

Assignment 3

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Download all python codes from

https://github.com/Taha-Adeel/AI1103/blob/main/Assignment_3/codes/assignment3.py

and latex-tikz codes from

https://github.com/Taha-Adeel/AI1103/tree/main/Assignment_3

1 PROBLEM (GATE 2008 (CS), Q.27)

Aishwarya studies either computer science or mathematics everyday. If she studies computer science on a day, then the probability she studies mathematics the next day is 0.6. If she studies mathematics on a day, then the probability she studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probability she studies computer science on Wednesday?

- (A) 0.24 (C) 0.4
(B) 0.36 (D) 0.6

2 SOLUTION

Let the random variable $X_i \in \{0, 1\}$, $i = 0, 1, 2, \dots$ represent her studying CS(Computer Science) or mathematics respectively on the i th day.

Subject	X_i	$\Pr(X_i X_{i-1} = 0)$	$\Pr(X_i X_{i-1} = 1)$
CS	0	x (Ref (2.0.1))	0.4
Maths	1	0.6	y (Ref (2.0.2))

As $X_i = 0$ and $X_i = 1$ are mutually exclusive, we can easily calculate x and y .

$$x = \Pr(X_i = 0 | X_{i-1} = 0) = 1 - \Pr(X_i = 1 | X_{i-1} = 0) = 0.4 \quad (2.0.1)$$

$$y = \Pr(X_i = 1 | X_{i-1} = 1) = 1 - \Pr(X_i = 0 | X_{i-1} = 1) = 0.6 \quad (2.0.2)$$

$$\begin{aligned} \therefore \Pr(X_i = 0) &= \Pr(X_i = 0 | X_{i-1} = 0) \times \Pr(X_{i-1} = 0) \\ &\quad + \Pr(X_i = 0 | X_{i-1} = 1) \times \Pr(X_{i-1} = 1) \\ &= 0.4 \times (\Pr(X_{i-1} = 0) + \Pr(X_{i-1} = 1)) \\ &= 0.4 \times 1 = 0.4 \end{aligned} \quad (2.0.3)$$

Therefore, the probability that Aishwarya studies computer science on any day is 0.4. So, the probability she studies computer science on Wednesday is also 0.4

(Ans: Option (C))

Alternatively, the probability she studies CS on Wednesday($i = 2$) can be found by($\because X_0 = 0$) :

$$\begin{aligned} \Pr(X_2 = 0) &= \Pr(X_1 = 0 | X_0 = 0) \times \Pr(X_2 = 0 | X_1 = 0) \\ &\quad + \Pr(X_1 = 1 | X_0 = 0) \times \Pr(X_2 = 0 | X_1 = 1) \end{aligned} \quad (2.0.4)$$

$$= 0.4 \times 0.4 + 0.6 \times 0.4 \quad (2.0.5)$$

$$= 0.4 \quad (2.0.6)$$

(Ans: Option (C))