Homework 2

Your Name

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We will now start reflecting on the coding questions of Homework 2. The code base that can be used to replicate the result can be found in the following link: `https://github.com/TagoreZhao/STAT260/tree/main/HW2`

1 Problem 6, 7, and 8

The code needed for generating these three matrices is given below:

```python
import numpy as np

def generate_covariance_matrix(d):
    indices = np.arange(d)
    Sigma = 2 * 0.5 ** np.abs(indices[:, None] - indices[None, :])
    return Sigma

def generate_gaussian_A(n, d, seed=1234):
    rng = np.random.default_rng(seed)
    Sigma = generate_covariance_matrix(d)
    mean = np.ones(d)
    A = rng.multivariate_normal(mean, Sigma, size=n)
    return A

def generate_t_distribution_A(n, d, df, seed=1234):
    rng = np.random.default_rng(seed)
    Sigma = generate_covariance_matrix(d)
    mean = np.ones(d)
    z = rng.multivariate_normal(mean, Sigma, size=n)
    chi2_samples = rng.chisquare(df, size=(n, 1))
    A = z / np.sqrt(chi2_samples / df)
    return A
```

Listing 1: Python code for generating matrices

Since numpy does not provide built in functions for generating t-distributed random variables, we have to generate the random variables ourselves. The Gaussian random variables are generated using the `multivariate_normal` function, while the t-distributed random variables are generated using the formula $A = Z/\sqrt{\chi^2/df}$, where Z is the Gaussian random variable, χ^2 is the chi-squared random variable, and df is the degrees of freedom.

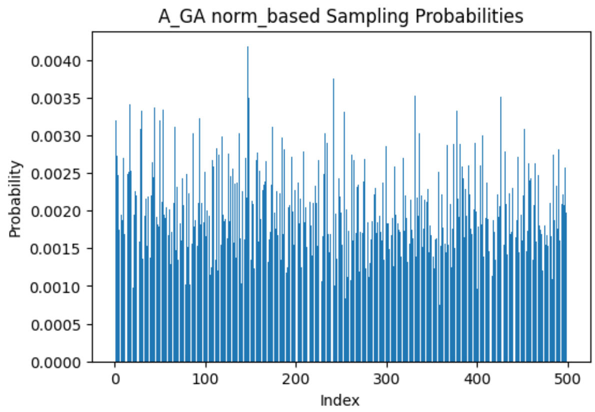

Figure 1: GA Norm based probability distribution

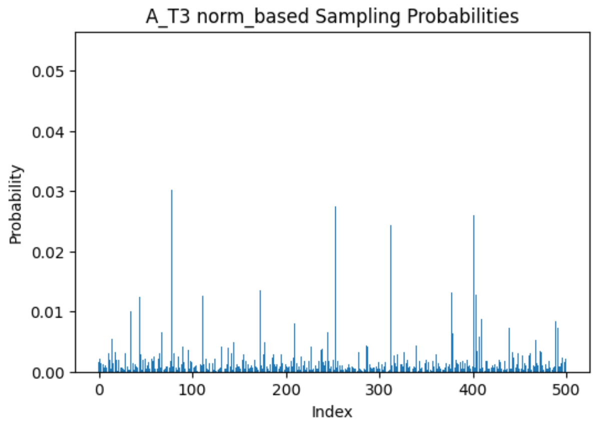

Figure 2: T3 Norm based probability distribution

Problem 9

We will first plot the norm based probability distribution for all three matrices that we generated using seed 1234.

We will now plot the frobenius and spectral error for all appoximations of the three matrices multiplication.

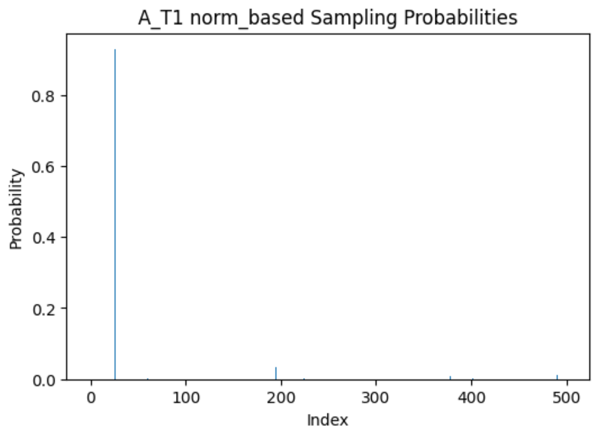

Figure 3: T1 Norm based probability distribution

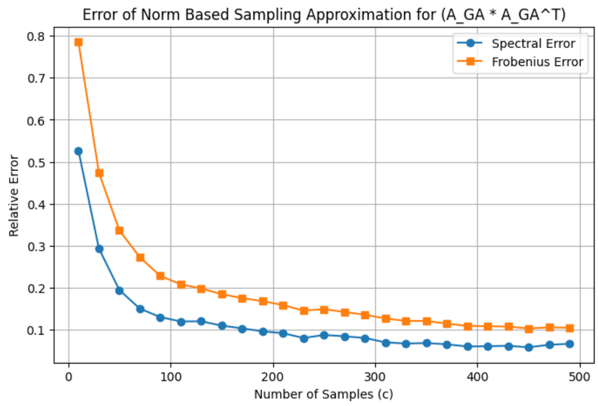

Figure 4: Q9 Error of Norm Based Sampling Approximation for (A_GAˆT * A_GA)

