Problem 1

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(1)

Given n1 = 150, n2 = 91, x1 = 74, x2 = 73
p1cap = x1 / n1 = 74 / 150 = 0.4933
p2cap = x2 / n2 = 73 / 91 = 0.8022
pcap = (74 + 73) / (150 + 91) = 0.61
α = 0.05

Step 1: Hypothesis

Null Hypothesis (Ho): p1cap - p2cap = 0

Alternative Hypothesis (Ha): p1cap - p2cap ≠ 0

Step 2: Formula for test statistic

Z = (p1cap - p2cap) / sqrt(pcap(1- pcap)((1/ n1) + (1 / n2)))

Step3 : Rejection Region

$|Z| > z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} \approx 0.4920$

Step 4: Compute the test statistic

Z = (0.4933 - 0.8022) / sqrt(0.61(1 - 0.61)((1 / 150) + (1 / 91)))

Z = -4.7663

Finding P value : 2 × P(Z > |z|) = 2 × P(Z > 4.7669) < extremely small < 0.001

Step 5: Conclusion and Interpretation

P-value is less than alpha value hence we reject the null hypothesis. The data provides statistical evidence that the new technology changes the probability of success on the first attempt.

(2)

Confidence Interval = $(p1cap - p2cap \pm z_{\alpha/2}$ sqrt(((p1cap(1 - p1cap)) / n1 + p2cap(1-p2cap) / n2))

CI = ((0.4933 - 0.8022) ± 0.4920 sqrt(0.4933(1 - 0.4933)/150 + 0.8022(1 - 0.8022)/91))

CI = [-0.3376, -0.2802]

-1

$Z_{0.025} = 1.96$

(3)

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prop.test(x = c(74, 73), n = c(150, 91), alternative = "two.sided", correct = FALSE, conf.level = 0.95)

2-sample test for equality of proportions without continuity correction

data:  c(74, 73) out of c(150, 91)
X-squared = 22.711, df = 1, p-value = 1.883e-06
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.4233181 -0.1944108
sample estimates:
   prop 1    prop 2
0.4933333 0.8021978
```

-2 need one-sided test

chi-square statistic: 22.711

P-value: 1.883e-06

confidence interval: [-0.4233, -0.1944]

The p value is extremely less than the alpha value and matches the conclusion from problem 1. We reject the null hypothesis. The data provides statistical evidence that the new technology changes the probability of success on the first attempt.

(4)

y1bar = 5.73, s_1 = 6.15, n_1 = 150

y2bar = 9.02, s_2 = 6.10, n_2 = 91

$s^2_p = [(n_1-1)s_1^2 + (n_2-1)s_2^2]/(n_1+n_2-2)$

$s^2_p = [(150-1)(6.15)^2 + (91-1)(6.10)^2]/(150+91-2)]$

$s^2_p = 37.5918$

$t = ((y1bar - y2bar) - \delta 0)/\sqrt{[s^2_p(1/n_1 + 1/n_2)]}$

$t = ((5.73 - 9.02)- 0)/\sqrt{[37.5918(1/150 + 1/91)]}$

$t = -4.0384$

P-value = $2 \times P(t_{239} > |t|) = 2 \times P(t_{239} > 4.0384)$ < very small < 0.0005

Since p-value < α , we reject the null hypothesis. The data provides strong statistical evidence that the mean age of children differs significantly between the two technology groups.

(5)

The significant age difference (part 4) suggests the groups are not comparable. If age affects success rates the observed improvement with the new technology (part 1) might be confounded by age. This weakens causal claims about the technology’s efficacy.