Problem 1 (8 points, 1 point for each part)

This problem consists of some true/false questions regarding concepts of statistical inference. Indicate if a statement is true or false. You do not need to explain why it is true or false.

1.1 In a hypothesis test, the p value is 0.043. This means that the null hypothesis would be rejected at $\alpha = 0.05$.

1.2 If the null hypothesis is rejected by a one-tailed hypothesis test at α, then it will also be rejected by a two-tailed test at α.

1.3 If a null hypothesis is rejected at the 0.01 level of significance, it will also be rejected at the 0.05 level of significance.

1.4 The probability of a type II error is controlled in a hypothesis test by establishing a specific significance level.

1.5 If we decrease the confidence coefficient for a fixed n, we decrease the width of the confidence interval.

1.6 If a 95% confidence interval on μ was from 50.5 to 60.6, we would reject the null hypothesis that $\mu = 60$ at the 0.05 level of significance.

1.7 If the sample size is increased and the level of confidence is decreased, the width of the confidence interval will increase.

1.8 A research article reports that a 95% confidence interval for mean reaction time is from 0.25 to 0.29 seconds. Therefore, about 95% of individuals will have reaction times in this interval.

Problem 2 (9 points, 1 point for each part)

Multiple choice questions. Only one of the statements is correct.

2.1 In a hypothesis test the p value is 0.043. This means that we can find statistical significance at:

a) both the 0.05 and 0.01 levels
b) the 0.05 but not at the 0.01 level
c) the 0.01 but not at the 0.05 level
d) neither the 0.05 or 0.01 levels
e) none of the above

2.2 A research report states: The differences between public and private school seventh graders' attitudes toward minority groups were statistically significant at the $\alpha = 0.05$ level. This means that:

a) It has been proven that the two groups are different.
b) There is a probability of 0.05 that the attitudes of the two groups are different.
c) There is a probability of 0.95 that the attitudes of the two groups are different.
d) If there is no difference between the groups, the difference observed in the sample would occur by chance with a probability of no more than 0.05.
e) None of the above is correct.

2.3 If the null hypothesis is really false, which of these statements characterizes a situation where the value of the test statistic falls in the rejection region?

a) The decision is correct.
b) A type I error has been committed.
c) A type II error has been committed.
d) Insufficient information has been given to make a decision.
e) None of the above is correct.

2.4 If the null hypothesis is really false, which of these statements characterizes a situation where the value of the test statistic does not fall in the rejection region?

a) The decision is correct.
b) A type I error has been committed.
c) A type II error has been committed.
d) Insufficient information has been given to make a decision.
e) None of the above is correct.

2.5 If the value of any test statistic does not fall in the rejection region, the decision is:

a) Reject the null hypothesis.
b) Reject the alternative hypothesis.
c) Fail to reject the null hypothesis.
d) Fail to reject the alternative hypothesis.
e) There is insufficient information to make a decision.

2.6 For a particular sample, the 95% two-sided confidence interval for the population mean is from 11 to 17. You are asked to test the hypothesis that the population mean is 18 against a two-sided alternative. Your decision is:

a) Fail to reject the null hypothesis, $\alpha = 0.05$.
b) Reject the null hypothesis, $\alpha = 0.05$.
c) There is insufficient information to decide.

2.7 If we decrease the confidence level, the width of the confidence interval will:

a) Increase
b) remain unchanged
c) decrease
d) double
e) none of the above

2.8 If the value of the test statistic falls in the rejection region, then:

a) We cannot commit a type I error.
b) We cannot commit a type II error.
c) We have proven that the null hypothesis is true.
d) We have proven that the null hypothesis is false.
e) None of the above is correct.

2.9 You are reading a research article that states that there is no significant evidence that the median income in the two groups differs, at $\alpha = 0.05$. You are interested in this conclusion, but prefer to use $\alpha = 0.01$.

a) You would also say there is no significant evidence that the medians differ.
b) You would say there is significant evidence that the medians differ.
c) You do not know whether there is significant evidence or not, until you know the *p* value.

Problem 3 (10 points, 5 points for each part)

Suppose that for a given population with $\sigma = 7.2$ we want to test $H_0: \mu = 80$ against $H_1: \mu < 80$ based on a sample of $n = 100$. For this problem, you can use R to find the probabilities associated with the standard normal and $z_\alpha$but you need to present sufficient details about your calculations.

(1) If the null hypothesis is rejected when $\bar{y} < 76$, what is the probability of a type I error?

(2) What would be the rejection region if we wanted to have a level of significance of exactly 0.05?

Problem 4 (25 points)

The family incomes in a certain city in 1970 had a mean of \$14,200 with a standard deviation of \$2600. A random sample of 75 families taken in 1975 produced $\bar{y} = \$15{,}300$ (adjusted for inflation). For this problem, you can use R to find the probabilities associated with the standard normal and z_α but you still need to present sufficient details about your calculations.

(1) **(11 points)** Assume the standard deviation has remained unchanged. Use the rejection region method to test if the mean family income has changed at a 0.05 level of significance. Please clearly specify 5 steps used in the test.

(2) **(6 points)** Calculate the power of the test from (1) for the mean income of \$15300 and \$15600.

(3) **(3 points)** Fine the p-value associated with this test.

(4) **(5 points)** Construct a 99% two-sided confidence interval on the mean family income in 1975. Based on this confidence interval, state if you would like to reject the null hypothesis from part (1) with a significance level of 0.01.