Problem 1 (40 points)

The following data gives the average pH in rain/sleet/snow for the two-year period 2004–2005 at 20 rural sites on the U.S. West Coast. (Source: National Atmospheric Deposition Program):

5.335	5.345	5.395	5.305	5.315
5.380	5.520	5.190	5.455	5.330
5.360	6.285	5.350	5.125	5.115
5.510	5.340	5.340	5.305	5.265

For all calculations, you can use the sample mean and the sample variance calculated from R. You can create a data in R with the following R script:

```
ph <-  c(5.335, 5.345, 5.395, 5.305, 5.315, 5.380, 5.520, 5.190, 5.455, 5.330,
    5.360, 6.285, 5.350, 5.125, 5.115, 5.510, 5.340, 5.340, 5.305, 5.265)
```

Note: Please use 4 decimal digits in your solution files and in your calculation.

(1) (**3 points**) Find the sample mean, sample variance, and sample standard deviation.

(2) (**11 points**) Perform a *t*-test to see if the mean pH is at least 5.40 with the significance level of 0.05. Please clearly specify 5 steps used in the test and use the rejection approach. You need to use the t-table to find t_{α} or $t_{\frac{\alpha}{2}}$.

(3) (**5 points**) Perform a *t*-test based on the confidence interval approach. You only need to calculate the corresponding confidence interval and present Step 5 for this part. You need to use the t-table to find t_{α} or $t_{\frac{\alpha}{2}}$.

(4) (**2 points**) Use R to calculate the *p*-value for this test from (2).

(5) (**4 points**) Find the two-sided 95% confidence interval for the pH. Use the t-table to find t_{α} or $t_{\frac{\alpha}{2}}$.

(6) (**5 points**) Draw the QQ-plot and then comment on its normality with one or two sentences.

(7) (**5 points**) Draw box plot of the pH and identify any extreme values. Would the sample mean or the sample median be a better descriptor of typical pH values?

(8) (**5 points**) Remove the most extreme value, then perform a t-test to see if the mean pH is at least 5.40 with the significance level of 0.05. You only need to present your p-value and Step 5 for this part.

Problem 1

1.1

Sample mean = 5.3782

Sample Variance = 0.0564

Sample Standard Deviation = 0.2375

37/40

1.2

Step 1: Hypothesis

Null Hypothesis (H_o) : $\boldsymbol{\mu}$ = 5.40

-2

Alternate Hypothesis (H_a) : $\boldsymbol{\mu}$ < 5.40

Step 2: Test Statistic

$\boldsymbol{\alpha}$ = 0.05, n = 20,

t = (xbar - $\boldsymbol{\mu}$) / (s / sqrt(n))

with degrees of freedom df = n - 1 = 19

t = (xbar - 5.40) / (0.0564 / sqrt(20))

Step 3: Rejection region

For a one-tailed (lower-tail) test at $\boldsymbol{\alpha}$ = 0.05 with 19 df, the critical value from the t-table is -1.7291

Step 4: Compute test statistic

t = (5.3782 - 5.40) / (0.0564 / sqrt(20))

t = -0.4096

Step 5: Decision and Interpretation

Since -0.4096 is not less than -1.7291

We fail to reject the null hypothesis H_o

At the 0.05 significance level, data provides insufficient evidence to conclude that the mean pH is less than 5.40

1.3

For the confidence interval approach we should construct a one-sided 95% confidence interval for $\boldsymbol{\mu}$ and then check whether 5.40 is contained in this interval.

The 95% confidence interval (CI) is given by:

$$(-\infty, x + t_{0.025,\ df} * s / \sqrt{n})$$

Lower bound = $-\infty$
Upper bound is $x + t_{0.025,\ df} * s / \sqrt{n}$

$t_{0.025,\ 19}$ is the critical value for two-tail test with $\boldsymbol{\alpha} = 0.05$, df = 19 , by using R we get

$$5.37825 + 1.729133 * (0.2374884 / 20)$$

CI lower = $-\infty$

CI upper = 5.4701

Step 5:

95% Confidence Interval for μ: ($-\infty$, 5.4701). Since 5.40 is inside this interval, we fail to reject H_0.

Data provides insufficient evidence to conclude that the mean pH is less than 5.40.

1.4

p value using R

p_value <- pt(t_stat, df = df)

p_value = 0.3433

1.5

we should construct a two-sided 95% confidence interval for $\boldsymbol{\mu}$ and then check whether 5.40 is contained in this interval.

The 95% confidence interval (CI) is given by:

$$x \pm t_{0.025,\ df} * s / \sqrt{n}$$

Lower bound = $x - t_{0.025,\ df} * s / \sqrt{n}$

Upper bound = $x + t_{0.025,\ df} * s / \sqrt{n}$

$t_{0.025,\,19}$ is the critical value for two-tail test with α / 2 = 0.025, df = 19 , by using R we get

5.37825 $\pm$ 5.37825 * (0.2374884 / 20)

