A Critical Review of ”An Algebraic Process for Visualization Design”

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***Abstract*—This is a review on this paper, which introduces a model for visualization design grounded in algebraic principles, aimed at enhancing the understanding, evaluation, and creation of visual encodings. The model comprises three key components: the mathematical structure of the data, its computational representation, and a mathematical approximation of human perception of the visualization. The practical utility of this model is demonstrated through three general principles for effective visualization design. The authors apply the model to analyze existing visualization methods, identifying their strengths and weaknesses, while also proposing new techniques. Additionally, the model is positioned as a framework for interpreting experimental user studies, offering actionable insights. This approach provides a foundation for future research in visualization.**

***Index Terms*—Visualization Design, Symmetries, Visualization Theory**

I. Introduction

As Visualization and the tools for it become incredibly vital for business, science and medicine, the research in visualization has made significant strides in improving or building algorithms, used by the tools for visualization of monumentally large datasets, revealing patterns that can be represented using them. However, these advances have also highlighted the necessity for more robust theoretical foundations that can guide the design and evaluation of visualizations.

Most research in visualization creates valuable frameworks through the development of taxonomies, categorizing data, tasks, and methods in a structured manner. These taxonomies provide essential insights into the scope and landscape of the field. However, theoretical research goes further by revealing the underlying mechanisms of how visualizations function. This type of research builds a deeper, more rigorous foundation, drawing on principles from psychology, vision, and cognition to understand the fundamental properties of visualization. While taxonomies are invaluable for classification, theoretical approaches offer a more profound understanding of the design and efficacy of visualizations, particularly when faced with new data types and complex tasks.

The authors introduce an algebraic framework for visualizations that goes beyond traditional guidelines such as those proposed by Tufte et al [2], remain valuable. The authors of the paper feel these guidelines summarizes the properties good visualizations have in common. This paper seeks to provide a formalized process with a well defined theoretical basis to describe visualizations, and act as a practical guide to designing data visualizations, in the form of three design principles, generalized in mathematical terms. The paper says there are three fundamental elements while creating visualizations or viewing them. In each of these aspects, mathematical structures inform both the computational representation and the visual design of data, ensuring the visualization is meaningful and coherent for its intended purpose. They are:

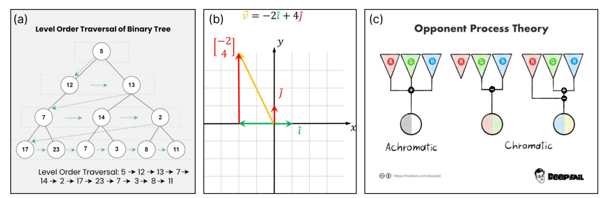

Fig. 1. Examples of elements in the visual design of data (a) Ordering of a tree data structure. Image courtesy: GeeksForGeeks (b) A vector in two-dimensional space that has undergone a linear transformation. Image courtesy: datahacker (c) A visual example of Opponent Process Theory. Image courtesy: DeepFail on Medium

Mathematical Structure in the Underlying Data: Data can be categorized based on its inherent mathematical properties, which can vary depending on the type of data and its organizational structure. This abstract characterization is critical for visualization design. For example, hierarchical structures, like trees, have an inherent partial ordering, as seen in child-parent relationships. Additionally, operations such as order-preserving mappings between these trees maintain their hierarchical integrity (Figure 1a). Similarly, data represented as vectors in Euclidean space includes not only magnitudes and directions but also properties of linear transformations acting upon these vectors, which are essential when interpreting relationships in visualizations (Figure 1b). The specific mathematical structures that are important will often depend on the objectives and context of the visualization being constructed.

Concrete Representation of Data in a Computer: In practice, data is represented computationally in ways that may introduce artifacts not present in the raw data itself. For example, a set of data points has no inherent order, but if stored as a list or table, an arbitrary ordering is imposed. Furthermore, for representations such as eigenvectors, the computer must choose between positive or negative orientations arbitrarily (Figure 1c). Another important consideration is that sometimes the data to be visualized is not directly accessible; instead, it is inferred from samples. For example, in statistical visualization, one works with samples of a population rather than the population itself, or with samples on a discrete grid when visualizing an underlying continuous field.

