DATA 1202
Spring 2024

Lecture 13

Chance

Announcements

- **HW 7** due Wednesday 3/6 at 11pm
- **Project 1**
  - Whole Project due Friday 3/8 at 11pm

Review:
Random Selection

Random Selection

`np.random.choice`

- Selects uniformly at random
- with replacement
- from an array,
- a specified number of times

```
np.random.choice(some_array, sample_size)
```

Review:
Appending Arrays

A Longer Array

- `np.append(array_1, value)`
  - new array with `value` appended to `array_1`
  - `value` has to be of the same type as elements of `array_1`
- `np.append(array_1, array_2)`
  - new array with `array_2` appended to `array_1`
  - `array_2` elements must have the same type as `array_1` elements

Review:
Iteration

`for` Statements

- `for` is a keyword that begins a multiline `for` statement.
- Executing a `for` statement performs a computation for every element in a list or array.
- A common special case is to perform a computation a fixed number of times.

Anatomy of a for loop

Example:

variable name

array of values

```
for item in some_array:
    print(item)
```

indent

code to evaluate in each iteration of for loop

Simulation

(Demo)

Chance

Basics

- **Lowest value**: 0
  - Chance of event that is impossible
- **Highest value**: 1 (or 100%)
  - Chance of event that is certain

- **Complement**: If an event has chance 70%, then the chance that it doesn't happen is
  - 100% - 70% = 30%
  - 1 - 0.7 = 0.3

Equally Likely Outcomes

Assuming all <u>outcomes are equally likely</u>, the chance of an event A is:

$$P(A) = \frac{\text{number of outcomes that make A happen}}{\text{total number of outcomes}}$$

A Question

- I have three cards: **ace of hearts**, **king of diamonds**, and **queen of spades**.
- I shuffle them and draw two cards *at random without replacement.*

- What is the chance that I get the Queen followed by the King?

(Demo)

Multiplication Rule

Chance that two events *A* and *B* both happen

= P(*A* happens) x P(*B* happens given that *A* has happened)

- The answer is *less than or equal to* each of the two chances being multiplied
- The more conditions you have to satisfy, the less likely you are to satisfy them all

Another Question

- I have three cards: **ace of hearts**, **king of diamonds**, and **queen of spades**.
- I shuffle them and draw two cards *at random without replacement.*

- What is the chance that one of the cards I draw is a King and the other is Queen?

(Demo)

Addition Rule

If event *A* can happen in *exactly one* of two ways, then

$$P(A) = P(\text{first way}) + P(\text{second way})$$

- The answer is *greater than or equal to* the chance of each individual way

Complement: At Least One Head

- In 3 tosses:
  - Any outcome *except* TTT
  - P(TTT) = (1/2) x (1/2) x (1/2) = 1/8
  - P(at least one head) = 1 - P(TTT) = 1 - (1/8) = 87.5%

- In 10 tosses:
  - 1 - (1/2)**10 ≅ 99.9%

(Demo)

Problem-Solving Method

Here’s a method that works widely:

Ask yourself what event must happen on the first trial.

- If there’s a clear answer (e.g. “not a six”) whose probability you know, you can most likely use the **multiplication rule**.
- If there’s no clear answer (e.g. “could be K or Q, but then the next one would have to be Q or K …”), list all the **distinct ways** your event could occur and **add up their chances**.
- If the list above is long and complicated, look at the **complement**. If the complement is simpler (e.g. the complement of “at least one” is “none”), you can find its chance and subtract that from 1.

Discussion Question

A population has 100 people, including Kendall and Roman. We sample two people at random without replacement.

(a) P(both Kendall and Roman are in the sample)

= P(first Kendall, then Roman) + P(first Roman, then Kendall)