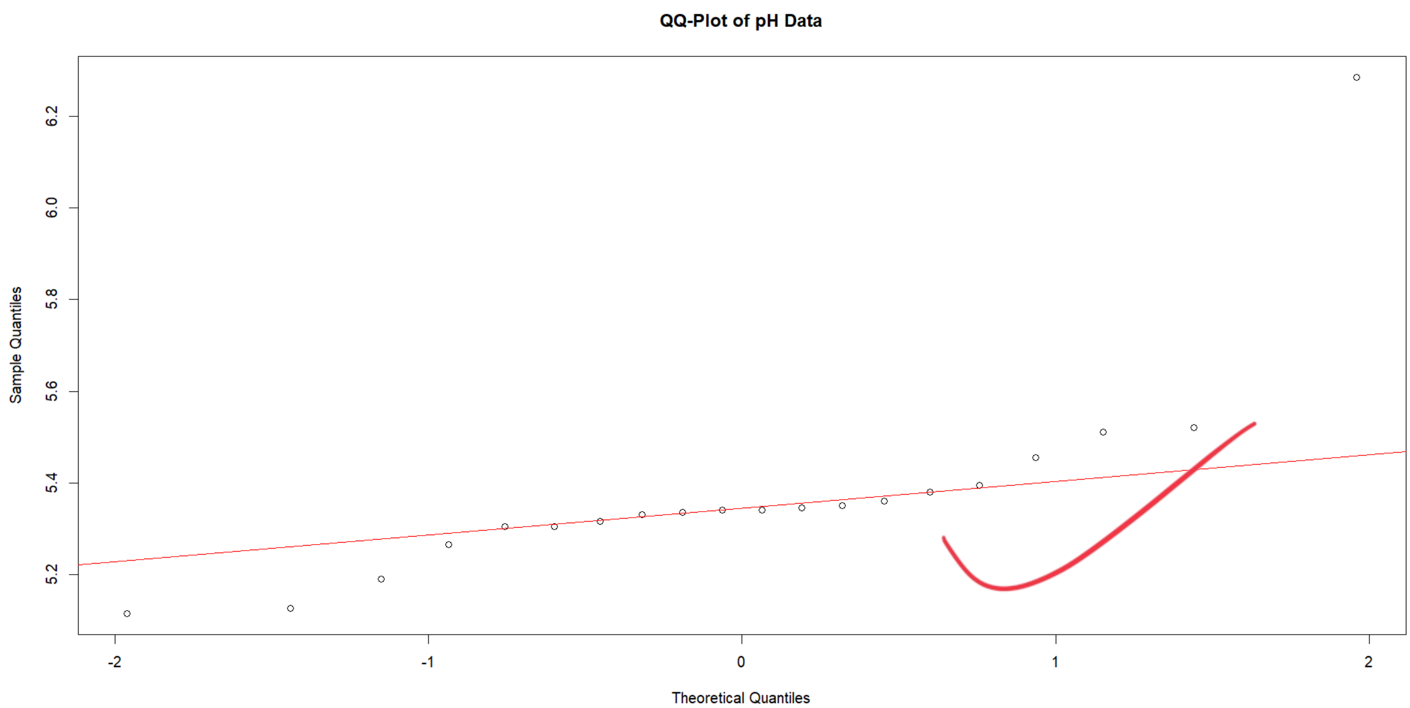
CI lower = 5.2671

CI upper = 5.4701

95% Confidence Interval for μ: (5.2671 , 5.4701)

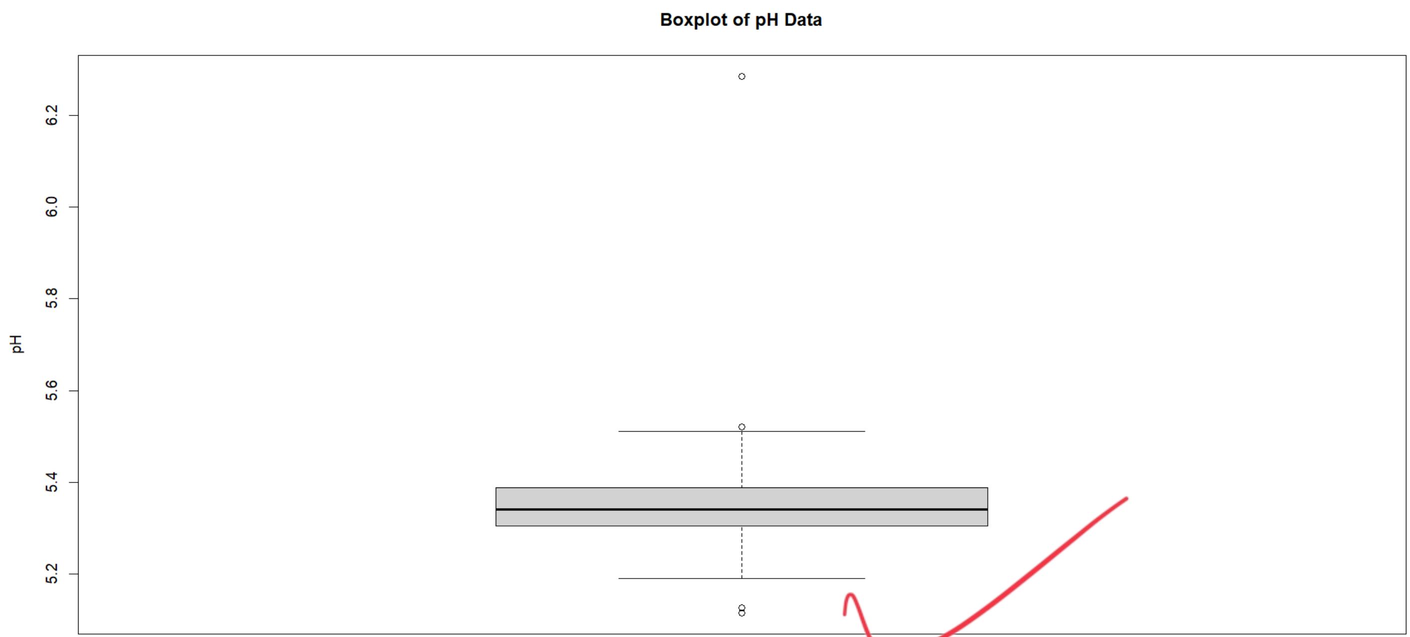
1.6

QQ plot



The QQ-plot appears approximately linear, which suggests that the pH data is roughly normally distributed. Although, there appears to be one outlier which can influence the normality.