Mathematical Structure in the Perception of Visualizations: The human visual system interprets visualizations using mathematical principles, as demonstrated by psychological models of perception. Opponent color theory, for instance, defines a relationship between colors such that combining certain pairs results in a neutral gray. This theory mathematically formalizes the concept of negation in perceptual color space, providing critical insights for designing color encodings that are easily distinguishable to the human eye. The process of visual perception, guided by such mathematical models, helps inform design choices that optimize how a visualization is processed by viewers.

The rest of the rules/guidelines for creating effective visualizations provided by the paper builds upon these mathematical foundations. The authors boil down their analysis of a good visualization to the adherence to three principals, and offer names for elements of visualizations that violate them.

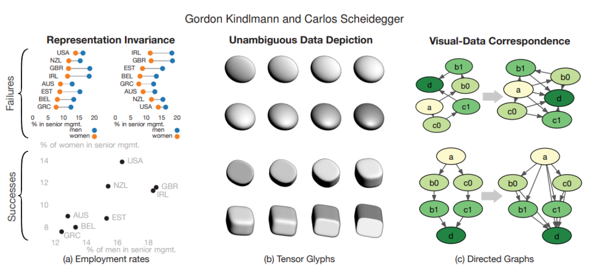

Fig. 2. Examples of visualizations violating the authors' three proposed principles. (a) The Principle of Representation Invariance shown using visualizations of Employment rates by gender by country. (b) The Principle of Unambiguous Data Depiction shown using representation of tensors with ellipsoid glyphs and superquadric glyphs (c) The Principle of Visual-Data Correspondence shown using a directed graph and node rank. Image courtesy: Screenshot from [1]

Principle of Representation Invariance: Visualizations should be invariant with respect to the choice of data representation; changing the representation should not change the visualization. A visualization that fails this principle has a hallucinator, creating a different impression from a different representation of the same data.

Principle of Unambiguous Data Depiction: Visualizations should be unambiguous; changing the underlying data should produce a change in the resulting visualization. A visualization that fails this principle has confusers, resulting in changes in the data that are effectively invisible to the viewer.

Principle of Visual-Data Correspondence: Significant changes in the data should meaningfully correspond with noticeable changes in the visual impression and vice versa. If an important change in data is not clearly manifested in the visualization, it has jumbled the data. Conversely, if a clear transformation of the visualization corresponds with an unimportant change in the data, the visualization is misleading.

II. Related Work

A. Previous Work in the Domain

Bertin et al.'s seminal work [3] on enumerating data types and retinal variables was laid as the foundation for visualization design, which was further refined by Mackinlay et al.'s APT system [4] into a set of expressiveness and effectiveness criteria, ultimately driving the automatic visualization generation found in modern tools like Tableau.

A mathematical basis for understanding data types through group symmetry was provided by Stevens et al. [5], categorizing categorical, ordinal, interval, and ratio data according to their respective symmetry groups. This approach was directly inspired by the paper's concept of "data symmetry" and "visualization symmetry," echoing the focus of Stevens et al. on identifying invariant properties for statistical measures such as medians for ordinal data.

A framework for understanding how visualizations are interpreted at a basic level was provided by Cleveland et al. [6], while Gestalt principles were applied by Wattenberg et al. [7] to relate the scale-space structure of visualization stimuli to the underlying data. This work is seen as a psychology-based precedent to the Correspondence Principle developed in the paper.

The notion of affordances was proposed by Gibson et al. [8], further generalized to visualization interfaces by Ware et al. [9], focusing on how potential actions within a visualization environment are perceived by users. This concept influenced the algebraic approach to affordances and their alignment with important low-level tasks, as represented in the paper's Correspondence Principle.

The work on injectivity, surjectivity, and bijectivity in visualizations was contributed to by Ziemkiewicz et al. [10], which contributed to the development of the Unambiguity Principle in this paper. Insight was provided by their focus on the syntactic "reading" of visualizations, but a broader understanding of how visualizations are perceived, particularly in the context of ordering and perceptual differences, is aimed for by the paper.

