

Problem 1

Part - 1 Testing the mean

Step1: Hypothesis

Null Hypothesis: $H_0: \mu = 0$

Alternative Hypothesis: $H_a: \mu \neq 0$

Step2: Test Statistic

One-sample t-statistic

$$t = (\bar{x} - \mu_0) / (s/\sqrt{n})$$

$$t = (0.02133 - 0) / (0.25626 / \sqrt{15})$$

Step3: Rejection Region

For a two-tailed test at $\alpha = 0.05$ with $n - 1$ degrees of freedom ($df = 14$), the critical t-values are $\pm t_{14, 0.025}$

Is ± 2.1448

Step4: Compute test statistic

$$t = (0.02133 - 0) / (0.25626 / \sqrt{15})$$

$$t = 0.3224$$

Step5: Decision and interpretation

As 0.3223 is less than the critical value 2.1448 we fail to reject the null hypothesis. The data provides insufficient evidence to prove that the mean gain/loss is zero.

P-value :

$$p = 2[1 - P(T \leq |t|)] \text{ or } p = 2\text{pr}(T > 0.3224)$$

$$p = 2 * (1 - \text{pt}(\text{abs}(t_stat), df))$$

$$p = 0.75$$

Confidence Interval:

$$95\% \text{ CI for } \mu: \bar{x} \pm t_{0.025} \cdot (s/\sqrt{n})$$

$$0.02134 \pm 2.1448 * (0.25626 / \sqrt{15})$$

95% CI for μ : (-0.1206 , 0.1632)

Based on p value and CI confirms we fail to reject H_0 . The data provides insufficient evidence to prove that the mean gain/loss is zero.

Part - 2

The manufacturer claims that at least 95% of watches are accurate to within ± 0.2 seconds per week. We'll test this claim using the sample proportion method at a 5% significance level ($\alpha=0.05$)

Hypothesis

Null Hypothesis H_0 : $p \geq 0.95$ (at least 95% of watches are within ± 0.2 seconds)

Alternative Hypothesis H_a : $p < 0.95$ (less than 95% of watches are within ± 0.2 seconds)

Sample proportion

$n = 15$, values outside $\pm 0.2 = 2$, values inside $\pm 0.2 = 13$

$\hat{p} = 13/15 = 0.8666$

binary outcome with in or without ± 0.2 , one-tailed binomial test, $\alpha = 0.05$

```
test_result <- binom.test(13, 15, p = 0.95, alternative = "less")
test_result
```

Exact binomial test

```
data: successes and n
number of successes = 13, number of trials = 15, p-value = 0.171
alternative hypothesis: true probability of success is less than 0.95
95 percent confidence interval:
 0.0000000 0.9757743
sample estimates:
probability of success
 0.8666667
```

P - value = 0.171

Confidence interval = [0, 0.9757]

p-value(0.171) > 0.05, we fail to reject the null hypothesis and data does not provide sufficient evidence to reject the manufacturer's claim that at least 95% of the watches are accurate to within ± 0.2 seconds per week at the 5% significance level.