CS5841/EE5841 Machine Learning

Lecture 13: Recurrent Neural Networks

Evan Lucas

Some last notes about CNNs

- CNNs are amazing, but not perfect
- Heavily biased towards textures

(a) Texture image
81.4% **Indian elephant**
10.3% indri
8.2% black swan

(b) Content image
71.1% **tabby cat**
17.3% grey fox
3.3% Siamese cat

(c) Texture-shape cue conflict
63.9% **Indian elephant**
26.4% indri
9.6% black swan

Figure 1: Classification of a standard ResNet-50 of **(a)** a texture image (elephant skin: only texture cues); **(b)** a normal image of a cat (with both shape and texture cues), and **(c)** an image with a texture-shape cue conflict, generated by style transfer between the first two images.

https://openreview.net/pdf?id=Bygh9j09KX
https://arxiv.org/abs/2312.16880
https://arxiv.org/abs/1712.09665
https://www.theverge.com/2021/3/8/22319173/openai-machine-vision-adversarial-typographic-attacka-clip-multimodal-neuron

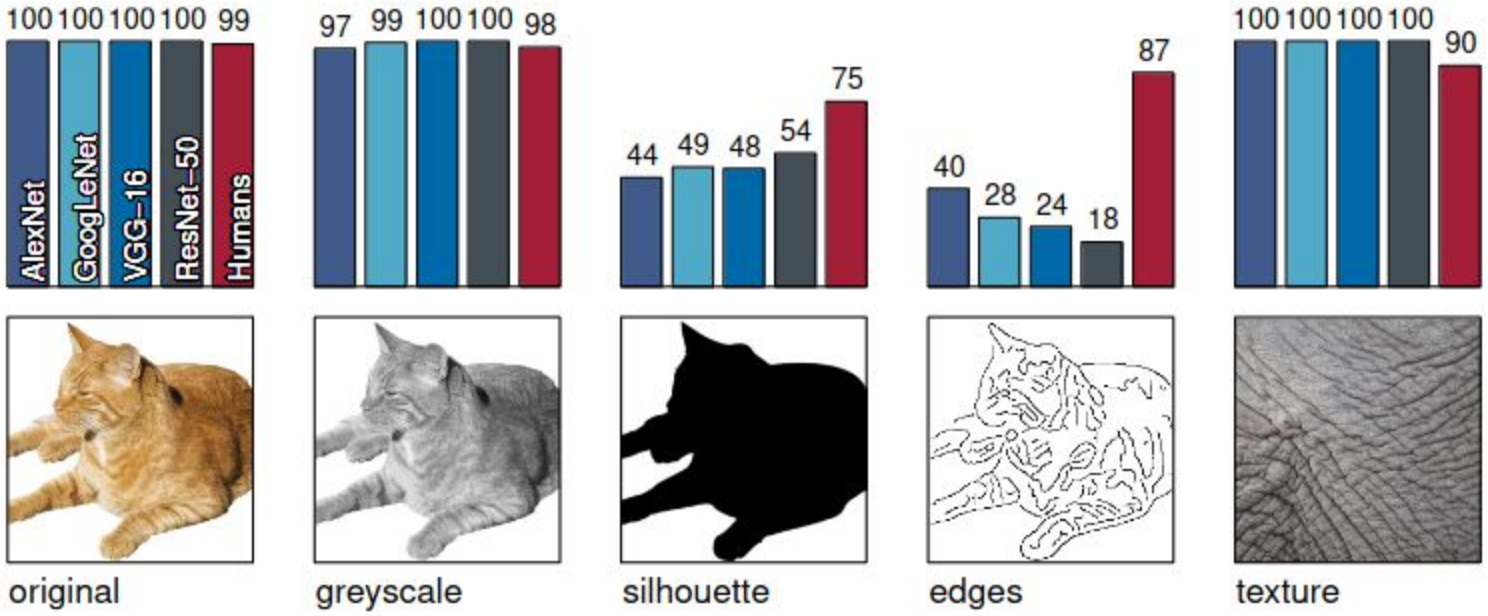


Figure 2: Accuracies and example stimuli for five different experiments without cue conflict.

Fig. 3. Five images used as patches: (i) saltshaker, (ii) soap dispenser, (iii) digital clock, (iv) piggy bank, and (v) toaster. The sizes for the patch images are 32*32, 48*48, and 64*64.

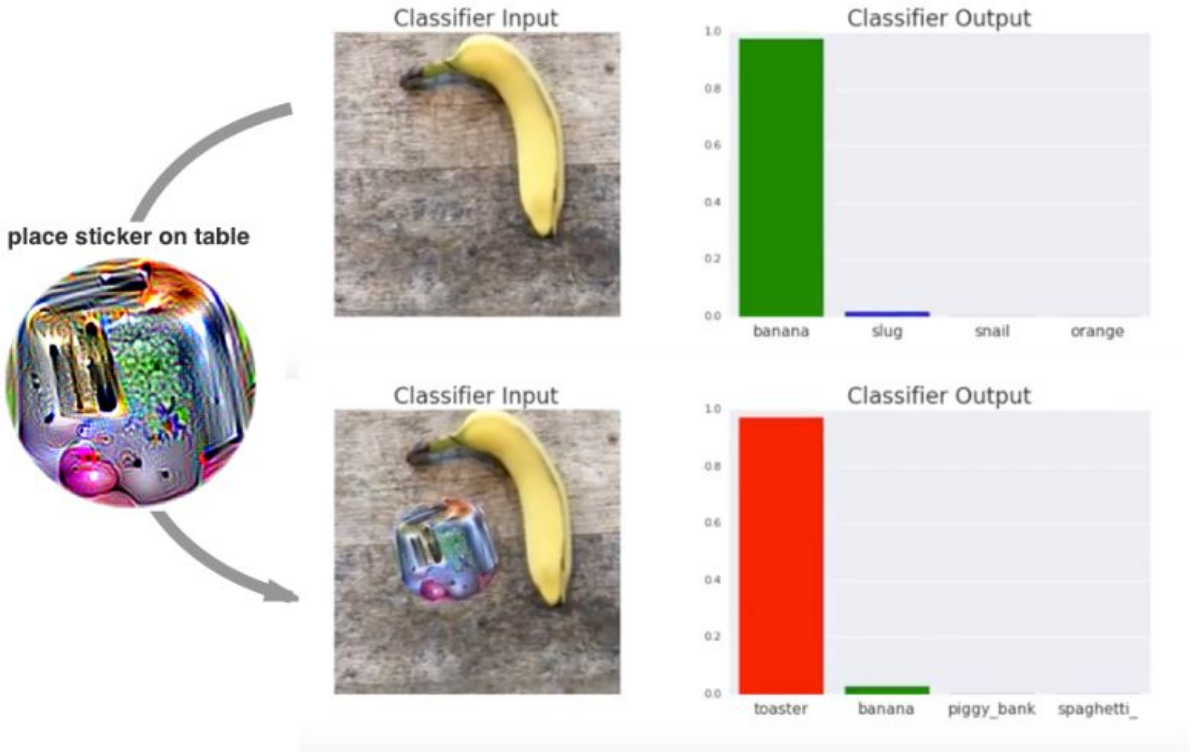


Figure 1: A real-world attack on VGG16, using a physical patch generated by the white-box ensemble method described in Section 3. When a photo of a tabletop with a banana and a notebook (top photograph) is passed through VGG16, the network reports class 'banana' with 97% confidence (top plot). If we physically place a sticker targeted to the class "toaster" on the table (bottom photograph), the photograph is classified as a toaster with 99% confidence (bottom plot). See the following video for a full demonstration: `https://youtu.be/i1sp4X57TL4`

Granny Smith	85.6%
iPod	0.4%
library	0.0%
pizza	0.0%
toaster	0.0%
dough	0.1%

Granny Smith	0.1%
iPod	99.7%
library	0.0%
pizza	0.0%
toaster	0.0%
dough	0.0%

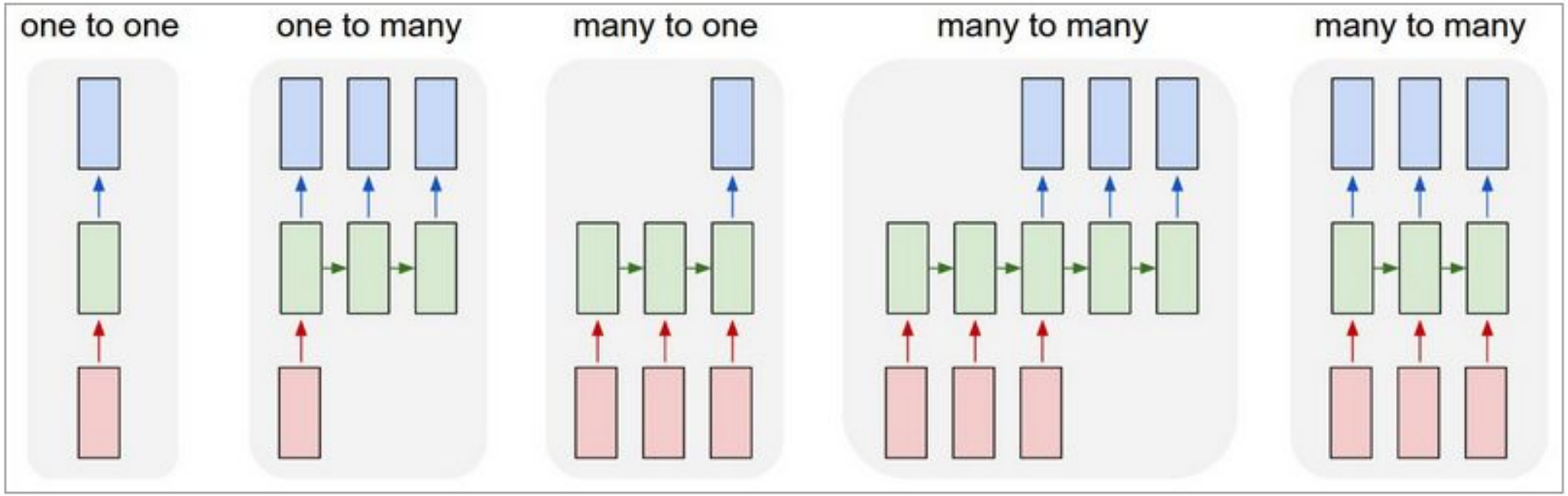
chainsaw	91.1%
lawn mower	7.0%
power drill	1.0%
vacuum cleaner	0.4%
wheelbarrow	0.1%
tractor	0.1%

piggy bank	70.1%
chainsaw	1.5%
slot machine	1.1%
wheelbarrow	0.9%
hammer	0.8%
mousetrap	0.6%

Why are ANNs/CNNs limited

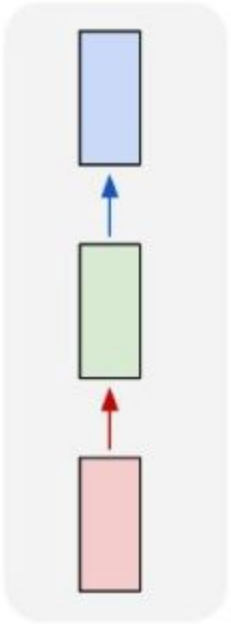
- One reason - fixed input/output
- Not optimized for sequences (CNNs are better)
-

Sequences



“Vanilla” Neural Network

one to one

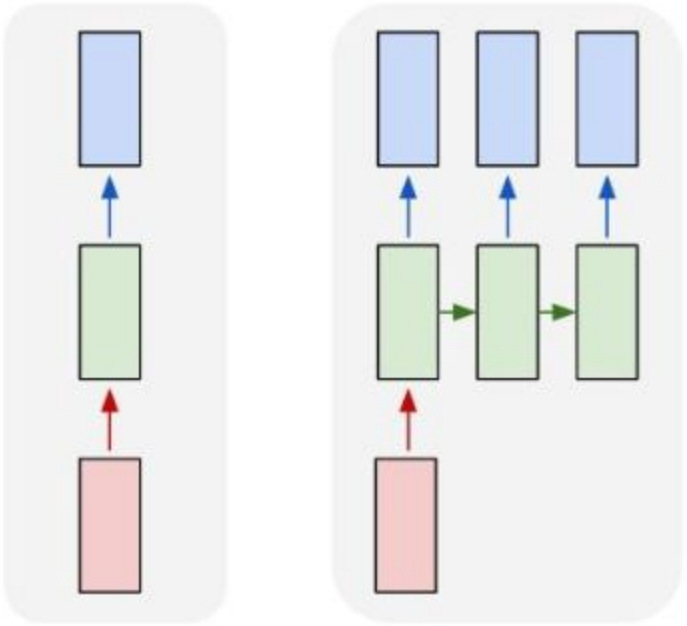


Vanilla Neural Networks

Recurrent Neural Networks: Process Sequences

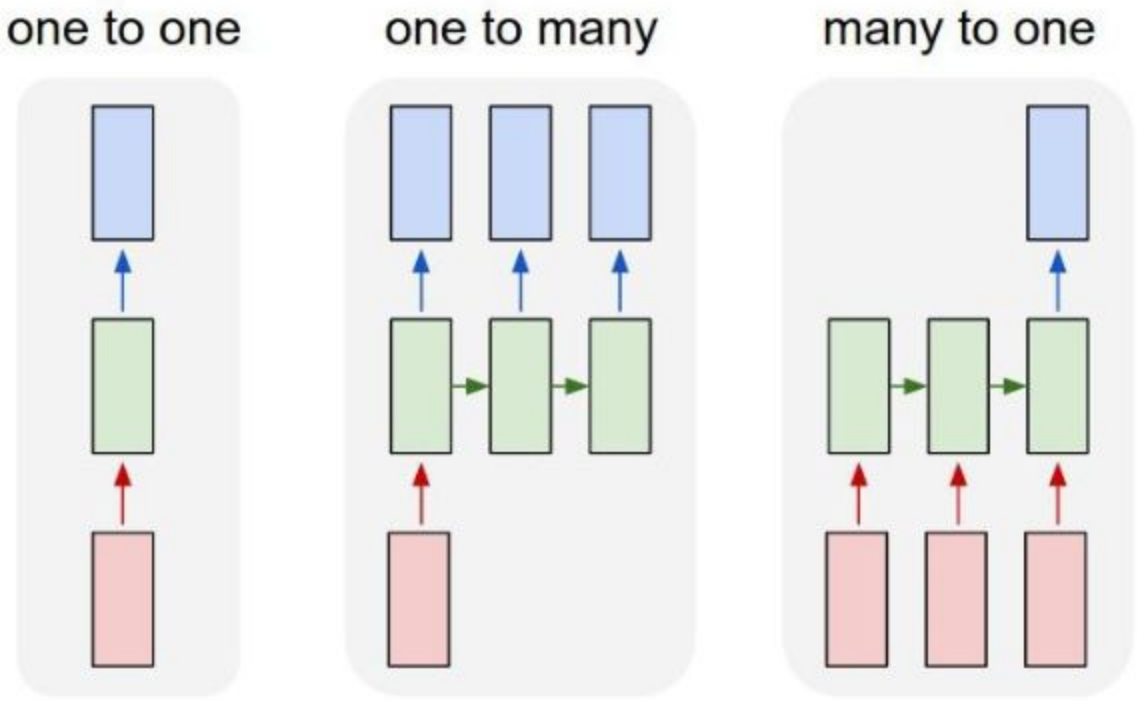
one to one

one to many



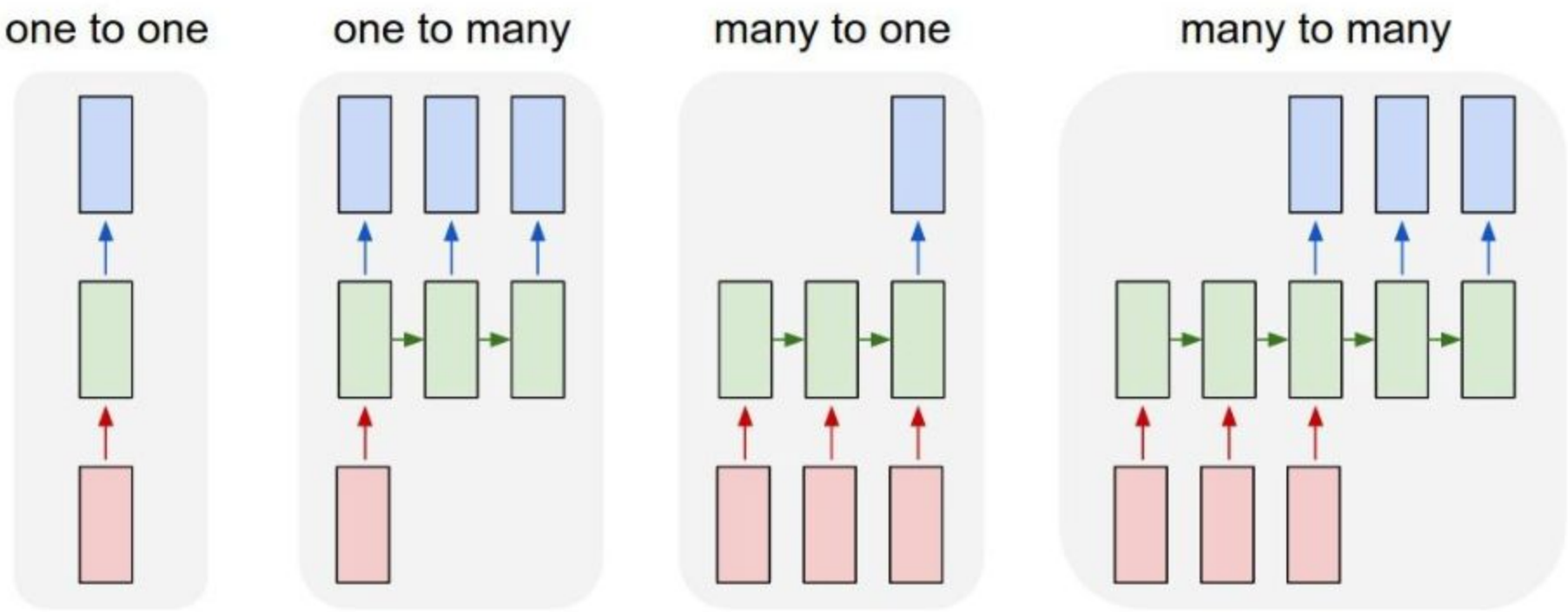
e.g. **Image Captioning**
image -> sequence of words

Recurrent Neural Networks: Process Sequences



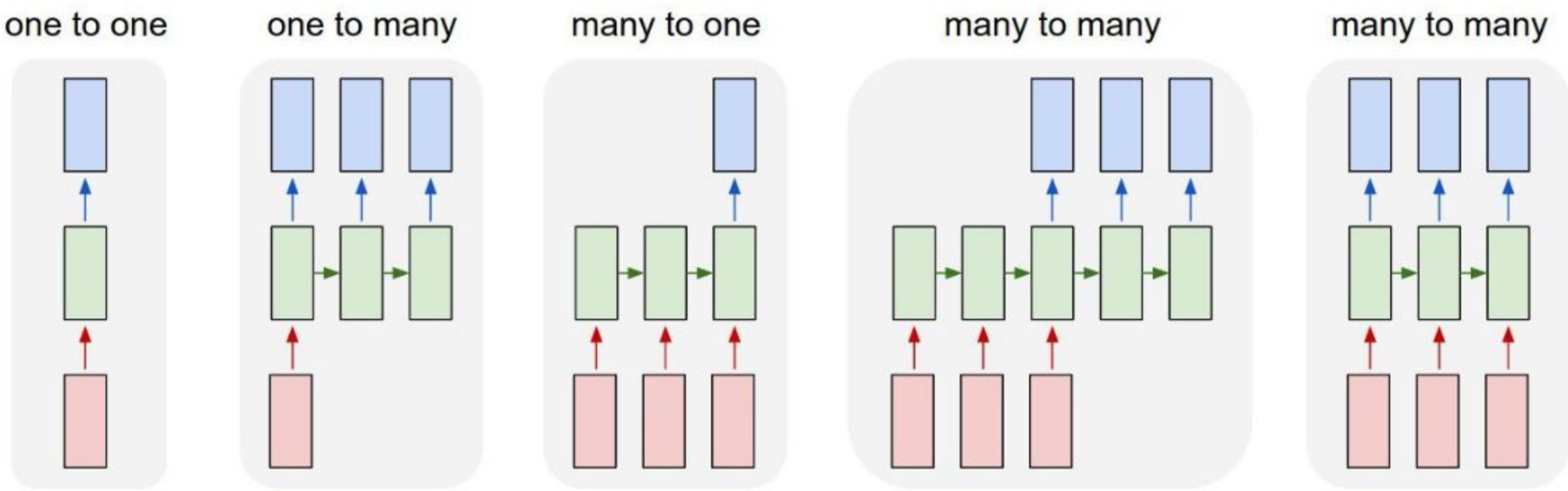
e.g. **action prediction**
sequence of video frames -> action class

Recurrent Neural Networks: Process Sequences



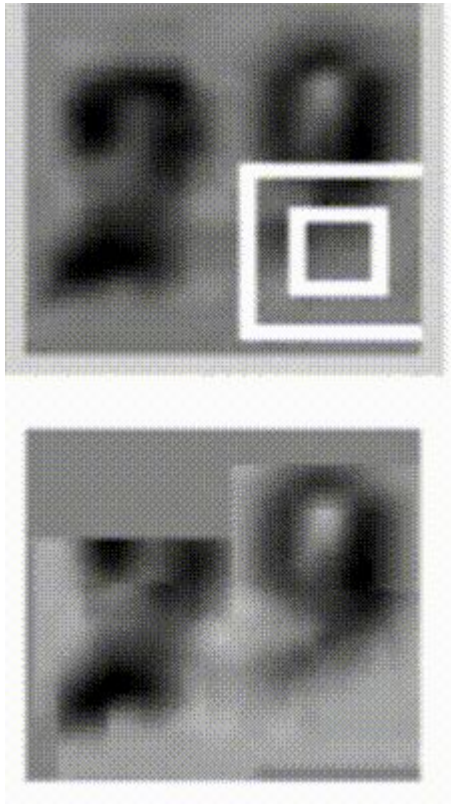
E.g. **Video Captioning**
Sequence of video frames -> caption

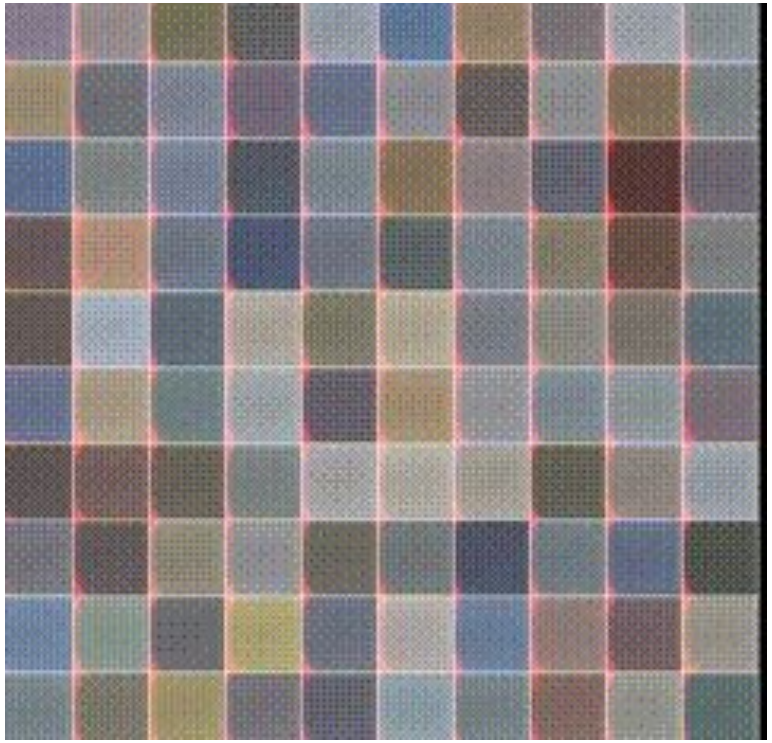
Recurrent Neural Networks: Process Sequences



e.g. **Video classification on frame level**

Can images be sequences?