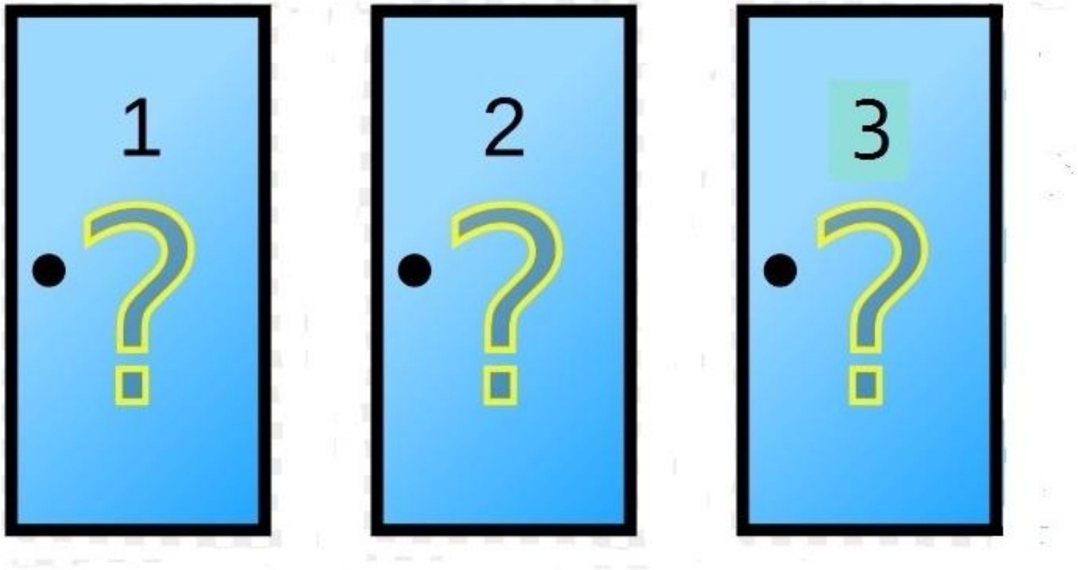
= (1/100) * (1/99) + (1/100) * (1/99) = 0.0002

(b) P(neither Kendall nor Roman is in the sample)

= (98/100) * (97/99) = 0.9602

The Monty Hall Problem

Monty Hall Problem



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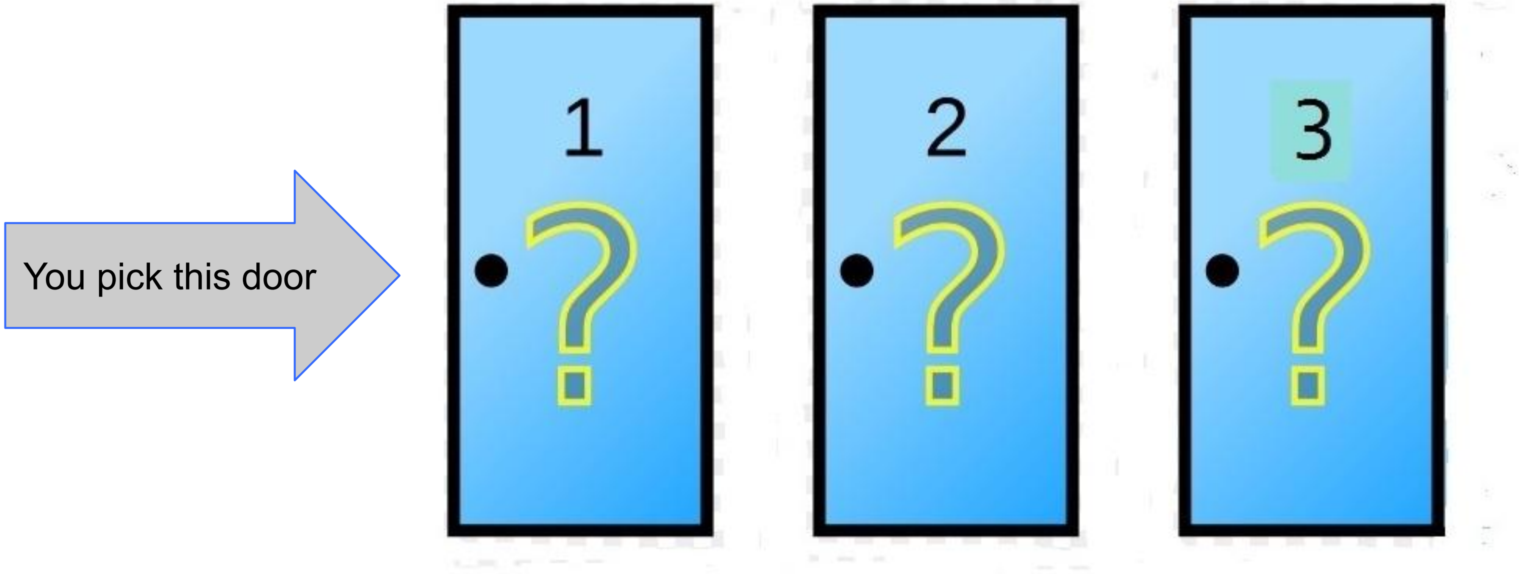
Monty Hall Problem



