# Actor Critic Methods

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# Overview

1. Review
2. Towards Actor-Critic Formulation
3. Actor Critic Algorithms
4. Towards Deterministic Policy Gradient Formulations

# Review

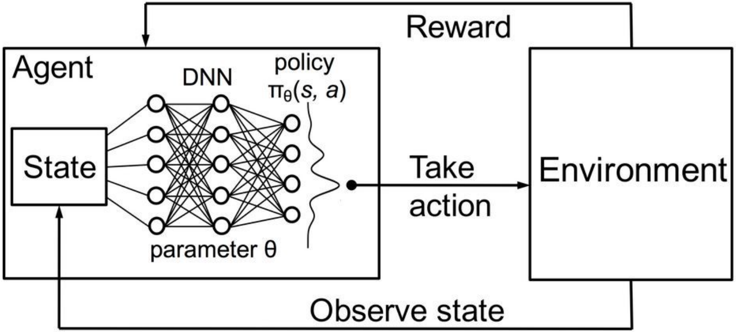
# Policy Based Reinforcement Learning

- We will directly parametrize the policy

$$\pi_\theta(a|s) = P(a|s, \theta)$$

- We will consider model free control with parametrized policies
  - ★ With state-value functions $Q$, computing $\arg\max$ over actions gets tricky when action space is large or continuous
  - ★ Better convergence properties
  - ★ Can learn stochastic policies

# Policy Using Function Approximators




- If action space is discrete
  - ★ Network could output a vector of probabilities (softmax)
- If action space is continuous
  - ★ Network could output the parameters of a distribution (For e.g.. mean and variance of a Gaussian)

# Policy Optimization

A policy $\pi(\cdot)$ is parametrized by parameter $\theta$ and denoted by $\pi_\theta$

Performance of a policy $\pi_\theta$ is given by

$$J(\theta) = V^{\pi_\theta}(s) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{\infty} \gamma^t r_{t+1} | s_0 = s\right]$$

Goal of RL is to find a policy

$$\pi_\theta^* = \arg\max_{\pi_\theta} V^{\pi_\theta}(s) = \arg\max_{\pi_\theta} \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{\infty} \gamma^t r_{t+1} | s_0 = s\right]$$

We will look for $\pi_\theta^*$ in class of stochastic policies by finding $\theta$ that maximizes $J(\theta)$

# Policy Gradient Estimate

- Gradient derivation yields the following estimate

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau\sim\pi_\theta}\left[\nabla_\theta \log P(\tau;\theta) G(\tau)\right]$$

- Sample based estimate is given by

$$\nabla_\theta J(\theta) \approx \frac{1}{K}\sum_{i=1}^{K} \nabla_\theta \log P(\tau^{(i)};\theta) G(\tau^{(i)})$$

# Policy Gradient : Model Free Formulation

Model free formulation of the policy gradient is given by

$$\nabla_\theta J(\theta) \approx \frac{1}{K} \sum_{i=1}^{K} \left[ \sum_{t=0}^{\infty} \nabla_\theta \log \pi(a_t^{(i)} | s_t^{(i)}) \right] \left[ \sum_{t=0}^{\infty} \gamma^t r_{t+1}^{(i)} \right]$$

# REINFORCE : Monte-Carlo based Policy Gradient

**Algorithm** REINFORCE : MC based Policy Gradient

1: Initialize policy network $\pi$ with parameters $\theta_1$ and learning rate $\alpha$
2: **for** $n = 1$ to $N$ **do**
3: Sample $K$ trajectories from $\pi_{\theta_n}$
4: Calculate gradient estimate

$$\nabla_{\theta_n} J(\theta_n) \approx \frac{1}{K} \sum_{i=1}^{K} \left[ \sum_{t=0}^{\infty} \nabla_{\theta_n} \log \pi(a_t^{(i)} | s_t^{(i)}) \right] \left[ \sum_{t=0}^{\infty} \gamma^t r_{t+1}^{(i)} \right]$$

