Problem 1 (10 points, 1 point for each part)

Simply indicate it as true if the statement is always true and state it as false otherwise. You do not need to explain why the statement is true or false.

(1) The probability that a continuous random variable lies in the interval 4 to 7, inclusively, is the sum of $\Pr(4) + \Pr(5) + \Pr(6) + \Pr(7)$. Here $\Pr(4)$ is the probability that the random variable equals to 4.

(2) The variance of the number of successes in a binomial experiment of n trials is $\sigma^2 = np(1-p)$.

(3) A normal distribution is characterized by its mean and its degrees of freedom.

(4) The standard normal distribution has mean zero and variance σ^2 .

(5) The standard deviation of the sample mean increases as the sample size increases.

(6) As α increases, the value of z_α will decrease.

Problem 2 (4 points, 1 point for each part)

(1) Use a normal distribution table to find $\Pr(Z > 0.374)$ where $Z \sim N(0,1)$.

(2) Use R to find $\Pr(Y > 0.374)$ where $Z \sim N(0,1)$.

(3) Use a normal distribution table to find $z_{0.12}$.

(4) Use R to find $z_{0.12}$.

Problem 3 (25 points, 5 points for each part)

Suppose that Y is normally distributed random variable with $\mu = 10$ and $\sigma = 2$ and X is also normally distributed with $\mu = 5$ and $\sigma = 5$. X and Y are independent. Calculate the following probabilities according to a **normal distribution table**. You need to show sufficient details. For example, if you want to calculate $\Pr(Z < 0.10)$, you cannot directly write out $\Pr(Z < 0.10) = 0.5398$. You should use $\Pr(Z < 0.10) = 1 - \Pr(Z > 0.10) = 1 - 0.4602 = 0.5398$. These steps show that the normal table is indeed used.

(1) $\Pr(Y > 12)$ and $\Pr(3 < X < 6)$.

(2) $\Pr(Y > 12 \text{ and } 3 < X < 6)$. [**Hint**: X and Y are independent]

(3) $\Pr(Y > 12 \text{ or } 3 < X < 6)$ (You can use the results from (1) and (2)).

(4) The value of C such that $\Pr(Y < C) = 0.94$.

(5) The value of D such that $\Pr(X > D) = 0.40$.

Problem 4 (10 points)

The average modulus of rupture (MOR) for a particular grade of pencil lead is known to be 6500 psi with a standard deviation of 250 psi. Again, you need to use calculation should be based on a normal table.

(1) (**8 points**) Find the probability that a random sample 16 pencil leads will have an average MOR between 6400 and 6550 psi.
(2) (**2 points**) What did you assume to in order to find this probability?

Problem 5 (15 points, 5 points for each part)

The Kaufman Assessment Battery for Children is designed to measure achievement and intelligence with a special emphasis on nonverbal intelligence. Its global measures, such as its Sequential Processing score, are scaled to have a mean of 100 and a standard deviation of 15. Assume that the Sequential Processing score has a normal distribution. You can use a normal table or R for your calculations.

(1) Find a value that divides the children with the highest 10% of the scores from those with the lower 90%.
(2) What proportion of children will have Sequential Processing scores between 90 and 110?
(3) In a sample of 20 children, what is the probability the sample mean will differ from the population mean by more than 3 points (either positive or negative)?