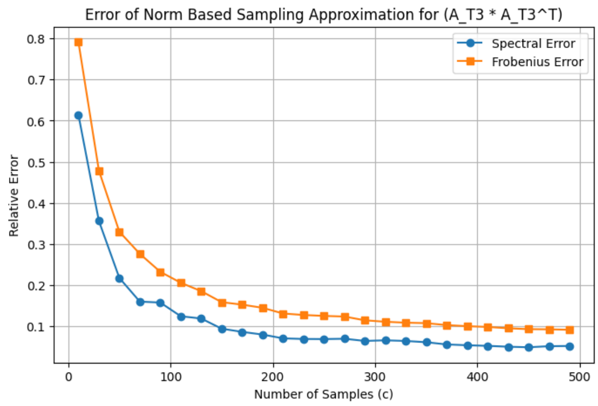

Figure 5: Q9 Error of Norm Based Sampling Approximation for (A_T3ˆT * A_T3)

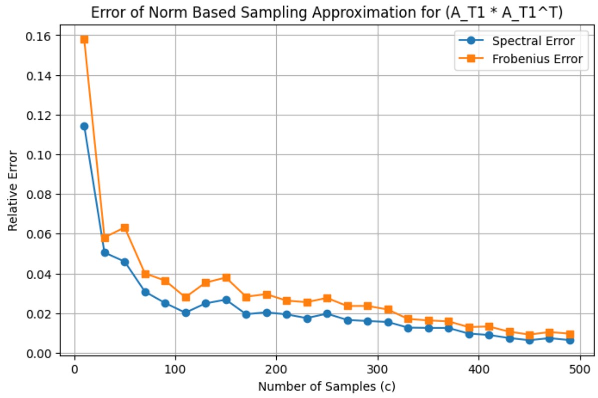

Figure 6: Q9 Error of Norm Based Sampling Approximation for (A_T1ˆT * A_T1)

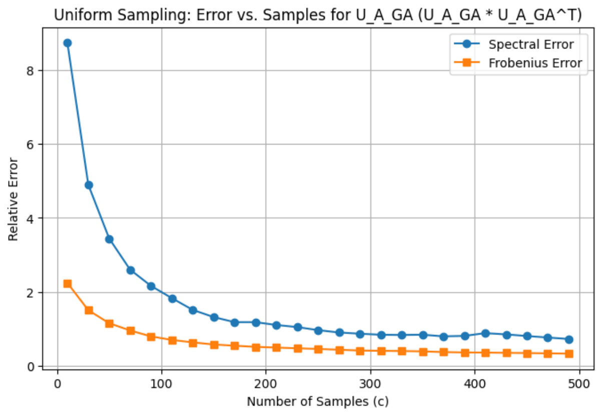

Figure 7: Q10 Error of Norm Based Sampling Approximation for (U_GAˆT * U_GA)

Problem 10

We will now plot the frobenius and spectral error for all appoximations of the three left singular matrices multiplication.

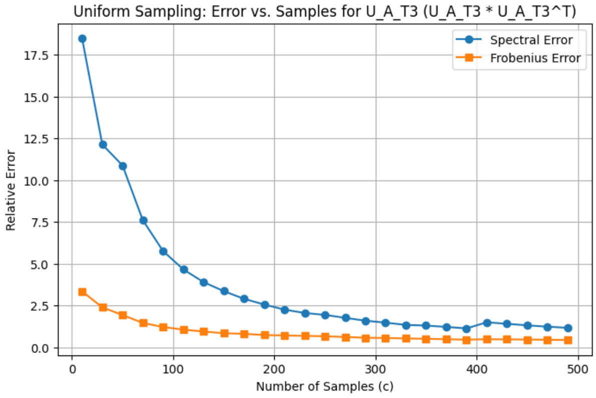

Figure 8: Q10 Error of Norm Based Sampling Approximation for (U_T3^T * U_T3)

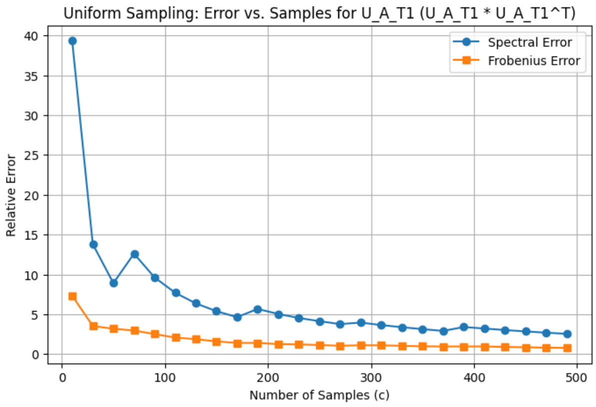

Figure 9: Q10 Error of Norm Based Sampling Approximation for (U_T1^T * U_T1)

Problem 11

We will now plot the frobenius and spectral error for all appoximations of $A^\top A$ using leverage score sampling.