

Homework 2

Your Name

February 27, 2025

We will now start reflecting on the coding questions of Homework 2. The code base that can be used to replicate the result can be found in the following link: <https://github.com/TagoreZhao/STAT260/tree/main/HW2>

1 Problem 6, 7, and 8

The code needed for generating these three matrices is given below:

```
1 import numpy as np
2
3 def generate_covariance_matrix(d):
4     indices = np.arange(d)
5     Sigma = 2 * 0.5 ** np.abs(indices[:, None] - indices[None, :])
6     return Sigma
7
8 def generate_gaussian_A(n, d, seed=1234):
9     rng = np.random.default_rng(seed)
10    Sigma = generate_covariance_matrix(d)
11    mean = np.ones(d)
12    A = rng.multivariate_normal(mean, Sigma, size=n)
13    return A
14
15 def generate_t_distribution_A(n, d, df, seed=1234):
16     rng = np.random.default_rng(seed)
17     Sigma = generate_covariance_matrix(d)
18     mean = np.ones(d)
19     z = rng.multivariate_normal(mean, Sigma, size=n)
20     chi2_samples = rng.chisquare(df, size=(n, 1))
21     A = z / np.sqrt(chi2_samples / df)
22     return A
```

Listing 1: Python code for generating matrices

Since numpy does not provide built in functions for generating t-distributed random variables, we have to generate the random variables ourselves. The Gaussian random variables are generated using the `multivariate_normal` function, while the t-distributed random variables are generated using the formula $A = Z / \sqrt{\chi^2 / df}$, where Z is the Gaussian random variable, χ^2 is the chi-squared random variable, and df is the degrees of freedom.

Problem 9

We will first plot the norm based probability distribution for all three matrices that we generated using seed 1234.