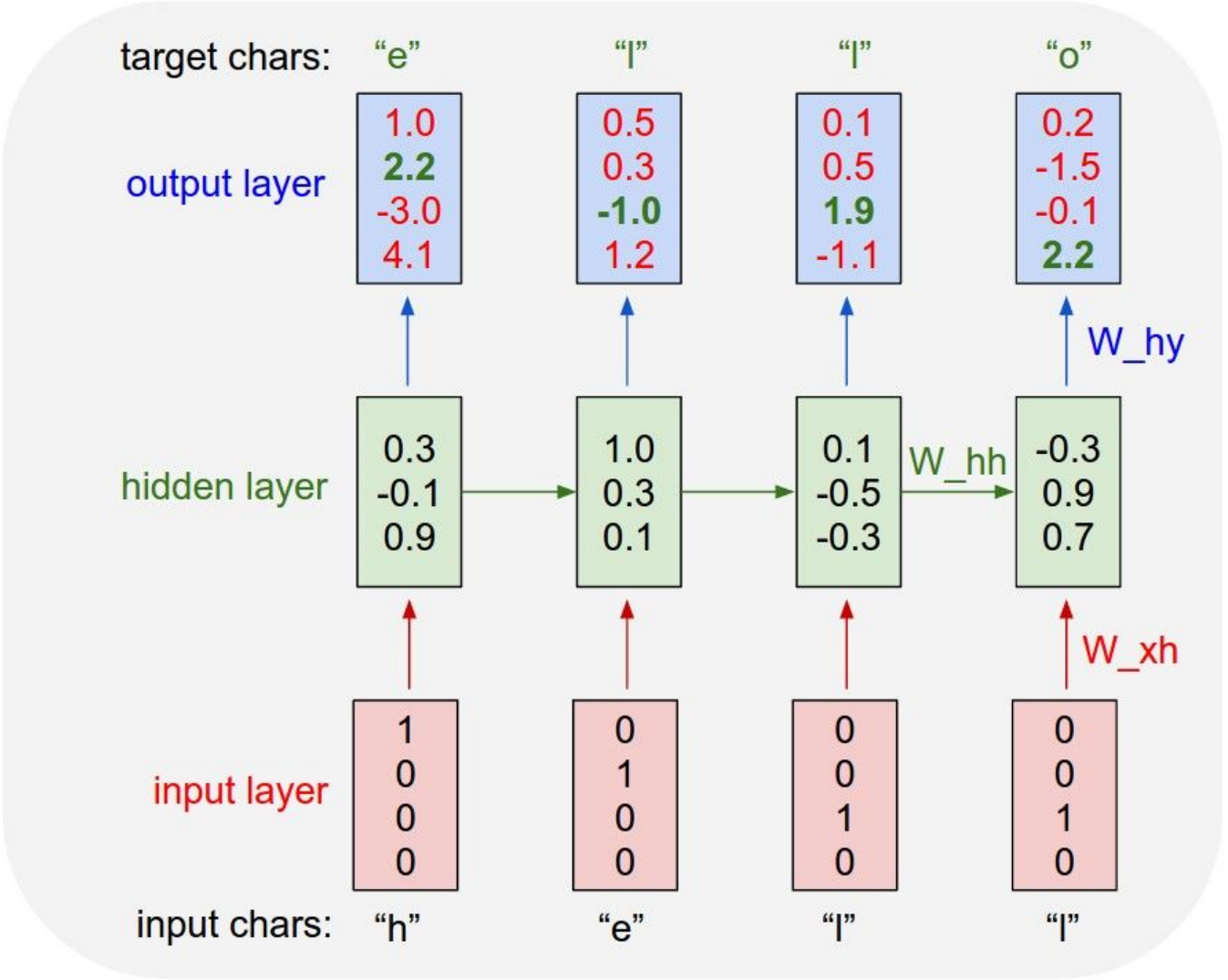


Sequence is an abstract term

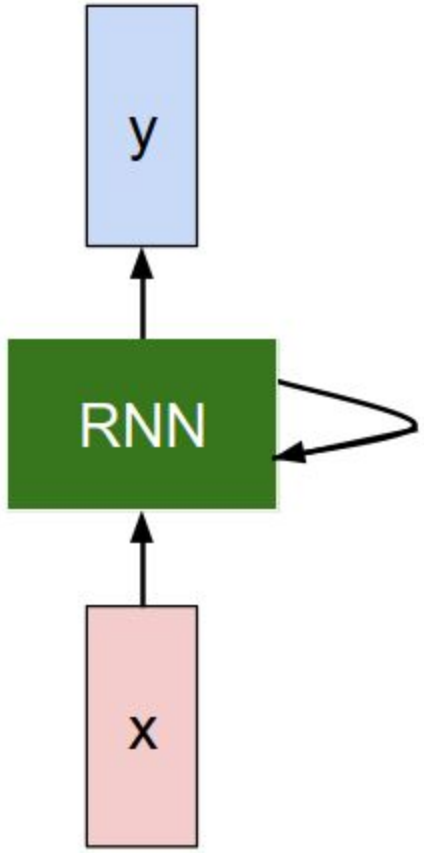
- Data can almost always be handled sequentially
- RNNs learn stateful programs to process data

High level

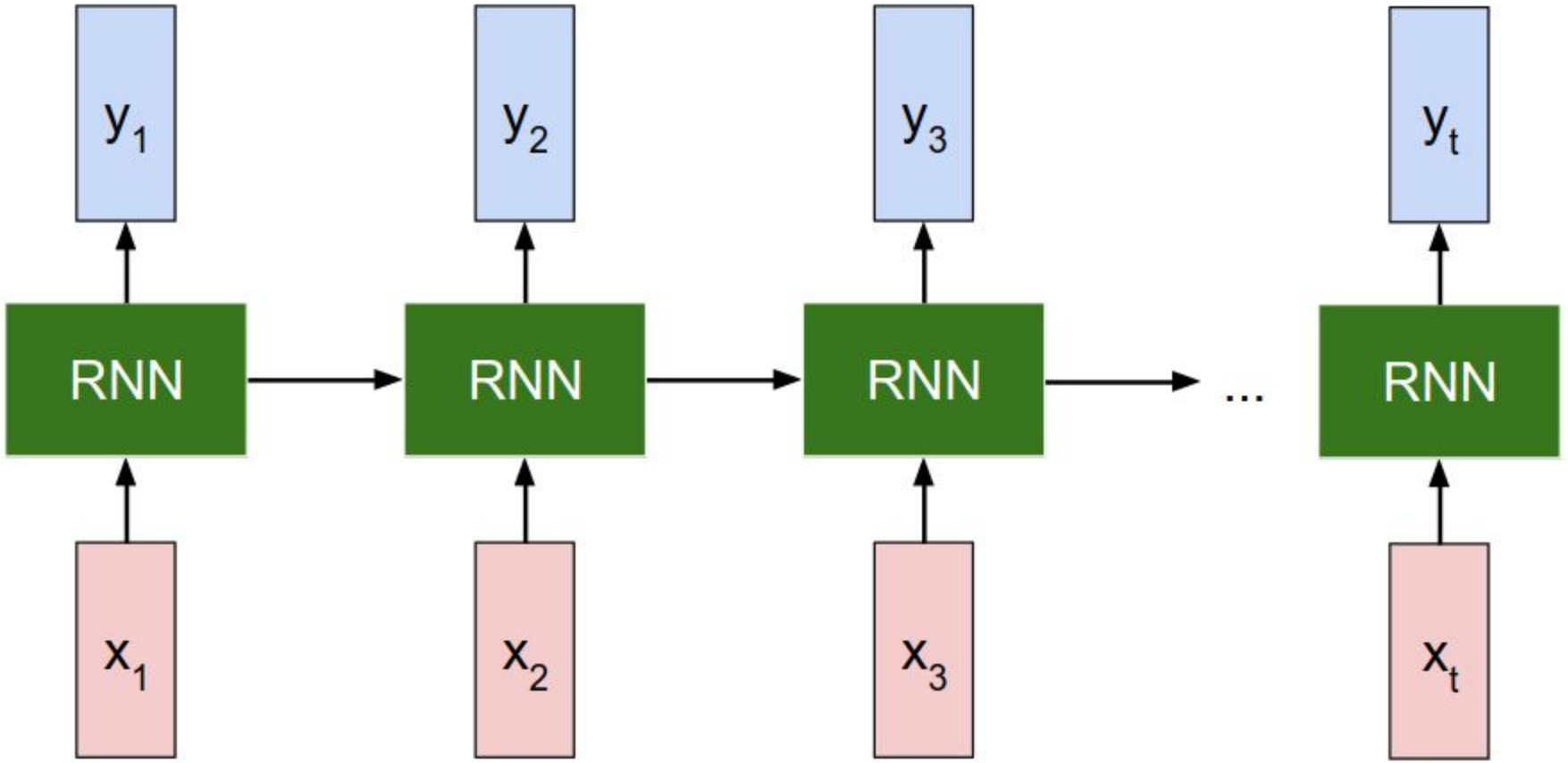
- Input vector - x
- Output vector - y
- Hidden state - h



Recurrent Neural Network



Unrolled RNN



RNN hidden state update

We can process a sequence of vectors **x** by applying a **recurrence formula** at every time step:

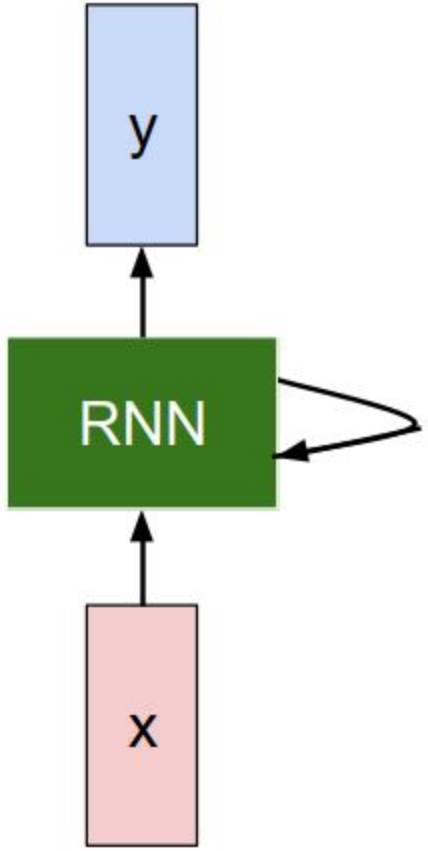
$$h_t = f_W(h_{t-1}, x_t)$$

new state

some function with parameters W

old state

input vector at some time step



RNN output generation

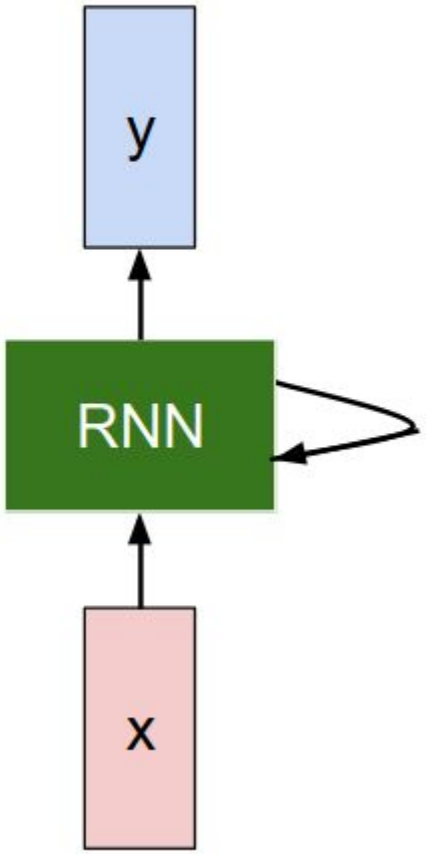
We can process a sequence of vectors **x** by applying a **recurrence formula** at every time step:

$$y_t = f_{W_{hy}}(h_t)$$

output

new state

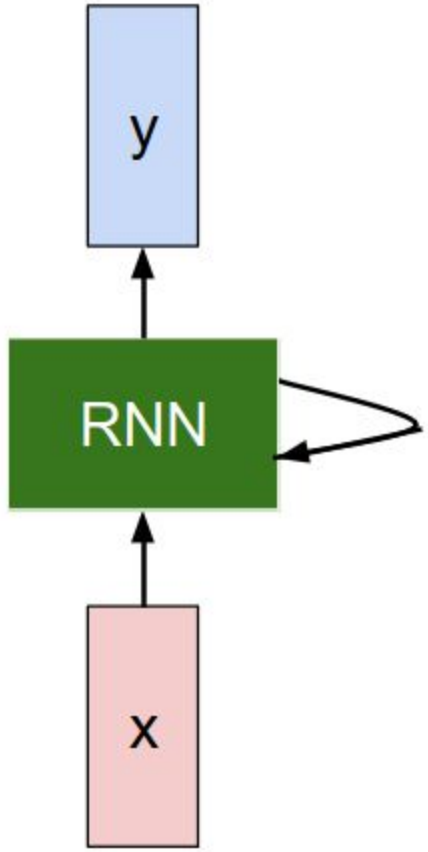
another function with parameters W_o



Recurrent Neural Network

We can process a sequence of vectors **x** by applying a **recurrence formula** at every time step:

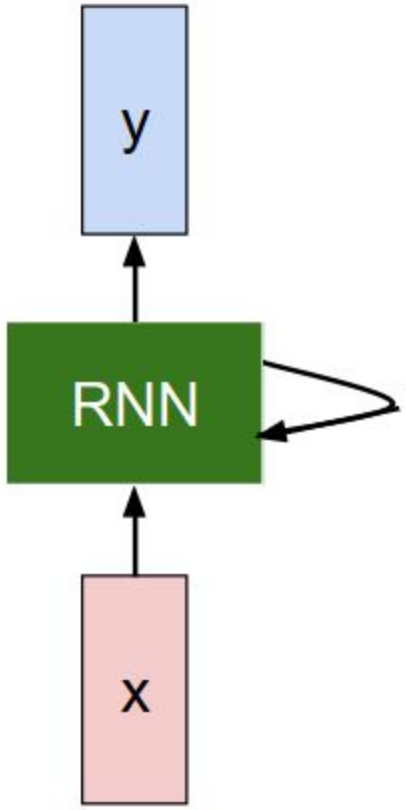
$$h_t = f_W(h_{t-1}, x_t)$$



Notice: the same function and the same set of parameters are used at every time step.

(Vanilla) Recurrent Neural Network

The state consists of a single *"hidden"* vector **h**:



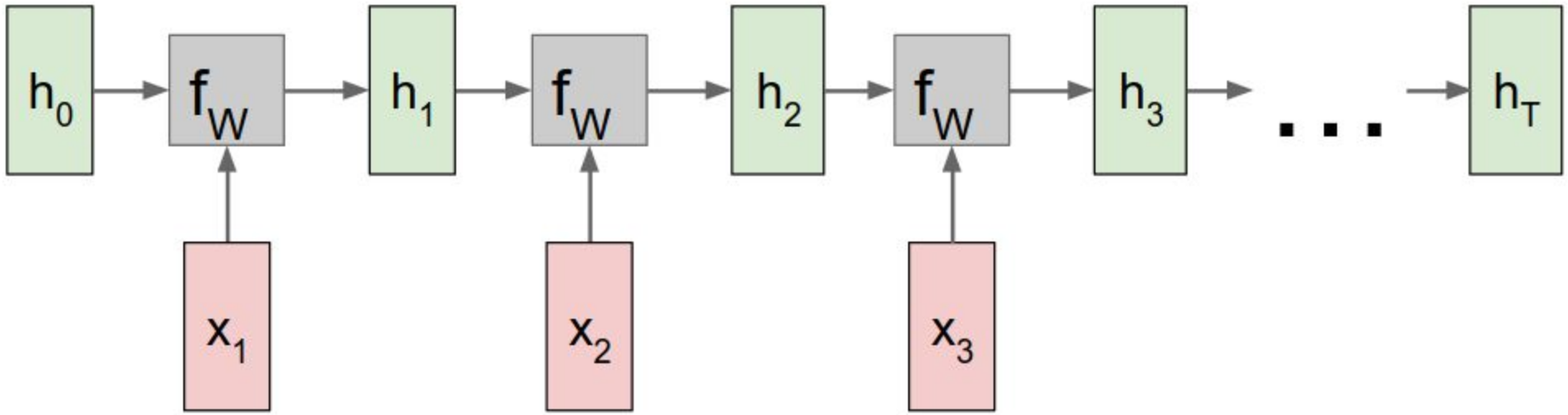
$$h_t = f_W(h_{t-1}, x_t)$$

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

$$y_t = W_{hy}h_t$$

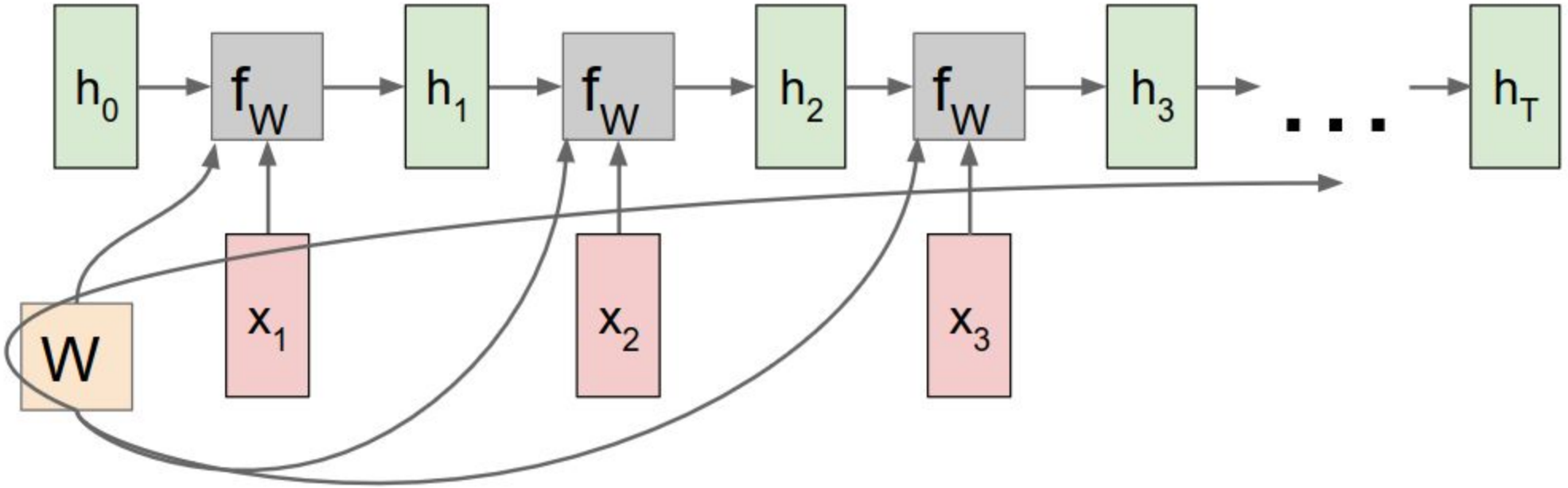
Sometimes called a "Vanilla RNN" or an "Elman RNN" after Prof. Jeffrey Elman

