

Homework 2

Your Name

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We will now start reflecting on the coding questions of Homework 2. The code base that can be used to replicate the result can be found in the following link: <https://github.com/TagoreZhao/STAT260/tree/main/HW2>

1 Problem 6, 7, and 8

The code needed for generating these three matrices is given below:

```
1 import numpy as np
2
3 def generate_covariance_matrix(d):
4     indices = np.arange(d)
5     Sigma = 2 * 0.5 ** np.abs(indices[:, None] - indices[None, :])
6     return Sigma
7
8 def generate_gaussian_A(n, d, seed=1234):
9     rng = np.random.default_rng(seed)
10    Sigma = generate_covariance_matrix(d)
11    mean = np.ones(d)
12    A = rng.multivariate_normal(mean, Sigma, size=n)
13    return A
14
15 def generate_t_distribution_A(n, d, df, seed=1234):
16     rng = np.random.default_rng(seed)
17     Sigma = generate_covariance_matrix(d)
18     mean = np.ones(d)
19     z = rng.multivariate_normal(mean, Sigma, size=n)
20     chi2_samples = rng.chisquare(df, size=(n, 1))
21     A = z / np.sqrt(chi2_samples / df)
22     return A
```

Listing 1: Python code for generating matrices

Since numpy does not provide built in functions for generating t-distributed random variables, we have to generate the random variables ourselves. The Gaussian random variables are generated using the `multivariate_normal` function, while the t-distributed random variables are generated using the formula $A = Z / \sqrt{\chi^2 / df}$, where Z is the Gaussian random variable, χ^2 is the chi-squared random variable, and df is the degrees of freedom.

Problem 9

We will first plot the norm based probability distribution for all three matrices that we generated using seed 1234.

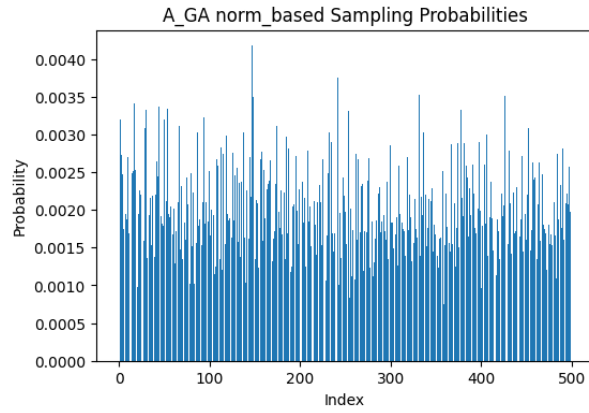


Figure 1: GA Norm based probability distribution

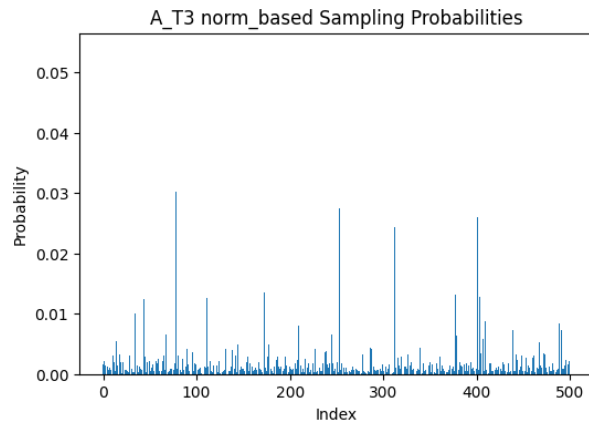


Figure 2: T3 Norm based probability distribution

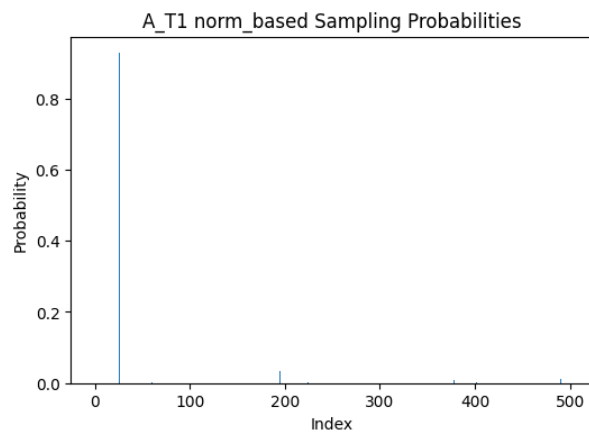


Figure 3: T1 Norm based probability distribution

We will now plot the Frobenius and spectral error for the approximations of the three matrix multiplications.