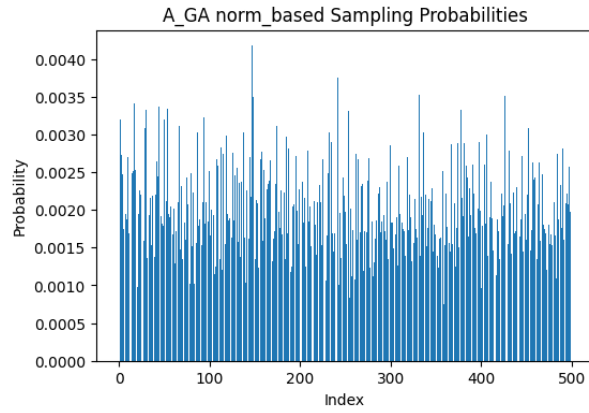


Figure 1: GA Norm based probability distribution

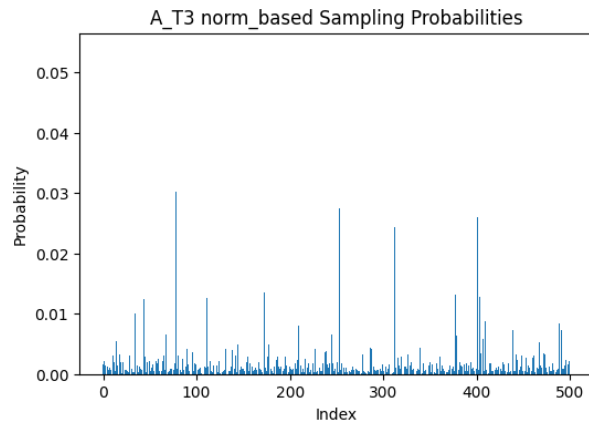


Figure 2: T3 Norm based probability distribution

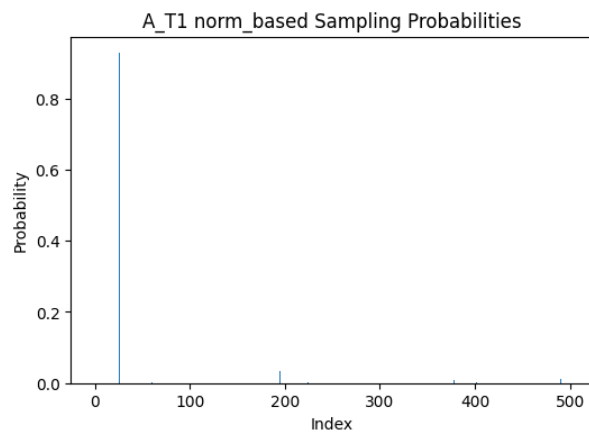


Figure 3: T1 Norm based probability distribution

We will now plot the Frobenius and spectral error for the approximations of the three matrix multiplications.

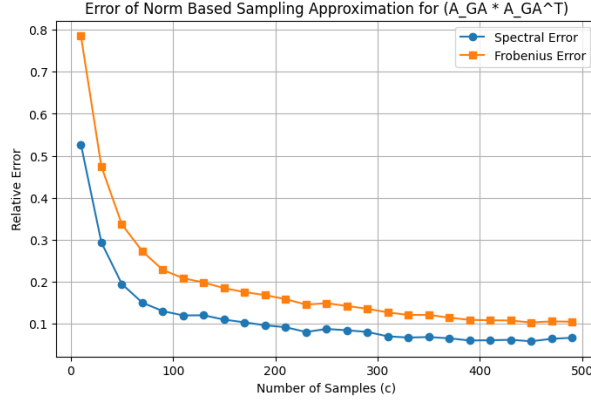


Figure 4: Error of Norm Based Sampling Approximation for $(A_{GA}^T A_{GA})$

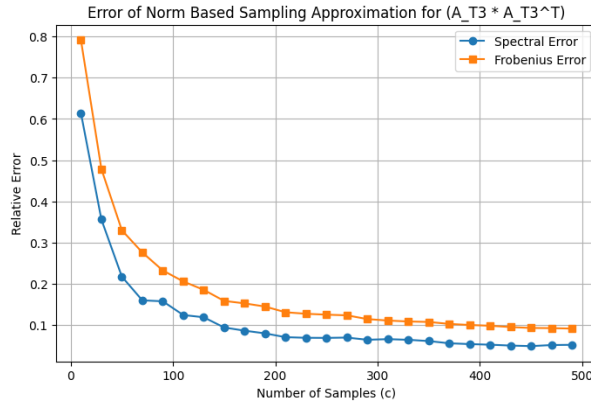


Figure 5: Error of Norm Based Sampling Approximation for $(A_{T3}^T A_{T3})$

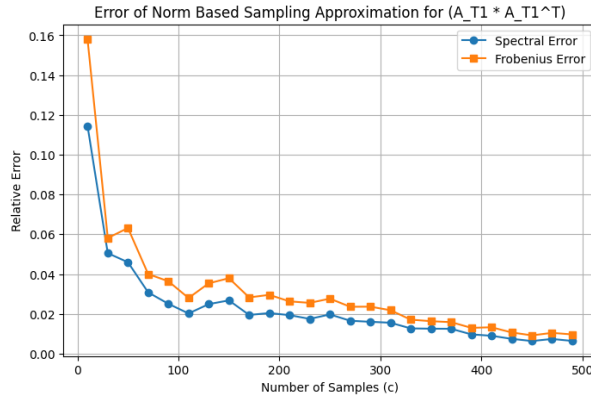


Figure 6: Error of Norm Based Sampling Approximation for $(A_{T1}^T A_{T1})$

Problem 10

We will now plot the Frobenius and spectral error for the approximations of the three left singular matrices multiplication.

