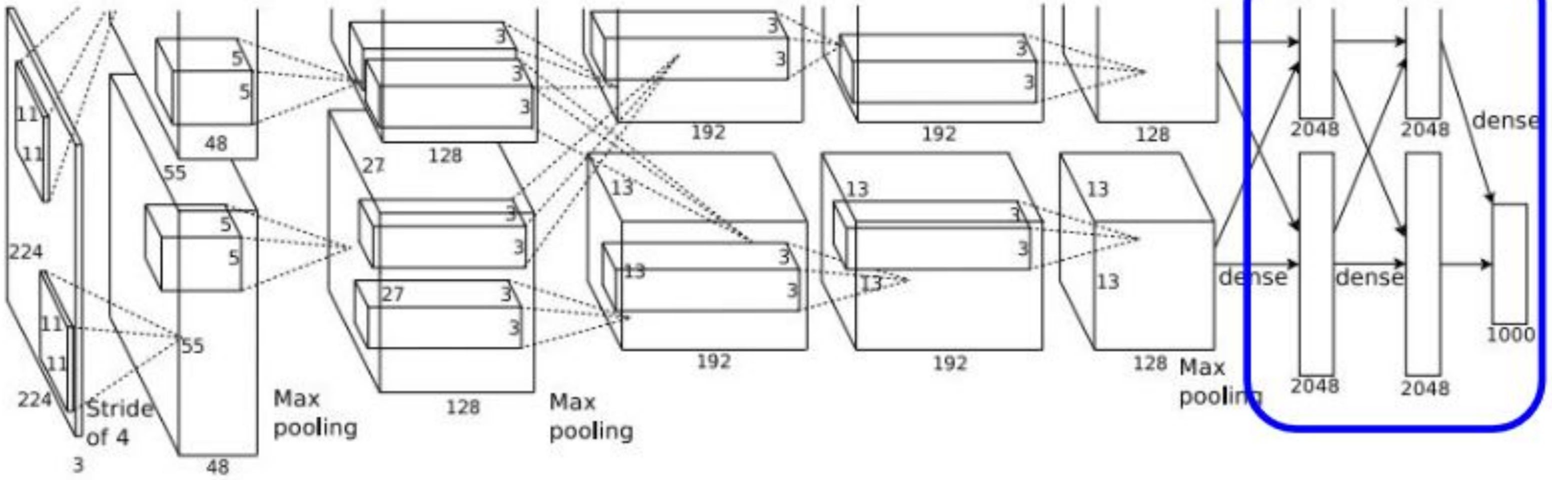
$f(\mathbf{x},\mathbf{W})$

10 numbers giving
class scores

W
parameters
or weights

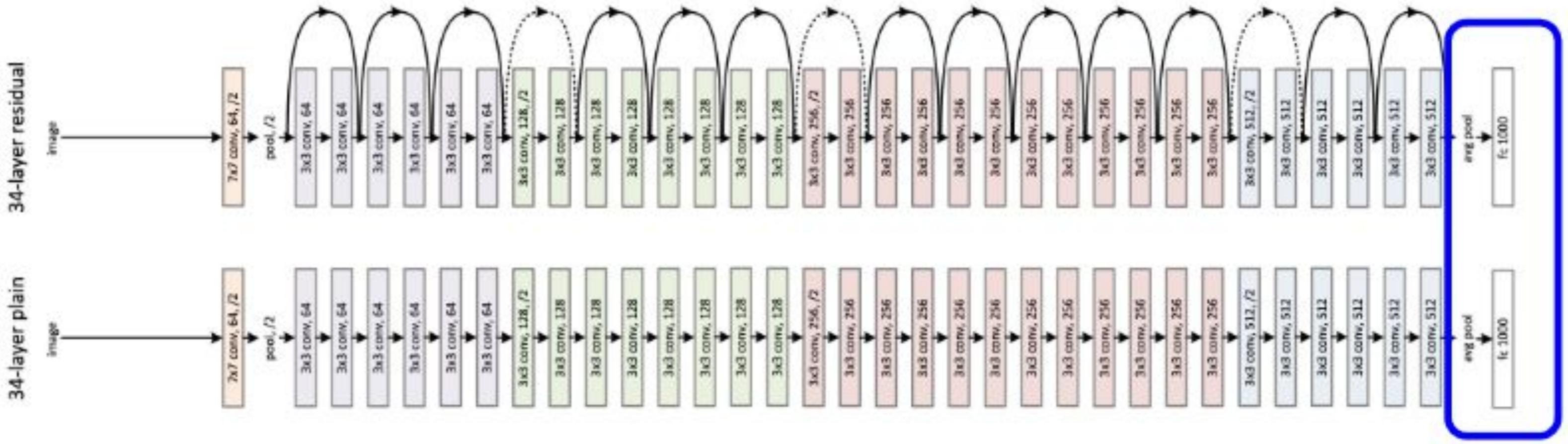
Neural Network

Linear classifiers



[Krizhevsky et al. 2012]

Linear layers

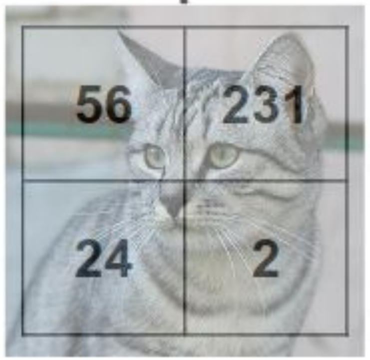


[He et al. 2015]

