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Assignment 1

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Download all python codes from

https://github.com/Taha-Adeel/AI1103/tree/main/ Assignment%201/codes

and latex-tikz codes from

https://github.com/Taha-Adeel/AI1103/tree/main/ Assignment%201

1 Problem (80)

Suppose X is a real-valued random variable. Which of the following values CANNOT be attained by E[X] and $E[X^2]$, respectively?

(C)
$$\frac{1}{2}$$
 and $\frac{1}{3}$

2 Solution (80)

The variance of a distribution is given by

$$\sigma = \sum_{X \in \mathbb{R}} \left((E[X - E[X]])^2 \Pr(X) \right)$$
 (2.0.1)
= $E[X^2] - E[X]^2$ (2.0.2)

As variance is always positive (We can easily see that from (2.0.1)),

$$E[X^2] - E[X]^2 \ge 0 (2.0.3)$$

is a necessary condition for any real valued random variable. Computing the value of $E[X^2] - E[X]^2$ for the options, we have

(A) 0 and 1

$$\implies E[X^2] - E[X]^2 = 1 - 0^2 = 1 > 0 \quad (2.0.4)$$

(B) 2 and 3

$$\implies E[X^2] - E[X]^2 = 3 - 2^2 = -1 \le 0$$
(2.0.5)

(C)
$$\frac{1}{2}$$
 and $\frac{1}{3}$

$$\implies E[X^2] - E[X]^2 = \frac{1}{3} - \frac{1}{2}^2 = \frac{1}{12} \ge 0$$
(2.0.6)

(D) 2 and 5

$$\implies E[X^2] - E[X]^2 = 5 - 2^2 = 1 \ge 0 \quad (2.0.7)$$

 \therefore E[X] and $E[X^2]$ cannot attain the values 2 and 3 (**Option B**) (As then the variance would be negative (Refer (2.0.5))