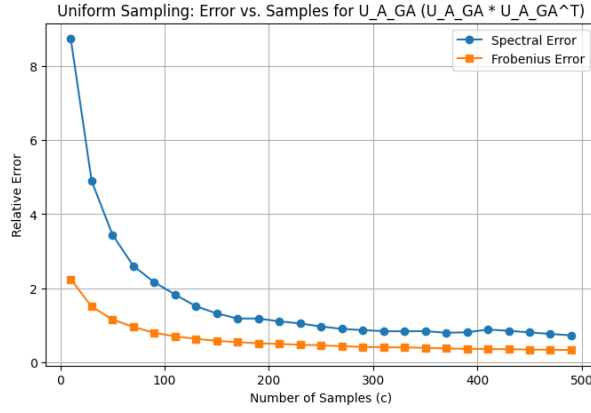


Figure 7: Error of Uniform Based Sampling Approximation for $(U_{GA}^T U_{GA})$

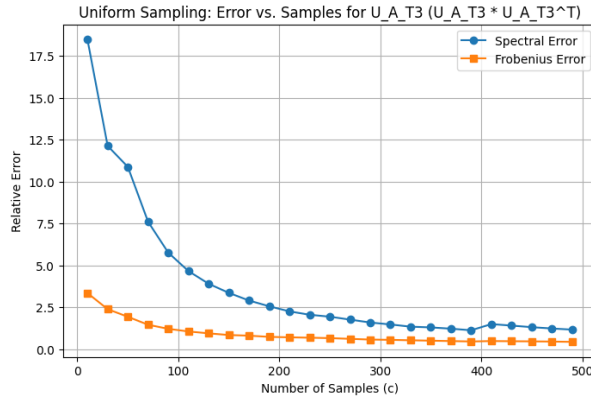


Figure 8: Error of Uniform Based Sampling Approximation for $(U_{T3}^T U_{T3})$

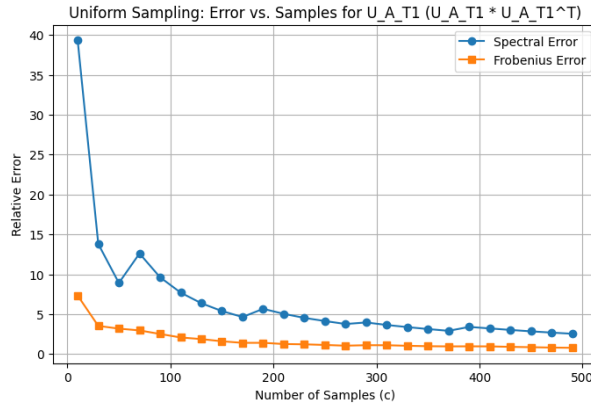


Figure 9: Error of Uniform Based Sampling Approximation for $(U_{T1}^T U_{T1})$

Problem 11

We will now plot the Frobenius and spectral error for the approximations of $A^\top A$ using leverage score sampling.

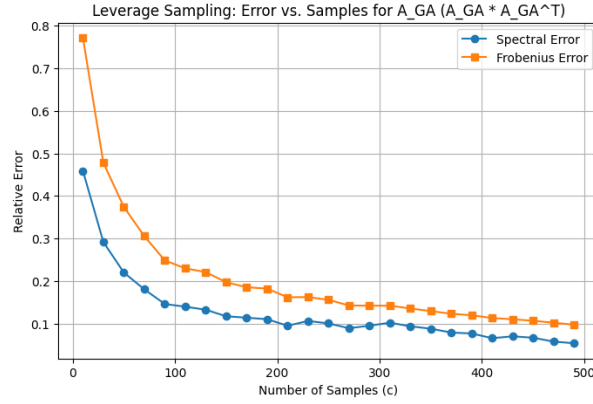


Figure 10: Error of Leverage Based Sampling Approximation for $(A_{GA}^\top A_{GA})$