1.7



From the box plot we can see that there appears to be three outliers. The median is below 5.4. Most values lie in the range between 5.3 to 5.4 indicating low variance.

4 of them, specify them and 6.285 is an extreme outlier

-1

1.8

outlier = 6.285, after removing

Sample mean = 5.3305

Sample variance = 0.0115

Sample standard deviation = 0.107

n = 19

t = (xbar - μ) / (s / sqrt(n))

t = (5.3305 - 5.40) / (0.107 / sqrt(19))

t = -2.8297

p_value = 0.0056

Because the p-value = 0.0056 is well below 0.05, we reject the null hypothesis at the 5% significance level. After removing the outlier, there is strong evidence that the true mean pH is less than 5.40.

```
# R code
# pH data
ph <- c(5.335, 5.345, 5.395, 5.305, 5.315,
        5.380, 5.520, 5.190, 5.455, 5.330,
        5.360, 6.285, 5.350, 5.125, 5.115,
        5.510, 5.340, 5.340, 5.305, 5.265)

# Sample size
n <- length(ph)

# (1)
xbar <- mean(ph)
s2   <- var(ph)
s    <- sd(ph)

#  rounded to 4 decimal places
cat("Sample Mean =", round(xbar, 4), "\n")
cat("Sample Variance =", round(s2, 4), "\n")
cat("Sample Standard Deviation =", round(s, 4), "\n")

t_stat <- (xbar - 5.40) / (s / sqrt(n))
cat("t-statistic =", round(t_stat, 4), "\n")


df <- n - 1
t_crit <- qt(0.05, df = df)
cat("t-critical =", round(t_crit, 4), "\n")


alpha <- 0.05
t_val <- qt(0.95, df = df)
margin <- t_val * (s / sqrt(n))
CI_lower <- -Inf
CI_upper <- xbar + margin
cat("95% Confidence Interval for μ: (", round(CI_lower, 4), ",", round(CI_upper, 4), ")\n")

# p_value <- pt(t_stat, df = df)
cat("One-tailed p-value =", round(p_value, 4), "\n")


# two sided
alpha <- 0.05
t_val <- qt(1 - alpha/2, df = df)
margin <- t_val * (s / sqrt(n))
```

```r
CI_lower <- xbar - margin
CI_upper <- xbar + margin
cat("95% Confidence Interval for μ: (", round(CI_lower, 4), ",", round(CI_upper, 4), ")\n")


t_val <- qt(1 - alpha/2, df = df)
margin <- t_val * (s / sqrt(n))
CI_lower <- xbar - margin
CI_upper <- xbar + margin
cat("95% Confidence Interval for μ: (", round(CI_lower, 4), ",", round(CI_upper, 4), ")\n")


qqnorm(ph, main = "QQ-Plot of pH Data")
qqline(ph, col = "red")

boxplot(ph, main = "Boxplot of pH Data", ylab = "pH")


ph_no_outlier <- ph[ph != 6.285]
n_no <- length(ph_no_outlier)
xbar_no <- mean(ph_no_outlier)
s_no <- sd(ph_no_outlier)
df_no <- n_no - 1

t_stat_no <- (xbar_no - mu0) / (s_no / sqrt(n_no))
p_value_no <- pt(t_stat_no, df = df_no)

cat("After removing the outlier:\n")
cat("Sample Mean =", round(xbar_no, 4), "\n")
cat("t-statistic =", round(t_stat_no, 4), "\n")
cat("p-value =", round(p_value_no, 4), "\n")
```

Problem 1 (20 Bonus Points)

A manufacturer of watches has established that on average his watches do not gain or lose. He also would like to claim that at least 95% of the watches are accurate to ±0.2 s per week. A random sample of 15 watches provided the following gains (+) or losses (−) in seconds in one week:

+0.17	-0.07	+0.13	+0.05	+0.23	+0.01	+0.06	+0.08
+0.14	+0.10	+0.08	+0.11	+0.05	-0.87	+0.05	

Can the claims be made with a 5% chance of being wrong? You can assume that the inaccuracies of these watches are normally distributed.

Note: There are two claims from the manufacturer.

The sample mean is: 0.02133; the sample standard deviation is: 0.25626

Perform two tests of hypothesis with the rejection region approach. Then calculate the p-value and construct the appropriate CI. Check if the assumptions for those tests are satisfied. All of your calculations should be based on the standard normal table and t-table provided by this course.

Problem 1

Part - 1 Testing the mean

Step1: Hypothesis

Null Hypothesis: H_0: $\mu = 0$

Alternative Hypothesis: H_a: $\mu \neq 0$

Step2: Test Statistic

One-sample t-statistic

$t = (\bar{x} - \mu_o) / (s/\sqrt{n})$

$t = (0.02133 - 0) / (0.25626/ \sqrt{15})$

Step3: Rejection Region

For a two–tailed test at $\alpha = 0.05$ with n – 1 degrees of freedom (df = 14), the critical t–values are $\pm t_{14,0.025}$

Is ±2.1448

Step4: Compute test statistic

$t = (0.02133 - 0) / (0.25626/ \sqrt{15})$

$t = 0.3224$

Ste5: Decision and interpretation

As 0.3223 is less than the critical value 2.1448 we fail to reject the null hypothesis. The data provides insufficient evidence to prove that the mean gain/loss is zero.