Matching distance functions between data space and perceptual space were proposed by Demiralp et al. [11] to evaluate visualizations. While alignment with the Correspondence Principle is found in their approach, limits are acknowledged by the paper, particularly in representing non-symmetric relationships like partial orders, and a more general mechanism for preserving structures in visualizations beyond metric spaces is sought.

The expressiveness and effectiveness criteria were used by Mackinlay et al. [12] to evaluate whether visualizations accurately represent all properties of the data without introducing misleading or accidental information. The Unambiguity and Correspondence principles introduced in the paper are built on these ideas to ensure that visualizations align with the structure and perception of the data.

The Congruence Principle was suggested by Tversky et al. [13], which states that visual structures should correspond to desired mental representations, and it is reinterpreted in the paper as the Correspondence Principle, emphasizing the mathematical relationship between data and visualizations. The Apprehension Principle, also proposed by Tversky et al. [14], relates to the Unambiguity and Correspondence principles by ensuring that visualizations are readily comprehensible.

A framework for understanding the relationship between data abstractions and encoding techniques was provided by Munzner et al. [15] and thier nested model for visualization. Mathematical guidelines for ensuring that visualizations accurately reflect the underlying data structure are proposed in the paper, predicting which visualizations will be most effective for specific tasks.

The extended nested model was similarly extended by Meyer et al. [16] through the paper's theory, offering mathematical guidelines for the abstraction layer based on the structure of the data. Predictions are made by the theory regarding which visual encodings will be successful, particularly when they preserve the metric structure of the dataset, aiding in tasks like similarity assessments.

Gaps Addressed by the Method

The algebraic model proposed in this paper contributes to the field of visualization by offering a systematic method for assessing the effectiveness of visualizations, both in terms of data accuracy and user perception. Previous research in visualization has often focused on descriptive taxonomies and heuristic-based guidance, such as the design rules provided by Tufte et al. [2], which are helpful but lack the formal rigor needed to handle complex datasets and modern visualization challenges. The model draws inspiration from fields such as algebra, cognitive psychology, and perceptual theory to bridge the gap between data structure and human perception. The algebraic framework presented in this paper fills this gap by providing a mathematical basis for understanding how data should be represented visually and how changes in data should translate into changes in the visualization.

1) **Formalization of Visualization Symmetries**: While previous work in visualization design has focused on perceptual and cognitive aspects, this method provides a *formal, mathematical structure* for analyzing visualizations. By introducing symmetries (α and ω) and the commutative diagram, the model gives designers a clear and rigorous way to reason about the relationship between data transformations and visual outputs.
2) **Error Identification**: The introduction of concepts like *hallucinators*, *confusers*, *jumblers*, and *misleaders* allows for a systematic classification of the types of errors or inconsistencies that can arise in visualizations. This goes beyond prior work that may have identified specific pitfalls in individual cases without providing a comprehensive framework for diagnosing these issues.
3) **Task-Dependent Visualization Design**: The *Visual-Data Correspondence Principle* is particularly novel in emphasizing the task-specific nature of visualization design. By modeling particular data properties (symmetries) relevant to a task and finding corresponding visual transformations, this method allows designers to tailor visualizations to specific user needs in a way that previous models, which focused more on general rules, do not.

III. Methodology

A. Basic Elements of the Model

In this paper, the authors propose an algebraic model for visualizations that unites three core spaces: the space of data (*D*), the space of representations (*R*), and the space of visualizations (*V*). The purpose of this algebraic framework is to model the process of generating visualizations from data and to define design principles that ensure clarity, consistency, and meaningful correspondence between data and the visual output.

Data Space (*D*): This is the space in which the raw data resides. It could be numbers, text, or any other form of data.

Representation Space (*R*): This is an intermediate space where the raw data is transformed into some form of representation that the computer can process, such as tables or graphs. It acts as a bridge between the data and the visual output.