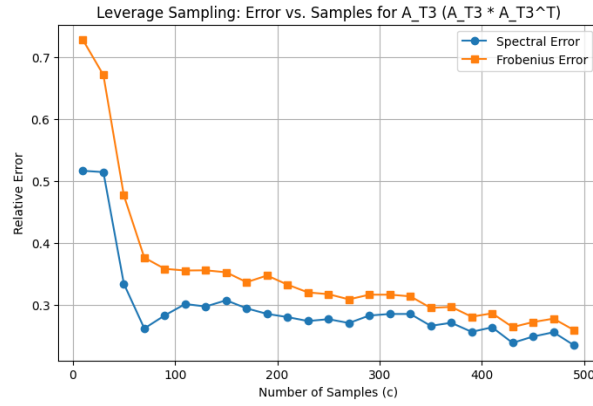


Figure 11: Error of Leverage Based Sampling Approximation for $(A_{T3}^\top A_{T3})$

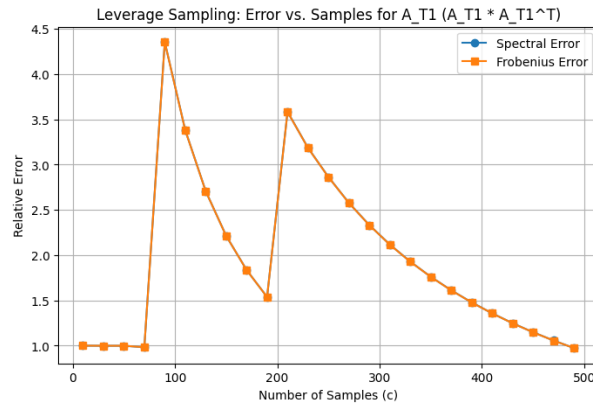


Figure 12: Error of Leverage Based Sampling Approximation for $(A_{T1}^\top A_{T1})$

The results look similar for A_{GA} and A_{T3} , but the error for A_{T1} is significantly higher than the other two matrices. This means that the leverage score sampling is not as effective for A_{T1} as it is for the other two matrices.

Problem 12

We will now plot the Frobenius and spectral error for the approximations of AA^\top using gaussian projection and $\{\pm 1\}$ projection.