RNN: Computational Graph

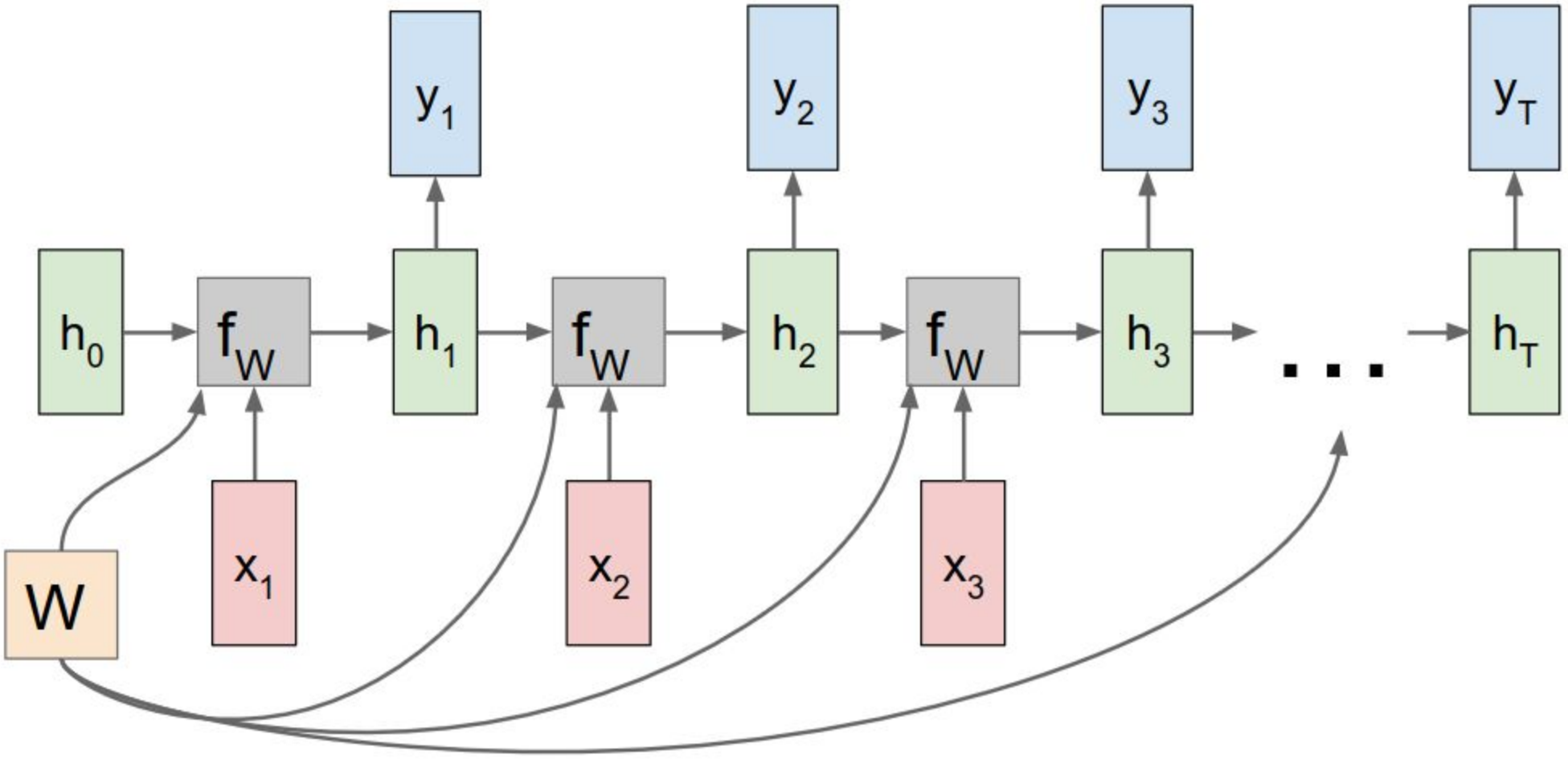


RNN: Computational Graph

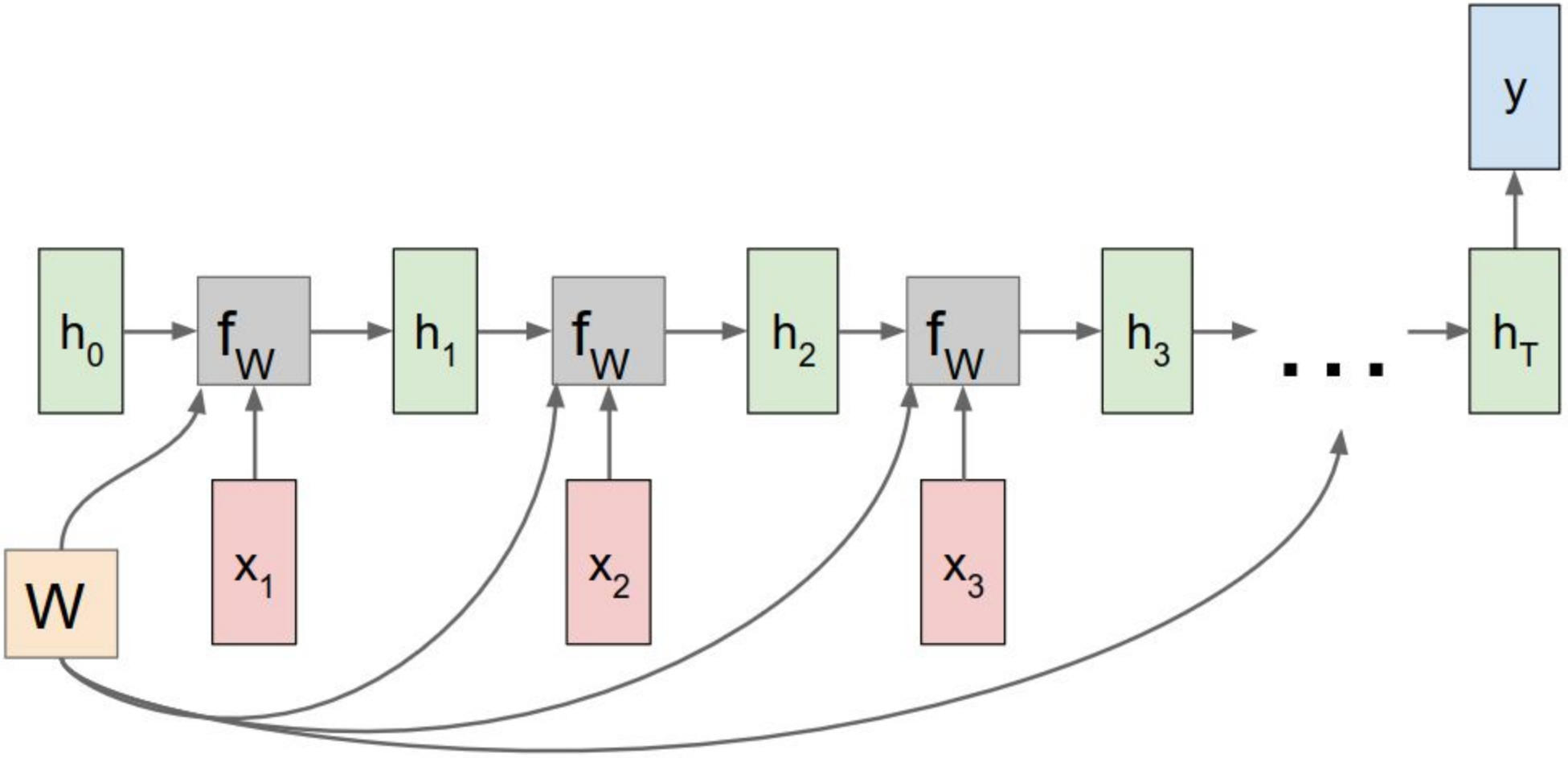
Re-use the same weight matrix at every time-step



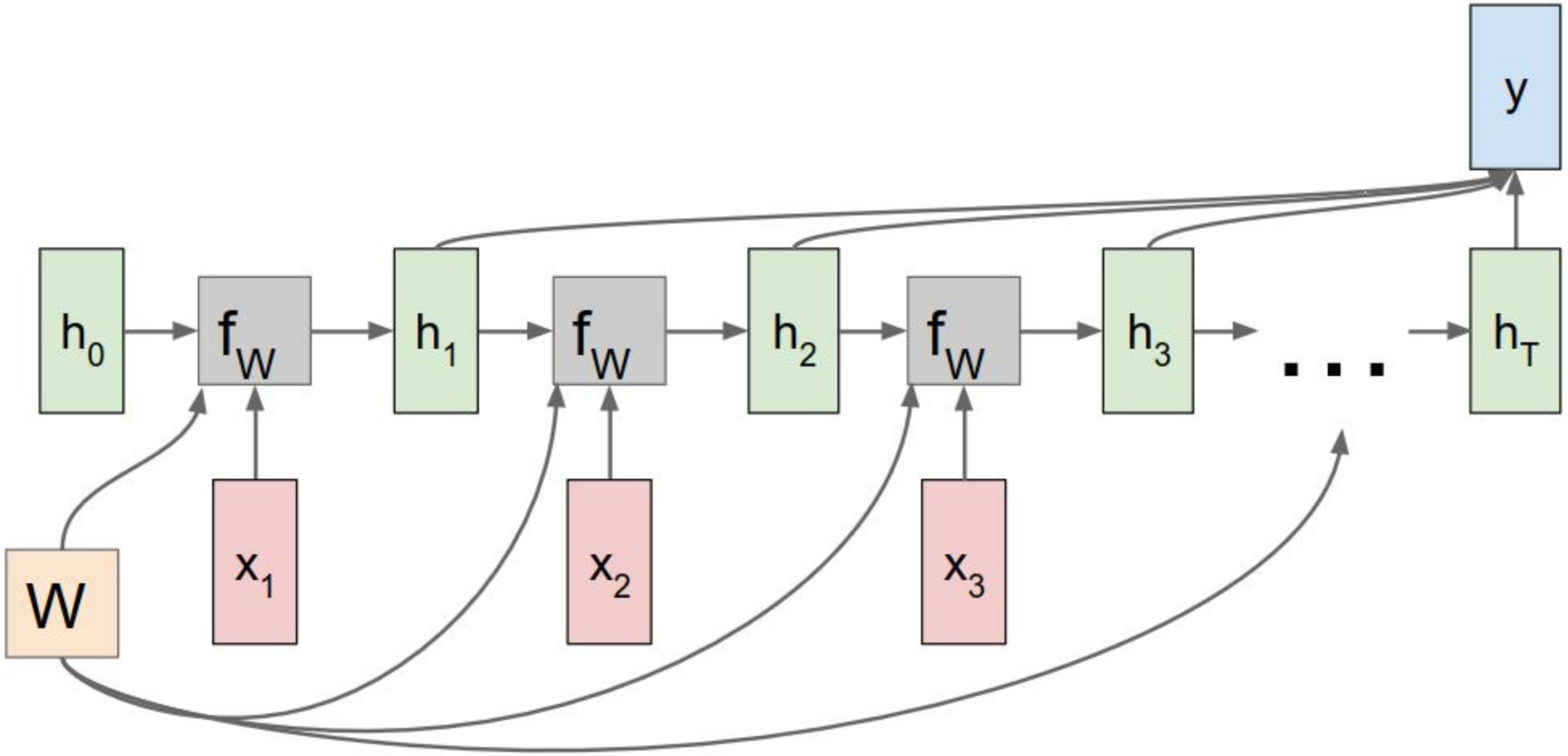
RNN: Computational Graph: Many to Many



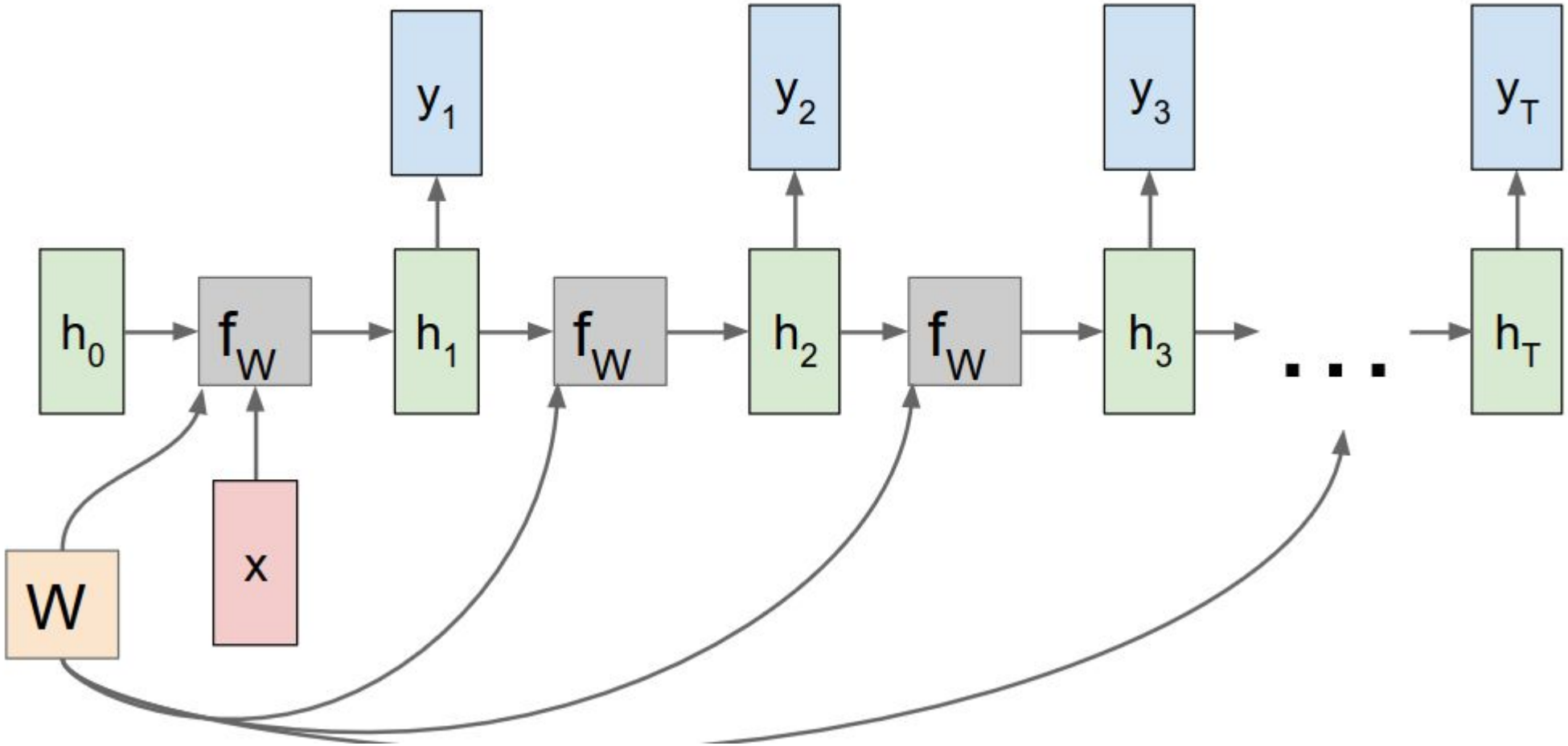
RNN: Computational Graph: Many to One



RNN: Computational Graph: Many to One



RNN: Computational Graph: One to Many



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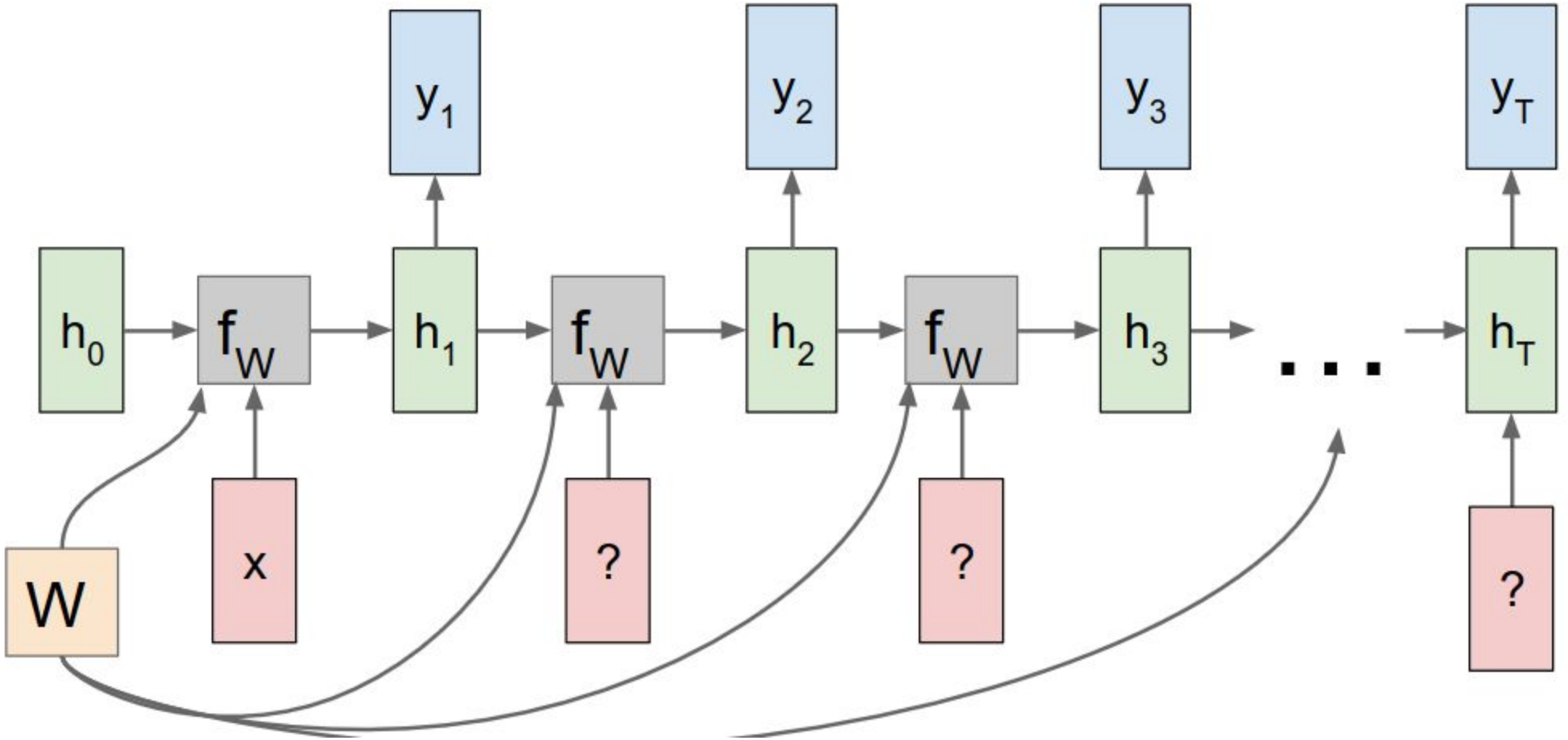

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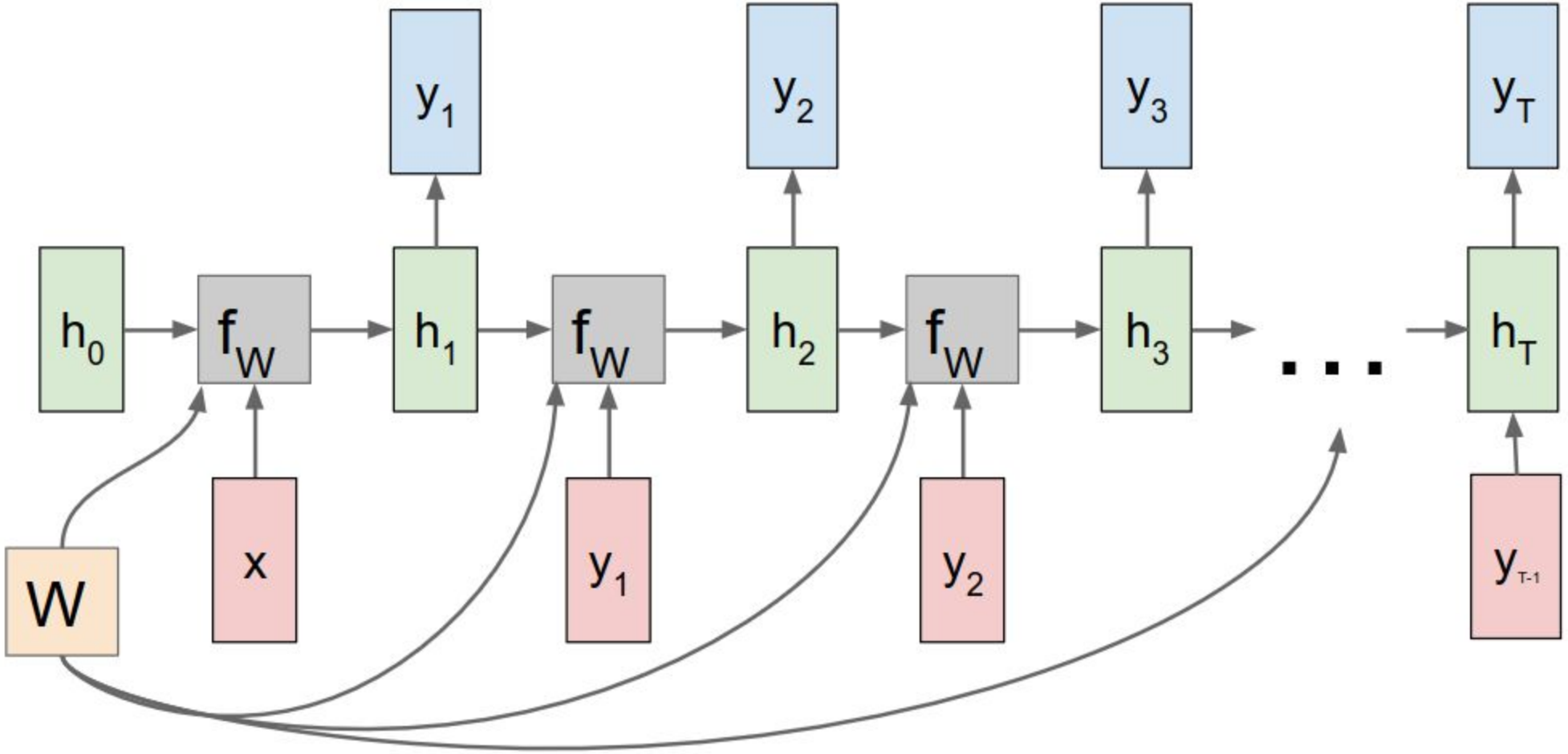
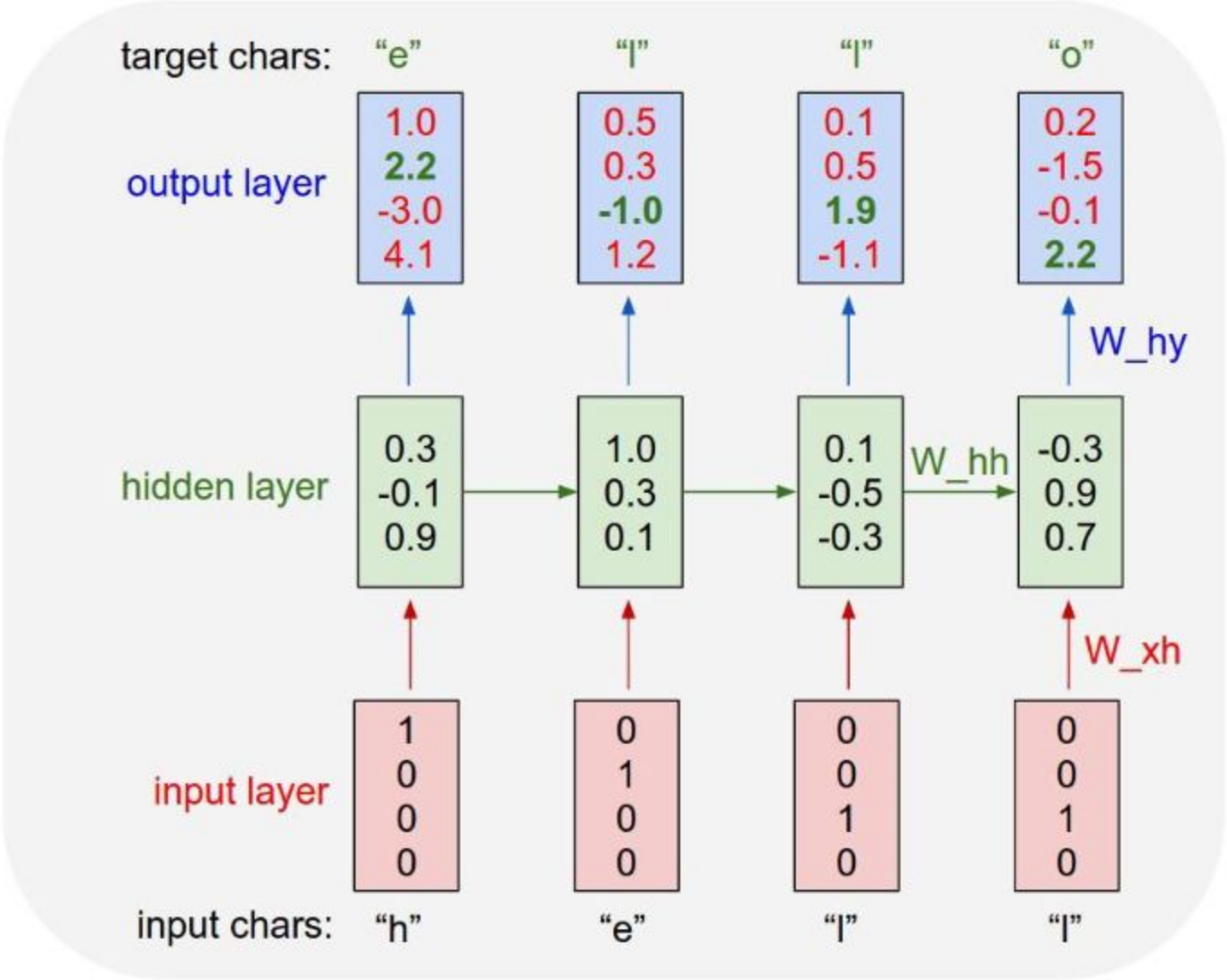