Example with an image with 4 pixels, and 3 classes (cat/dog/ship

Algebraic Viewpoint

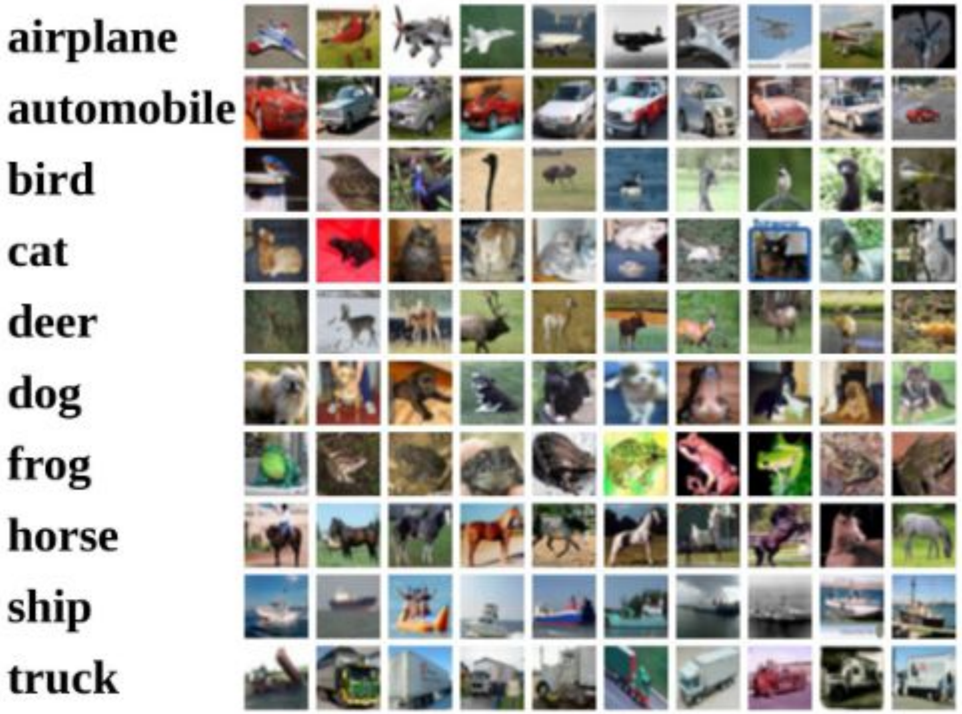
Flatten tensors into a vector

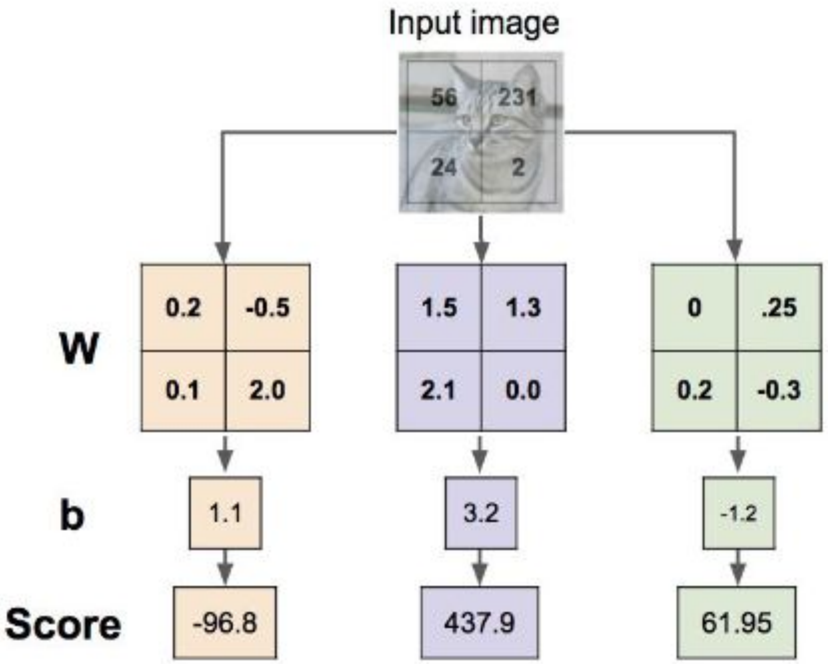


Input image

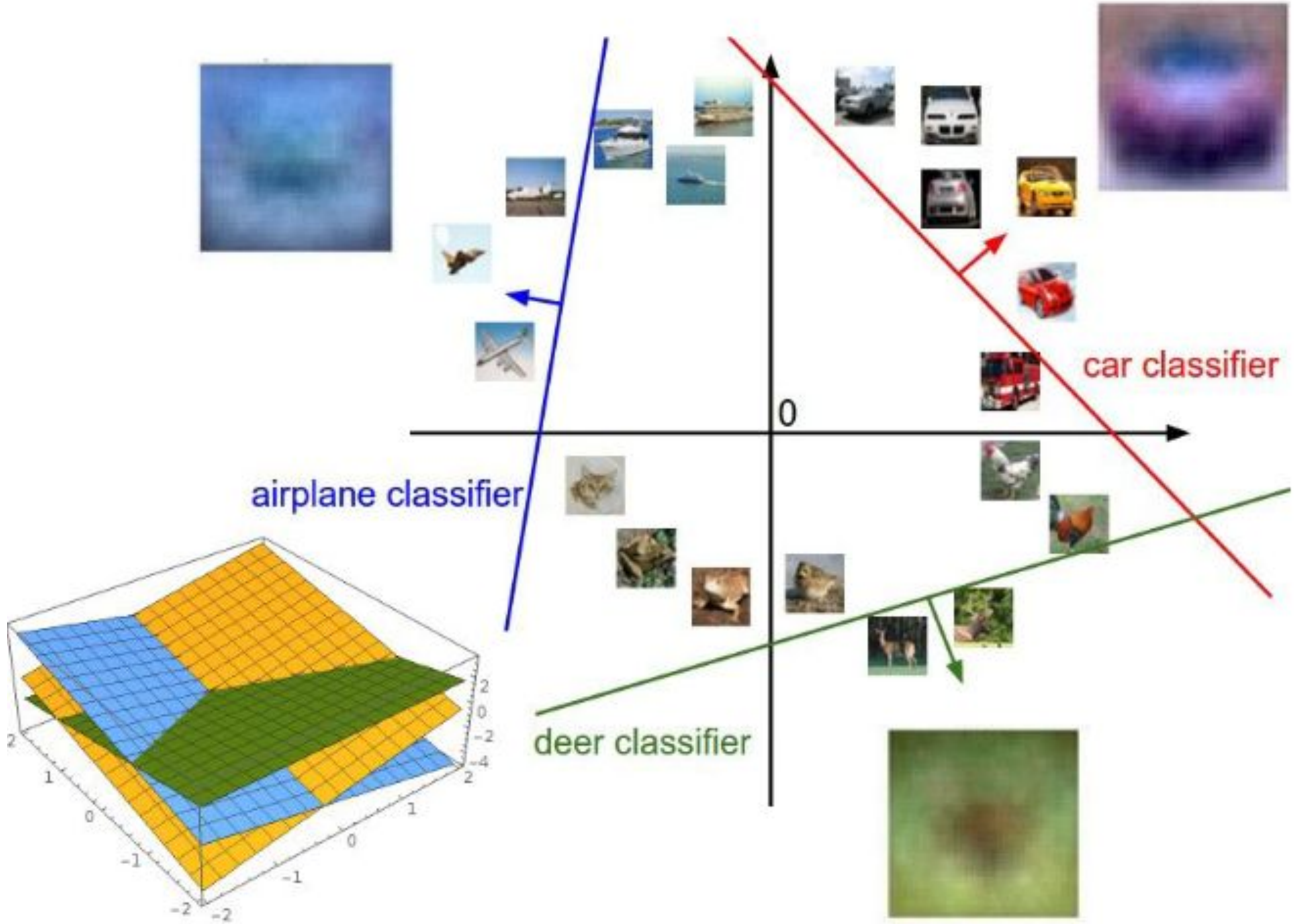