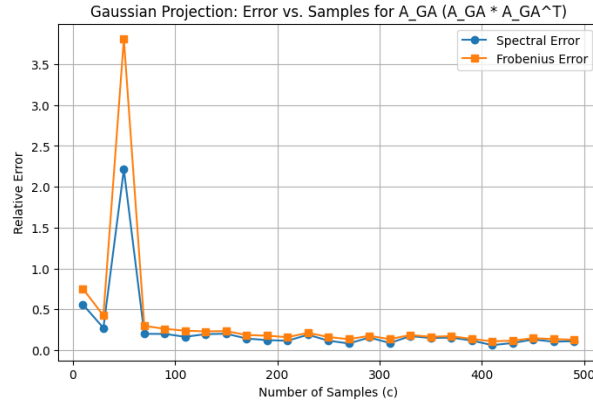


Figure 13: Error of Gaussian Projection Approximation for $(A_{GA}A_{GA}^\top)$

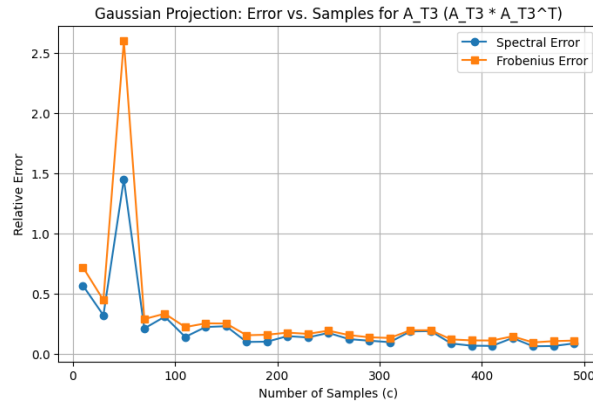


Figure 14: Error of Gaussian Projection Approximation for $(A_{T3}A_{T3}^\top)$

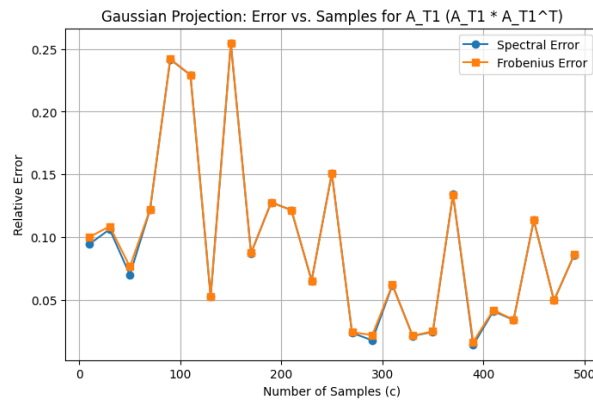


Figure 15: Error of Gaussian Projection Approximation for $(A_{T1}A_{T1}^\top)$

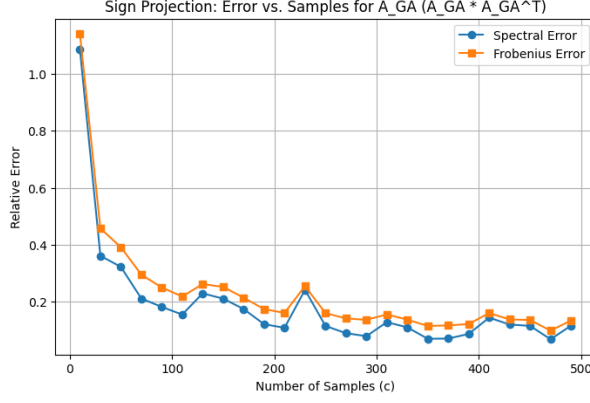


Figure 16: Error of $\{\pm 1\}$ Projection Approximation for $(A_{GA}A_{GA}^\top)$

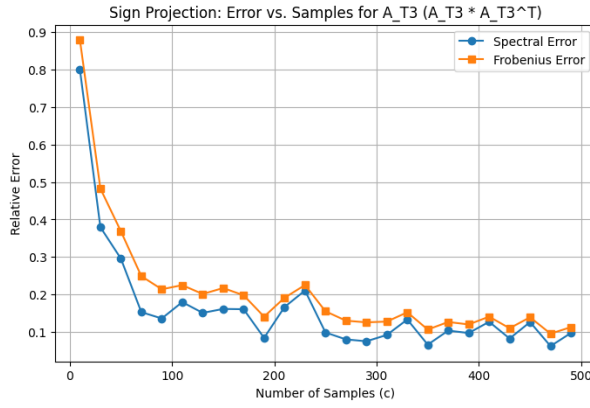


Figure 17: Error of $\{\pm 1\}$ Projection Approximation for $(A_{T3}A_{T3}^\top)$

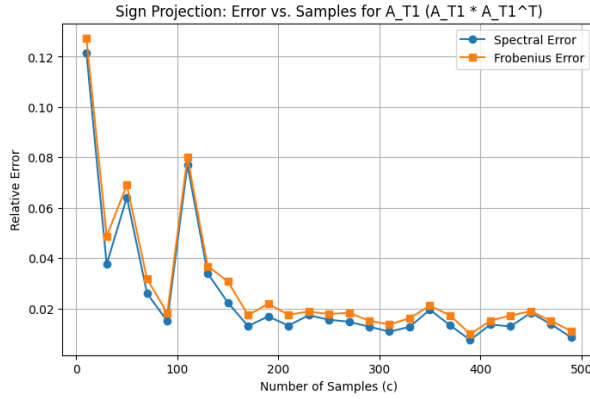


Figure 18: Error of $\{\pm 1\}$ Projection Approximation for $(A_{T1}A_{T1}^\top)$

Comparing the results of the Gaussian projection and $\{\pm 1\}$ projection: The gaussian projection produces more stable results and has slightly smaller error for projecting A_{GA} and A_{T3} when there is reasonable amount of dimensions. The gaussian projection performs extremely well for A_{T1} , while the $\{\pm 1\}$ projection has a higher error for all three matrices. The only extreme case is that sign projection seems to outperform gaussian projection for A_{T1} when the number of dimensions is small.