Example: Character-level Language Model

Vocabulary:
[h,e,l,o]

Example training sequence:
“hello”



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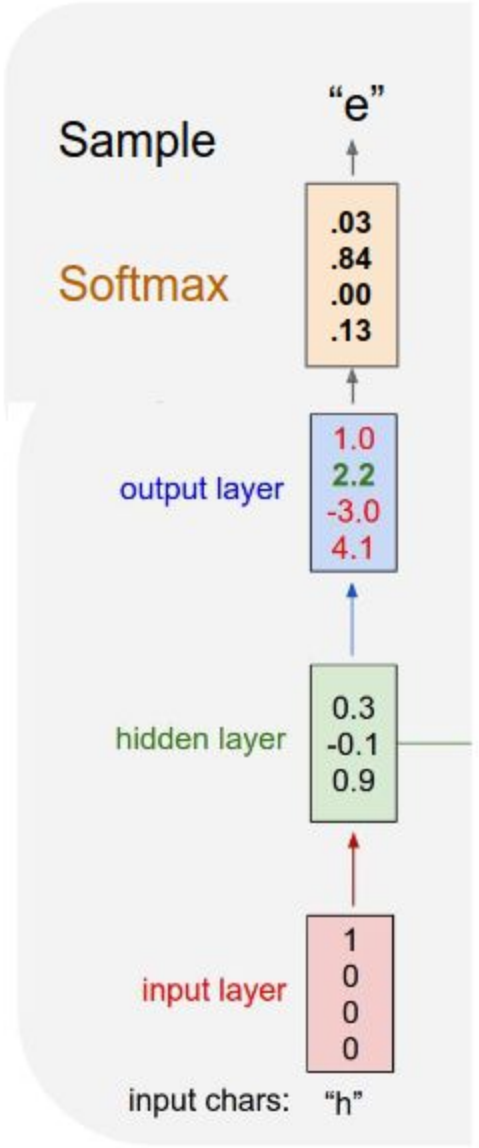
Example: Character-level Language Model Sampling

Vocabulary:
[h,e,l,o]

At test-time sample characters one at a time, feed back to model



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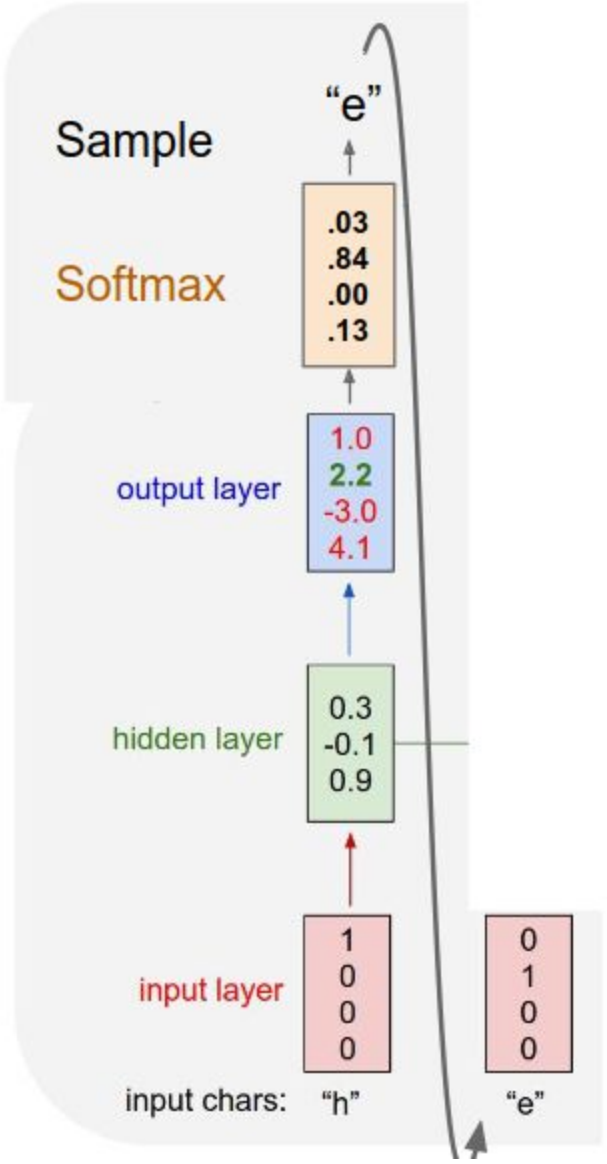

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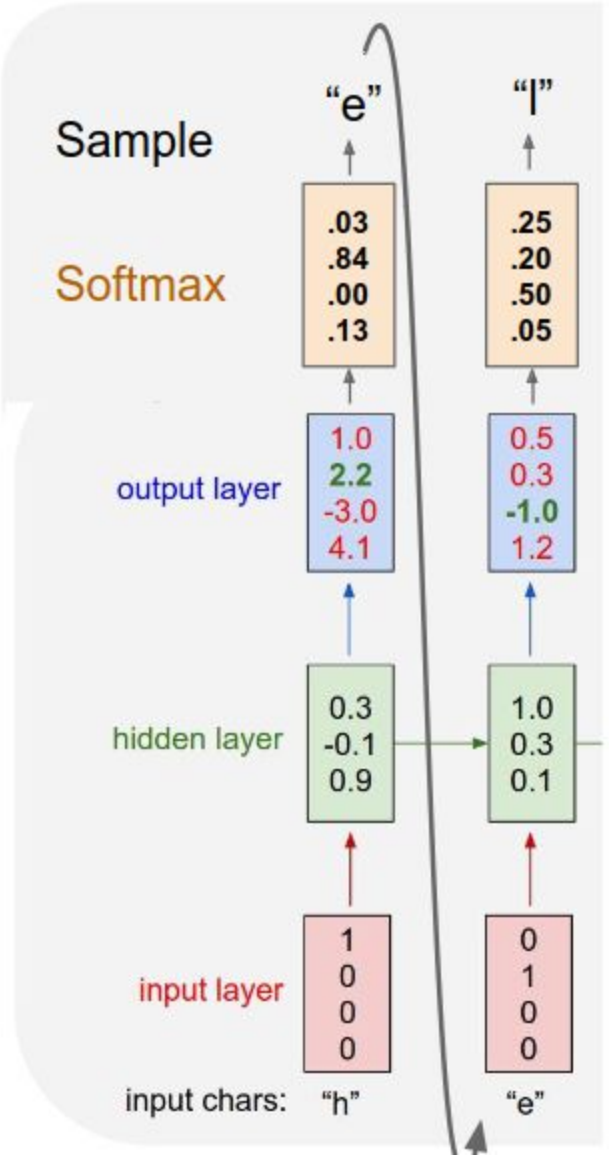

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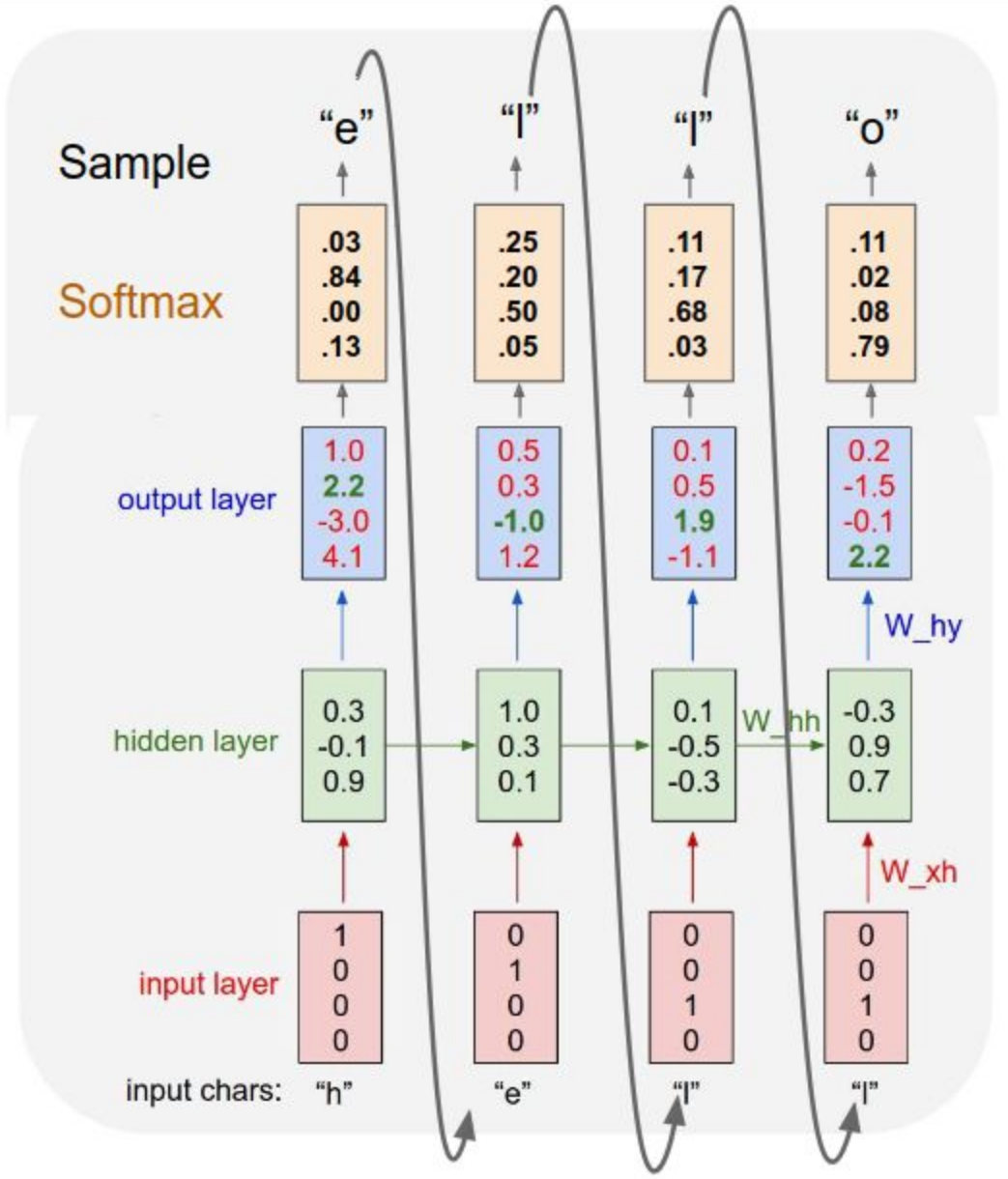
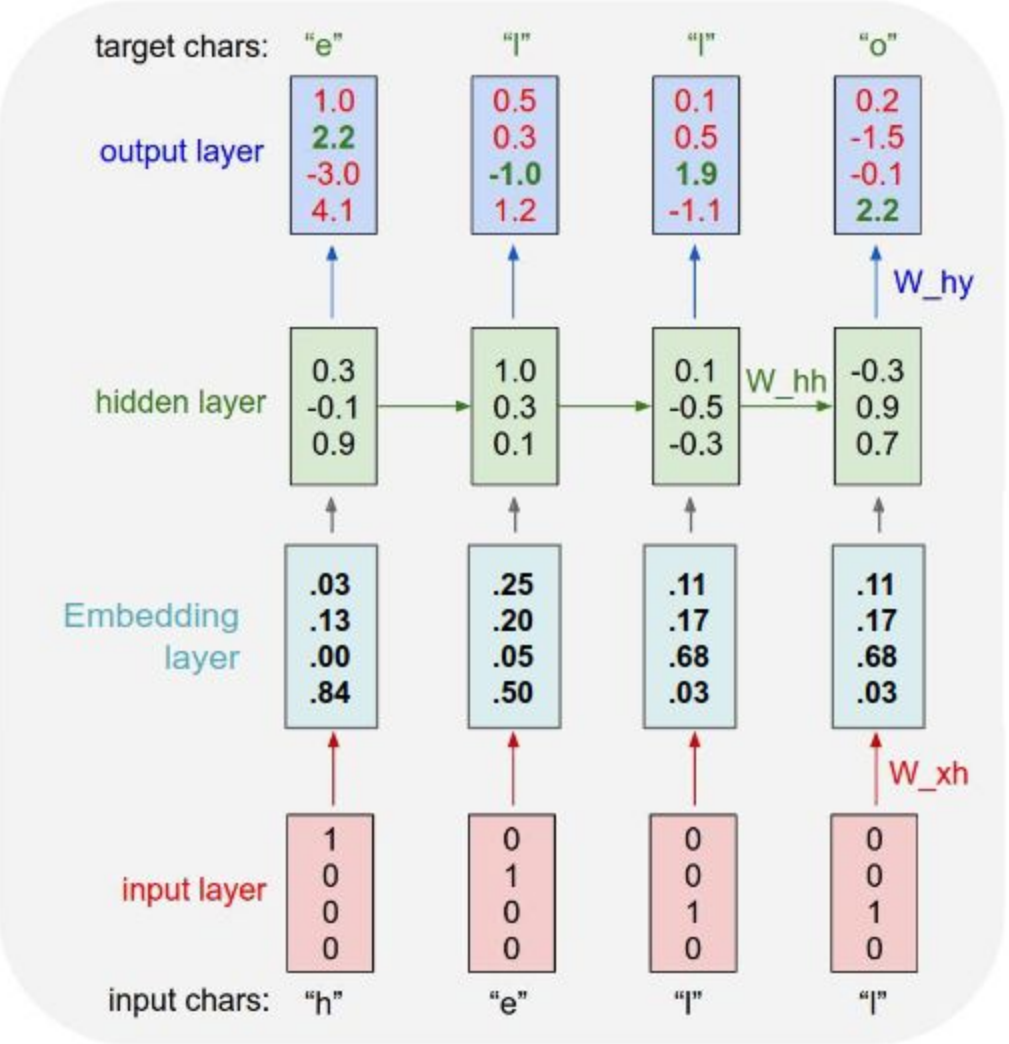


Example: Character-level Language Model Sampling

$$\begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} \\ w_{21} & w_{22} & w_{23} & w_{14} \\ w_{31} & w_{32} & w_{33} & w_{14} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} w_{11} \\ w_{21} \\ w_{31} \end{bmatrix}$$

Matrix multiply with a one-hot vector just extracts a column from the weight matrix. We often put a separate **embedding** layer between input and hidden layers.



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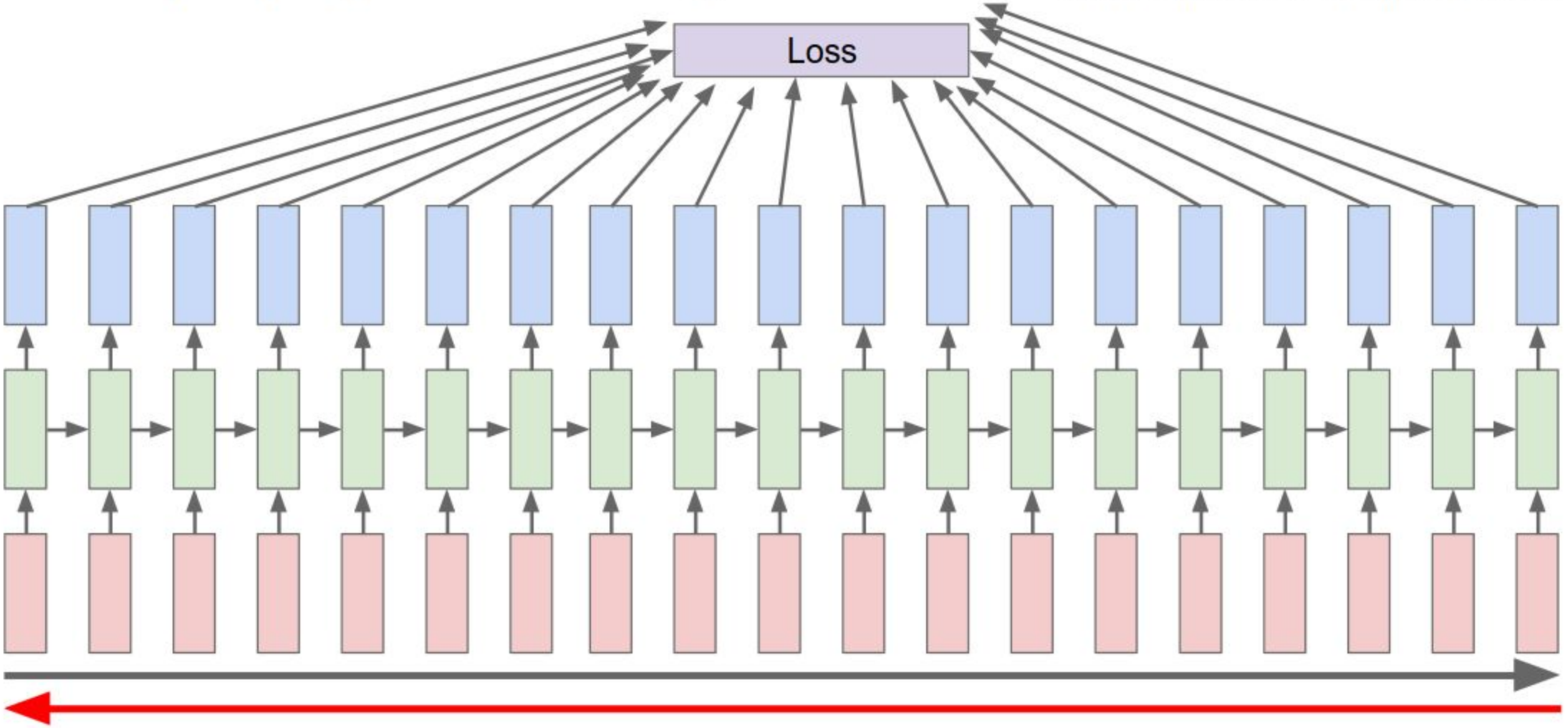
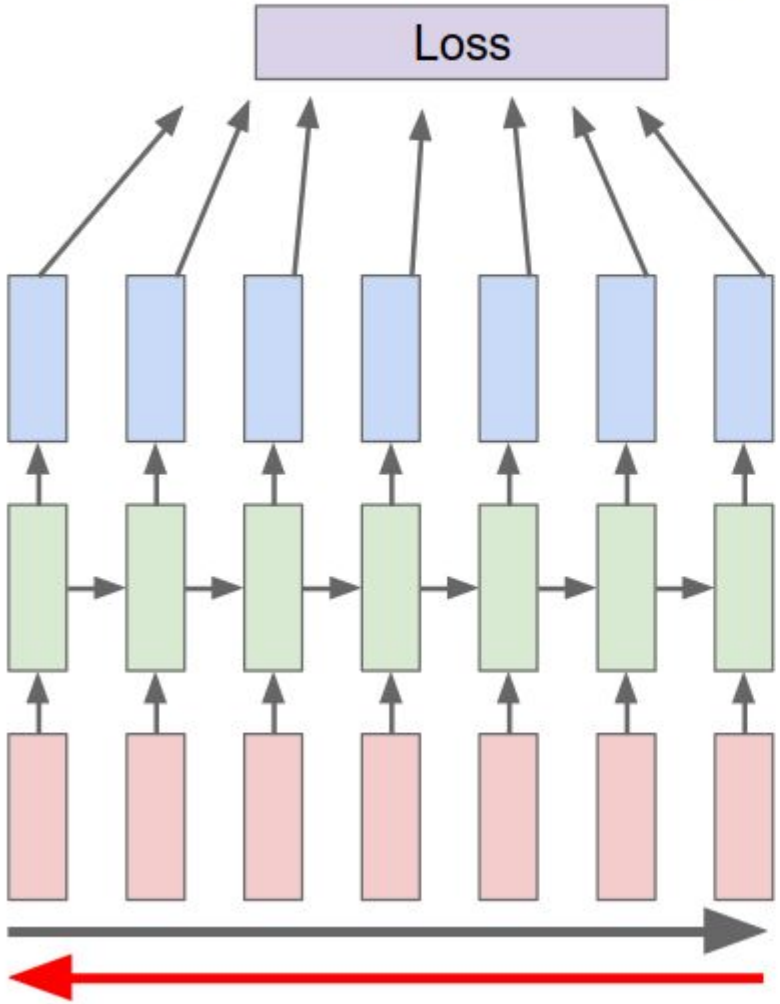
Backpropagation through time

Forward through entire sequence to compute loss, then backward through entire sequence to compute gradient

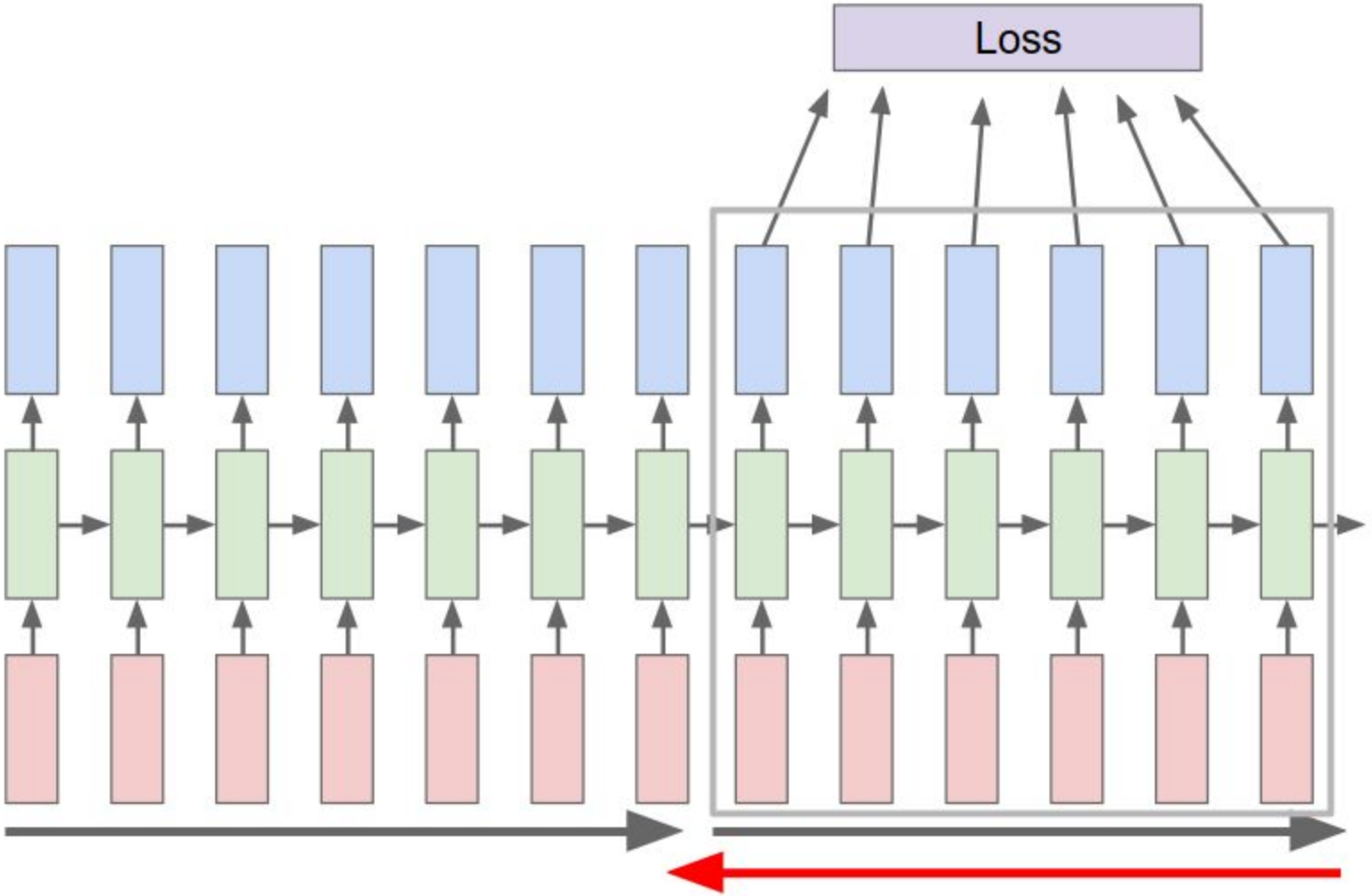


Truncated Backpropagation through time



Run forward and backward through chunks of the sequence instead of whole sequence