Visualization Space (*V*): This is the space of visual stimuli or the final visualization that the viewer perceives.

B. Relations between the Elements

In this model, visualizations do not act directly on the data (*D*) but rather on the representation of the data in the computer (*R*). The core idea is that the relationship between the data and its visual representation can be expressed algebraically, making it possible to analyze whether the visualization preserves important data properties.

Data Mapping (r): This is a function that maps data (D) to its representation (R). Specifically, $r_1 : D \to R$ is the initial mapping from data to representation, and $r_2 : D \to R$ represents transformations or alternative representations of the same data.

Visualization Mapping (v): This is a function that maps a representation (R) to its visualization (V), i.e., $v : R \to V$.

C. Symmetries

The framework introduces two types of **symmetries**, which are crucial to understanding how data and visualization transformations correspond to each other:

Data Symmetries (α): These are transformations within the data space (D) that represent certain properties or tasks applied to the data. For example, a data symmetry might involve scaling or rotating a dataset, depending on the domain.

Visualization Symmetries (ω): These are transformations within the visualization space (V) that modify the visual representation. For example, a visualization symmetry could change the color of a chart or move a point on a graph.

Both symmetries are connected via the core equation, expressed as follows:

$$v \circ r_2 \circ \alpha = \omega \circ v \circ r_1$$

This equation, shown through a **commutative diagram**, describes the relationship between transformations in the data and how they are reflected in the visualization. The **commutative diagram** ensures that any two different paths from the upper-left node (D) to the lower-right node (V) yield the same result.

In this equation:

On the **left-hand side**, the data symmetry α is applied to the data, followed by the mapping to representation r_2, and finally mapped to the visualization via v.

On the **right-hand side**, the same data is first mapped to a representation r_1, visualized through v, and then transformed by a visualization symmetry ω.

For a successful visualization, this equation must "commute", meaning both sides of the equation must be equal. Designers are tasked with finding appropriate pairs of symmetries (α, ω) that ensure this commutation. The model is particularly useful in ensuring that visual transformations correspond meaningfully to data transformations.

IV. Impact On The Research Community

Dragicevic et al. [17] also handle traditional statistical practice, like p-values and binary testing, challenges in the field of Human-Computer Interaction (HCI). It is argued that all these statistical tools, though widely used, are largely ineffective in promoting transparent and communicative reporting of research results. Such methods could potentially lead to misinterpretations and over-reliance on broad notions. In view of this, an approach toward more informative methods, particularly estimation techniques, is in order. Such methods generally emphasize the report of effect sizes & interval estimates, and instead of significance testing, point toward nuances in interpretations. Guidance for representing empirical results in a clear and accurate way is also offered in the paper, motivating HCI researchers to use more visual and subtle ways of statistical presentation that better communicate the robustness of findings.

Quotes are further made towards the argument to enhance it with visual robustness through Kindlmann et al. [1], presenting the relation introduced within our paper on visual data correspondence. Specifically, the conversation on clear data presentation (or unambiguity) is based on it and is referred to as the heart of ensuring that visualizations accurately represent variability in data. Underpinning the reality of creating visualization designs that not only clearly present statistical findings but also avoid ambiguity about how they might present data, the notion of unambiguity draws its legitimacy through our work. Actually, this is much better at result communication and aligns it with the overall goal of the paper-determining ways that statistical practices in HCI may be improved.

Ge et al. [18] investigate a systematic approach to the determination of people's ability to identify and understand misleading visualizations. Growing misuse in visualization is highlighted, and the authors argue that the assessment has to move beyond the classical tests which focus on proper visualizations and only them. A test, developed based on carefully developed definitions of "misleaders"—visualization decisions leading to conclusions not supported by the data—is proposed. The steps in constructing and refining test items using a design space of misleading chart types are presented, and tested on 497 participants. Through analysis, including Item Response Theory and qualitative feedback, the final bank of test items is tuned to assess how well people can detect errors in flawed visualizations.