P-value :

$p = 2[1 – P(T \leq |t|)]$ or $p = 2pr(T > 0.3224)$

p = 2 * (1 - pt(abs(t_stat), df))

p = 0.75

Confidence Interval:

95% CI for μ: $\bar{x} \pm t_{0.025} \cdot (s/\sqrt{n})$

$0.02134 \pm 2.1448 * (0.25626/ \sqrt{15})$

95% CI for μ: (-0.1206 , 0.1632)

Based on p value and CI confirms we fail to reject H_0. The data provides insufficient evidence to prove that the mean gain/loss is zero.

Part - 2

The manufacturer claims that at least 95% of watches are accurate to within ±0.2 seconds per week. We'll test this claim using the sample proportion method at a 5% significance level ($\alpha=0.05$)

Hypothesis

Null Hypothesis H_0: $p \geq 0.95$ (at least 95% of watches are within ±0.2 seconds)

Alternative Hypothesis H_a : $p < 0.95$ (less than 95% of watches are within ±0.2 seconds)

Sample proportion

n = 15, values outside ±0.2 = 2, values inside ±0.2 = 13

phat = 13/15 = 0.8666

binary outcome with in or without ±0.2, one-tailed binomial test, alpha = 0.05

```
test_result <- binom.test(13, 15, p = 0.95, alternative = "less")
test_result
```

```
Exact binomial test

data:  successes and n
number of successes = 13, number of trials = 15, p-value = 0.171
alternative hypothesis: true probability of success is less than 0.95
95 percent confidence interval:
 0.0000000 0.9757743
sample estimates:
probability of success
             0.8666667
```

P - value = 0.171

Confidence interval = [0, 0.9757]

p-value(0.171) > 0.05, we fail to reject the null hypothesis and data does not provide sufficient evidence to reject the manufacturer's claim that at least 95% of the watches are accurate to within ±0.2 seconds per week at the 5% significance level.

Problem 1 (6 points, 1 point for each part)

This section consists of some true/false questions regarding concepts of statistical inference. Indicate if a statement is true if it is always true or false otherwise. You do not need to explain why it is true or false.

(1) The quantity $(\bar{y} - \mu)/\sqrt{\sigma^2/n}$ has the t-distribution with $n - 1$ degrees of freedom.

(2) When the test statistic is t and the number of degrees of freedom is > 30, the critical value of t ($t_{n,\alpha}$) is very close to that of z (the standard normal) (z_α).

(3) The variance of a binomial proportion is npq [or $np(1 - p)$].

(4) The sampling distribution of a proportion is approximated by the t distribution.

(5) The t-test can be applied with absolutely no assumptions about the distribution of the population.

(6) The degrees of freedom for the t-test do not necessarily depend on the sample size used in computing the sample mean.

Problem 2 (18 points)

A local congressman indicated that he would support the building of a new dam on the Yahoo River if more than 60% of his constituents supported the dam. His legislative aide sampled 225 registered voters in his district and found 145 favored the dam.

(1) (**11 points**) At the level of significance of 0.10 should the congressman support the building of the dam? Specify five steps for the hypothesis testing. For this part, you can use either the rejection region approach, the p-value approach, or the confidence interval approach.

(2) (**5 points**) Find the two-sided 95% confidence interval of the proportion of his constituents who supported the dam. Show sufficient details about your calculations.

(3) (**2 points**) Check if the normal assumption is satisfied here.

Problem 3 (8 points)

Use the function `t.test()` to answer Part (2) and Part (5) of Problem 1 in Week 10 HW. You do not need to specify 5 steps of hypothesis testing but need to report the test statistic, the p-value, and the appropriate confidence interval.

Problem 4 (12 points)

A certain soft drink bottler claims that less than 10% of its customers drink another brand of soft drink on a regular basis. A random sample of 100 customers yielded 18 who did in fact drink another brand of soft drink on a regular basis. Do these sample results support the bottler's claim? (Use a level of significance of 0.05).

(1) (**5 points**) Use the R function `prop.test()` to solve this problem. You do not need to specify 5 steps of hypothesis testing but need to specify the null and alternative hypothesis, report the statistic (the chi-square statistic and the z statistic), the p-value, and the confidence interval.

(2) (**5 points**) Construct the appropriate 95% CI of the proportion of its customers who drink another brand of soft drink on a regular basis. Show sufficient details about your calculations. Your calculations should be based on formulas covered in the class.

(3) (**2 points**) Check if the normal assumption is satisfied here.

42/44 great!

Problem 1

(1) False

(2) True

(3) False

-1

(4) False

(5) False

(6) False

Problem 2

(1)

A congressman will support building a dam if more than 60% of his constituents support it. A sample of 225 registered voters shows that 145 favor the dam.

Step 1: State the Hypothesis

Null Hypothesis (H_0): p = 0.60

Alternative Hypothesis (H_a): p > 0.60

Step 2: Formula for Test statistic

For proportion test, the test statistic is

z = $(phat \;-\; po) \,/\, sqrt(po(1 \;-\; po) \,/\, n)$

sample proportion

phat = 145 / 225 = 0.6444 , po = 0.60, n = 225

Step 3: Find the rejection region/ critical value/ p value

$Pr(Z > z_{\alpha}) = 0.10$

Using normal table we get

$z_{0.10}$ = 1.2815

Step 4: Compute the test statistic

$z = (0.6444 - 0.60) / \sqrt{0.60(1 - 0.60) / 225}$

$z = 1.3594$

p - value :

p value = $Pr(Z > 1.3594)$

$Pr(Z < 1.36) = 0.9131$

p - value = $1 - 0.9131$

p - value = 0.0869

Step 5: Conclusion and Interpretation

The computed test statistic value z (1.36) exceeds the test critical value (1.28), and the p-value (0.087) is less than α (0.10), we reject Null Hypothesis H_0. We can conclude that the data provides enough evidence that at alpha = 0.10 to suggest that p > 0.60 and the congressman should the dam.