5: Perform gradient update

$$\theta_{n+1} = \theta_n + \alpha \nabla_{\theta_n} J(\theta_n)$$

6: **end for**

# Issues with Gradient Estimate

- The gradient estimate, thus calculated, is unbiased but has high variance (reason : we are sampling stochastic paths)
- Hence the gradient descent is slow to converge
- Some variance reduction techniques are required in practice

# Different Policy Gradient Formulations

Gradient of the performance measure is given by

$$\nabla_\theta J(\theta) = \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \nabla_\theta \log \pi_\theta(a_t|s_t) \Psi_t \right]$$

1. $\Psi_t = \sum_{k=0}^{\infty} \gamma^k r_{k+1} = G_0$, Total reward of the trajectory
2. $\Psi_t = \sum_{t'=t}^{\infty} \gamma^{t'} r_{t'+1} = G_{t:\infty}$ ,Total reward following action $a_t$
3. $\Psi_t = \sum_{t'=t}^{\infty} \gamma^{t'} r_{t'+1} - b(s_{t'}) = G_{t:\infty} - b(s_t)$, Baseline version of the previous formula

# Vanilla Policy Gradient Algorithm

**Algorithm** Vanilla Policy Gradient Algorithm

1: Initialize policy network $\pi$ with parameters $\theta_1$ learning rate $\alpha$ and baseline $b$
2: **for** $n = 1$ to $N$ **do**
3: Sample $K$ trajectories by executing the policy $\pi_{\theta_n}$
4: At each time step of each trajectory compute $G_t = \sum_{t'=t}^{\infty} \gamma^{t'-t} r_{t+1}$ and advantage estimate $A_t = G_t - b(s_t)$
5: Calculate gradient estimate

$$\nabla_{\theta_n} J(\theta_n) \approx \frac{1}{K} \sum_{i=1}^{K} \left[ \sum_{t=0}^{\infty} \nabla_{\theta_n} \log \pi(a_t^{(i)} | s_t^{(i)}) A_t \right]$$

6: Perform gradient update

$$\theta_{n+1} = \theta_n + \alpha \nabla_{\theta_n} J(\theta_n)$$

7: **end for**

# Improvements to Vanilla Policy Gradient

- The REINFORCE and Vanilla policy gradient as described above is on-policy
  - ★ There is an off-policy way to do policy gradient algorithms
- We do learning by Monte-Carlo roll-outs
  - ★ Will be addressed by Actor-Critic method

# Towards Actor-Critic Formulation

# Temporal Structure and Actor-Critic Algorithms

The policy gradient estimate with temporal structure (takes causality into account) is given by

$$\nabla_\theta J(\theta) \quad = \quad \mathbb{E}_{\tau \sim \pi_\theta}\left[\sum_{t=0}^{\infty} \nabla_\theta \log \pi(a_t|s_t) G_{t:\infty}(\tau)\right]$$

where

$$G_{a:b}(\tau) = \sum_{t=a}^{b} \gamma^t r_{t+1}$$

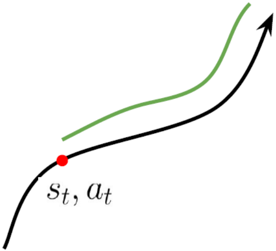
Sample estimate is given by

$$\nabla_\theta J(\theta) \approx \frac{1}{K}\sum_{i=1}^{K}\left[\sum_{t=0}^{\infty} \nabla_\theta \log \pi(a_t^{(i)}|s_t^{(i)}) \cdot \left[\sum_{t'=t}^{\infty} \gamma^{t'-t} r_{t'+1}^{(i)}\right]\right]$$

- This gradient estimate is the starting point of actor-critic algorithms

# Temporal Structure and Actor-Critic Algorithms

$$\nabla_\theta J(\theta) \approx \frac{1}{K}\sum_{i=1}^{K}\left[\sum_{t=0}^{\infty}\nabla_\theta \log \pi(a_t^{(i)}|s_t^{(i)}) \cdot \left[\sum_{t'=t}^{\infty}\gamma^{t'-t} r_{t'+1}^{(i)}\right]\right]$$

