Assignment 3

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Download all python codes from

```
https://github.com/Taha-Adeel/AI1103/blob/main/
    Assignment_3/codes/assignment3.py
```

and latex-tikz codes from

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https://github.com/Taha-Adeel/AI1103/tree/main/
    Assignment_3
```

1 Problem (GATE 2008 (CS), Q.27)

Aishwarya studies either computer science or mathematics everyday. If she studies computer science on a day, then the probability she studies mathematics the next day is 0.6. If she studies mathematics on a day, then the probability she studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probablity she studies computer science on Wednesday?

(A) 0.24	(C) 0.4
(B) 0.36	(D) 0.6

2 Solution

Consider the state space $S = \{1, 2\}$, where 1 represents her studying CS and 2 represents her studying maths on a particular day.
Let $\{X_0, X_1, \dots\}$ be a series of random variables. So we have the Markov chain

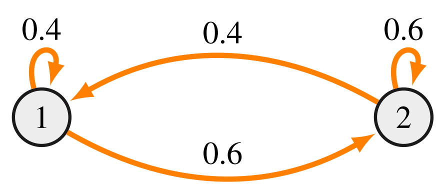
$$\{X_n \,|\, X_n \in S, n \geq 0\}, \tag{2.0.1}$$

with initial distribution $\alpha = (\alpha_1, \alpha_2) = (1, 0)$ ($\because \alpha_i = \Pr(X_0 = i)$).
The state transition matrix $P = (p_{ij})$ for the markov chain (where $p_{ij} = \Pr(X_{n+1} = j \,|\, X_n = i)$) is :-

$$P = \begin{array}{c c} & \overbrace{\begin{array}{cc} 1 & 2 \end{array}}^{X_{n+1}} \\ X_n \left\{ \begin{array}{c} 1 \\ 2 \end{array} \right. & \begin{bmatrix} x & 0.6 \\ 0.4 & y \end{bmatrix} \end{array} \tag{2.0.2}$$

As $X_n = 0$ and $X_n = 1$ are mutually exclusive, we can easily calculate x and y.

$$x = \Pr(X_{n+1} = 0 | X_n = 0) = 1 - \Pr(X_{n+1} = 1 \,| X_n = 0)$$
$$= 0.4 \tag{2.0.3}$$
$$y = \Pr(X_{n+1} = 1 | X_n = 1) = 1 - \Pr(X_{n+1} = 0 \,| X_n = 1)$$
$$= 0.6 \tag{2.0.4}$$



Markov Diagram

The $\Pr(X_{n+t} = j | X_n = i)$ is the (i, j)th position of P^t. Therefore $\Pr(X_2 = 1 | X_0 = 1)$(As $n = 2$ for Wednesday and $n = 0$ for Monday) is the $(1, 1)$th position of P^2.

$$P^2 = \begin{bmatrix} 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix} \times \begin{bmatrix} 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix} \tag{2.0.5}$$

$\therefore$ The probability she studies computer science on Wednesday is $P^2_{11} = 0.4$.
(**Ans: Option (C)**)