$$\underbrace{\begin{bmatrix} 0.2 & -0.5 & 0.1 & 2.0 \\ 1.5 & 1.3 & 2.1 & 0.0 \\ 0 & 0.25 & 0.2 & -0.3 \end{bmatrix}}_{W} \begin{bmatrix} 56 \\ 231 \\ 24 \\ 2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1.1 \\ 3.2 \\ -1.2 \end{bmatrix}}_{b} = \begin{bmatrix} -96.8 \\ 437.9 \\ 61.95 \end{bmatrix} \begin{array}{l} \text{Cat score} \\ \text{Dog score} \\ \text{Ship score} \end{array}$$

Interpreting a Linear Classifier





Interpreting a Linear Classifier: Geometric Viewpoint



Plot created using Wolfram Cloud

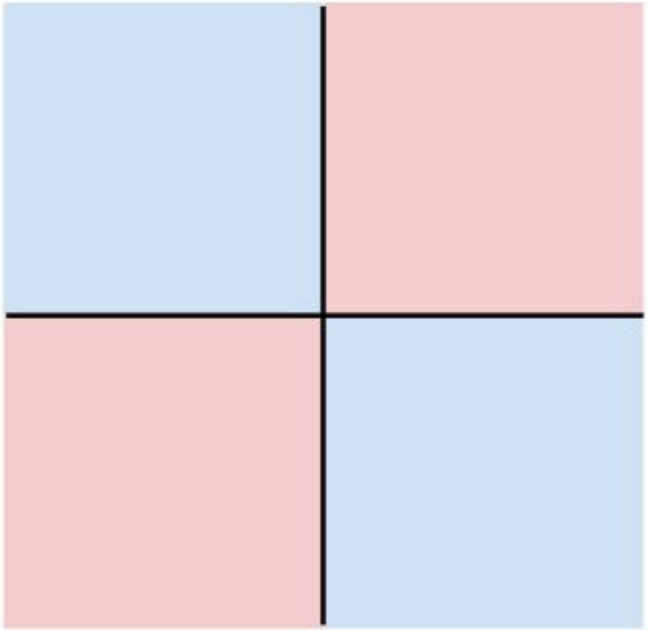
$$f(x,W) = Wx + b$$

Array of **32x32x3** numbers
(3072 numbers total)