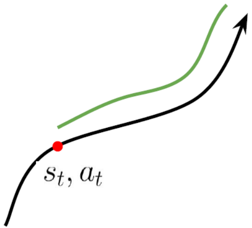



The green curve represents the entity in the inner summation

Figure Source: Jie-Han-Chen:SlideShare

# The Critic

$$\nabla_\theta J(\theta) \approx \frac{1}{K}\sum_{i=1}^{K}\left[\sum_{t=0}^{\infty}\nabla_\theta \log \pi(a_t^{(i)}|s_t^{(i)}) \cdot \left[\sum_{t'=t}^{\infty}\gamma^{t'-t} r_{t'+1}^{(i)}\right]\right]$$

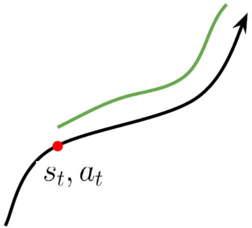



The inner summation is an estimate of $Q^{\pi_\theta}(s_t, a_t)$ !!

Figure Source: Jie-Han-Chen:SlideShare

# The Critic

$$\nabla_\theta J(\theta) \approx \frac{1}{K} \sum_{i=1}^{K} \left[ \sum_{t=0}^{\infty} \nabla_\theta \log \pi(a_t^{(i)} | s_t^{(i)}) \cdot \left[ \sum_{t'=t}^{\infty} \gamma^{t'-t} r_{t'+1}^{(i)} \right] \right]$$




The inner summation is an estimate of $Q(s_t, a_t)$ and it gives an estimate of how 'good' the action $a_t$ was in state $s_t$ (and hence the name '**critic**')

Figure Source: UCB : Sergey Levine

# Policy Gradient Theorem

$$
\begin{aligned}
\nabla_\theta J(\theta) &= \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{\infty}\left\{\nabla_\theta \log \pi_\theta(a_t|s_t) G_{t:\infty}(\tau)\right\}\right] \\
&= \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{\infty}\mathbb{E}_{\tau\sim\pi_\theta}\left(\left\{\nabla_\theta \log \pi_\theta(a_t|s_t) G_{t:\infty}(\tau)\right\}\Big| s_t, a_t\right)\right] \\
&= \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{\infty}\left\{\nabla_\theta \log \pi_\theta(a_t|s_t)\underbrace{\mathbb{E}_{\tau\sim\pi_\theta}\left(G_{t:\infty}(\tau)\Big| s_t, a_t\right)}_{??}\right\}\right]
\end{aligned}
$$

## Policy Gradient Theorem

For suitable objective function $J(\theta)$, we have,

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{\infty}\left\{\nabla_\theta \log \underbrace{\pi_\theta(a_t|s_t)}_{\text{Actor}} \underbrace{Q^{\pi_\theta}(s_t, a_t)}_{\text{Critic}}\right\}\right]$$

- Replacaes the single path reward by $Q^{\pi_\theta}(s_t, a_t)$
- Policy gradient theorem applies to start state objective, average reward objective and average value objective

(More on this later !!)

# Policy Gradient with Baseline

$$
\begin{aligned}
\nabla_\theta J(\theta) &= \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{\infty}\left\{\nabla_\theta \log \pi_\theta(a_t|s_t)(G_{t:\infty} - b(s_t))\right\}\right] \\
&= \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{\infty}\left\{\nabla_\theta \log \pi_\theta(a_t|s_t)(G_{t:\infty} - \underbrace{\mathbb{E}_{\pi_\theta}(G_{t:\infty}|s_t)}_{??})\right\}\right] \\
&= \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{\infty}\left\{\nabla_\theta \log \pi_\theta(a_t|s_t)(Q^{\pi_\theta}(s_t,a_t) - V^{\pi_\theta}(s_t))\right\}\right]
\end{aligned}
$$

# Advantage Function

- Advantage function

$$A^{\pi_\theta}(s,a) \overset{\text{def}}{=} Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s)$$

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{\infty}\{\nabla_\theta \log \pi_\theta(a_t|s_t) A^{\pi_\theta}(s_t,a_t)\}\right]$$

How can we estimate the advantage function using samples ?