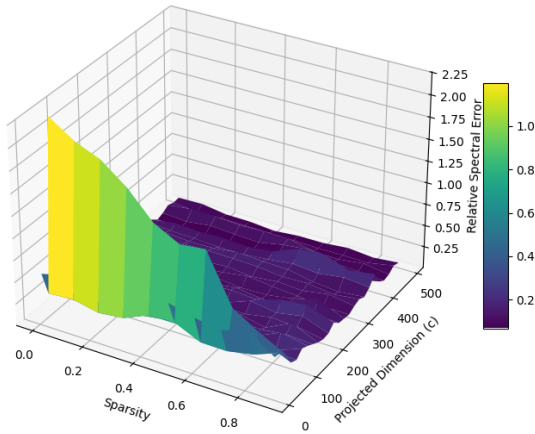
Comparing the results of projection and sampling: The projection method has much lower error compare to uniform sampling and overall slight lower error when comparing to norm based sampling and leverage score sampling. The projection method is more stable and has lower error for all three

matrices. However, there are times where the projection method shows outlier errors, such as the gaussian projection for A_{GA} when the number of dimensions is small.

Problem 13

We will now plot 3D plot of the error of the sparse approximations of AA^\top using gaussian projection and $\{\pm 1\}$ projection. The two axes are the sparsity and the number of dimensions, and the third axis is the error.

Q_13_A_GA (gaussian): Spectral Error vs. c and Sparsity



Q_13_A_GA (gaussian): Frobenius Error vs. c and Sparsity

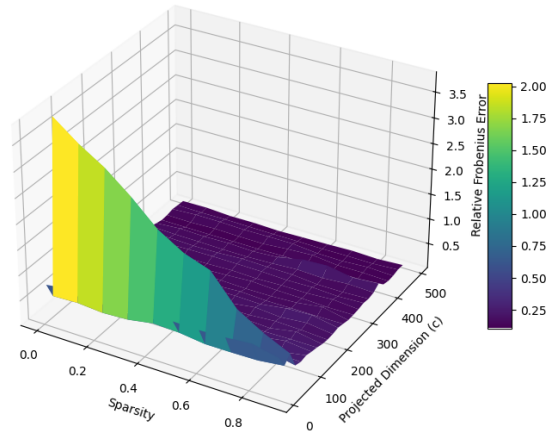
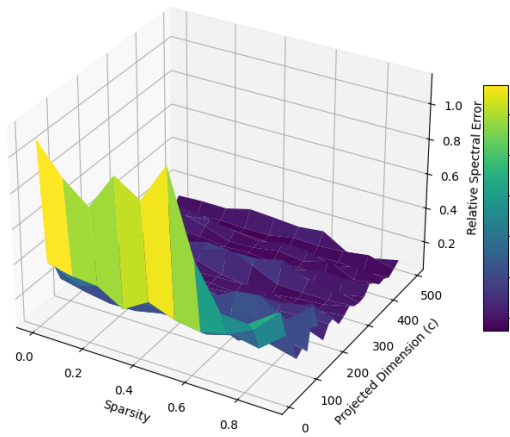