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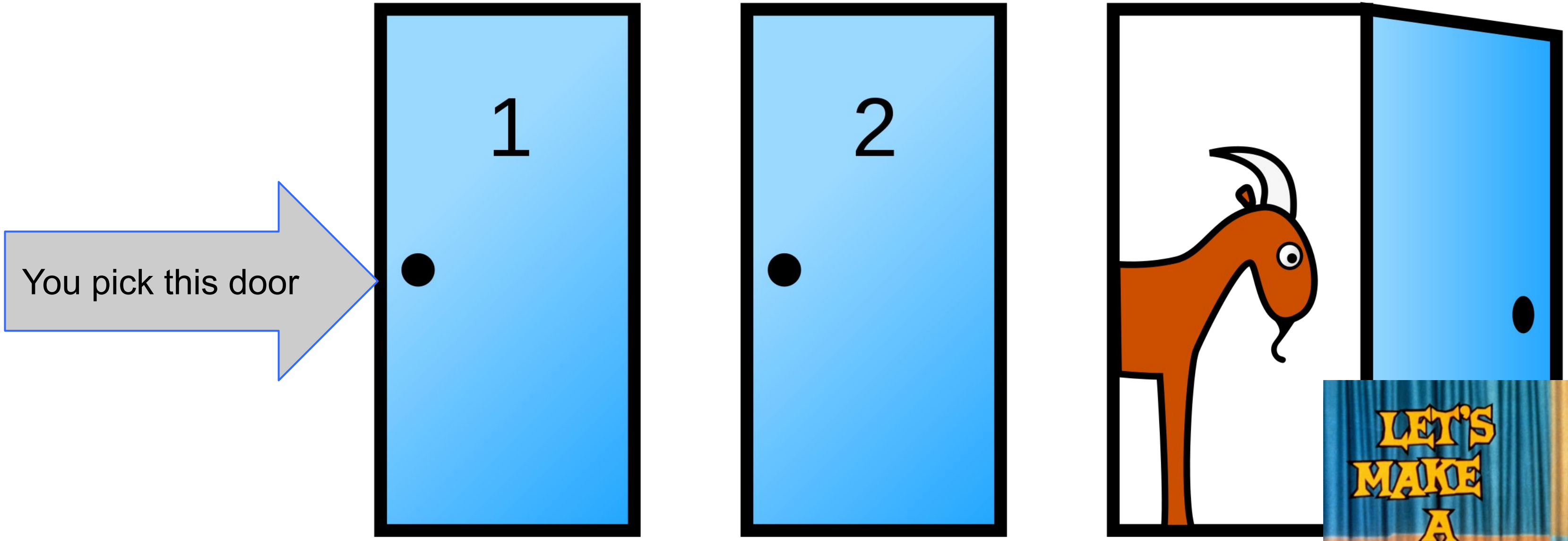
The Final Choice



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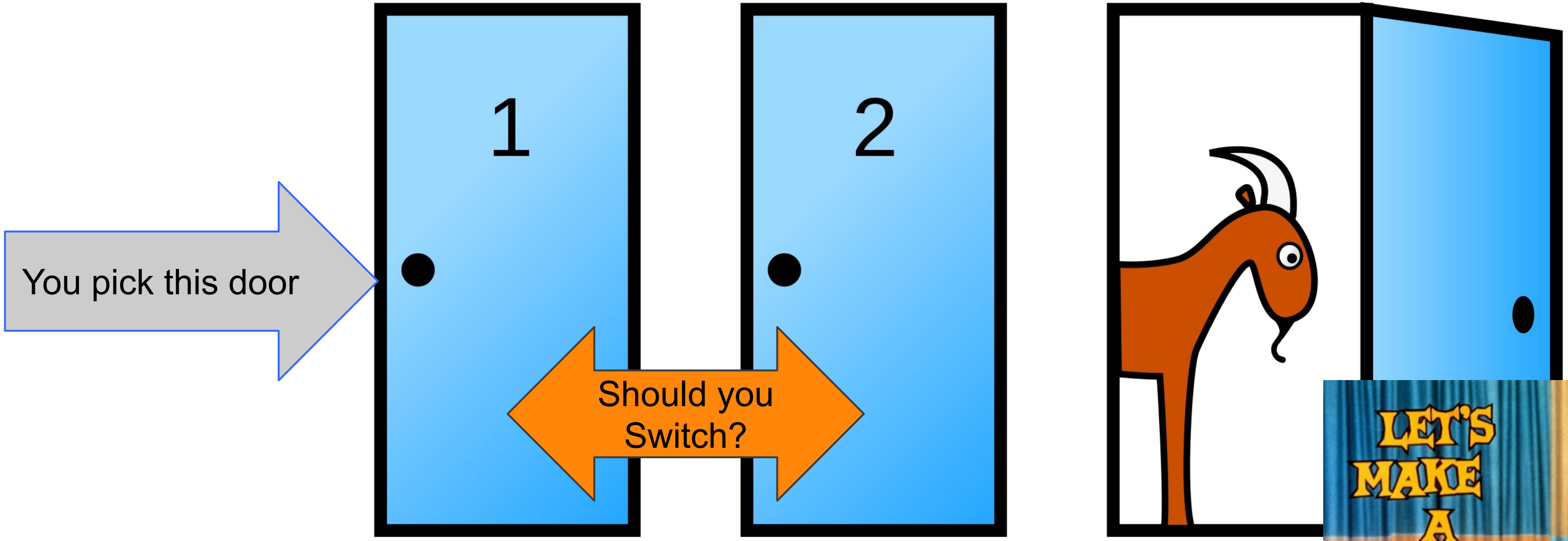