Ge et al. [18] integrate the hallucinators and confusers from our paper to enhance the framework for understanding visualization misleaders. An example of such an error is the hallucinators-an instance in which various renderings of the same data result in varying incorrect impressions-and the confusers-an instance in which the visualizations fail to reflect changes in the data. These definitions therefore helped refine test items through clear criteria for determining how users interpret visual errors and thus align with the intent of measuring critical thinking in the context of faulty visualizations. A theoretical underpinning of this kind allows the author to create structure in the assessment by using grounds to classify the errors and measure participant responses effectively.

Qu et al. [19] discuss consistency in preserving or trading off the process of creating multiple related visualizations, which can be dashboards or small multiples. Challenges of consistent visual encodings across multiple views are focused, arguing that current guidelines to design a visualization typically fail to address considerations of how to keep them for multiple related views. Inconsistent visual encodings using different scales for the same data field across different charts can cause a user to take much more time for interpretation and may impose additional cognitive burdens on a user. Towards this, an exploration is carried out to see how consistency is managed by a user within the process of visualization authoring and how such automated systems can assist better in the task.

Building on the theoretical foundations of Kindlmann et al. [1] they provide principles such as invariance and unambiguity. These principles claim that the impression created by a visualization should be data-led, and any changes within the data should reflect corresponding changes in the visualization. The principles refer to this context to frame the perception that consistency in the multiple visualizations is essential for data integrity and user understanding. In contrast, while the algebraic model hinges on evaluating a single visualization, Qu et al. [19] take these ideas to be extended toward consistency across multiple visualizations. Consistency is presented as something that can be implemented in automated systems whereby these principles are used within the design process in such a way that consistency is treated as a constraint satisfaction problem to be checked and validated at the same time during the creation of multiple views.

The approach Bach et al. [20] suggest is to visualize the patterns in time series data through curves in time: This method folds the timeline in such a way that similar time points are brought close to each other so that users can visually identify patterns such as steady progressions, abrupt changes, or reversion to earlier states. This technique is designed to be flexible to all types of temporal datasets, given that there exists a similarity metric between different time snapshots. Examples that demonstrate how this technique of time curves can be exploited over datasets of all kinds, domains ranging from collaborative document editing to dynamic network analysis, are presented.

They cite Kindlmann et al. [1], on the issue of reproducibility, stability and robustness of time curves. They indicate that "The concepts presented in this paper from the algebraic model, specifically with respect to invariance and ambiguity, are crucial in ensuring that time curves faithfully represent underlying data.". It specifically calls attention to the requirement that time curves be consistent (reproducible) under different runs of the algorithm and still robust to updates or noise in datasets. In line with this, the need is emphasized to base the algebraic process model of requiring time curves to reliably catch the most important temporal patterns without causing distortions that might deceive the users. This theoretical background allows us to estimate how well the curves of time reflect data and which limitations their visualization has.

The proposed methodology given by Kindlmann et al. [1] has significantly influenced the research community by challenging traditional statistical practices in Human-Computer Interaction (HCI) and advocating for more effective methods of data representation and visualization. By critiquing the reliance on p-values and binary testing, it has prompted researchers to adopt estimation techniques that focus on effect sizes and interval estimates, leading to clearer and more nuanced interpretations of data. The emphasis on visual robustness and unambiguity has resonated with subsequent studies, including those by Ge et al. [18], which utilize the concepts of hallucinators and confusers to enhance the understanding of misleading visualizations. The work of Qu et al. [19] builds on these principles to address the consistency of visual encodings across multiple related visualizations, thereby improving user comprehension and reducing cognitive load. Furthermore, Bach et al.'s [20] introduction of time curves as a method for visualizing temporal patterns illustrates the paper's impact on expanding the toolkit available for effective data representation. Overall, this research has catalyzed discussions on improving statistical practices in HCI and has provided a theoretical foundation that supports the development of more transparent and communicative visualization methods.

V. Conclusion

The work by Kindlmann et al. [1] proposes a novel, mathematically grounded approach to visualization design that addresses both the structure of data and human perception. By focusing on invariance, unambiguity, and correspondence, the authors offer a framework that not only aids in designing new visualizations but also helps evaluate and improve existing ones.

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