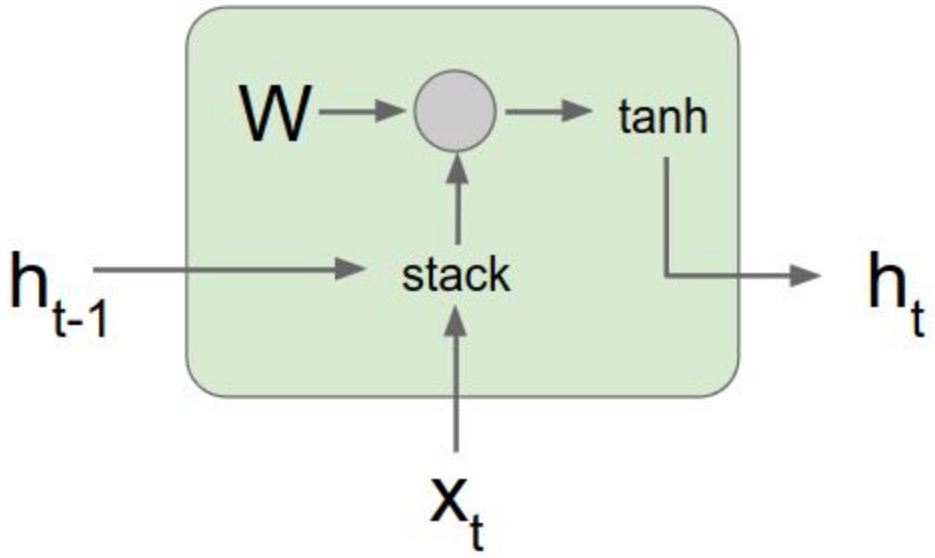
Truncated Backpropagation through time



Carry hidden states forward in time forever, but only backpropagate for some smaller number of steps

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$
$$= \tanh\left((W_{hh} \quad W_{hx})\begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$
$$= \tanh\left(W\begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$

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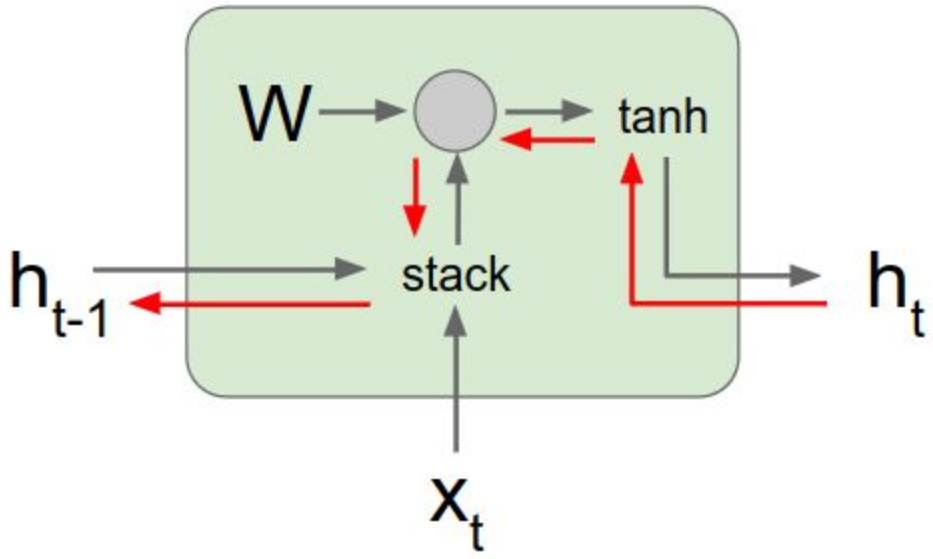
Backpropagation from h_t to h_{t-1} multiplies by W (actually W_{hh}^T)



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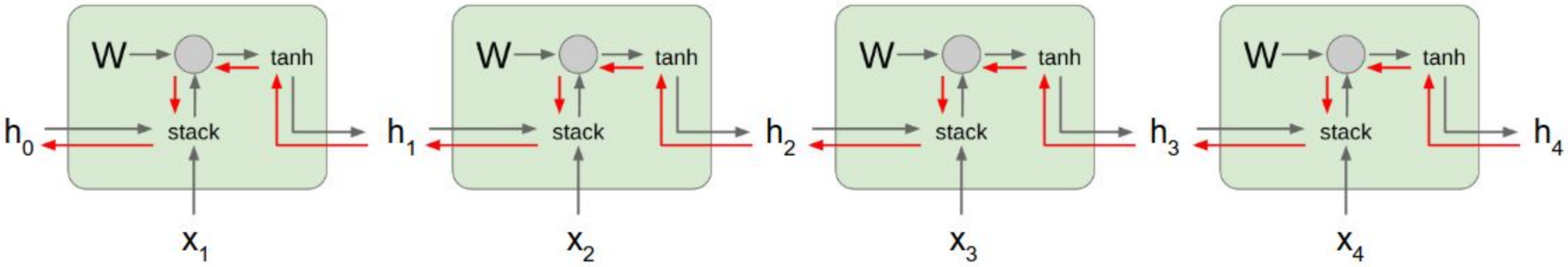

Computing gradient of h_0 involves many factors of W (and repeated tanh)

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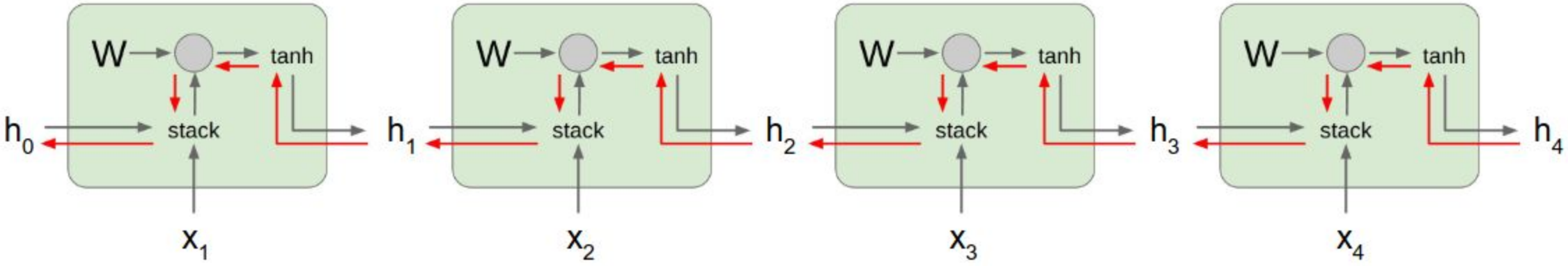

Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

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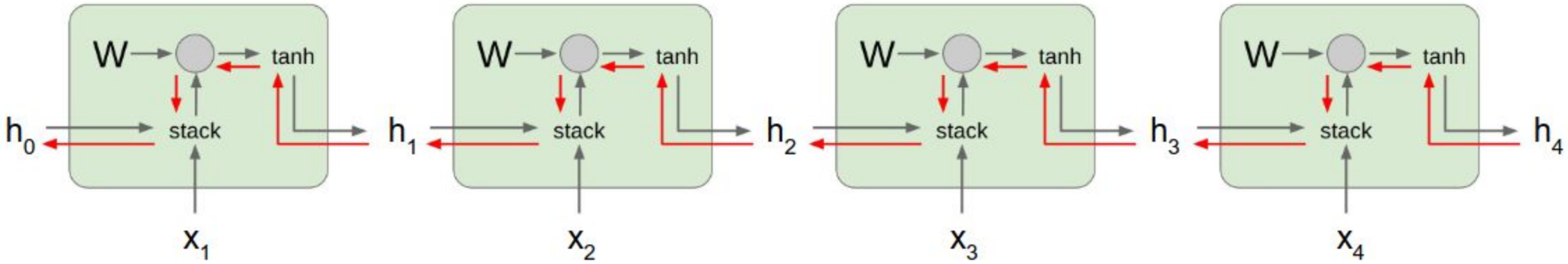
Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

→ **Gradient clipping**: Scale gradient if its norm is too big

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
  grad *= (threshold / grad_norm)
```

Largest singular value < 1:
Vanishing gradients

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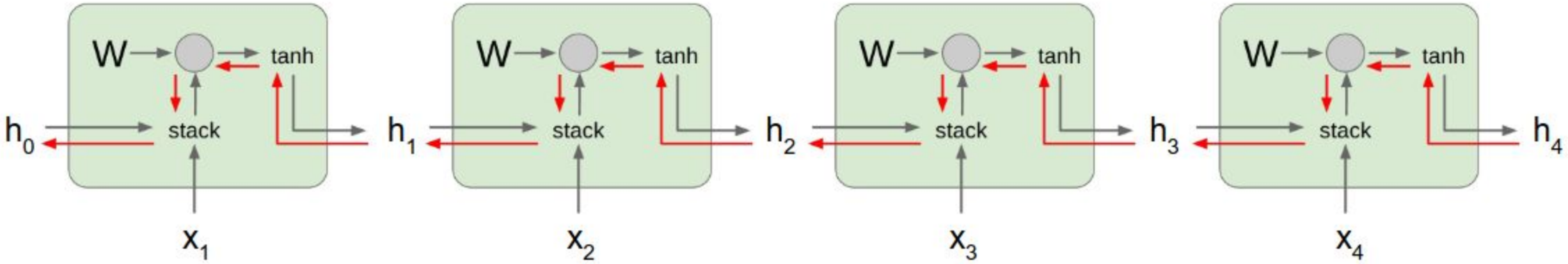

Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

→ Change RNN architecture

min-char-rnn.py gist: 112 lines of Python

```python
"""
Minimal character-level Vanilla RNN model. Written by Andrej Karpathy (@karpathy)
BSD License
"""
import numpy as np

# data I/O
data = open('input.txt', 'r').read() # should be simple plain text file
chars = list(set(data))
data_size, vocab_size = len(data), len(chars)
print 'data has %d characters, %d unique.' % (data_size, vocab_size)
char_to_ix = { ch:i for i,ch in enumerate(chars) }
ix_to_char = { i:ch for i,ch in enumerate(chars) }

# hyperparameters
hidden_size = 100 # size of hidden layer of neurons
seq_length = 25 # number of steps to unroll the RNN for
learning_rate = 1e-1

# model parameters
Wxh = np.random.randn(hidden_size, vocab_size)*0.01 # input to hidden
Whh = np.random.randn(hidden_size, hidden_size)*0.01 # hidden to hidden
Why = np.random.randn(vocab_size, hidden_size)*0.01 # hidden to output
bh = np.zeros((hidden_size, 1)) # hidden bias
by = np.zeros((vocab_size, 1)) # output bias

def lossFun(inputs, targets, hprev):
  """
  inputs,targets are both list of integers.
  hprev is Hx1 array of initial hidden state
  returns the loss, gradients on model parameters, and last hidden state
  """
  xs, hs, ys, ps = {}, {}, {}, {}
  hs[-1] = np.copy(hprev)
  loss = 0
  # forward pass
  for t in xrange(len(inputs)):
    xs[t] = np.zeros((vocab_size,1)) # encode in 1-of-k representation
    xs[t][inputs[t]] = 1
    hs[t] = np.tanh(np.dot(Wxh, xs[t]) + np.dot(Whh, hs[t-1]) + bh) # hidden state
    ys[t] = np.dot(Why, hs[t]) + by # unnormalized log probabilities for next chars
    ps[t] = np.exp(ys[t]) / np.sum(np.exp(ys[t])) # probabilities for next chars
    loss += -np.log(ps[t][targets[t],0]) # softmax (cross-entropy loss)
  # backward pass: compute gradients going backwards
  dWxh, dWhh, dWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
  dbh, dby = np.zeros_like(bh), np.zeros_like(by)
  dhnext = np.zeros_like(hs[0])
  for t in reversed(xrange(len(inputs))):
    dy = np.copy(ps[t])
    dy[targets[t]] -= 1 # backprop into y
    dWhy += np.dot(dy, hs[t].T)
    dby += dy
    dh = np.dot(Why.T, dy) + dhnext # backprop into h
    dhraw = (1 - hs[t] * hs[t]) * dh # backprop through tanh nonlinearity
    dbh += dhraw
    dWxh += np.dot(dhraw, xs[t].T)
    dWhh += np.dot(dhraw, hs[t-1].T)
    dhnext = np.dot(Whh.T, dhraw)
  for dparam in [dWxh, dWhh, dWhy, dbh, dby]:
    np.clip(dparam, -5, 5, out=dparam) # clip to mitigate exploding gradients
  return loss, dWxh, dWhh, dWhy, dbh, dby, hs[len(inputs)-1]

def sample(h, seed_ix, n):
  """
  sample a sequence of integers from the model
  h is memory state, seed_ix is seed letter for first time step
  """
  x = np.zeros((vocab_size, 1))
  x[seed_ix] = 1
  ixes = []
  for t in xrange(n):
    h = np.tanh(np.dot(Wxh, x) + np.dot(Whh, h) + bh)
    y = np.dot(Why, h) + by
    p = np.exp(y) / np.sum(np.exp(y))
    ix = np.random.choice(range(vocab_size), p=p.ravel())
    x = np.zeros((vocab_size, 1))
    x[ix] = 1
    ixes.append(ix)
  return ixes

n, p = 0, 0
mWxh, mWhh, mWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
mbh, mby = np.zeros_like(bh), np.zeros_like(by) # memory variables for Adagrad
smooth_loss = -np.log(1.0/vocab_size)*seq_length # loss at iteration 0
while True:
  # prepare inputs (we're sweeping from left to right in steps seq_length long)
  if p+seq_length+1 >= len(data) or n == 0:
    hprev = np.zeros((hidden_size,1)) # reset RNN memory
    p = 0 # go from start of data
  inputs = [char_to_ix[ch] for ch in data[p:p+seq_length]]
  targets = [char_to_ix[ch] for ch in data[p+1:p+seq_length+1]]

  # sample from the model now and then
  if n % 100 == 0:
    sample_ix = sample(hprev, inputs[0], 200)
    txt = ''.join(ix_to_char[ix] for ix in sample_ix)
    print '----\n %s \n----' % (txt, )

  # forward seq_length characters through the net and fetch gradient
  loss, dWxh, dWhh, dWhy, dbh, dby, hprev = lossFun(inputs, targets, hprev)
  smooth_loss = smooth_loss * 0.999 + loss * 0.001
  if n % 100 == 0: print 'iter %d, loss: %f' % (n, smooth_loss) # print progress

  # perform parameter update with Adagrad
  for param, dparam, mem in zip([Wxh, Whh, Why, bh, by],
                                [dWxh, dWhh, dWhy, dbh, dby],
                                [mWxh, mWhh, mWhy, mbh, mby]):
    mem += dparam * dparam
    param += -learning_rate * dparam / np.sqrt(mem + 1e-8) # adagrad update

  p += seq_length # move data pointer
  n += 1 # iteration counter
```

(https://gist.github.com/karpathy/d4dee566867f8291f086)

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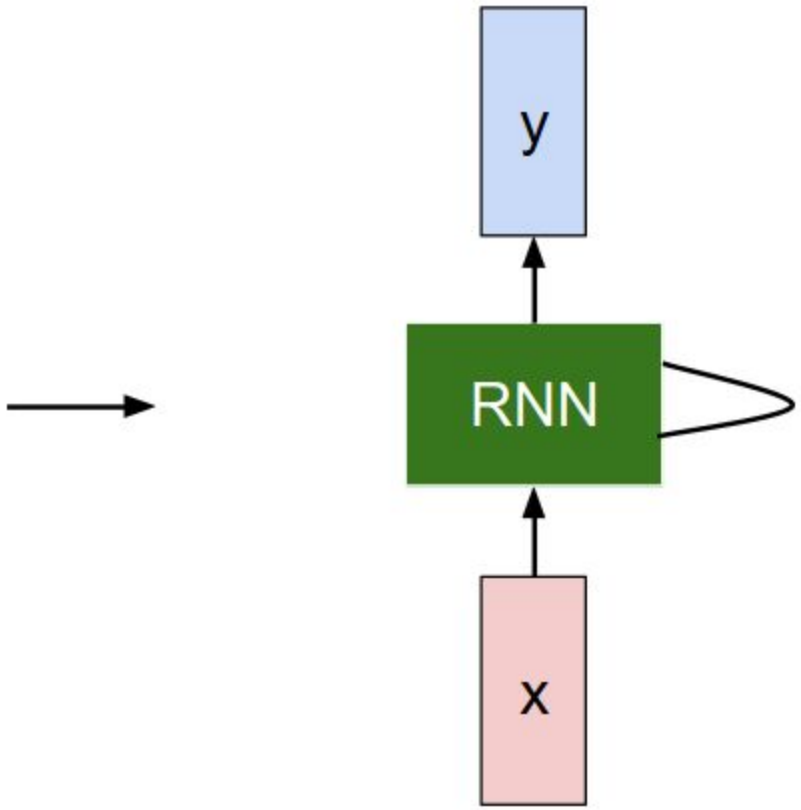
THE SONNETS

by William Shakespeare

From fairest creatures we desire increase,
That thereby beauty's rose might never die,
But as the riper should by time decease,
His tender heir might bear his memory:
But thou, contracted to thine own bright eyes,
Feed'st thy light's flame with self-substantial fuel,
Making a famine where abundance lies,
Thyself thy foe, to thy sweet self too cruel:
Thou that art now the world's fresh ornament,
And only herald to the gaudy spring,
Within thine own bud buriest thy content,
And tender churl mak'st waste in niggarding:
    Pity the world, or else this glutton be,
    To eat the world's due, by the grave and thee.

When forty winters shall besiege thy brow,
And dig deep trenches in thy beauty's field,
Thy youth's proud livery so gazed on now,
Will be a tatter'd weed of small worth held:
Then being asked, where all thy beauty lies,
Where all the treasure of thy lusty days;
To say, within thine own deep sunken eyes,
Were an all-eating shame, and thriftless praise.
How much more praise deserv'd thy beauty's use,
If thou couldst answer 'This fair child of mine
Shall sum my count, and make my old excuse,'
Proving his beauty by succession thine!
    This were to be new made when thou art old,
    And see thy blood warm when thou feel'st it cold.



at first:

```
tyntd-iafhatawiaoihrdemot  lytdws  e ,tfti, astai f ogoh eoase rrranbyne 'nhthnee e
plia tklrgd t o idoe ns,smtt   h ne etie h,hregtrs nigtike,aoaenns lng
```

train more

```
"Tmont thithey" fomesscerliund
Keushey. Thom here
sheulke, anmerenith ol sivh I lalterthend Bleipile shuwy fil on aseterlome
coaniogennc Phe lism thond hon at. MeiDimorotion in ther thize."
```

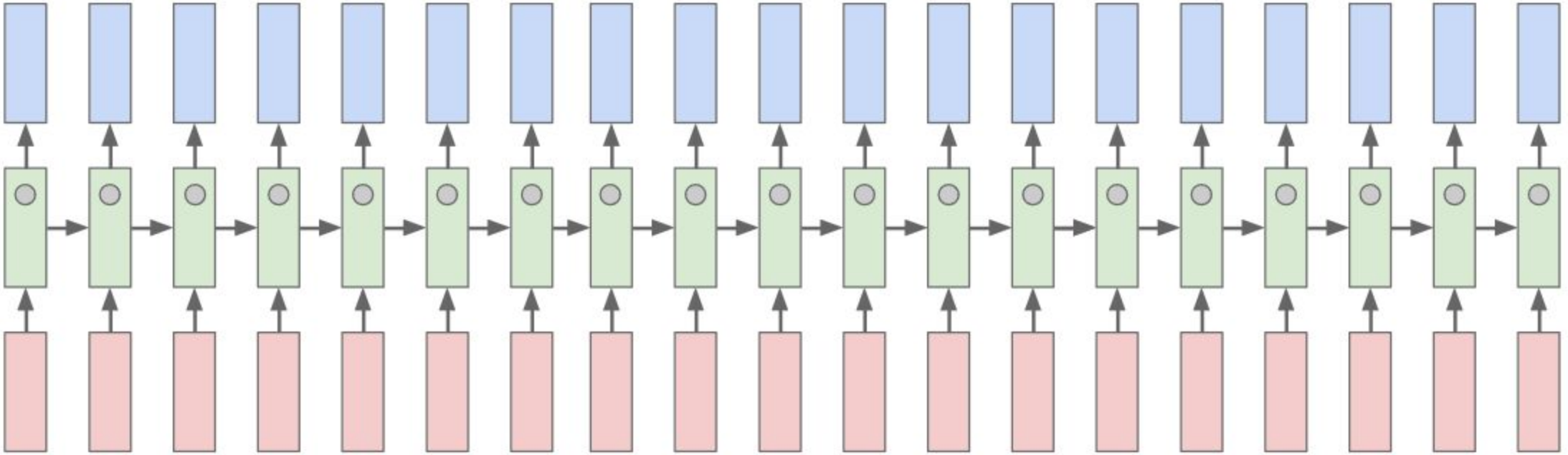
train more

```
Aftair fall unsuch that the hall for Prince Velzonski's that me of
her hearly, and behs to so arwage fiving were to it beloge, pavu say falling misfort
how, and Gogition is so overelical and ofter.
```

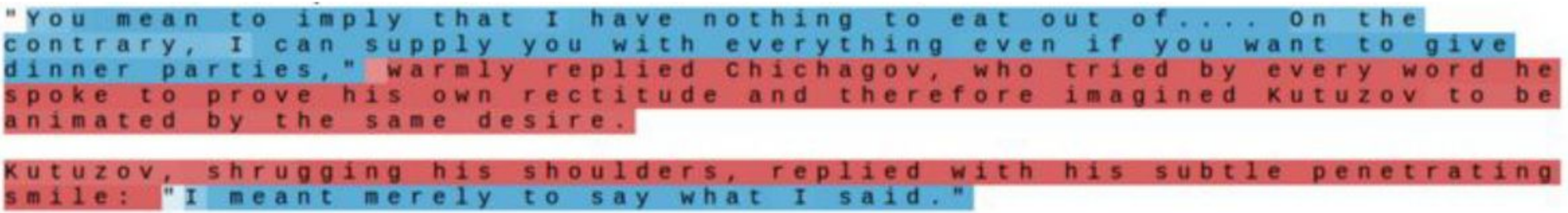
train more