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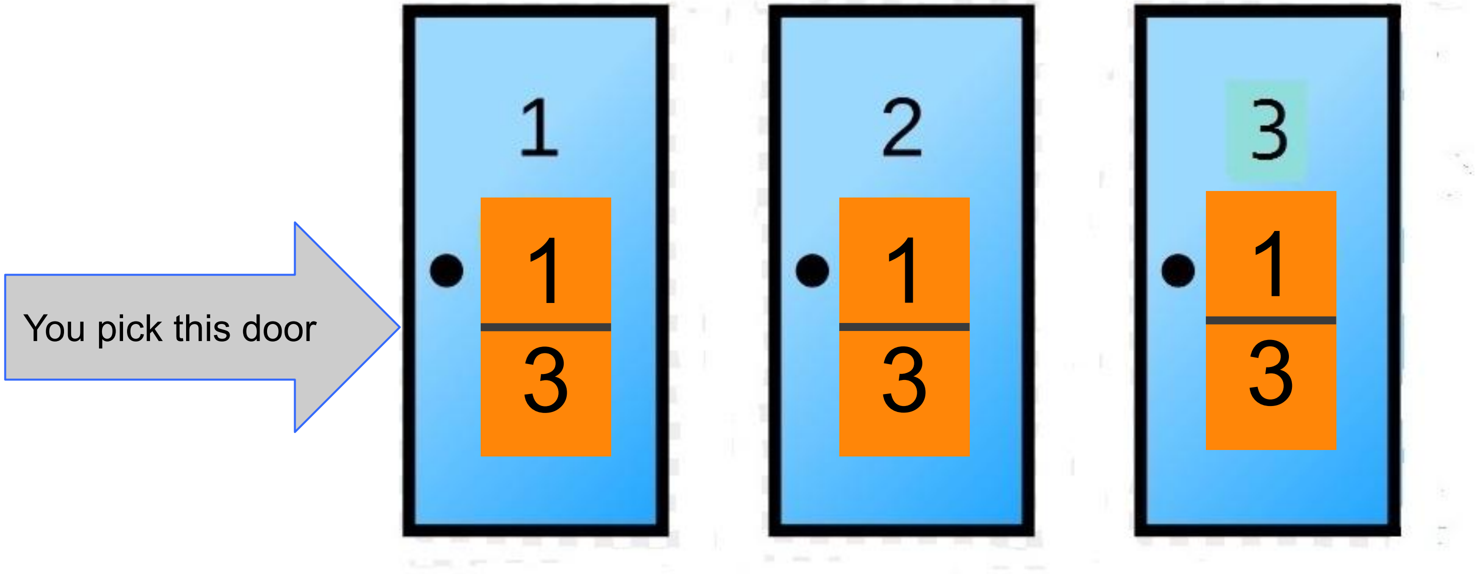
Monty Hall Problem

Chance of having a car:



