Problem 1

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1.1 True

1.2 False

1.3 True

1.4 False

1.5 True

1.6 False

1.7 False

1.8 False

Problem 2

2.1 b

2.2 d

2.3 a

2.4 c

2.5 c

2.6 b

2.7 c

-1 **2.8** e

2.9 a

Problem 3

1

we have $H_o : \mu = 80$ vs $H_1 : \mu < 80$, $n = 100$, $\sigma = 7.2$

A type I error is rejecting the null hypothesis (H_o) when the null hypothesis (H_o) is true.
Under H_o , Y_bar has mean 80 and standard deviation

SE = σ / sqrt(n) = 7.2 / 10 = 0.72

P(Type I error) = P(Y_bar < 76 | μ = 80)
P(Y_bar < 76) = P(Z < (76 - 80) / 0.72)

z = (76 - 80) / 0.72 = -5.556

pnorm(-5.5556) = 1.383299e-08

The probability of Type I error is 1.383299e-08

2

Rejection region for α = 0.05

For one-tailed test at level α = 0.05, we want the cutoff c such that

P(Y_bar < c | μ = 80) = 0.05

P(z < (c - 80) / 0.72) = 0.05

z_α = qnorm(0.05) = -1.645

c - 80) / 0.72 = -1.645

c = -1.645 * 0.72 + 80

c = 78.8156

So the rejection region (for a 5% left-tailed test) is y_bar < 78.8156

Problem 4

1

μ_o = \$14,200, σ = \$2,600,

A random sample of n = 75 , y_bar = \$15,300 , assume still σ = \$2,600

We want to test Ho : μ = 14200 vs H1 : μ ≠ 14200 at α = 0.05

Step 1: Hypothesis testing

Ho : μ = 14200 vs H1 : μ ≠ 14200

Step 2: Significance level: this is step 1

α = 0.05

two-sided test, α / 2 = 0.025

Step 3: test statistic

Under Ho, the test statistic is

$z = (y\_bar - \mu_o) / (\sigma / sqrt(n))$

z = (15300 - 14200) / (2600/sqrt(75))

z = 3.6639

Step 4: Rejection Region

It is two-sided at $\alpha = 0.05$, we reject if $|z| > z_{0.025}$

$z_{0.025} = 1.96$

The rejection region is z < -1.96 or z > 1.96

Step 5: Conclusion

z = 3.6639 greater than 1.96. Therefore we reject the null hypothesis (Ho).

There is significant evidence at the 5% level that the mean family income in 1975 differs from $14,200.

2

Power of the test $\mu = 15300$ and $\mu = 15600$

power = $1 - \boldsymbol{\beta}(\mu)$ = P(Reject H_o | μ is the true mean)

Reject H_o if

$z = (y\_bar - 14200) / (\sigma / sqrt(n)) > 1.96$ or $z < -1.96$

the rejection region in terms of y_bar

$y\_bar < 14200 - 1.96 * (\sigma / sqrt(n))$ or $y\_bar > 14200 + 1.96 * (\sigma / sqrt(n))$

SE = $(\sigma / sqrt(n))$ = 2600 / sqrt(75) = 300.2221

Cutoffs are

L = 14200 - 1.96 * 300.2221 = 13611.56

$R = 14200 + 1.96 * 300.2221 = 14788.44$

We reject the null hypothesis if y_bar < 13611.56 or y_bar > 14788.44

Power for $\mu = 15300$

Under true mean $\mu = 15300$,

P(reject) = P(y_bar < 13611.56 or y_bar > 14788.44 | $\mu = 15300$)

P(y_bar < 13611.56) = 0

P(y_bar > 14788.44) = P(Z > (14788.44 - 15300) / 300

P(Z > -1.7052) = 0.9560

The power is about 0.9560

Power for $\mu = 15600$

P(y_bar > 14788 | $\mu = 15600$)

P(z > (14788.44 - 15600) / 300

P(z > -2.7052) = 0.9967

power is about = 0.9966

3

p - value

Test statistic z = 3.6639. Two sided test

p = 2 * P(Z > 3.6639)

p = 0.00024

4

99% confidence Interval

CI = 15300 +- (2.576 * 300.2221)

CI = 15300 + 773.3721, 15300 - 773.3721

CI = [14526.63, 16073.37]

Since 14200 is outside the 99% interval, we reject the null hypothesis H_o : $\mu = 14200$ at $\boldsymbol{\alpha} = 0.01$