```
"Why do what that day," replied Natasha, and wishing to himself the fact the
princess, Princess Mary was easier, fed in had oftened him.
Pierre aking his soul came to the packs and drove up his father-in-law women.
```

Searching for interpretable cells



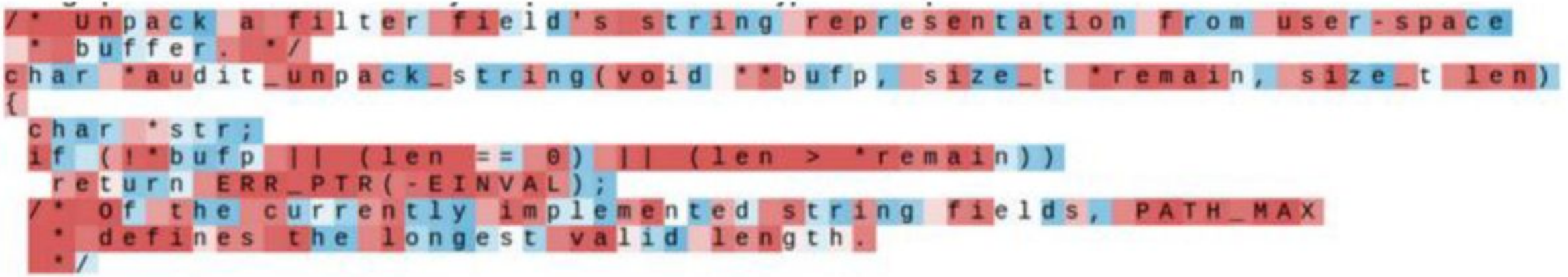
Searching for interpretable cells



quote detection cell

Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016

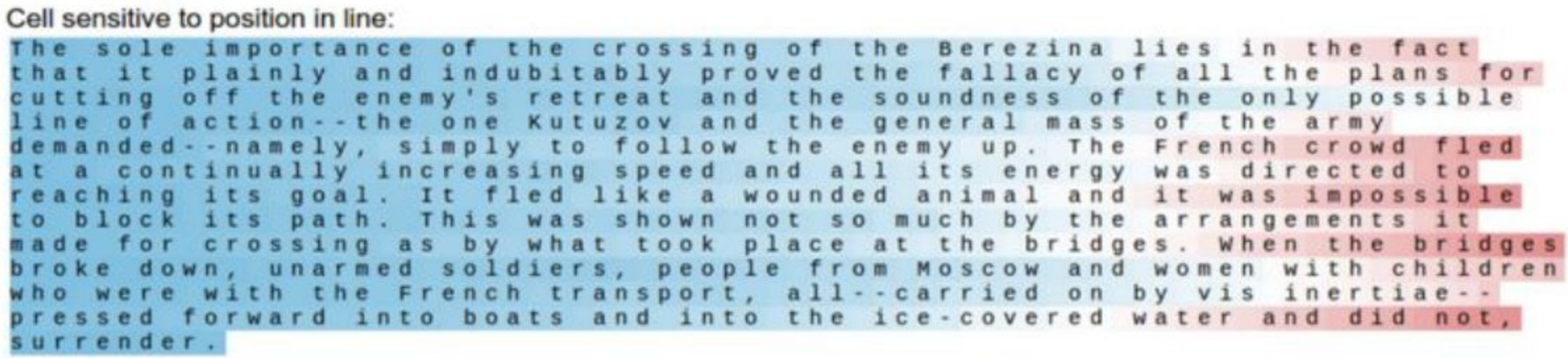
Searching for interpretable cells



```
/* Unpack a filter field's string representation from user-space
 * buffer. */
char *audit_unpack_string(void **bufp, size_t *remain, size_t len)
{
  char *str;
  if (!*bufp || (len == 0) || (len > *remain))
    return ERR_PTR(-EINVAL);
  /* Of the currently implemented string fields, PATH_MAX
   * defines the longest valid length.
   */
```

Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016

Searching for interpretable cells



line length tracking cell

Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016

Searching for interpretable cells

```
static int __dequeue_signal(struct sigpending *pending, sigset_t *mask,
        siginfo_t *info)
{
    int sig = next_signal(pending, mask);
    if (sig) {
        if (current->notifier) {
            if (sigismember(current->notifier_mask, sig)) {
                if (!(current->notifier)(current->notifier_data)) {
                    clear_thread_flag(TIF_SIGPENDING);
                    return 0;
                }
            }
        }
        collect_signal(sig, pending, info);
    }
    return sig;
}
```

if statement cell

Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016

Searching for interpretable cells