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Monty Hall Problem

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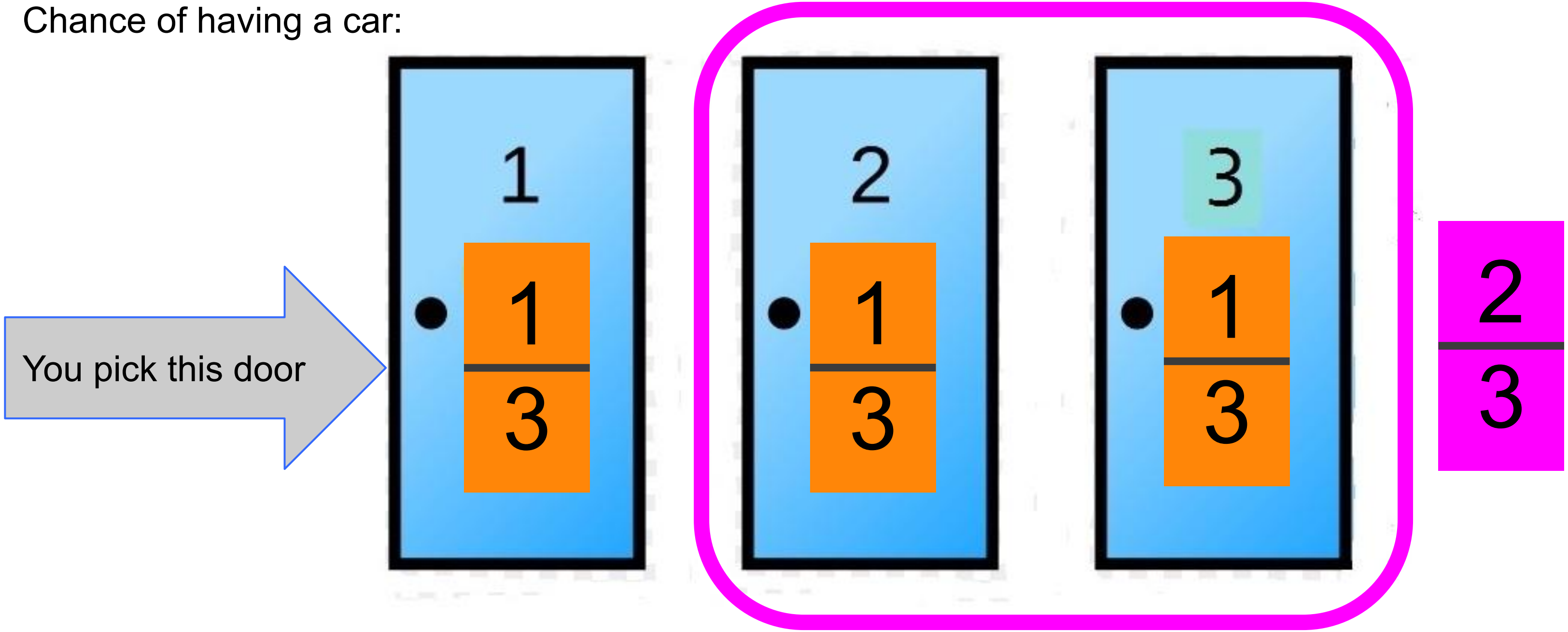

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The Final Choice

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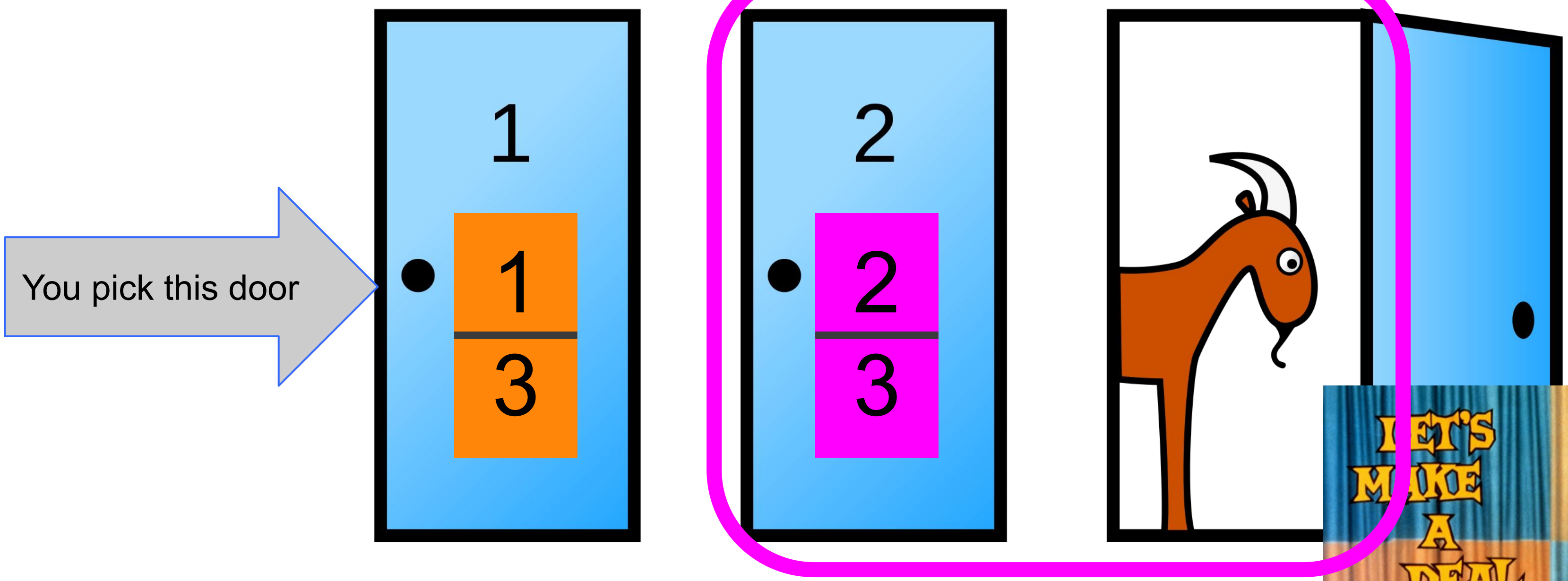

