44/44 great!

Problem 1

(1) True

(2) False. To make Pr(A and B) = Pr(A) * Pr(B) holds True only if A and B independent events. To make that statement true we need to explicitly mention that A and B are independent.

(3) True .

(4) False. The probability distribution function (PDF) of a continuous random variable must be non-negative. To make the statement true, it should state that the PDF can take any non-negative value.

Problem 2

Probability of a snowstorm at Houghton this Wednesday is 70% , Pr(A) = 0.7
Probability of a snowstorm at at Yellowstone Park this Wednesday is 40%, Pr(B) = 0.4

The distance between these two cities, Houghton and Yellowstone, is long so we can assume the event that there will be a snowstorm at Houghton and the event that there will be a snowstorm at Yellowstone are independent meaning A and B are independent events.

(1) Probability of there will be a snowstorm at Houghton this Wednesday and there will be a snowstorm at Yellowstone this Wednesday

$Pr(A \cap B) = Pr(A) * P(B)$ A, B independent events
$Pr(A \cap B) = 0.7 * 0.4$
$Pr(A \cap B) = 0.28$

(2) Probability of there will be a snowstorm at Houghton this Wednesday or there will be a snowstorm at Yellowstone this Wednesday

$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$
$Pr(A \cup B) = 0.7 + 0.4 - (0.7 * 0.4)$
$Pr(A \cup B) = 0.82$

(3) Probability of there will be a snowstorm at Houghton this Wednesday but there will not be a snowstorm at Yellowstone this Wednesday

$Pr(A \cap B^C) = Pr(A) * (1 - Pr(B))$
$Pr(A \cap B^C) = 0.7 * (1 - 0.4)$
$Pr(A \cap B^C) = 0.42$

Problem 3

(1)
Pr(A) - Probability of getting an odd number from tossing D1

$Pr(A) = Pr(D1 = 1) + Pr(D1 = 3) + Pr(D1 = 5)$
$Pr(A) = 0.2 + 0.1 + 0.2$
$Pr(A) = 0.5$

Pr(B) - Probability of getting an even number from tossing D2
$Pr(B) = Pr(D2 = 2) + Pr(D2 = 4) + Pr(D2 = 6)$
$Pr(B) = 0.1 + 0.3 + 0.1$
$Pr(B) = 0.5$

Pr(C) - Probability of getting 1 or 2 from tossing D2
$Pr(C) = Pr(D2 = 1) + Pr(D2 = 2)$
$Pr(C) = 0.1 + 0.1$
$Pr(C) = 0.2$

Pr(D) - Probability of getting the sum of two numbers from tossing D1 and D2 equal to 6
Pairs that equal to 6
(1, 5) : $Pr(D1 = 1) * Pr(D2 = 5) = 0.2 * 0.2 = 0.04$
(2, 4) : $Pr(D1 = 2) * Pr(D2 = 4) = 0.1 * 0.3 = 0.03$
(3, 3) : $Pr(D1 = 3) * Pr(D2 = 3) = 0.1 * 0.2 = 0.02$
(4, 2) : $Pr(D1 = 4) * Pr(D2 = 2) = 0.2 * 0.1 = 0.02$
(5, 1) : $Pr(D1 = 5) * Pr(D2 = 1) = 0.2 * 0.1 = 0.02$

$Pr(D) = 0.04 + 0.03 + 0.02 + 0.02 + 0.02$
$Pr(D) = 0.13$

(2)

$Pr(A \cap C)$, since A and C are independent events
$Pr(A \cap C) = Pr(A) * Pr(C)$
$Pr(A \cap C) = 0.5 * 0.2$
$Pr(A \cap C) = 0.1$

$Pr(A \cup C) = P(A) + P(C) - P(A \cap C)$
$Pr(A \cup C) = 0.5 + 0.2 - 0.1$
$Pr(A \cup C) = 0.6$

(3)

$Pr(A \cap D)$, Probability of D1 being odd and sum is 6. Valid Pairs (1, 5), (3, 3), (5, 1)
$Pr(A \cap D) = 0.2 * 0.2 + 0.1 * 0.2 + 0.2 * 0.1$
$Pr(A \cap D) = 0.04 + 0. 02 + 0.02$
$Pr(A \cap D) = 0.08$

$Pr(A \cup D) = Pr(A) + Pr(D) - Pr(A \cap D)$
$Pr(A \cup D) = 0.5 + 0.13 - 0.08$
$Pr(A \cup D) = 0.55$

(4)

Y be the number obtained by tossing D1.

Mean - Expected Value of Y

$E[Y] = \Sigma^6_{i=1}\ x_i * p(Y = x_i)$

$E[Y] = (1)(0.2) + (2) * (0.1) + (3)(0.1) + (4)(0.2) + (5)(0.2) + (6)(0.2)$

$E[Y] = 0.2 + 0.2 + 0.3 + 0.8 + 1 + 1.2$

$E[Y] = 3.7$

Variance of Y

$Var(Y) = \Sigma^6_{i=1}\ (x_i - u)^2 * p(Y = x_i)$ i.e $E[Y^2] - (E[Y])^2$

$Var(Y) = (1 - 3.7)^2 * (0.2) + (2 - 3.7)^2 * (0.1) + (3 - 3.7)^2 * (0.1) + (4 - 3.7)^2 * (0.2) + (5 - 3.7)^2 * (0.2) + (6 - 3.7)^2 * (0.2)$

$Var(Y) = 3.21$