Assignment 3

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Download all python codes from

```
https://github.com/Taha−Adeel/AI1103/blob/main/
    Assignment_3/codes/assignment3.py
```

and latex-tikz codes from

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https://github.com/Taha−Adeel/AI1103/tree/main/
    Assignment_3
```

1 Problem (GATE 2008 (CS), Q.27)

Aishwarya studies either computer science or mathematics everyday. If she studies computer science on a day, then the probability she studies mathematics the next day is 0.6. If she studies mathematics on a day, then the probability she studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probablity she studies computer science on Wednesday?

(A) 0.24 (C) 0.4
(B) 0.36 (D) 0.6

2 Solution (GATE 2008 (CS), Q.27)

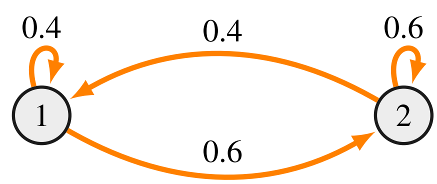
Consider the following parameters

Parameter	Definition	Value
S	State space (i.e possible states she can be in.)	$S = \{1, 2\}$, where 1 and 2 represents her studying CS or maths respectively on that day.
$\{X_0, X_1, \dots\}$	Random variables(which form a markov chain) where $X_i \in S$ represents her studying CS or maths on the ith day(i=0 for Monday)	
P	The one step state transition matrix (The elements $p_{ij} = \Pr(X_{n+1} = j \mid X_n = i)$)	$P = {\scriptstyle X_n}\begin{array}{c} \\ \left\{\begin{array}{c} 1 \\ 2 \end{array}\right. \end{array}\begin{array}{c} \overbrace{\begin{array}{cc} 1 & 2 \end{array}}^{X_{n+1}} \\ \begin{bmatrix} x & 0.6 \\ 0.4 & y \end{bmatrix} \end{array}$ (2.0.1)

As $X_n = 0$ and $X_n = 1$ are mutually exclusive, we can easily calculate x and y.

$$x = \Pr(X_{n+1} = 0 \mid X_n = 0) = 1 - \Pr(X_{n+1} = 1 \mid X_n = 0) = 0.4 \tag{2.0.2}$$

$$y = \Pr(X_{n+1} = 1 \mid X_n = 1) = 1 - \Pr(X_{n+1} = 0 \mid X_n = 1) = 0.6 \tag{2.0.3}$$



Markov Diagram

Given that her initial state is $X_0 = 1$ ($\because$ she studies CS on Monday(n=0)).
The $\Pr(X_{n+t} = j \mid X_n = i)$ is given by the (i, j)th position of P^t. Therefore $\Pr(X_2 = 1 \mid X_0 = 1)$ ($\because$ n=2 for Wednesday) is the $(1, 1)$th position of P^2.

$$P^2 = \begin{bmatrix} 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix} \times \begin{bmatrix} 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix} \tag{2.0.4}$$

$\therefore$ The probability she studies computer science on Wednesday is $P^2_{1\,1} = 0.4$.
(**Ans: Option (C)**)