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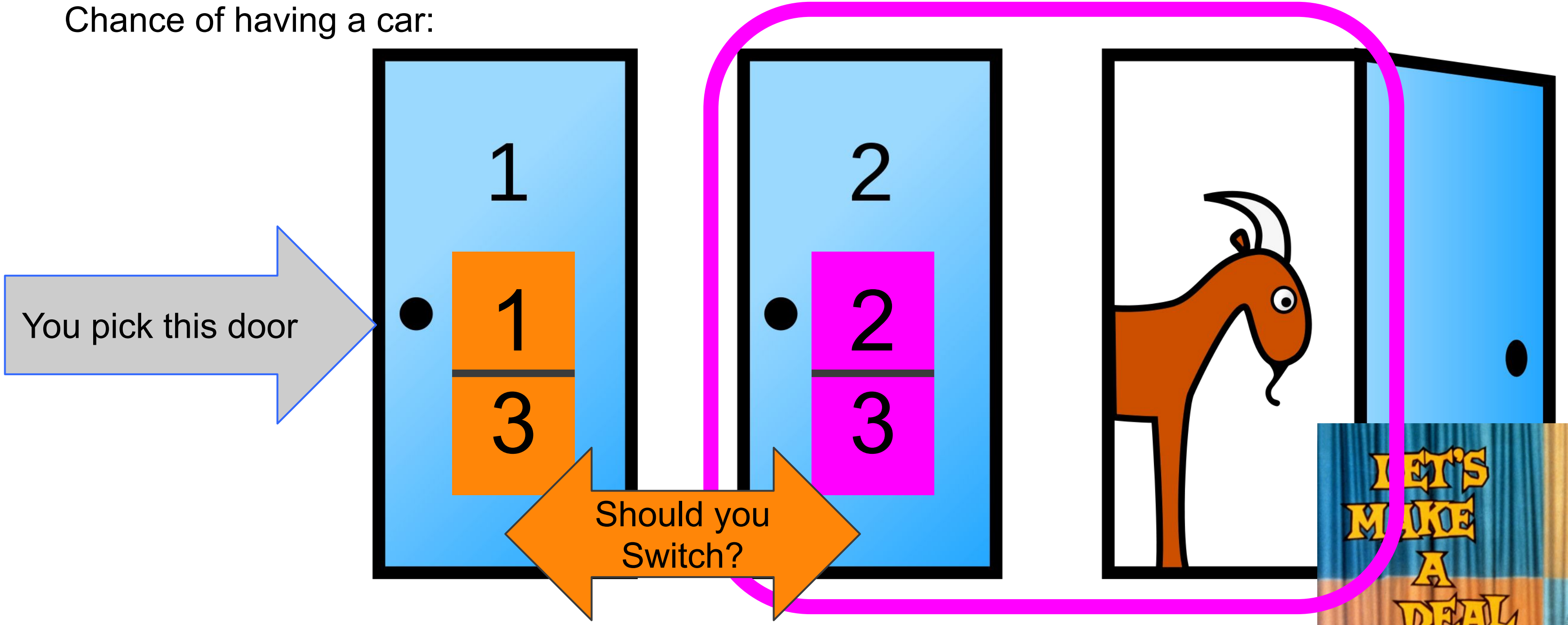

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(Demo)