Hard cases for a linear classifier

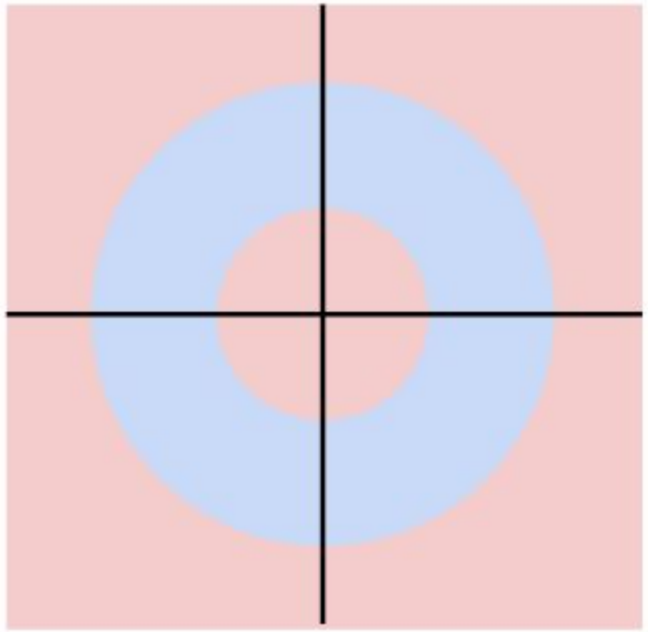
Class 1:
First and third quadrants

Class 2:
Second and fourth quadrants



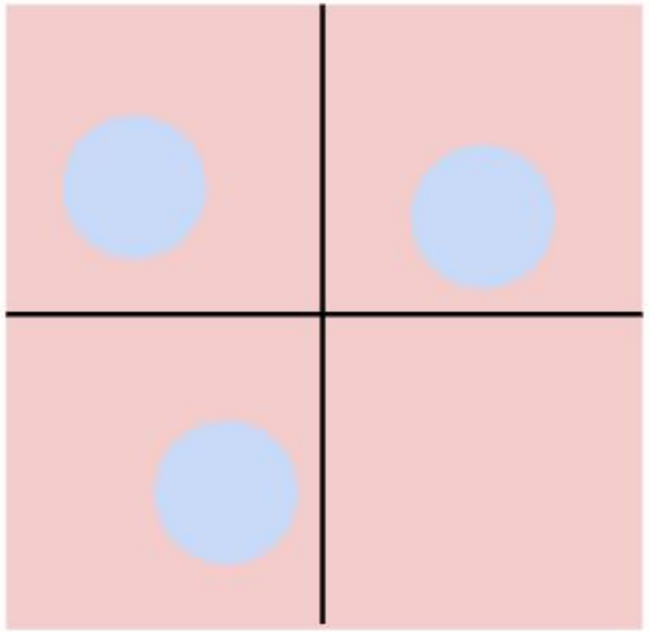
Class 1:
1 <= L2 norm <= 2

Class 2:
Everything else



Class 1:
Three modes

Class 2:
Everything else



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Linear Classifier – Choose a good W



airplane	-3.45	-0.51	3.42
automobile	-8.87	**6.04**	4.64
bird	0.09	5.31	2.65
cat	**2.9**	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	**-4.34**
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

TODO:

1. Define a **loss function** that quantifies our unhappiness with the scores across the training data.

2. Come up with a way of efficiently finding the parameters that minimize the loss function. **(optimization)**

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

$= \max(0, 2.2 - (-3.1) + 1)$
$\quad + \max(0, 2.5 - (-3.1) + 1)$
$= \max(0, 6.3) + \max(0, 6.6)$
$= 6.3 + 6.6$
$= 12.9$