We already had a way in the lecture on policy gradient !

# Vanilla Policy Gradient Algorithm

**Algorithm** Vanilla Policy Gradient Algorithm

1: Initialize policy network $\pi$ with parameters $\theta_1$ learning rate $\alpha$ and baseline $b$
2: **for** $n = 1$ to $N$ **do**
3: Sample $K$ trajectories by executing the policy $\pi_{\theta_n}$
4: At each time step of each trajectory compute $G_t = \sum_{t'=t}^{\infty} \gamma^{t'-t} r_{t+1}$ and advantage estimate $A_t = G_t - b(s_t)$
5: Calculate gradient estimate

$$\nabla_{\theta_n} J(\theta_n) \approx \frac{1}{K} \sum_{i=1}^{K} \left[ \sum_{t=0}^{\infty} \nabla_{\theta_n} \log \pi(a_t^{(i)} | s_t^{(i)}) A_t \right]$$

6: Perform gradient update

$$\theta_{n+1} = \theta_n + \alpha \nabla_{\theta_n} J(\theta_n)$$

7: **end for**

# Advantage Function Estimate

$$A^{\pi_\theta} = \sum_{t'=t}^{\infty} \gamma^{t'-t} r_{t+1} - b(s_t)$$

where

$$b(s_t) = \frac{1}{K} \sum_{i=1}^{K} G_{t:\infty}(\tau^{(i)})$$

(time dependent baseline)

- Unbiased estimate but variance is high due to the fact that it is a single sample estimate
- But, we can't roll out trajectories from state $s_t$ as we also need a the algorithm to be online

# Estimator for Advantage Function

- Consider the definition of advantage function

$$A^{\pi}(s,a) \stackrel{\text{def}}{=} Q^{\pi}(s,a) - V^{\pi}(s)$$

- Try having function approximator $V_\phi$ for $V^{\pi_\theta}$
- Consider one-step TD error for $V^{\pi_\theta}$

$$\delta_t^{\pi_\theta} = r_{t+1} + \gamma V^{\pi_\theta}(s_{t+1}) - V^{\pi_\theta}(s_t)$$

$$\begin{aligned} \mathbb{E}_{\pi_\theta}(\delta^{\pi_\theta}|s_t,a_t) &= \underbrace{E_{\pi_\theta}(r_{t+1} + \gamma V^{\pi_\theta}(s_{t+1})|s_t,a_t)}_{??} - V^{\pi_\theta}(s_t) \\ &= Q^{\pi_\theta}(s_t,a_t) - V^{\pi_\theta}(s_t) = A^{\pi_\theta}(s_t,a_t) \end{aligned}$$

- The one-step TD error is an unbiased estimate of the advantage function

$$\therefore \quad \nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{\infty}\left\{\nabla_\theta \log \pi_\theta(a_t|s_t)\delta_t^{\pi_\theta}\right\}\right]$$

# Estimator for Advantage Function

- In practice, use the approximate TD error using the function approximator $V_\phi$ (for $V^{\pi_\theta}$) as an estimate of the advantage function

$$A^{\pi_\theta}(s_t, a_t) \approx r_{t+1} + \gamma V_\phi(s_{t+1}) - V_\phi(s_t)$$

- Note : If we fit $V_\phi$ using Fitted $V$ iteration, the approximator is biased

# Actor Critic Algorithms

# Batch Actor Critic Algorithm

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**Algorithm** Batch Actor-Critic Algorithm