Figure 19: Error of Gaussian Projection Approximation for $(A_{GA}A_{GA}^\top)$

Q_13_A_GA (sign): Spectral Error vs. c and Sparsity



Q_13_A_GA (sign): Frobenius Error vs. c and Sparsity

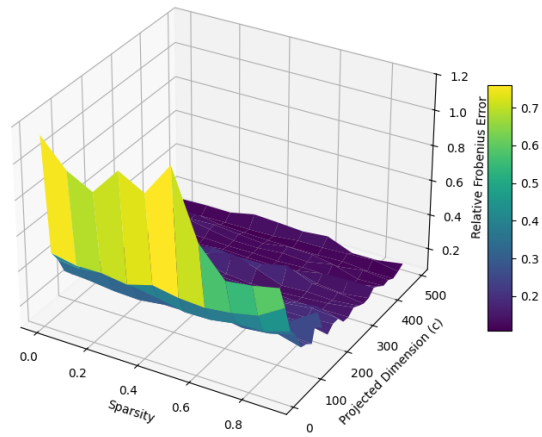
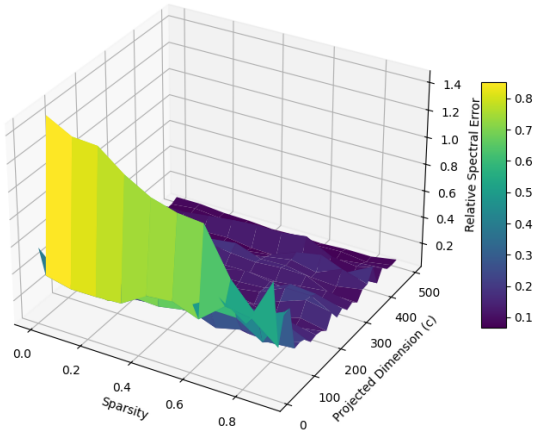


Figure 20: Error of $\{\pm 1\}$ Projection Approximation for $(A_{GA}A_{GA}^\top)$

Q_13_A_T3 (gaussian): Spectral Error vs. c and Sparsity



Q_13_A_T3 (gaussian): Frobenius Error vs. c and Sparsity

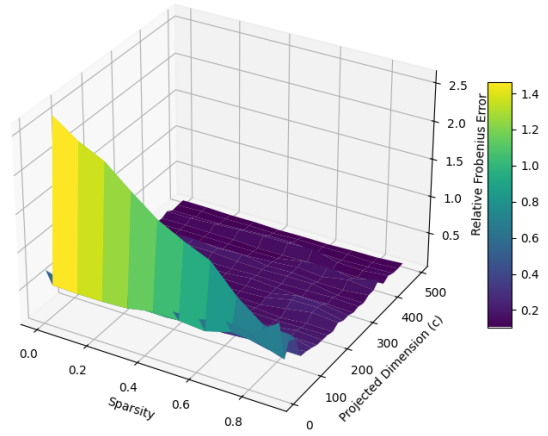
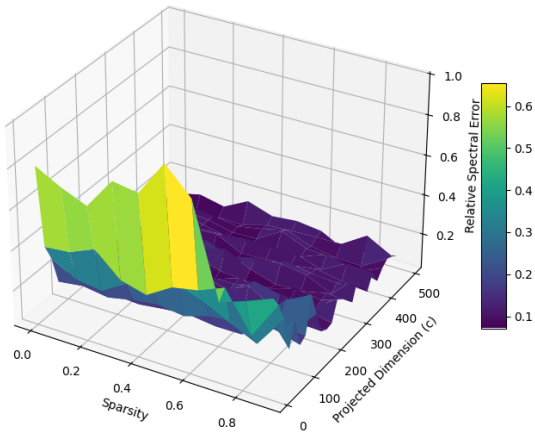