```
#ifdef CONFIG_AUDITSYSCALL
static inline int audit_match_class_bits(int class, u32 *mask)
{
  int i;
  if (classes[class]) {
    for (i = 0; i < AUDIT_BITMASK_SIZE; i++)
      if (mask[i] & classes[class][i])
        return 0;
  }
  return 1;
}
```

code depth cell

Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016

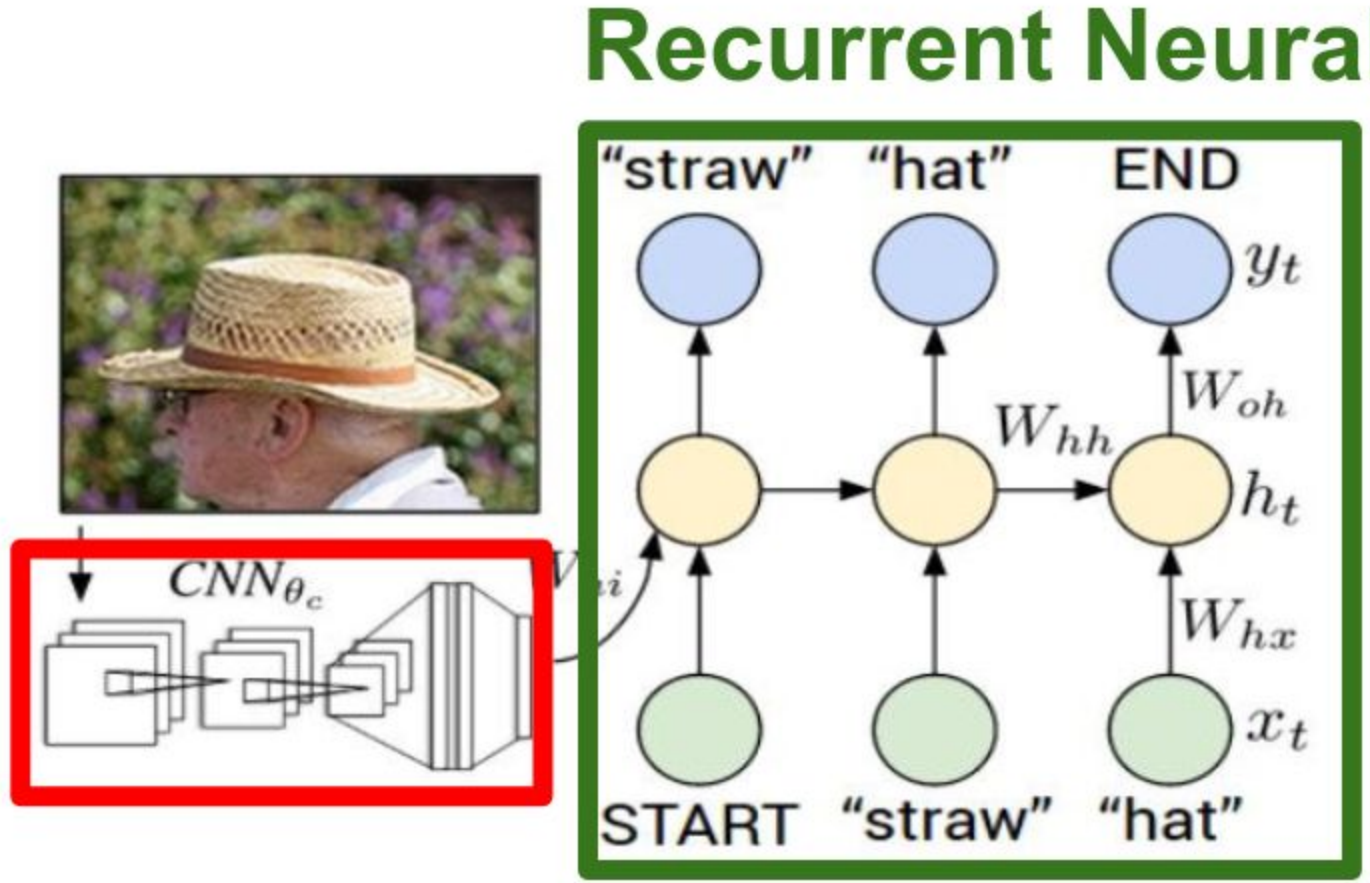
RNN tradeoffs

RNN Advantages:

- Can process any length input
- Computation for step *t* can (in theory) use information from many steps back
- Model size doesn't increase for longer input
- Same weights applied on every timestep, so there is symmetry in how inputs are processed.

RNN Disadvantages:

- Recurrent computation is slow
- In practice, difficult to access information from many steps back


Recurrent Neural Network
"straw"
"hat"
END
y_t
W_{oh}
W_{hh}
h_t
W_{hx}
x_t
START
"straw"
"hat"
CNN_{θ_c}
Convolutional Neural Network

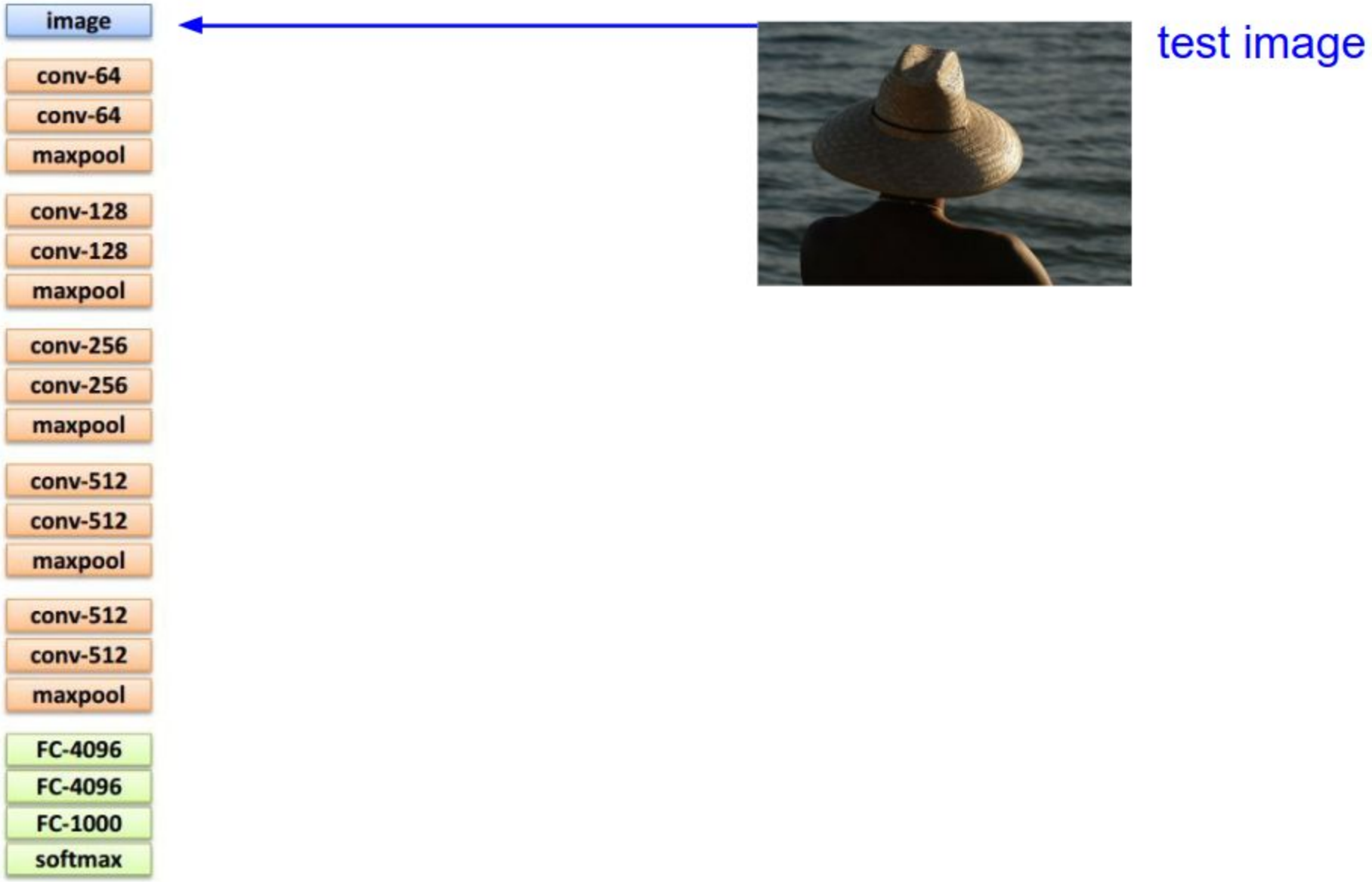


image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096
FC-1000
softmax
test image

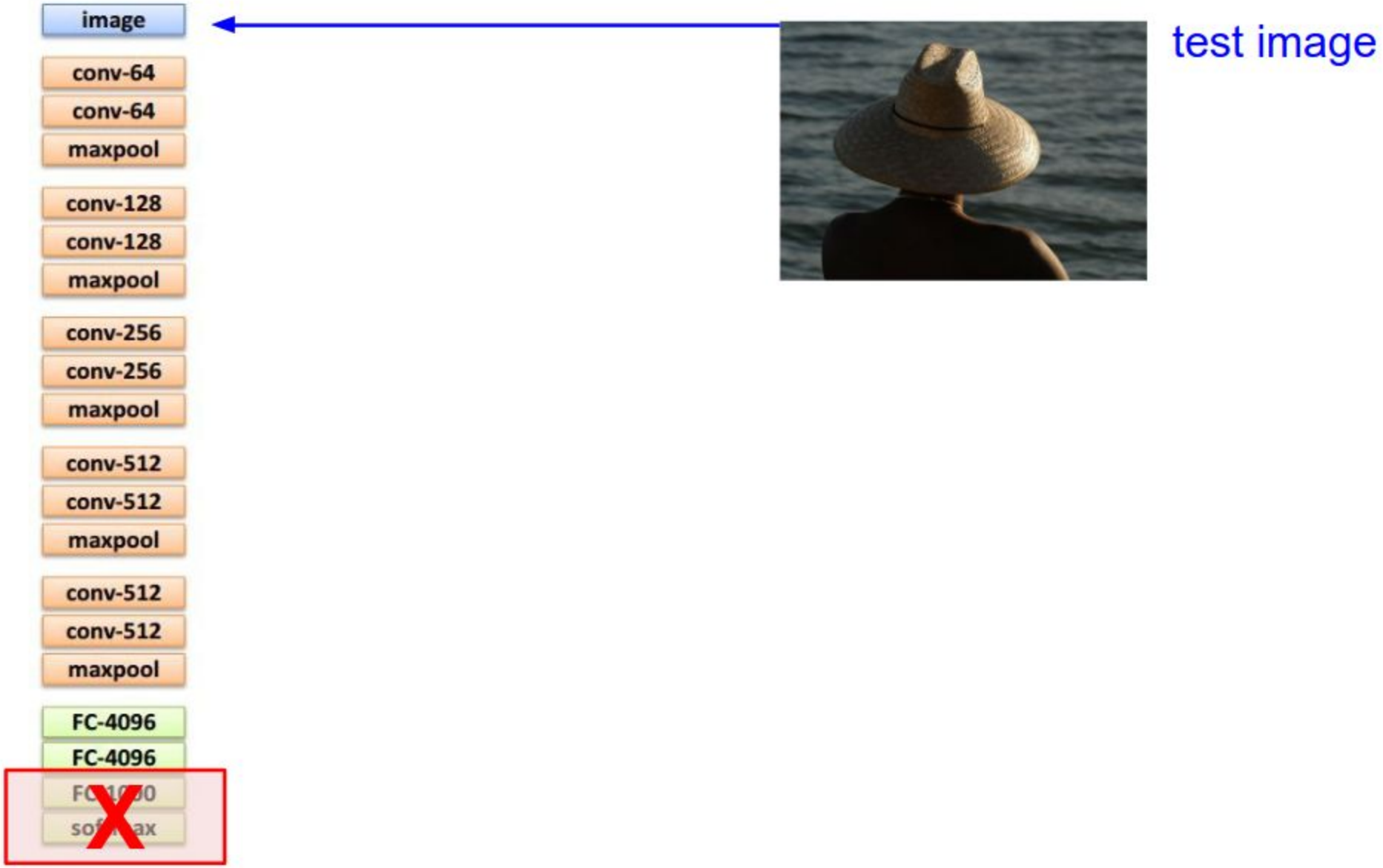
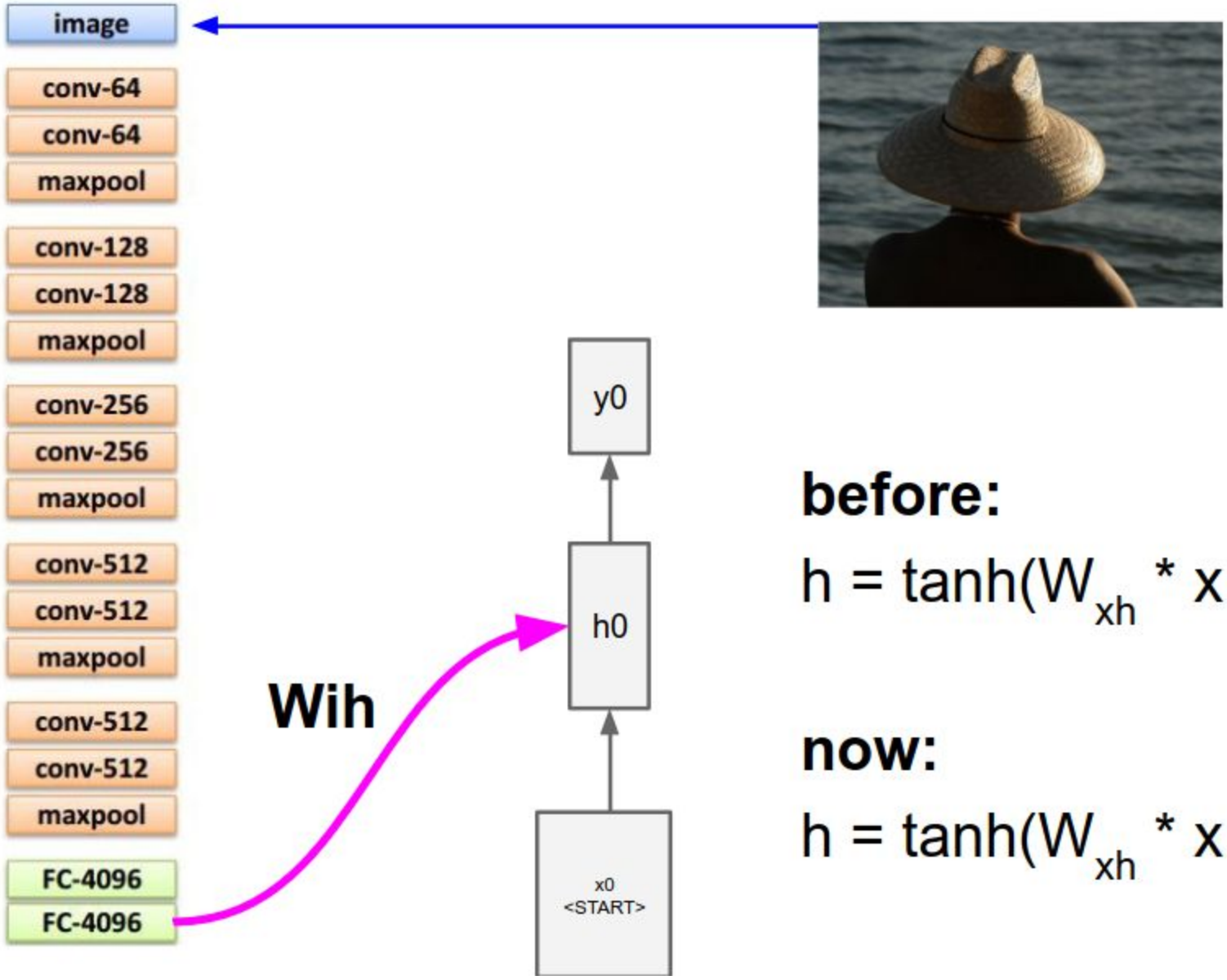


image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096
X
test image



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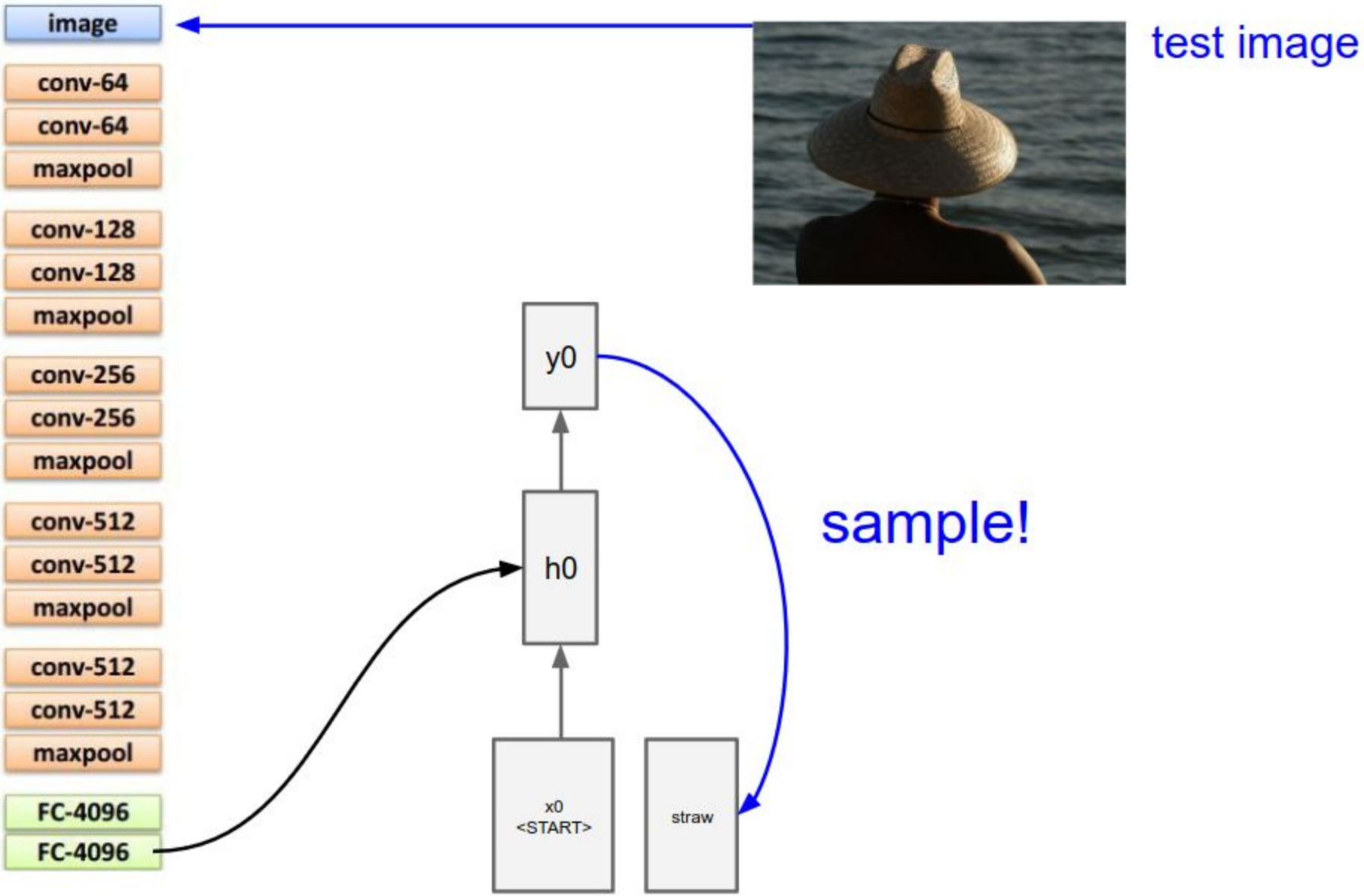
image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096
test image
y0
h0
x0
<START>
straw
sample!

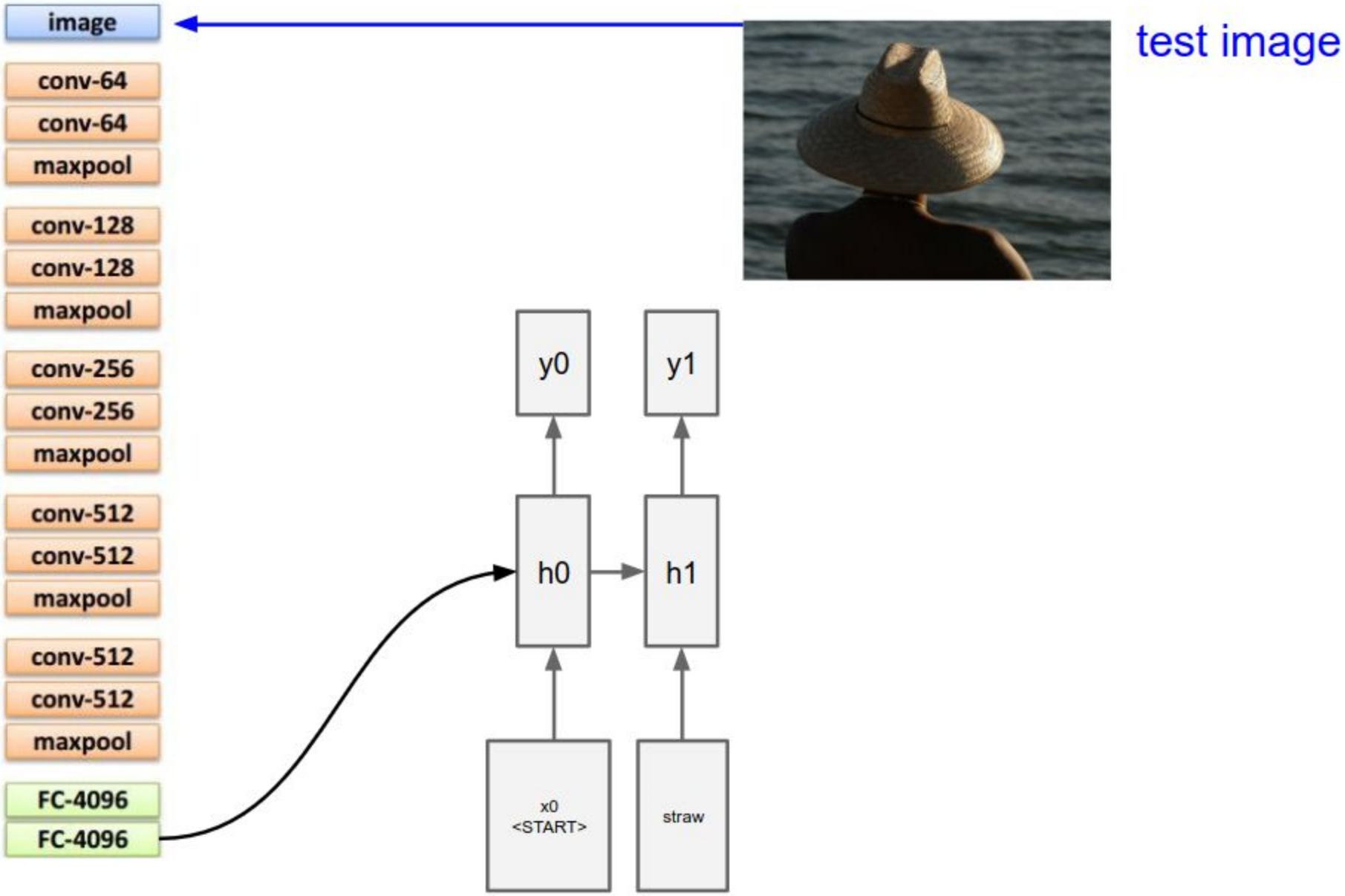
image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096
test image
y0
y1
h0
h1
x0
<START>
straw

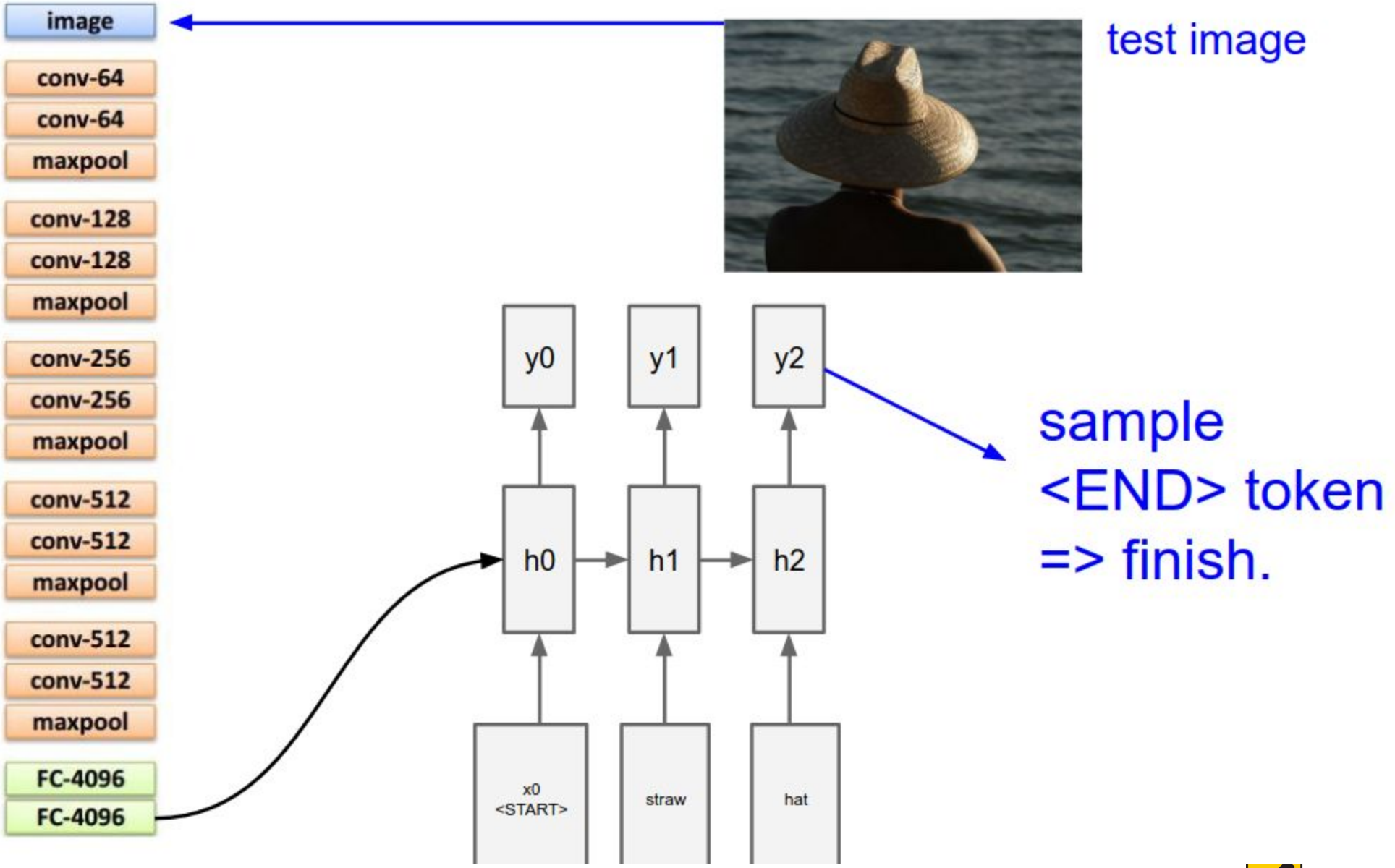
image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096
test image
y0
y1
y2
h0
h1
h2
x0
<START>
straw
hat
sample
<END> token
=> finish.

Image Captioning: Example Results

Captions generated using neuraltalk2
All images are CC0 Public domain:
cat suitcase, cat tree, dog, bear,
surfers, tennis, giraffe, motorcycle

A cat sitting on a suitcase on the floor

A cat is sitting on a tree branch

A dog is running in the grass with a frisbee

A white teddy bear sitting in the grass

Two people walking on the beach with surfboards

A tennis player in action on the court

Two giraffes standing in a grassy field

A man riding a dirt bike on a dirt track

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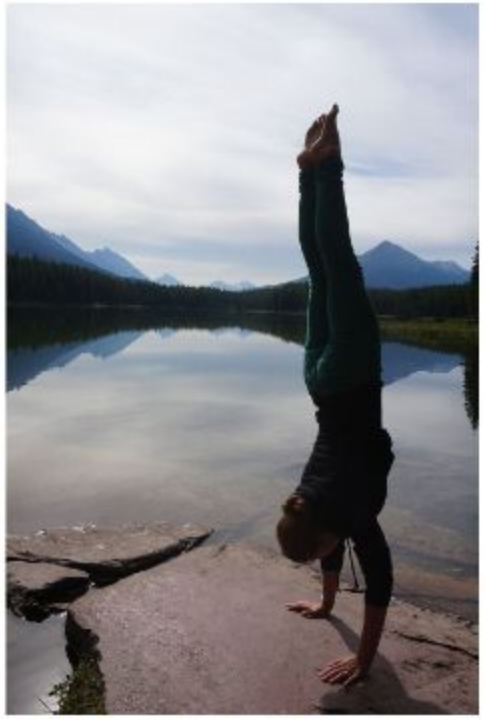
Image Captioning: Failure Cases

Captions generated using neuraltalk2
All images are CC0 Public domain: fur coat, handstand, spider web, baseball

A woman is holding a cat in her hand

A person holding a computer mouse on a desk



A woman standing on a beach holding a surfboard

A bird is perched on a tree branch

A man in a baseball uniform throwing a ball

Visual Question Answering (VQA)

Q: What endangered animal is featured on the truck?

A: **A bald eagle.**
A: A sparrow.
A: A humming bird.
A: A raven.

Q: Where will the driver go if turning right?

A: **Onto 24 ¾ Rd.**
A: Onto 25 ¾ Rd.
A: Onto 23 ¾ Rd.
A: Onto Main Street.

Q: When was the picture taken?

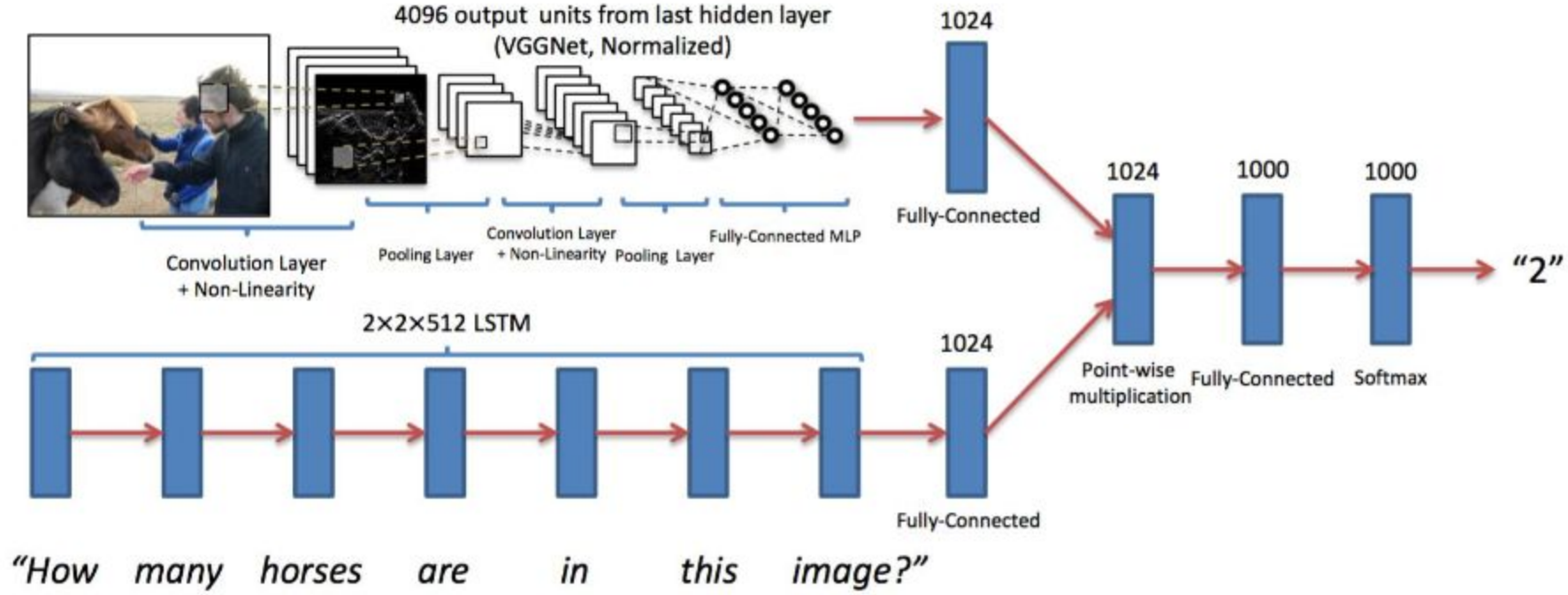
A: **During a wedding.**
A: During a bar mitzvah.
A: During a funeral.
A: During a Sunday church service

Q: Who is under the umbrella?

A: **Two women.**
A: A child.
A: An old man.
A: A husband and a wife.

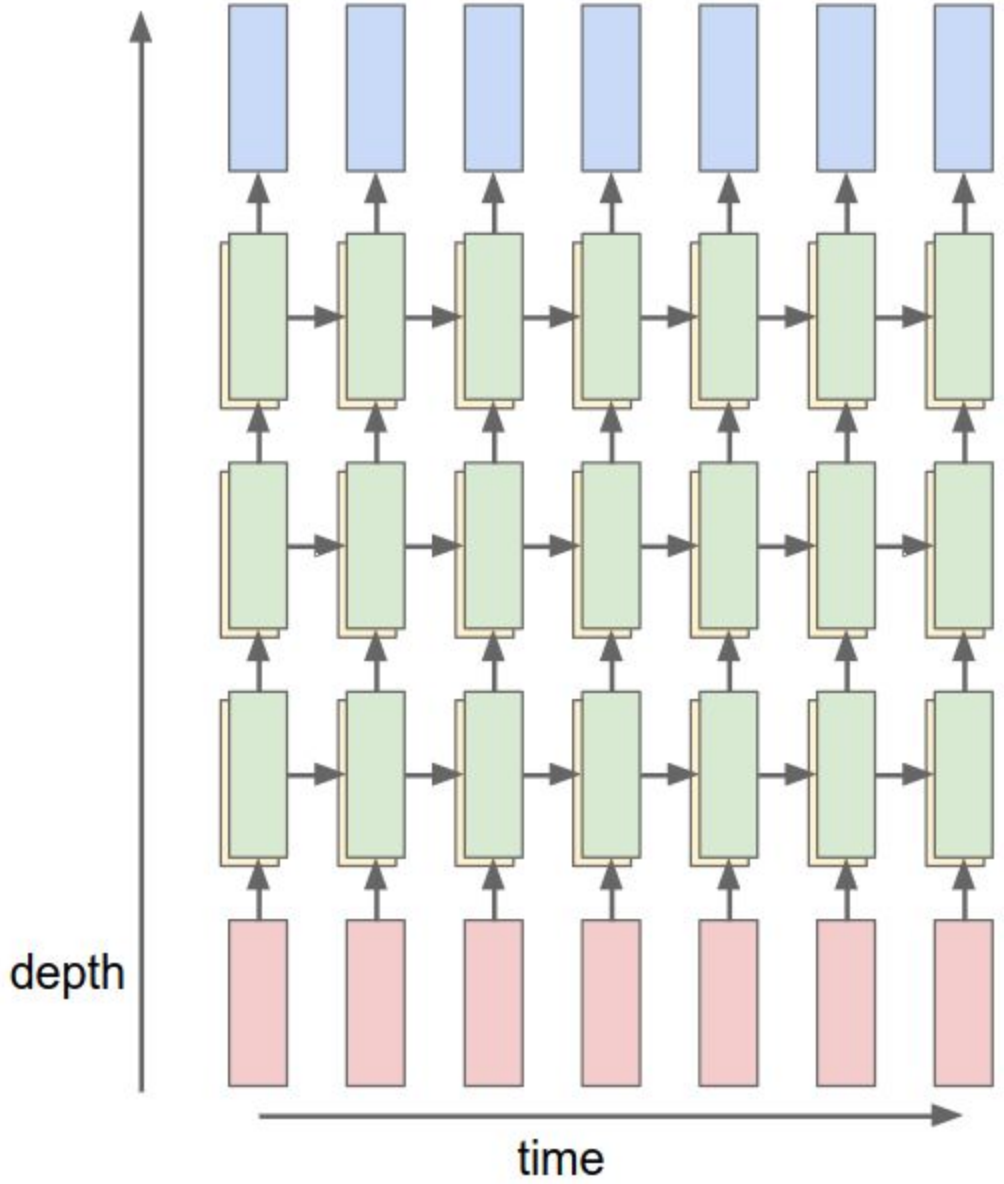
Agrawal et al, "VQA: Visual Question Answering", ICCV 2015
Zhu et al, "Visual 7W: Grounded Question Answering in Images", CVPR 2016

Visual Question Answering (VQA)



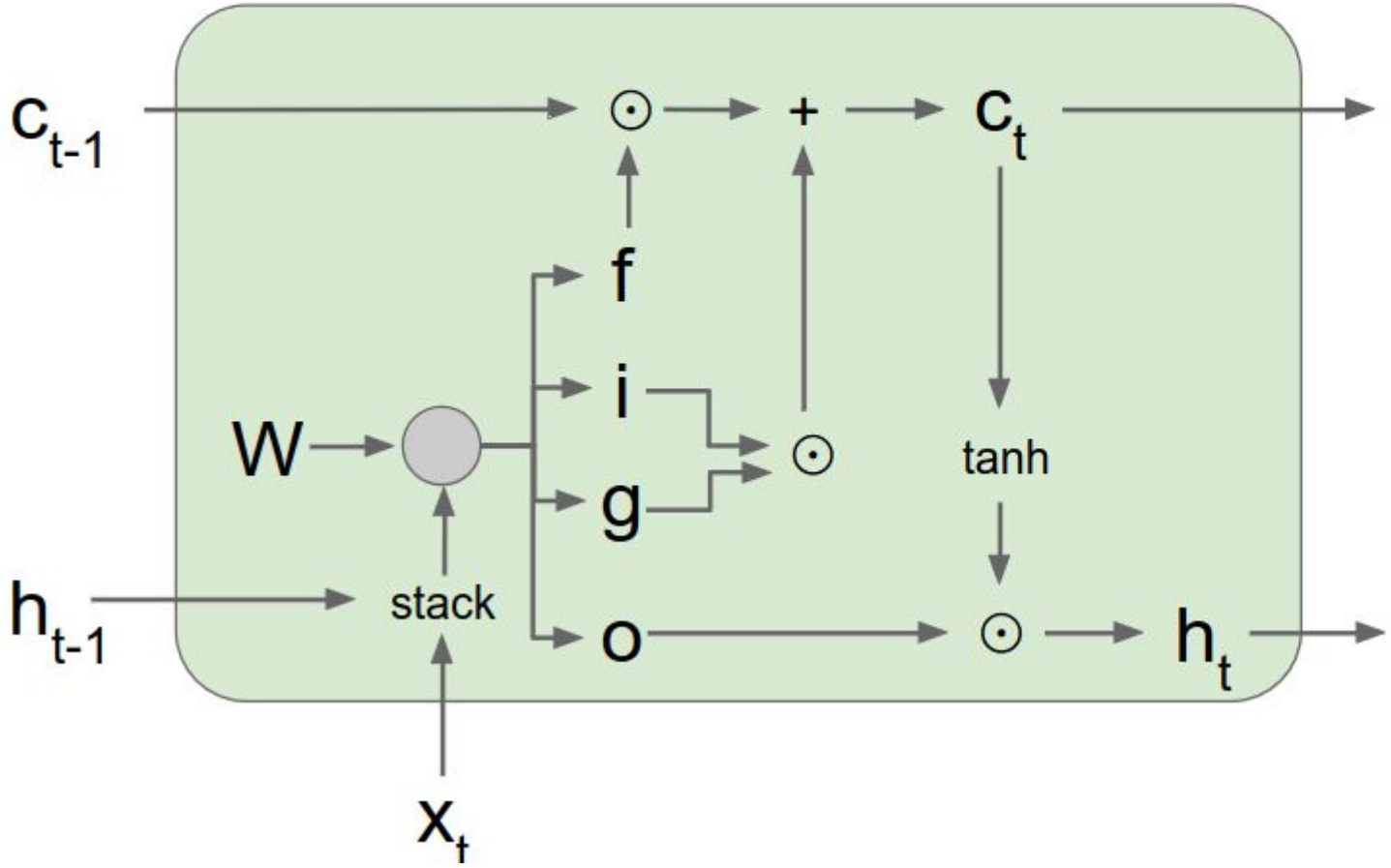
Agrawal et al, "Visual 7W: Grounded Question Answering in Images", CVPR 2015

Multilayer RNNs



Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]



$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Long Short Term Memory (LSTM)

Vanilla RNN

$$h_t = \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$

LSTM

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

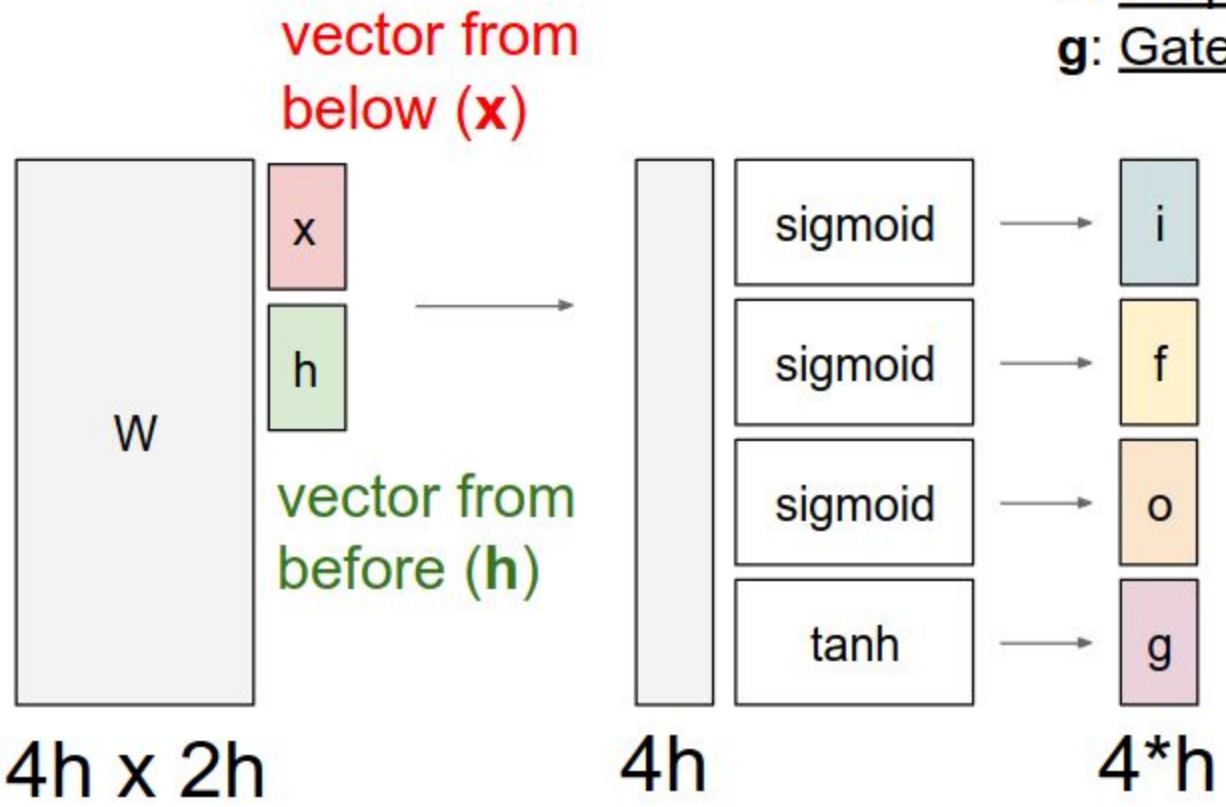
$$h_t = o \odot \tanh(c_t)$$