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1: Initialize critic $\phi$, actor $\theta$
2: **for** Repeat over several transitions **do**
3: Sample $K$ transitions $(s_i, a_i, r_i, s'_i)$ using $\pi_\theta$
4: Fit $V_\phi(s_i)$ to sampled reward sums from $s_i$
5: Evaluate the advantage function (for all $K$ samples) using

$$A^{\pi_\theta}(s_i, a_i) \approx r_i + \gamma V_\phi(s'_i) - V_\phi(s_i)$$

6: Update actor $\theta \leftarrow \theta + \alpha \sum_{i=1}^{K} \nabla_\theta \log \pi_\theta(a_i|s_i) A^{\pi_\theta}(s_i, a_i)$
7: **end for**

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The $V$ function can be fitted using fitted $V$ iteration

# Online Actor Critic Algorithm

**Algorithm** Online Actor-Critic Algorithm

1: Initialize state $s$, critic $\phi$, actor $\theta$
2: **for** Repeat over several transitions **do**
3: Let $a$ be the action suggested by policy $\pi_\theta$ at state $s$
4: Take action $a$, observe reward $r$ and next state $s'$ and get a transition $(s, a, r, s')$
5: Fit $V_\phi(s)$ using target $r + V_\phi(s')$
6: Evaluate the advantage function using

$$A^{\pi_\theta}(s, a) \approx r + \gamma V_\phi(s') - V_\phi(s)$$

7: Compute $\nabla_\theta J(\theta) \leftarrow \nabla_\theta \log \pi_\theta(a|s) A^{\pi_\theta}(s, a)$
8: Update actor $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$
9: **end for**

- Fitting $V_\phi$ has moving target and data correlation problem
- The gradient update of the actor in Step 6 has lot of variance (single sample estimate)

# Advantage Actor Critic Algorithms

Steps 5 and 7 works best with a batch (parallel workers)

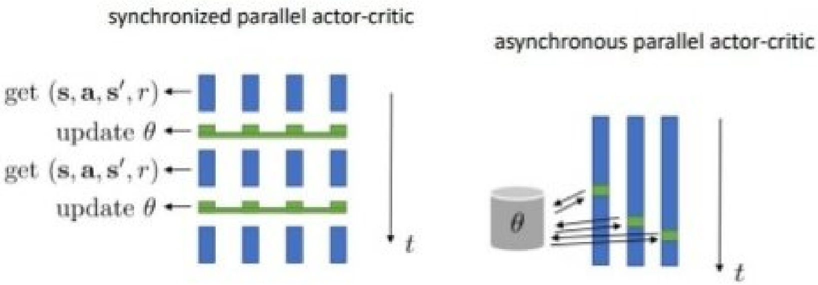



Figure Source: UCB : Sergey Levine

# On Applicabilty of A3C Algorithms

- The A3C (with its synchronous version) requires multiple worker threads to simulate samples for gradient computation
- Useful when simulators are available. Instantiate multiple copies of simulator
- In many real applications, this can be an expensive step
  - ★ Navigation of physical robots (require many physical robots)
  - ★ Driving a car (requires samples generated from multiple cars)

# Towards $n$-step returns

- One step TD error based Advantage estimate

$$A_{\mathrm{C}}^{\pi_\theta}(s,a) \approx r + \gamma V_\phi(s') - V_\phi(s)$$

  - ★ Low variance
  - ★ Biased due to the use of function approximators

- Monte Carlo based Advantage estimate

$$A_{\mathrm{MC}}^{\pi_\theta}(s,a) = \sum_{t'=t}^{\infty} \gamma^{t'-t} r_{t+1} - b(s)$$

  - ★ High variance
  - ★ No bias

# Towards $n$-step returns

- We considered the critic who provides one-step TD error

$$\delta_t^{(1)} = r_{t+1} + \gamma V(s_{t+1}) - V(s_t)$$

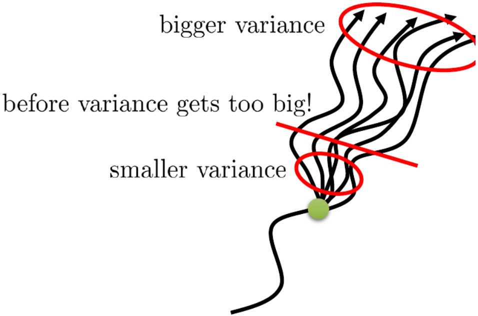
as feedback to the actor

- We could also consider a critic that provides $n$-step TD error as feedback to the actor where the $n$-step TD error

$$\delta_t^{(n)} = r_{t+1} + \gamma r_{t+2} + \cdots + \gamma^{n-1} r_{t+n} + \gamma^n V(s_{t+n}) - V(s_t)$$