(2) Two-sided 95% Confidence Interval

$\text{phat} \pm z_{\alpha/2} \sqrt{\text{phat}(1 - \text{phat}) / n}$

phat = 0.6444, n = 225 , alpha = 0.05 , alpha / 2 = 0.025, $P(Z \le z_{\alpha/2}) = 1 - 0.025 = 0.975$

$z_{0.975} = 1.959964 = 1.96$

0.6444 ± 1.96 sqrt(0.6444 (1 - 0.6444) / 225)

The Two-sided 95% confidence interval of the proportion of his constituents
who supported the dam is: (0.5818 , 0.7069)

(3) Normal Assumption

For the normal approximation to be valid when testing proportions, both np_o and $n(1 - p_o)$ should be atleast 10

$np_o = 225 * 0.60 = 135$

$n(1 - p_o) = 225 * (1 - 0.60) = 90$

Both are well above 10, the normal approximation is satisfied here.

Problem 3

```
ph <- c(5.335, 5.345, 5.395, 5.305, 5.315, 5.380, 5.520, 5.190,
        5.455, 5.330, 5.360, 6.285, 5.350, 5.125, 5.115, 5.510,
        5.340, 5.340, 5.305, 5.265)
```

part - 2

```
t_test_one_sided <- t.test(ph, mu = 5.40, alternative = "great", conf.level = 0.95)
t_test_one_sided
```

greater

```
One Sample t-test

data:  ph
t = -0.40957, df = 19, p-value = 0.6567
alternative hypothesis: true mean is greater than 5.4
95 percent confidence interval:
 5.286426      Inf
sample estimates:
mean of x
```

test statistic = -0.4095

P-value = 0.6567

95% confidence interval = [5.2864, Inf]

part - 5

```
t_test_two_sided <- t.test(ph, conf.level = 0.95)
t_test_two_sided
```

```
One Sample t-test

data:  ph
t = 101.28, df = 19, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 5.267102 5.489398
sample estimates:
mean of x
  5.37825
```

test statistic = 101.28

P-value < 2.2e-16

95% confidence interval = [5.2671, 5.4893]

Problem 4

(1)

The bottler's claim is that less than 10% drink another brand.

Null Hypothesis (H_0): $p = 0.10$

Alternative Hypothesis (H_a): $p < 0.10$

n = 100, x = 18, p = 0.1, phat = 18/100 = 0.18

```
prop.test(x = 18, n = 100, p = 0.1, alternative = "less", correct = FALSE)

1-sample proportions test without continuity correction

data:  18 out of 100, null probability 0.1
X-squared = 7.1111, df = 1, p-value = 0.9962
alternative hypothesis: true p is less than 0.1
95 percent confidence interval:
 0.0000000 0.2513522
sample estimates:
   p
0.18
```

Chi-square Statistic: 7.1111

Z - value = sqrt(Chi-square Statistic) = sqrt(7.1111) = 2.6666

P- value = 0.9962

95% Confidence Interval: (0, 0.2513)

P value greater than alpha value. We fail to reject the null hypothesis. We conclude that the data provide insufficient evidence to support the bottler's claim that less than 10% of its customers drink another brand.

(2)

95% Confidence Interval

$\text{phat} \pm z_{alpha/2} \sqrt{\text{phat}(1 - \text{phat}) / n}$

phat = 0.18, alpha = 0.05 , alpha / 2 = 0.025, $P(Z \leq z_{\alpha/2}) = 1 - 0.025 = 0.975$, $z_{0.975} = 1.96$, n = 100

$0.18 \pm 1.96 (\sqrt{0.18(1 - 0.18)} / 100$

95% Confidence Interval: (0.1047 , 0.2553)

(3)

For the normal approximation to be valid when testing proportions, both np_o and $n(1 - p_o)$ should be atleast 10

$np_o = 100 * 0.18 = 18$

$n(1 - p_o) = 100 * (1 - 0.18) = 82$

Both are well above 10, the normal approximation is satisfied here.

$p_o = 0.10$
not 0.18

-1

Problem 1 (6 points, 1 point for each part)

Indicate if a statement is **true** if it is always or **false** otherwise. You do not need to explain why it is true or false.

(1) One of the assumptions underlying the use of the pooled estimate of population variance in the two sample t-test is that the samples are drawn from populations having equal variances.

(2) In the two sample t-test, the number of degrees of freedom for the test statistic increases as sample sizes increase.

(3) If every observation is multiplied by 2, then the t statistic is multiplied by 2.

(4) The standard normal distribution can be used for inferences concerning proportions of success from binomial populations.

(5) The pooled variance estimate is used when comparing means of two populations using independent samples.

(6) It is not necessary to have equal sample sizes for the paired t-test.