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Probabilities must be >= 0

Probabilities must sum to 1

	Unnormalized log-probabilities / logits	$\xrightarrow{\text{exp}}$	unnormalized probabilities	$\xrightarrow{\text{normalize}}$	probabilities
cat	**3.2**		**24.5**		**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$ Softmax Function

$$L_i = -\log P(Y = y_i | X = x_i)$$

Probabilities must be >= 0

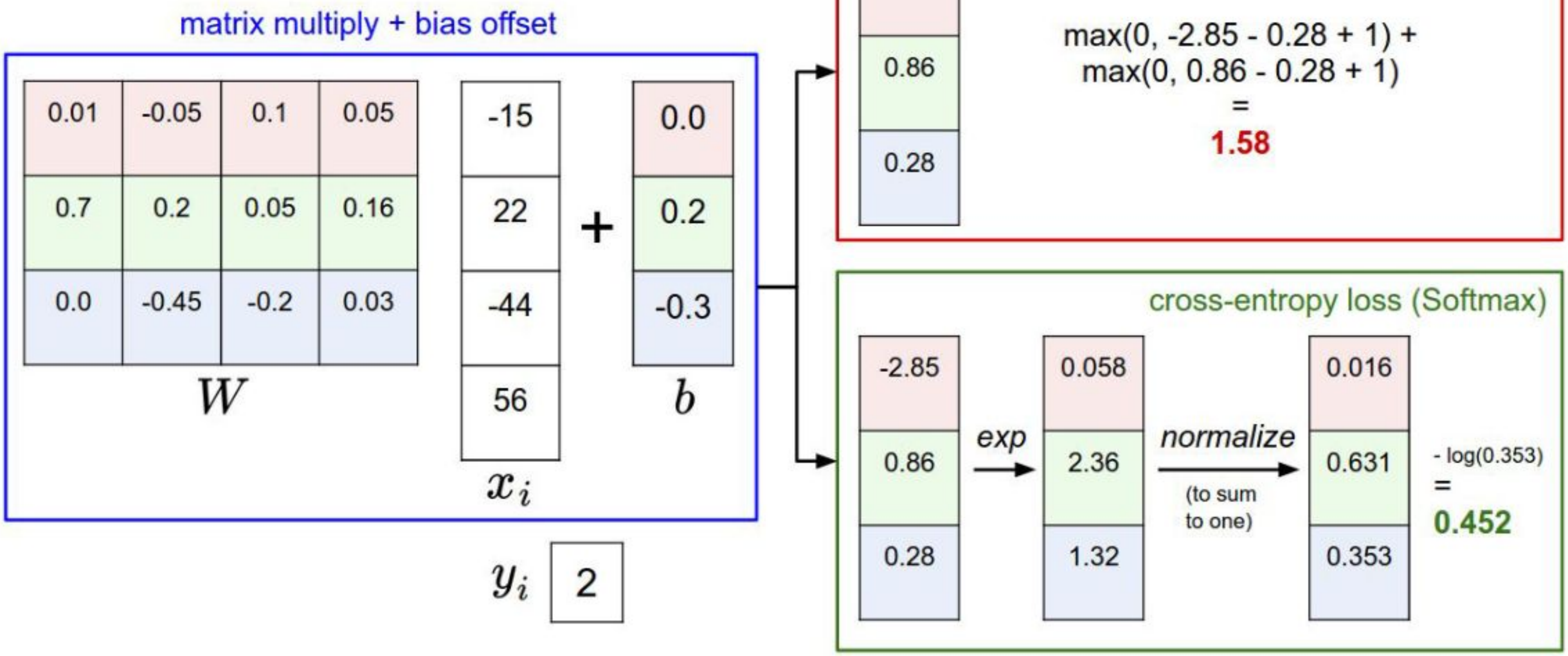
Probabilities must sum to 1

	Unnormalized log-probabilities / logits		unnormalized probabilities		probabilities		Correct probs
cat	**3.2**	exp →	**24.5**	normalize →	**0.13**	→ compare ←	**1.00**
car	5.1		164.0		0.87		0.00
frog	-1.7		0.18		0.00		0.00

Cross Entropy

$$H(P, Q) = H(p) + D_{KL}(P \| Q)$$

Softmax vs. SVM



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Questions + Comments?

Resources used

http://cs231n.stanford.edu/slides/2023/lecture_2.pdf