- In theory, $\delta_t^{(n)}$ is also an unbiased estimate of $A^{\pi_\theta}$ if $V = V^{\pi_\theta}$
- Gives rise to a method called Generalized Advantage Estimation (GAE)

# Towards $n$-step returns




$$A_n^{\pi_\theta}(s_t, a_t) \approx \sum_{t'=t}^{t+n} \gamma^{t'-t} r(s'_t, a'_t) + \gamma^n V_\phi(s_{t+n}) - V_\phi(s_t)$$

Figure Source: UCB : Sergey Levine

# TD-$\lambda$ as Targets

- We could also consider the TD($\lambda$) error given by

$$\delta_t^{(\lambda)} = (1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1}\delta_t^{(n)}$$

  for the critic formulation (again unbiased in theory)
- The critic itself can be updated using TD($\lambda$)
- Both TD($\lambda$) (critic and the feedback) updates can be implemented using eligibility traces

# A3C

- Asynchronous methods for deep Asynchronous methods for deep reinforcement learning (2016)
- Online actor crtic and paralleized batch
- $N$-step returns with N = 4 steps
- Single network for actor and critic

# Different Policy Gradient Formulations

Gradient of the performance measure is given by

$$\nabla_\theta J(\theta) = \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \nabla_\theta \log \pi_\theta(a_t|s_t)\Psi_t \right]$$

1. $\Psi_t = \sum_{k=0}^{\infty} \gamma^k r_{k+1} = G_0$, Total reward of the trajectory
2. $\Psi_t = \sum_{t'=t}^{\infty} \gamma^{t'} r_{t'+1} = G_{t:\infty}$ ,Total reward following action $a_t$
3. $\Psi_t = \sum_{t'=t}^{\infty} \gamma^{t'} r_{t'+1} - b(s_{t'}) = G_{t:\infty} - b(s_t)$, Baseline version of the previous formula
4. $\Psi_t = \gamma^t Q^{\pi_\theta}(s_t, a_t)$, State action value function
5. $\Psi_t = \gamma^t A^{\pi_\theta}(s_t, a_t) = \gamma^t \left[Q^{\pi_\theta}(s_t, a_t) - V^{\pi_\theta}(s_t)\right]$ , Advantage function
6. $\Psi_t = \gamma^t \left[r_{t+1} + \gamma V^{\pi_\theta}(s_{t+1}) - V^{\pi_\theta}(s_t)\right]$, TD residual

# Towards Deterministic Policy Gradient Formulations

# Stationary Distribution of Markov Chain

- Given a MDP $< \mathcal{M} = \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma >$ and a policy $\pi_\theta$, we have an induced Markov chain given by $< \mathcal{S}, \mathcal{P}^{\pi_\theta} >$
- Imagine that you can travel along the Markov chain's states forever, and eventually, as the time progresses, the probability of you ending up with at state $s$ from state $s_0$ (start state) becomes unchanged and is given by

$$d^{\pi_\theta}(s) = \lim_{t \to \infty} \mathbb{P}(s_t = s | s_0, \pi_\theta)$$

- The entity $d^{\pi_\theta}(s)$ is the limiiting (stationary as well) distribution of Markov chain and is assumed to independent of $s_0$
- Existence of such stationary distribution can be guaranteed under certain some conditions on the Markov chain

# Objective Function Formulations

- In episodic environments, we can use the value of the start state as the objective function given by

$$J_1(\theta) = V^{\pi_\theta}(s) = \mathbb{E}_{\tau \sim \pi_\theta}\left[\sum_{t=0}^{\infty} \gamma^t r_{t+1} | s_0 = s\right]$$