Problem 2 (22 points)

The manager of a large office building needs to buy a large shipment of light bulbs. After reviewing specifications and prices from a number of suppliers, the choice is narrowed to two brands whose specifications with respect to price and quality appear identical. The manager intends to buy the bulbs with a longer mean life. To test the life of bulbs of each brand, he purchases some bulbs of each brand and subjects them to an accelerated life test, recording hours to burnout, the following are the data obtained from the manager's experiment:

Brand A: sample size = 38; sample mean = 1532; sample variance = 128008

Brand B: sample size = 40; sample mean = 1390; sample variance = 92564

Based on the above data, the manager consults you about which brand of bulbs he should buy. As a student taking a statistical course, you try to answer his question with the following calculations:

(1) (**13 points**) You test if the mean life of bulbs from two brands differs with a significance level of 0.05 and the equal variance assumption. Clearly specify 5 steps with the p-value approach. You need to provide sufficient details about your calculations.

(2) (**5 points**) Construct a 95% confidence interval on the mean difference in the life of bulbs for two brands. Here you still assume the equal variance of life of bulbs from two brands. Based on this confidence interval, do you reject the null hypothesis in (1)?

(3) (**4 points**) The complete data can be found in data file bulb.csv. Use R function `t.test()` to find the corresponding test statistic, the p-value, and the 95% confidence interval and report them in your solution file. You can use bulb$life[bulb$brand == "a"] and bulb$life[bulb$brand == "b"] to get the life of brand A and brand B bulbs. Once you create a data named "bulb" with R function `read.csv()`.

Problem 3 (22 points)

A maintenance manager must test if a new repair method can increase the expected time between repairs. For each machine used in the study, she recorded the last time between failures prior to using the new method, which she called "Current", and the first time between failures after using the new method, which she called "New". These are the times (in hours):

Machine	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Current	155	222	346	287	115	389	183	451	140	252
New	211	345	419	274	244	420	319	505	396	222
Difference	56	123	73	-13	129	31	136	54	256	-30

Sample means: Current = 254.0; New = 333.5; Difference = 81.5

Sample variances: Current = 12752.667; New = 9711.89; Difference = 6976.722

(1) (**11 points**) Conduct the most appropriate hypothesis test using a 0.05 significance level and the p-value approach. You must clearly specify 5 steps for your test and provide sufficient details about your calculations.

(2) (**5 points**) Construct a 95% two-sided confidence interval for the difference of time between repairs before and after using the new repair method. You must provide sufficient details about your calculations.

(3) (**3 points**) Use R function `t.test()` based on the data of "Current" and "New" to find the test statistic, the p-value, and confidence interval for the test from (1) and report them in your solution file. In other words, you need to perform a two sample t-test with R function `t.test()` and the option "paired = TRUE".

(4) (**3 points**) Use R function `t.test()` based on the data of "Difference" to find the test statistic, the p-value, and confidence interval for the test from (1) and report them in your solution file.

Problem 1

(1) True

49/50

(2) True

(3) False

(4) True

(5) True

(6) False

Problem 2

(1)

Given $n1 = 38$, $y1bar = 1532$, $s1^2 = 128008$

$n2 = 40$, $y2bar = 1390$, $s2^2 = 92564$

$\alpha = 0.05$

Step1 : Hypothesis , let mu1 = mean life for Brand A, mu2 = mean life for Brand B

Null Hypothesis (Ho) : $mu1 - mu2 = \delta_0 = 0$

Alternative Hypothesis (Ha) : $mu1 - mu2 \neq 0$

Step 2: Formula for Test statistic

Test statistic for two sample t - test (equal variances) is

$$t = (y1bar - y2bar) - \delta_0 \ / \ S_p \, sqrt(1 / n_1 + 1 / n_1)$$

$$t = (y1bar - y2bar) - 0 \ / \ S_p \, sqrt(1 / n_1 + 1 / n_2)$$

Step 3: Find the Rejection Region

$| t | > t_{df, \alpha / 2} = t_{n1 + n2 - 2, 0.05 / 2} = t_{38 + 40 - 2, 0.025} = t_{76, 0.025} \approx 1.9921$

Step 4: Compute the test statistic

$S_p = sqrt(((n_1 - 1) s1^2 + (n_2 - 1) s2^2) / (n_1 + n_2 - 2))$

$S_p = sqrt(((38 - 1)128008 + (40 - 1)92564) / ((38 - 1) + (40 - 1) - 2))$ 38+40-2

-1

S_p = sqrt(112787.7)

S_p = 335.8388

t = (1532 - 1390) / (335.8388 * sqrt(1/38 + 1/40))

t = 1.8665

Find P - value

P- value = 2 * Pr(T > | t |)

P- value = 2 * Pr(t_{76} > | 1.86|)

P- value = 2×0.031=0.062

Step 5: Conclusion and Interpretation

The p–value (0.062) is greater than the significance level (0.05), we fail to reject the null hypothesis. The data provides insufficient evidence to conclude that the mean life of bulbs differs between the two brands.

(2)

Confidence interval = y1bar - y2bar $\pm$ $t_{n1 + n2 - 2,\ \alpha / 2}$ * S_p (√(1 / n1 + 1 / n2))

= (38 - 40 ± 1.9921 * 335.8388√(1/38 + 1/40))

= [-9.554, 293.554]

Because the interval includes 0, it aligns with our conclusion in part (1) that we do not have significant evidence of a difference. Based on the 95% CI, since it contains 0, we do not reject the null hypothesis.