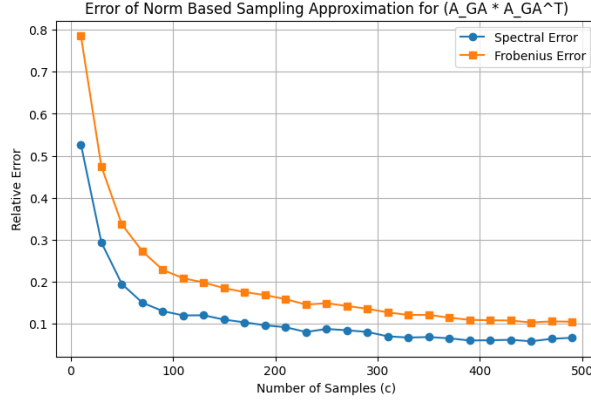


Figure 4: Error of Norm Based Sampling Approximation for $(A_{GA}^T A_{GA})$

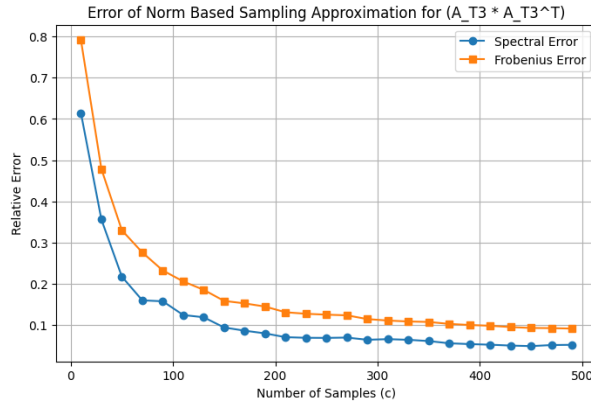


Figure 5: Error of Norm Based Sampling Approximation for $(A_{T3}^T A_{T3})$

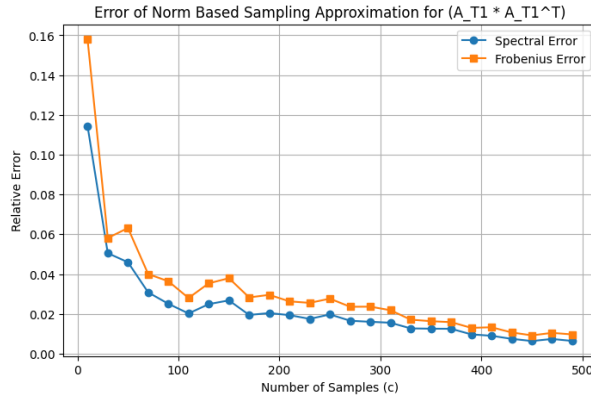


Figure 6: Error of Norm Based Sampling Approximation for $(A_{T1}^T A_{T1})$

Problem 10

We will now plot the Frobenius and spectral error for the approximations of the three left singular matrices multiplication.

