

Problem 1

(1) False

(2) True

(3) False

(4) False

(5) False

(6) False

42/44 great!

Problem 2

(1)

A congressman will support building a dam if more than 60% of his constituents support it. A sample of 225 registered voters shows that 145 favor the dam.

Step 1: State the Hypothesis

Null Hypothesis (H_0): $p = 0.60$

Alternative Hypothesis (H_a): $p > 0.60$

Step 2: Formula for Test statistic

For proportion test, the test statistic is

$$z = (\hat{p} - p_0) / \sqrt{p_0(1 - p_0) / n}$$

sample proportion

$$\hat{p} = 145 / 225 = 0.6444, p_0 = 0.60, n = 225$$

Step 3: Find the rejection region/ critical value/ p value

$$\Pr(Z > z_\alpha) = 0.10$$

Using normal table we get

$$z_{0.10} = 1.2815$$

Step 4: Compute the test statistic

$$z = (0.6444 - 0.60) / \sqrt{0.60(1 - 0.60) / 225}$$

$$z = 1.3594$$

p - value :

$$p \text{ value} = \Pr(Z > 1.3594)$$

$$\Pr(Z < 1.36) = 0.9131$$

$$p \text{ - value} = 1 - 0.9131$$

$$p \text{ - value} = 0.0869$$

Step 5: Conclusion and Interpretation

The computed test statistic value z (1.36) exceeds the test critical value (1.28), and the p -value (0.087) is less than α (0.10), we reject Null Hypothesis H_0 . We can conclude that the data provides enough evidence that at $\alpha = 0.10$ to suggest that $p > 0.60$ and the congressman should the dam.

(2) Two-sided 95% Confidence Interval

$$\hat{p} \pm z_{\alpha/2} \sqrt{\hat{p}(1 - \hat{p}) / n}$$

$$\hat{p} = 0.6444, n = 225, \alpha = 0.05, \alpha / 2 = 0.025, P(Z \leq z_{\alpha/2}) = 1 - 0.025 = 0.975$$

$$z_{0.975} = 1.959964 = 1.96$$

$$0.6444 \pm 1.96 \sqrt{0.6444(1 - 0.6444) / 225}$$

The Two-sided 95% confidence interval of the proportion of his constituents who supported the dam is: (0.5818 , 0.7069)

(3) Normal Assumption

For the normal approximation to be valid when testing proportions, both np_0 and $n(1 - p_0)$ should be atleast 10

$$np_0 = 225 * 0.60 = 135$$

$$n(1 - p_0) = 225 * (1 - 0.60) = 90$$

Both are well above 10, the normal approximation is satisfied here.

Problem 3

```
ph <- c(5.335, 5.345, 5.395, 5.305, 5.315, 5.380, 5.520, 5.190,  
        5.455, 5.330, 5.360, 6.285, 5.350, 5.125, 5.115, 5.510,  
        5.340, 5.340, 5.305, 5.265)
```

part - 2

```
t_test_one_sided <- t.test(ph, mu = 5.40, alternative = "great", conf.level = 0.95)  
t_test_one_sided
```

One Sample t-test

data: ph

t = -0.40957, df = 19, p-value = 0.6567

alternative hypothesis: true mean is greater than 5.4

95 percent confidence interval:

5.286426 Inf

sample estimates:

mean of x

test statistic = -0.4095

P-value = 0.6567

95% confidence interval = [5.2864, Inf]

part - 5

```
t_test_two_sided <- t.test(ph, conf.level = 0.95)  
t_test_two_sided
```

One Sample t-test

data: ph

t = 101.28, df = 19, p-value < 2.2e-16

alternative hypothesis: true mean is not equal to 0

95 percent confidence interval:

5.267102 5.489398

sample estimates:

mean of x

5.37825

test statistic = 101.28

P-value < 2.2e-16

95% confidence interval = [5.2671, 5.4893]

Problem 4

(1)

The bottler's claim is that less than 10% drink another brand.

Null Hypothesis (H_0): $p = 0.10$

Alternative Hypothesis (H_a): $p < 0.10$

$n = 100$, $x = 18$, $p = 0.1$, $\hat{p} = 18/100 = 0.18$

`prop.test(x = 18, n = 100, p = 0.1, alternative = "less", correct = FALSE)`

1-sample proportions test without continuity correction

data: 18 out of 100, null probability 0.1

X-squared = 7.1111, df = 1, p-value = 0.9962

alternative hypothesis: true p is less than 0.1

95 percent confidence interval:

0.0000000 0.2513522

sample estimates:

p
0.18

Chi-square Statistic: 7.1111

Z - value = $\sqrt{\text{Chi-square Statistic}} = \sqrt{7.1111} = 2.6666$

P- value = 0.9962

95% Confidence Interval: (0, 0.2513)

P value greater than alpha value. We fail to reject the null hypothesis. We conclude that the data provide insufficient evidence to support the bottler's claim that less than 10% of its customers drink another brand.

(2)

95% Confidence Interval

$\hat{p} \pm z_{\alpha/2} \sqrt{\hat{p}(1 - \hat{p}) / n}$

$\hat{p} = 0.18$, $\alpha = 0.05$, $\alpha / 2 = 0.025$, $P(Z \leq z_{\alpha/2}) = 1 - 0.025 = 0.975$, $z_{0.975} = 1.96$, $n = 100$

$0.18 \pm 1.96 (\sqrt{0.18(1 - 0.18)}) / 100$

95% Confidence Interval: (0.1047 , 0.2553)

(3)

For the normal approximation to be valid when testing proportions, both np_o and $n(1 - p_o)$ should be atleast 10

✓ $np_o = 100 * 0.18 = 18$

$n(1 - p_o) = 100 * (1 - 0.18) = 82$

Both are well above 10, the normal approximation is satisfied here.

$p_o \approx 0.18$
not 0.18

— 1