(3)

```
bulb <- read.csv("bulb.csv")
t.test(bulb$life[bulb$brand == "a"],
    bulb$life[bulb$brand == "b"],
    alternative = "two.sided",
    var.equal = TRUE)
```

```
Two Sample t-test

data:  bulb$life[bulb$brand == "a"] and bulb$life[bulb$brand == "b"]
t = 2.4079, df = 78, p-value = 0.01841
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 24.51247 258.53753
```

sample estimates:
mean of x mean of y
1531.975 1390.450

Test Statistic (t): 2.4079

Degrees of Freedom (df): 78

p–value: 0.01841

95% Confidence Interval: [24.51247, 258.53753]

Problem 3

(1) Given $y1 = 254$, $y2 = 335.5$, dbar = 81.5 , $s1^2 = 12752.667$, $s2^2 = 9711.89$, $S_d^{2.} = 6976.722$, n = 10

Step 1: Hypothesis

Null Hypothesis (Ho) : $\mu_2 - \mu_1 = \delta_0 = 0$

Alternative Hypothesis (Ha) : $\mu_2 - \mu_1 > 0$

Step 2: Formula for test statistic

test statistic for a paired t–test

$t = (dbar - \delta_0) / (S_d / sqrt(n))$

$t = dbar - 0 / (S_d / sqrt(n))$

Step 3: Find the rejection region but we use p value here

Step 4: Compute the test statistic

t = 81.5 / (83.5268 / sqrt(10))

t = 3.0855

Find P- value

P-value = $Pr(T_{df} > t) = Pr(T_9 > 3.0855) < 0.01$

Step 5: Conclusion and Interpretation

P-value is less than alpha value so we reject the null hypothesis. The data provides that there is statistically significant evidence at the 5% level that the new repair method increases the mean time between failures.

(2)

95% Two-Sided Confidence Interval:

$$\text{dbar} \pm t_{\alpha/2,\, df} * S_d / \text{sqrt}(n)$$

$$81.5 \pm t_{0.025,\, 9} * 83.5268 / \text{sqrt}(10)$$

$$81.5 \pm 2.262 * 83.5268 / \text{sqrt}(10)$$

[21.7527, 141.2473]

(3)

```
Current <- c(155, 222, 346, 287, 115, 389, 183, 451, 140, 252)
New <- c(211, 345, 419, 274, 244, 420, 319, 505, 396, 222)
t.test(New, Current, paired = TRUE, alternative = "greater")

        Paired t-test

data:  New and Current
t = 3.0855, df = 9, p-value = 0.006511
alternative hypothesis: true mean difference is greater than 0
95 percent confidence interval:
 33.0811     Inf
sample estimates:
mean difference
          81.5
```

Test Statistic (t): 3.0855

Degrees of Freedom (df): 9

p–value: 0.006511

95% Confidence Interval: [33.0811,   Inf]

(4)

```
Diff <- c(56, 123, 73, -13, 129, 31, 136, 54, 256, -30)
t.test(Diff, mu = 0, alternative = "greater")

	One Sample t-test

data:  Diff
t = 3.0855, df = 9, p-value = 0.006511
alternative hypothesis: true mean is greater than 0
95 percent confidence interval:
 33.0811     Inf
sample estimates:
mean of x
     81.5
```

Test Statistic (t): 3.0855

Degrees of Freedom (df): 9

p–value: 0.006511

95% Confidence Interval: [33.0811,   Inf]

Problem 1 (32 points)

Researchers at Wolfson Children's Hospital, Jacksonville, FL tested a new technology meant to reduce the number of attempts needed to draw blood from children. They collected data on the number of successes on the first attempt using the new technology and on a historical comparison group using standard technology. This data is summarized in the following table:

	Standard Technology	**New Technology**
Successful on 1st	74	73
Unsuccessful on 1st	76	18
Total	150	91

Please use a significant level of 0.05 for your tests. For (2), please perform the t-test with the equal variance assumption.

(1) (**11 points**) Is there evidence that the new technology changes the probability of success on the first attempt? Please specify 5 steps of hypothesis testing with the p-value approach.

(2) (**5 points**) Construct the two-side confidence interval of the difference of the probability of success on the first attempt between the standard technology and the new technology. Please provide sufficient details about your calculations here.

(3) (**5 points**) Use R function `prop.test()` to test if the new technology improves the probability of success on the first attempt. You only need to report the chi-square statistic, the p-value, and the confidence interval and state your conclusion in your solution file.

(4) (**8 points**) The researchers also recorded the ages of children. In the standard technology group, the 150 children had a mean age of 5.73 and a standard deviation of 6.15. In the new technology group, the mean age was 9.02 with a standard deviation of 6.10. Does the mean age of the children in the two groups differ significantly? Just provide the calculation of test statistics and the p-value with sufficient details and state your conclusion.

(5) (**3 points**) How do the results of part (2) complicate the interpretation of part (1)?

Problem 1

$\frac{29}{32}$

(1)

Given n1 = 150, n2 = 91, x1 = 74, x2 = 73
p1cap = x1 / n1 = 74 / 150 = 0.4933
p2cap = x2 / n2 = 73 / 91 = 0.8022
pcap = (74 + 73) / (150 + 91) = 0.61
α = 0.05

Step 1: Hypothesis

Null Hypothesis (Ho): p1cap - p2cap = 0

Alternative Hypothesis (Ha): p1cap - p2cap ≠ 0

Step 2: Formula for test statistic

Z = (p1cap - p2cap) / sqrt(pcap(1- pcap)((1/ n1) + (1 / n2)))

Step3 : Rejection Region

$|Z| > z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} \approx 0.4920$

Step 4: Compute the test statistic

Z = (0.4933 - 0.8022) / sqrt(0.61(1 - 0.61)((1 / 150) + (1 / 91)))

Z = -4.7663

Finding P value : 2 × P(Z > |z|) = 2 × P(Z > 4.7669) < extremely small < 0.001

Step 5: Conclusion and Interpretation

P-value is less than alpha value hence we reject the null hypothesis. The data provides statistical evidence that the new technology changes the probability of success on the first attempt.