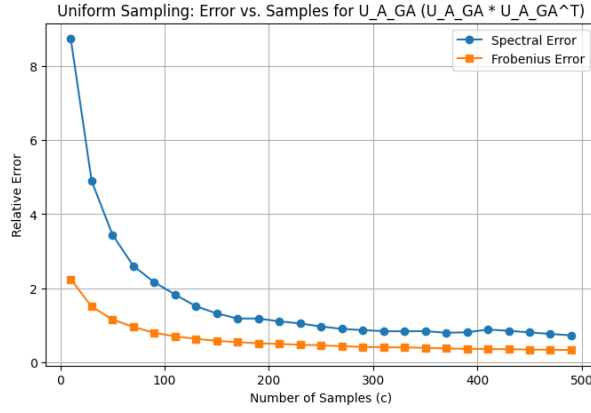


Figure 7: Error of Uniform Based Sampling Approximation for $(U_{GA}^T U_{GA})$

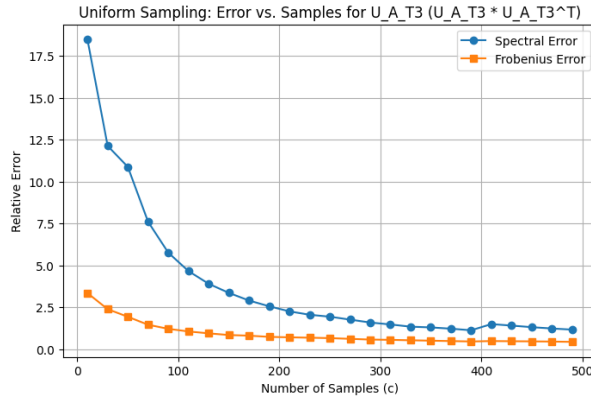


Figure 8: Error of Uniform Based Sampling Approximation for $(U_{T3}^T U_{T3})$

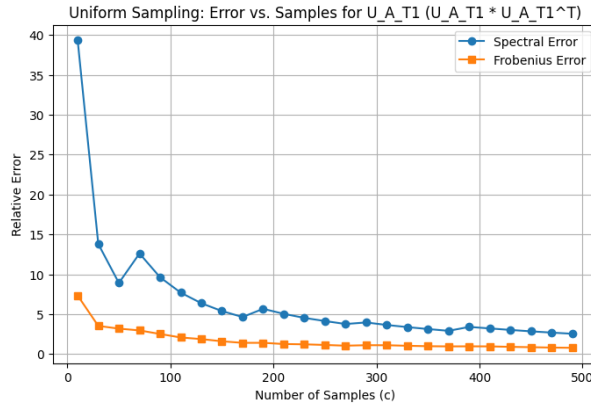


Figure 9: Error of Uniform Based Sampling Approximation for $(U_{T1}^T U_{T1})$

Problem 11

We will now plot the Frobenius and spectral error for the approximations of $A^\top A$ using leverage score sampling.

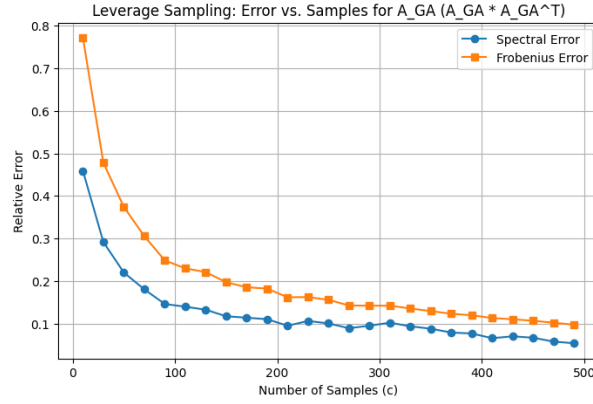


Figure 10: Error of Leverage Based Sampling Approximation for $(A_{GA}^\top A_{GA})$

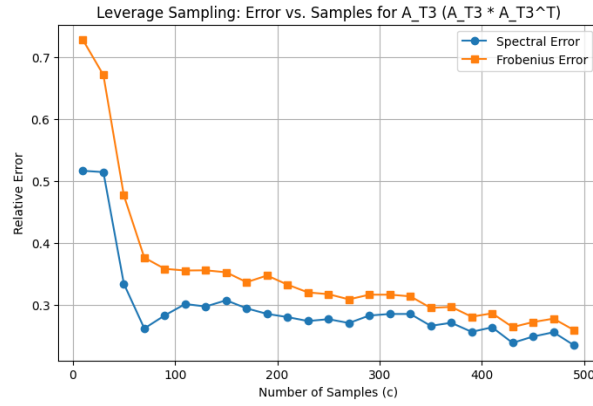


Figure 11: Error of Leverage Based Sampling Approximation for $(A_{T3}^\top A_{T3})$

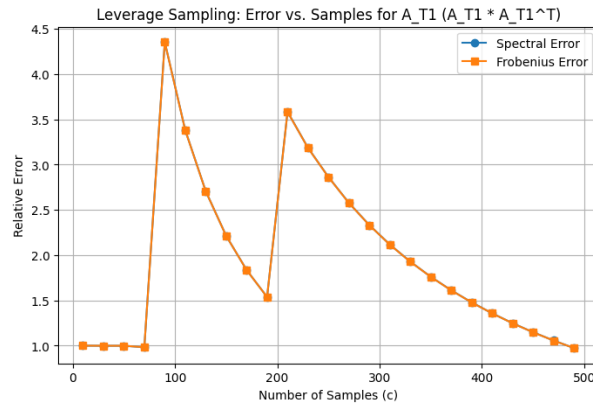


Figure 12: Error of Leverage Based Sampling Approximation for $(A_{T1}^\top A_{T1})$

The results look similar for A_{GA} and A_{T3} , but the error for A_{T1} is significantly higher than the other two matrices. This means that the leverage score sampling is not as effective for A_{T1} as it is for the other two matrices.

Problem 12