Hochreiter and Schmidhuber, "Long Short Term Memory", Neural Computation 1997

Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]

i: Input gate, whether to write to cell
f: Forget gate, Whether to erase cell
o: Output gate, How much to reveal cell
g: Gate gate (?), How much to write to cell



$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

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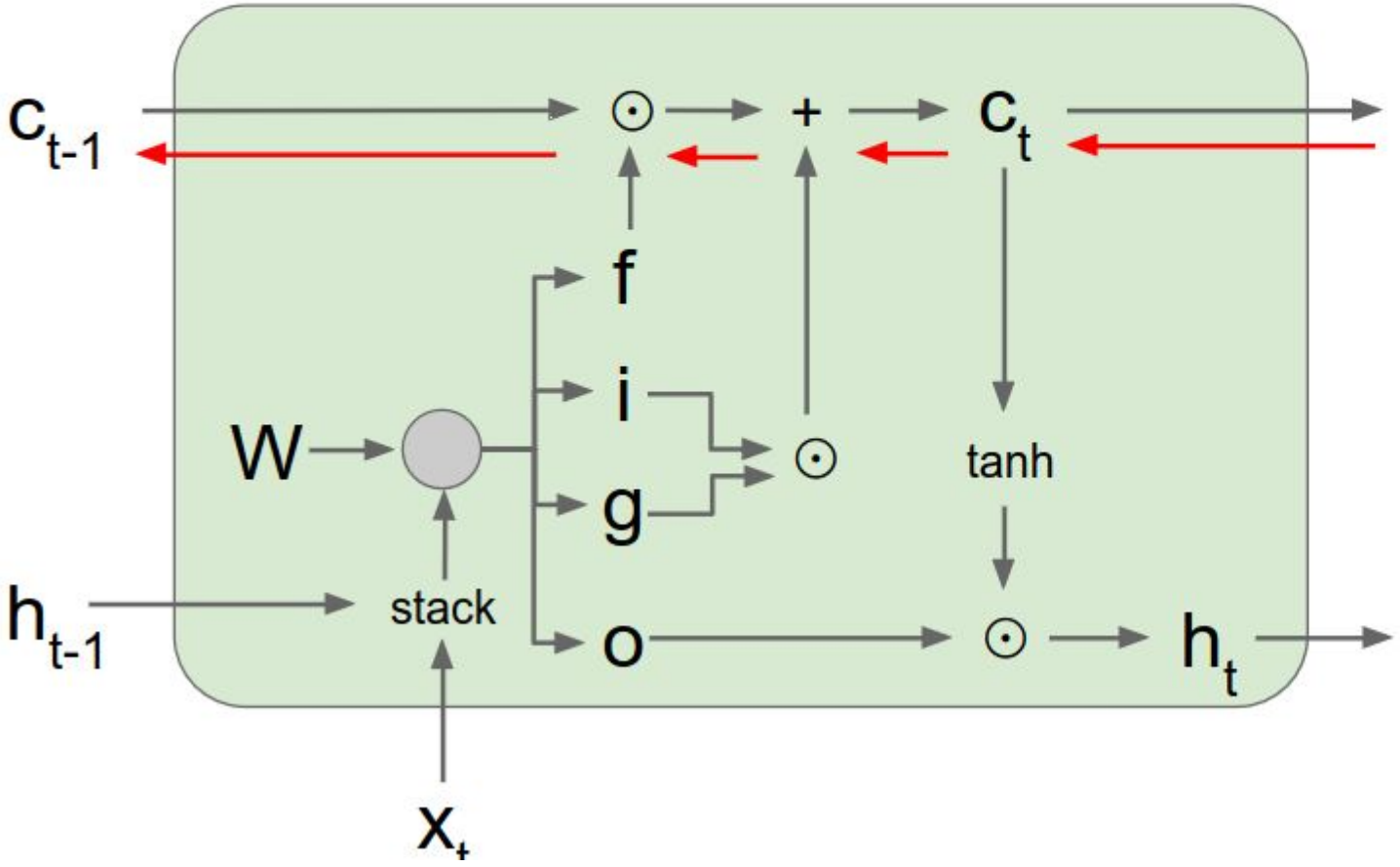
Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]



Backpropagation from c_t to c_{t-1} only elementwise multiplication by f, no matrix multiply by W

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

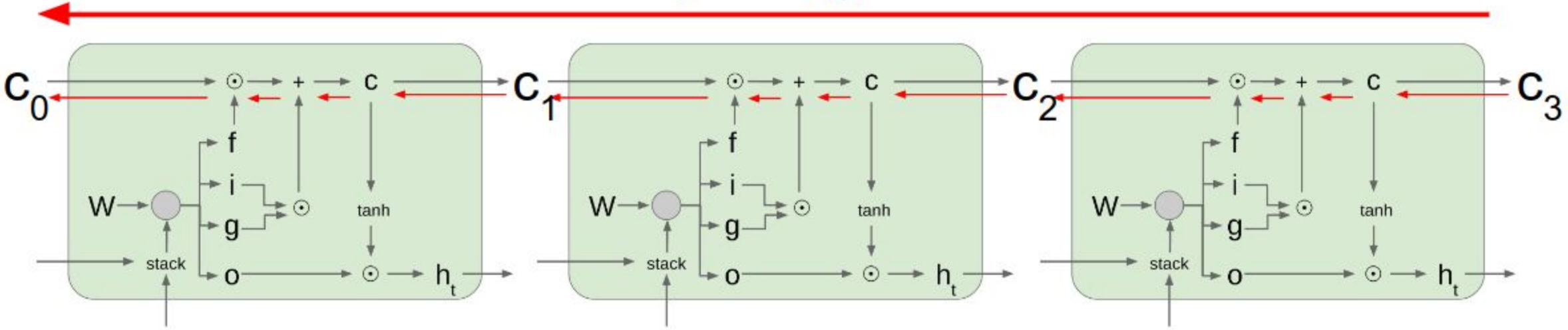
$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

Uninterrupted gradient flow!



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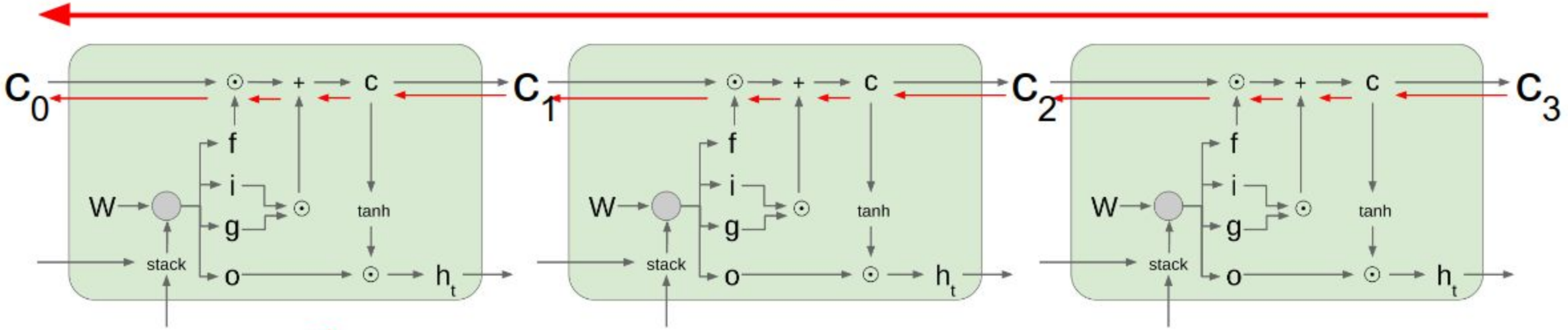
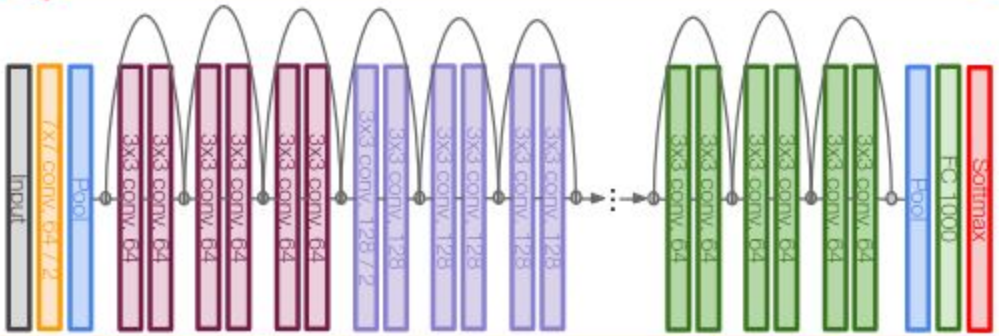


Similar to ResNet!



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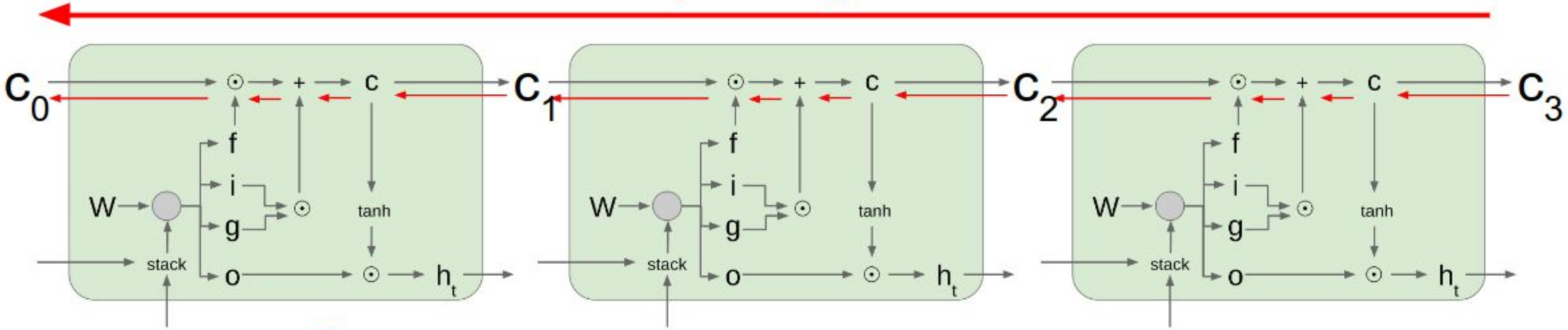

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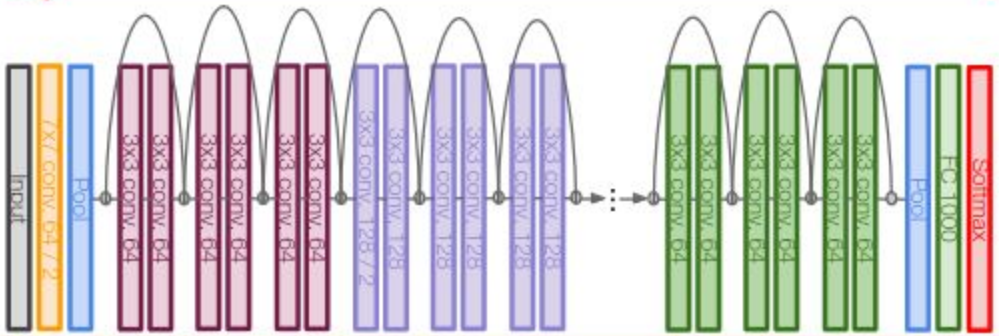


In between:
Highway Networks

$$g = T(x, W_T)$$

$$y = g \odot H(x, W_H) + (1 - g) \odot x$$

Srivastava et al, "Highway Networks", ICML DL Workshop 2015

Other RNN Variants

GRU [*Learning phrase representations using rnn encoder-decoder for statistical machine translation*, Cho et al. 2014]

$$r_t = \sigma(W_{xr}x_t + W_{hr}h_{t-1} + b_r)$$
$$z_t = \sigma(W_{xz}x_t + W_{hz}h_{t-1} + b_z)$$
$$\tilde{h}_t = \tanh(W_{xh}x_t + W_{hh}(r_t \odot h_{t-1}) + b_h)$$
$$h_t = z_t \odot h_{t-1} + (1 - z_t) \odot \tilde{h}_t$$

[*LSTM: A Search Space Odyssey*, Greff et al., 2015]

[*An Empirical Exploration of Recurrent Network Architectures*, Jozefowicz et al., 2015]

MUT1:

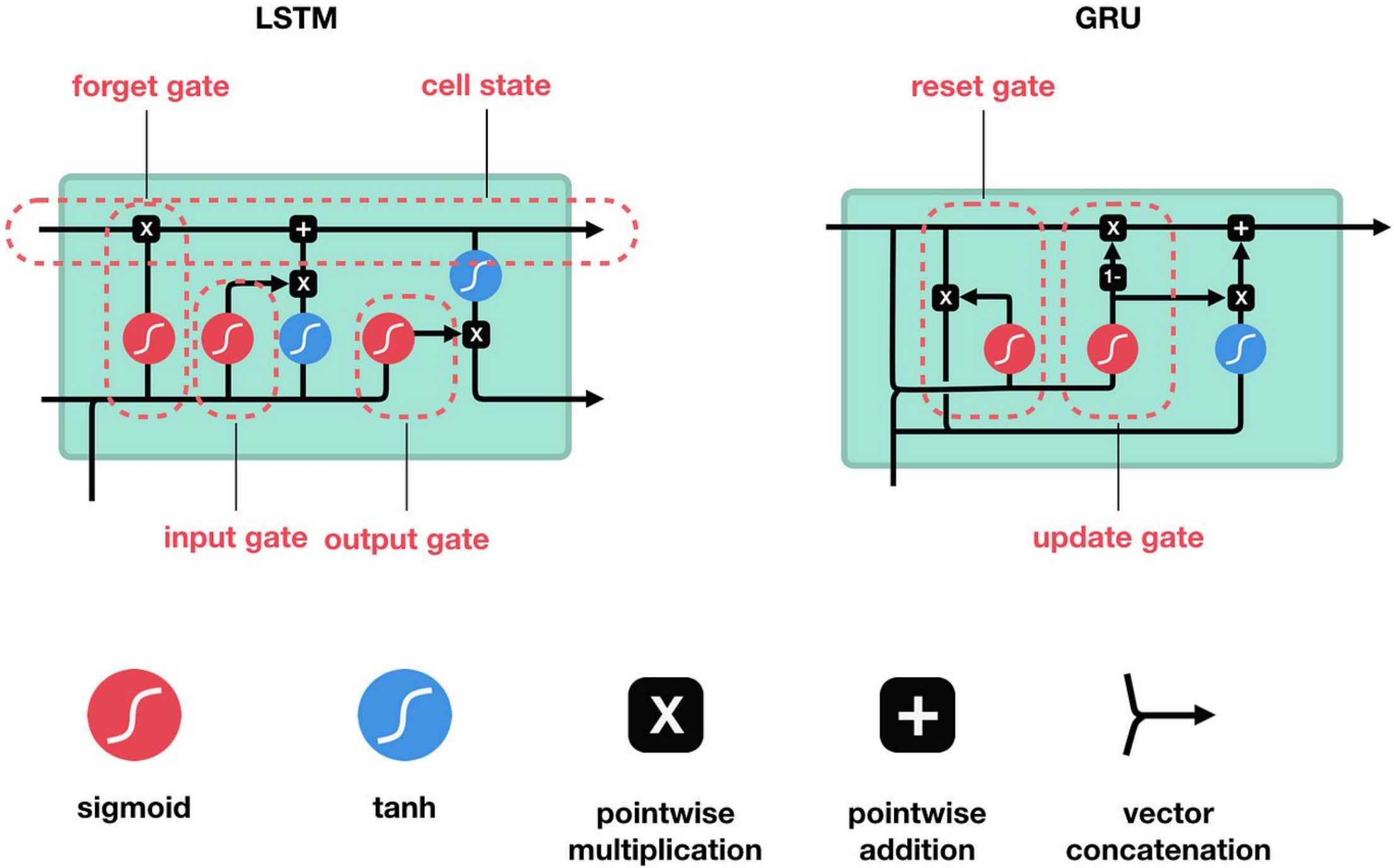
$$\begin{aligned} z &= \text{sigm}(W_{xz}x_t + b_z) \\ r &= \text{sigm}(W_{xr}x_t + W_{hr}h_t + b_r) \\ h_{t+1} &= \tanh(W_{hh}(r \odot h_t) + \tanh(x_t) + b_h) \odot z \\ &+ h_t \odot (1 - z) \end{aligned}$$

MUT2:

$$\begin{aligned} z &= \text{sigm}(W_{xz}x_t + W_{hz}h_t + b_z) \\ r &= \text{sigm}(x_t + W_{hr}h_t + b_r) \\ h_{t+1} &= \tanh(W_{hh}(r \odot h_t) + W_{xh}x_t + b_h) \odot z \\ &+ h_t \odot (1 - z) \end{aligned}$$

MUT3:

$$\begin{aligned} z &= \text{sigm}(W_{xz}x_t + W_{hz}\tanh(h_t) + b_z) \\ r &= \text{sigm}(W_{xr}x_t + W_{hr}h_t + b_r) \\ h_{t+1} &= \tanh(W_{hh}(r \odot h_t) + W_{xh}x_t + b_h) \odot z \\ &+ h_t \odot (1 - z) \end{aligned}$$

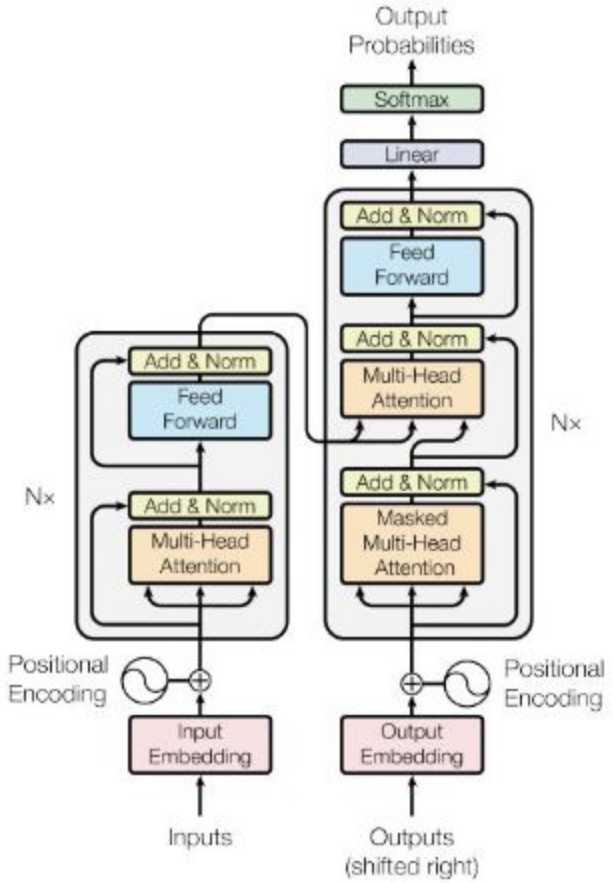

LSTM
forget gate
cell state
input gate
output gate
GRU
reset gate
update gate
1-
sigmoid
tanh
pointwise multiplication
pointwise addition
vector concatenation

Recently in Natural Language Processing…
New paradigms for reasoning over sequences

["Attention is all you need", Vaswani et al., 2018]

- New "Transformer" architecture no longer processes inputs sequentially; instead it can operate over inputs in a sequence in parallel through an attention mechanism

- Has led to many state-of-the-art results and pre-training in NLP, for more interest see e.g.
  - "BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding", Devlin et al., 2018
  - OpenAI GPT-2, Radford et al., 2018



Summary

- RNNs allow a lot of flexibility in architecture design
- Vanilla RNNs are simple but don't work very well
- Common to use LSTM or GRU: their additive interactions improve gradient flow
- Backward flow of gradients in RNN can explode or vanish. Exploding is controlled with gradient clipping. Vanishing is controlled with additive interactions (LSTM)
- Better/simpler architectures are a hot topic of current research, as well as new paradigms for reasoning over sequences
- Better understanding (both theoretical and empirical) is needed.

Simple RNN

Questions + Comments?

Resources used

http://cs231n.stanford.edu/slides/2023/lecture_8.pdf

http://karpathy.github.io/2015/05/21/rnn-effectiveness/