(2)

Confidence Interval = (p1cap - p2cap $\pm z_{\alpha/2}$ sqrt(((p1cap(1 - p1cap)) / n1 + p2cap(1-p2cap) / n2))

CI = ((0.4933 - 0.8022) ± 0.4920 sqrt(0.4933(1 - 0.4933)/150 + 0.8022(1 - 0.8022)/91))

CI = [-0.3376, -0.2802]

-1

$z_{0.025} = 1.96$

(3)

```
prop.test(x = c(74, 73), n = c(150, 91), alternative = "two.sided", correct = FALSE, conf.level = 0.95)

2-sample test for equality of proportions without continuity correction

data:  c(74, 73) out of c(150, 91)
X-squared = 22.711, df = 1, p-value = 1.883e-06
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.4233181 -0.1944108
sample estimates:
   prop 1    prop 2
0.4933333 0.8021978
```

−2 need one-sided test

chi-square statistic: 22.711

P-value: 1.883e-06

confidence interval: [-0.4233, -0.1944]

The p value is extremely less than the alpha value and matches the conclusion from problem 1. We reject the null hypothesis. The data provides statistical evidence that the new technology changes the probability of success on the first attempt.

(4)

y1bar = 5.73, s_1 = 6.15, n_1 = 150

y2bar = 9.02, s_2 = 6.10, n_2 = 91

$s^2_p = [(n_1-1)s_1^2 + (n_2-1)s_2^2]/(n_1+n_2-2)$

$s^2_p = [(150-1)(6.15)^2 + (91-1)(6.10)^2]/(150+91-2)]$

$s^2_p = 37.5918$

$t = ((\text{y1bar} - \text{y2bar}) - \delta 0)/\sqrt{[s^2_p(1/n_1 + 1/n_2)]}$

$t = ((5.73 - 9.02)- 0)/\sqrt{[37.5918(1/150 + 1/91)]}$

$t = -4.0384$

P-value = $2 \times P(t_{239} > |t|) = 2 \times P(t_{239} > 4.0384)$ < very small < 0.0005

Since p-value < α , we reject the null hypothesis. The data provides strong statistical evidence that the mean age of children differs significantly between the two technology groups.

(5)

The significant age difference (part 4) suggests the groups are not comparable. If age affects success rates the observed improvement with the new technology (part 1) might be confounded by age. This weakens causal claims about the technology's efficacy.

Problem 1 (20 bonus points)

A local bank has three branch offices. The bank has a liberal sick leave policy, and a vice-president was concerned about employees taking advantage of this policy. She thought that the tendency to take advantage depended on the branch at which the employee worked. To see whether there were differences in the time employees took for sick leave, she asked each branch manager to sample employees randomly and record the number of days of sick leave taken during 2008. Ten employees were chosen, and the summary of data are listed in the following table.

	Sample Size	Sample Mean	Sample Variance
Branch 1	4	17.0	8.67
Branch 2	3	12.3	5.33
Branch 3	3	20.0	7.00
All Data	10	16.5	15.61

Construct the following analysis of variance table. Show how you calculate those numbers. Based on this table, what is your conclusion? Use a level of significance of 0.05.

Source	Degrees of Freedom	Sum of Squares	Mean Square	F Value	Pr > F
Between Groups					
Within Groups				N/A	N/A
Total			N/A	N/A	N/A

Note: Show sufficient details on calculations for this problem, otherwise no credit will be given. Partial credit will be given for the correct parts of your work.

Problem 1

Branch 1: n1 = 4, y1bar = 17.0 , S_1^2 = 8.67

Branch 2: n2 = 3, y2bar = 12.3, S_2^2 = 5.33

Branch 3: n3 = 3, y3bar = 20.0, S_3^2 = 7.0

All Data: N = 10, ybar = 16.5, S^2 = 15.61

df1 = k - 1 = 3 - 1 = 2

df2 = N - k = 10 - 3 = 7

df_total = N - 1 = 10 - 1 = 9

Sum of Squares between groups (SSB):

SSB = $\Sigma[n_i(y_i\text{bar} - \text{ybar})^2]$

SSB = 4*(17.0-16.5)^2 + 3*(12.3-16.5)^2 + 3*(20.0-16.5)^2

SSB = 90.67

Sum of Squares Within Groups (SSW):

SSW = $\Sigma[(n_i - 1)s^2_i]$

SSW = (4 - 1)(8.67) + (3 - 1)(5.33) + (3 - 1)(7.00)

SSW = 50.67

Total Sum of Squares (SST):

SST = SSB + SSW = 90.67 + 50.67 = 141.34

Mean Square Between (MSB):

MSB = SSB / df1 = 90.67 / 2 = 45.335

Mean Square Within (MSW):

MSW = SSW / df2 = 50.67 / 7 = 7.2385

F = MSB / MSW = 45.335 / 7.239 = 6.263

P - value = F(2,7) at α = 0.05   = 4.74

$P(F(2,7) > 6.263) \approx 0.027$

Source	Degrees of Freedom	Sum of Squares	Mean Square	F Value	Pr > F
Between Groups	2	90.67	45.335	6.263	0.027
Within Groups	7	50.67	7.239		
Total	9	141.34			

Since the p-value (0.027) is less than the significance level ($\alpha = 0.05$), we reject the null hypothesis.
There is sufficient statistical evidence to conclude that the mean number of sick leave days differs significantly among the three bank branches.