Figure 21: Error of Gaussian Projection Approximation for $(A_{T3}A_{T3}^T)$

Q_13_A_T3 (sign): Spectral Error vs. c and Sparsity



Q_13_A_T3 (sign): Frobenius Error vs. c and Sparsity

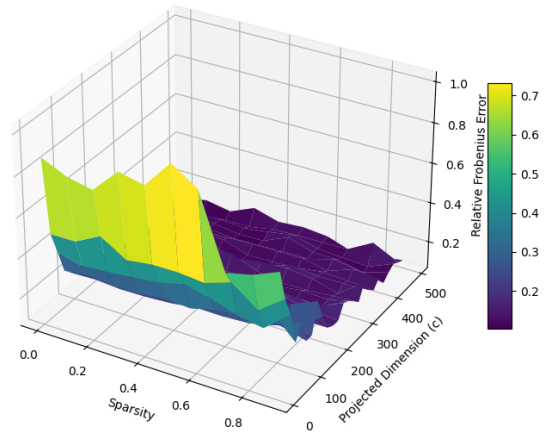
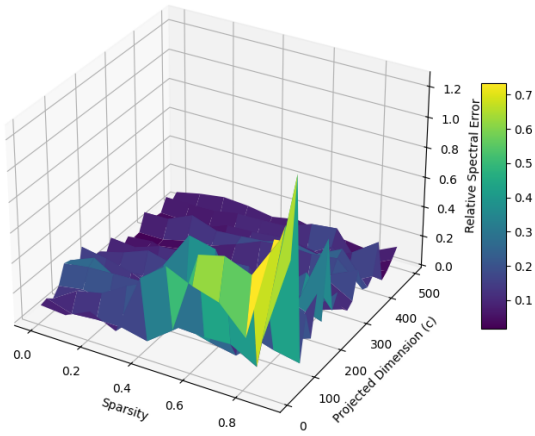


Figure 22: Error of $\{\pm 1\}$ Projection Approximation for $(A_{T3}A_{T3}^T)$

Q_13_A_T1 (gaussian): Spectral Error vs. c and Sparsity



Q_13_A_T1 (gaussian): Frobenius Error vs. c and Sparsity

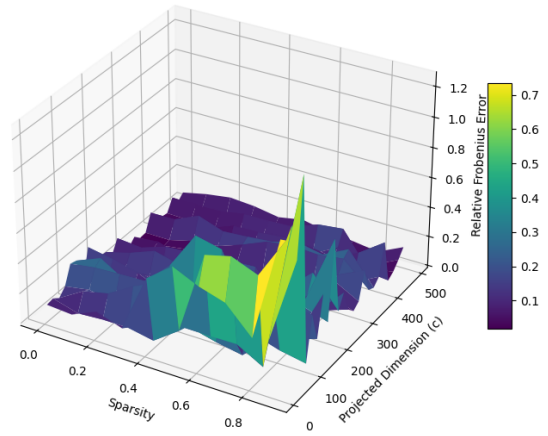
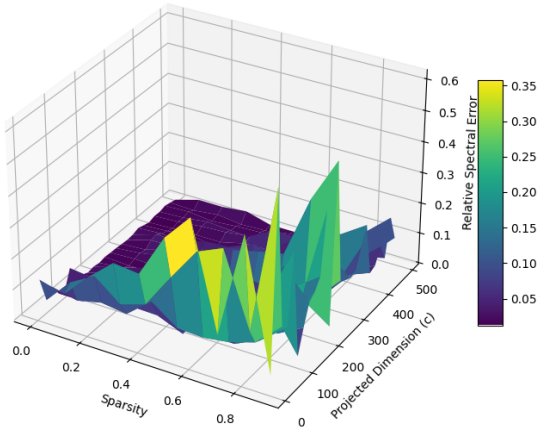


Figure 23: Error of Gaussian Projection Approximation for $(A_{T1}A_{T1}^T)$

Q_13_A_T1 (sign): Spectral Error vs. c and Sparsity



Q_13_A_T1 (sign): Frobenius Error vs. c and Sparsity

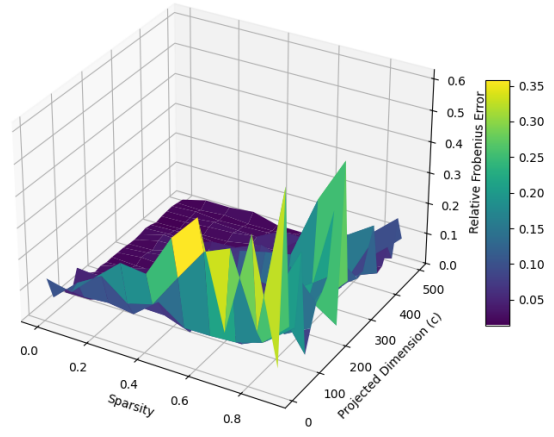


Figure 24: Error of $\{\pm 1\}$ Projection Approximation for $(A_{T1}A_{T1}^\top)$