

Assignment 3

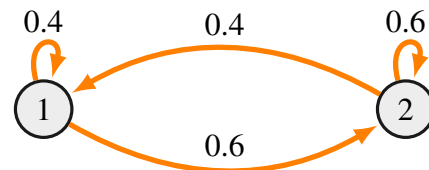
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Download all python codes from

https://github.com/Taha-Adeel/AI1103/blob/main/Assignment_3/codes/assignment3.py

and latex-tikz codes from

https://github.com/Taha-Adeel/AI1103/tree/main/Assignment_3



Markov Diagram

1 PROBLEM (GATE 2008 (CS), Q.27)

Aishwarya studies either computer science or mathematics everyday. If she studies computer science on a day, then the probability she studies mathematics the next day is 0.6. If she studies mathematics on a day, then the probability she studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probability she studies computer science on Wednesday?

- (A) 0.24 (C) 0.4
(B) 0.36 (D) 0.6

2 SOLUTION

Let the random variables $X_i \in \{1, 2\}$, $i = 0, 1, 2, \dots$ represent her studying CS(Computer Science) or mathematics respectively on the i th day. The transition matrix P for the markov chain $\{X\}$ (where $P_{ij} = \Pr(X_t = j | X_{t-1} = i)$) is :-

$$P = \begin{bmatrix} x & 0.6 \\ 0.4 & y \end{bmatrix} \quad (2.0.1)$$

As $X_i = 0$ and $X_i = 1$ are mutually exclusive, we can easily calculate x and y .

$$\begin{aligned} x &= \Pr(X_i = 0 | X_{i-1} = 0) = 1 - \Pr(X_i = 1 | X_{i-1} = 0) \\ &= 0.4 \end{aligned} \quad (2.0.2)$$

$$\begin{aligned} y &= \Pr(X_i = 1 | X_{i-1} = 1) = 1 - \Pr(X_i = 0 | X_{i-1} = 1) \\ &= 0.6 \end{aligned} \quad (2.0.3)$$

The $\Pr(X_{0+t} = i | X_0 = j)$ is the (i, j) th position of P^t . Therefore $\Pr(X_2 = 1 | X_0 = 1)$ is the $(1, 1)$ th position of P^2 .

$$P^2 = \begin{bmatrix} 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix} \times \begin{bmatrix} 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix} \quad (2.0.4)$$

\therefore The probability she studies computer science on Wednesday is $P_{11}^2 = 0.4$.

(Ans: Option (C))