Problem 1

37/37 great!

(1) $Pr(Y > 1)$

$Y > 1$ meaning Y can take 2 and 3 values.
$Pr(Y > 1) = Pr(Y = 2) + Pr(Y = 3)$
$Pr(y > 1) = 0.02 + 0.01 = 0.03$

(2) The mean of Y

Mean of Y = E[Y]
$E[Y] = \Sigma^3_{i=0}\ x_i * p(Y = x_i)$
$E[Y] = 0 * 0.94 + 1 * 0.03 + 2 * 0.02 + 3 * 0.01$
$E[Y] = 0.10$

(3) The variance of Y

$Var(Y) = \Sigma^3_{i=0}\ (x_i - u)^2 * p(Y = x_i)$ i.e $E[Y^2] - (E[Y])^2$
$Var(Y) = (0 - 0.1)^2 * 0.94 + (1 - 0.1)^2 * 0.03 + (2 - 0.1)^2 * 0.02 + (3 - 0.1)^2 * 0.01$
$Var(Y) = 0.19$

Problem 2

(1) Define the random variable X as the number of tails.

We toss a coin two times. Let the probability of getting a head be p (so the probability of a tail is 1–p), and assume the tosses are independent.

The sample space for two tosses is {HH, HT, TH, TT}. We count tails as follows:

- For HH: 0 tails Probability = $p \times p = p^2$.
- For HT: 1 tail Probability = $p \times (1-p)$.
- For TH: 1 tail Probability = $(1-p) \times p$.
- For TT: 2 tails Probability = $(1-p) \times (1-p) = (1-p)^2$.

The probability mass function (pmf) for X is:
$Pr(X = 0) = p^2$,
$Pr(X = 1) = p(1-p) + (1-p)p = 2p(1-p)$,
$Pr(X = 2) = (1-p)^2$.

Since X is the count of tails in two independent tosses, it follows a binomial distribution with parameters n = 2 and success probability 1–p.
The expected value is $E(X) = 2(1-p)$,
The variance is $Var(X) = 2(1-p) \cdot p$.

(2)

Define the random variable Y such that

$Y = 1$ if the outcome of the second toss is the same as the outcome of the first toss, and
$Y = 0$ otherwise.

The outcomes:

- HH: both tosses are the same, so $Y = 1$; probability = $p \times p = p^2$.
- TT: both tosses are the same, so $Y = 1$; probability = $(1-p) \times (1-p) = (1-p)^2$.
- HT or TH: tosses differ, so $Y = 0$; combined probability = $p(1-p) + (1-p)p = 2p(1-p)$.

Thus, the pmf of Y is:

$\Pr(Y = 1) = p^2 + (1-p)^2$,

$\Pr(Y = 0) = 2p(1-p)$.

The expected value of Y is

$E(Y) = 1 * [p^2 + (1-p)^2] + 0 * [2p(1-p)] = p^2 + (1-p)^2$.

Since Y is a binary random variable (taking values 0 and 1), its variance is given by

$\mathrm{Var}(Y) = \Sigma (x_i - u)^2 * p(Y = x_i)$ i.e $E[Y^2] - (E[Y])^2$

$\mathrm{Var}(Y) = [p^2 + (1-p)^2] \times [1 - (p^2 + (1-p)^2)]$.

Problem 3

(1)

Target population: All adults who recently filed for unemployment benefits in the county

(2)

Let Y be the number of people in the sample who experience these feelings of sadness. Assuming that the proportion of adults with these feelings in the target population is the same as the national rate (10%), and that each person's response is independent, Y follows a binomial distribution with parameters n = 68 and p = 0.10.

$Y \sim$ Binomial(68, 0.10).

(3)

Pr(Y >= 12) = 1 - pbinom(11, size = 68, prob = 0.10)
Pr(Y >= 12) = 0.0362

(4)

Pr(Y <= 8) = pbinom(8, size = 68, prob = 0.10)
Pr(Y <= 8) = 0.763

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# R Code
1 - pbinom(11, size = 68, prob = 0.10)
 pbinom(8, size = 68, prob = 0.10)
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