49/50

Problem 1

(1) True

(2) True

(3) False

(4) True

(5) True

(6) False

Problem 2

(1)

Given $n1 = 38$, $y1bar = 1532$, $s1^2 = 128008$

$n2 = 40$, $y2bar = 1390$, $s2^2 = 92564$

$\alpha = 0.05$

Step1 : Hypothesis , let mu1 = mean life for Brand A, mu2 = mean life for Brand B

Null Hypothesis (Ho) : $mu1 - mu2 = \delta_0 = 0$

Alternative Hypothesis (Ha) : $mu1 - mu2 \neq 0$

Step 2: Formula for Test statistic

Test statistic for two sample t - test (equal variances) is

$$t = (y1bar - y2bar) - \delta_0 \;/\; S_p \, sqrt(1 / n_1 + 1 / n_1)$$

$$t = (y1bar - y2bar) - 0 \;/\; S_p \, sqrt(1 / n_1 + 1 / n_2)$$

Step 3: Find the Rejection Region

$| t | > t_{df, \alpha / 2} = t_{n1 + n2 - 2, 0.05 / 2} = t_{38 + 40 - 2, 0.025} = t_{76, 0.025} \approx 1.9921$

Step 4: Compute the test statistic

$S_p = sqrt(((n_1 - 1) s1^2 + (n_2 - 1) s2^2) / (n_1 + n_2 - 2))$

$S_p = sqrt(((38 -1)128008 + (40 -1)92564) / (\underline{(38 -1) + (40 -1) - 2}))$ 38+40-2

-1

S_p = sqrt(112787.7)

S_p = 335.8388

t = (1532 - 1390) / (335.8388 * sqrt(1/38 + 1/40))

t = 1.8665

Find P - value

P- value = 2 * Pr(T > | t |)

P- value = 2 * Pr(t_{76} > | 1.86|)

P- value = 2×0.031=0.062

Step 5: Conclusion and Interpretation

The p–value (0.062) is greater than the significance level (0.05), we fail to reject the null hypothesis. The data provides insufficient evidence to conclude that the mean life of bulbs differs between the two brands.

(2)

Confidence interval = y1bar - y2bar $\pm\ t_{n1+n2-2,\ \alpha/2}$ * S_p (√(1 / n1 + 1 / n2))

= (38 - 40 ± 1.9921 * 335.8388√(1/38 + 1/40))

= [-9.554, 293.554]

Because the interval includes 0, it aligns with our conclusion in part (1) that we do not have significant evidence of a difference. Based on the 95% CI, since it contains 0, we do not reject the null hypothesis.

(3)

```
bulb <- read.csv("bulb.csv")
t.test(bulb$life[bulb$brand == "a"],
    bulb$life[bulb$brand == "b"],
    alternative = "two.sided",
    var.equal = TRUE)
```

```
Two Sample t-test

data:  bulb$life[bulb$brand == "a"] and bulb$life[bulb$brand == "b"]
t = 2.4079, df = 78, p-value = 0.01841
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 24.51247 258.53753
```

sample estimates:
mean of x mean of y
1531.975 1390.450

Test Statistic (t): 2.4079

Degrees of Freedom (df): 78

p–value: 0.01841

95% Confidence Interval: [24.51247, 258.53753]

Problem 3

(1) Given $y1 = 254$, $y2 = 335.5$, dbar = 81.5 , $s1^2 = 12752.667$, $s2^2 = 9711.89$, $S_d^{2} = 6976.722$, $n = 10$

Step 1: Hypothesis

Null Hypothesis (Ho) : $\mu_2 - \mu_1 = \delta_0 = 0$

Alternative Hypothesis (Ha) : $\mu_2 - \mu_1 > 0$

Step 2: Formula for test statistic

test statistic for a paired t–test

$t = (dbar - \delta_0) / (S_d / sqrt(n))$

$t = dbar - 0 / (S_d / sqrt(n))$

Step 3: Find the rejection region but we use p value here

Step 4: Compute the test statistic

$t = 81.5 / (83.5268 / sqrt(10))$

$t = 3.0855$

Find P- value

P-value = $Pr(T_{df} > t) = Pr(T_9 > 3.0855) < 0.01$

Step 5: Conclusion and Interpretation

P-value is less than alpha value so we reject the null hypothesis. The data provides that there is statistically significant evidence at the 5% level that the new repair method increases the mean time between failures.

(2)

95% Two-Sided Confidence Interval:

$$\bar{d} \pm t_{\alpha/2,\ df} * S_d / \text{sqrt}(n)$$

$$81.5 \pm t_{0.025,\ 9} * 83.5268 / \text{sqrt}(10)$$

$$81.5 \pm 2.262 * 83.5268 / \text{sqrt}(10)$$

[21.7527, 141.2473]

(3)

```
Current <- c(155, 222, 346, 287, 115, 389, 183, 451, 140, 252)
New <- c(211, 345, 419, 274, 244, 420, 319, 505, 396, 222)
t.test(New, Current, paired = TRUE, alternative = "greater")

        Paired t-test

data:  New and Current
t = 3.0855, df = 9, p-value = 0.006511
alternative hypothesis: true mean difference is greater than 0
95 percent confidence interval:
 33.0811     Inf
sample estimates:
mean difference
           81.5
```

Test Statistic (t): 3.0855

Degrees of Freedom (df): 9

p–value: 0.006511

95% Confidence Interval: [33.0811,   Inf]

(4)

```
Diff <- c(56, 123, 73, -13, 129, 31, 136, 54, 256, -30)
t.test(Diff, mu = 0, alternative = "greater")

        One Sample t-test

data:  Diff
t = 3.0855, df = 9, p-value = 0.006511
alternative hypothesis: true mean is greater than 0
95 percent confidence interval:
 33.0811     Inf
sample estimates:
mean of x
     81.5
```

Test Statistic (t): 3.0855

Degrees of Freedom (df): 9

p–value: 0.006511

95% Confidence Interval: [33.0811,   Inf]