- In **continuing** environments we have a slightly different formulation for the objective function given by,

$$J_{avV}(\theta) = \sum_{s} d^{\pi_\theta}(s) V^{\pi_\theta}(s) = \sum_{s \in \mathcal{S}} d^{\pi_\theta}(s) \sum_{a \in \mathcal{A}} \pi_\theta(a|s) Q^{\pi_\theta}(s, a)$$

where

$$d^{\pi_\theta}(s) = \lim_{t \to \infty} \mathbb{P}(s_t = s | s_0, \pi_\theta)$$

- **Idea** : Average of $V^{\pi_\theta}(s)$ computed using $d^{\pi_\theta}(s)$ as weights (for all $s \in \mathcal{S}$).
- Average is computed from the tail of episodic sequence starting at state $s_0$
- Second equality uses the relationship between $V^{\pi_\theta}$ and $Q^{\pi_\theta}$

# Stochastic Policy Gradient Theorem

**Stochastic Policy Gradient Theorem**
For any differentiable policy $\pi_\theta$, for any of the policy objective functions $J(\theta) = J_1(\theta)$, $\frac{1}{1-\gamma} J_{avV}(\theta)$, the gradient estimate of the objective function with respect to the parameter $\theta$, under some conditions, is given by,

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{\infty}\left\{\nabla_\theta \log \underbrace{\pi_\theta(a_t|s_t)}_{\text{Actor}} \underbrace{Q^{\pi_\theta}(s_t, a_t)}_{\text{Critic}}\right\}\right]$$

# Deterministic Policy Gradient Algorithm : Key Ideas

- Thus far, considered the policy function $\pi(\cdot|s)$ as a probability distribution over actions space and thus considered stochastic policies
- **Deterministic policy gradient algorithms (DPG)** instead models the policy as a deterministic decision : $a = \pi(s)$
- Specifically DDPG, an off-policy actor-critic algoirthm, can be thought of as DQN for continuous action space setting
- Interleaves between learning optimal action-value function $Q^*(s,a)$ and learning optimal policy $\pi^*(s)$
- Uses Bellman equation to learn $Q^*(s,a)$ and policy gradients to learn $\pi^*(s)$

# Deterministic Policy Gradient Algorithm : Key Ideas

- Bellman equation is the starting point for learning optimal action-value function $Q^*(s, a)$.
- Optimal action-value function int the DQN setting is learnt using the following MSBE function

$$L_i(\phi_i) = \left[\mathbb{E}_{(s,a,r,s')\in D}\left(Q_{\phi_i}(s,a) - \underbrace{r + \max_{a'} Q_{\phi_i'}(s', a')}_{\text{target}}\right)^2\right]$$

- However, in the DDPG setting, we calculate the max over actions using the policy netwoek as follows,

$$L_i(\phi_i) = \left[\mathbb{E}_{(s,a,r,s')\in D}\left(Q_{\phi_i}(s,a) - \underbrace{r + Q_{\phi_i'}(s', \pi_\theta(s'))}_{\text{target}}\right)^2\right]$$

# Deterministic Policy Gradient Algorithm : Key Ideas

- Policy is learnt by recoginizing that we are looking for a deterministic policy $\pi_\theta(s)$ that gives an action that maximizes $Q_\phi(s, a)$. Achieved by, performing gradient ascent on the following objective function

$$\max_\theta \mathbb{E}_{s \in \mathcal{D}} Q_\phi(s, \pi_\theta(s))$$

- Because the policy that is being learnt is deterministic, to make DDPG policies explore better, we add noise to their actions at training time.
  - ★ OU noise
  - ★ zero-mean Gaussian noise
- Target networks are updated using Polyak averaging
- The idea of deterministic policy gradient has connections to the stochastic policy gradient setting (in the limiting case)

# Deep Deterministic Policy Gradient (DDPG)

**Algorithm** Deep Deterministic Policy Gradient

Initialize state $s$, critic $\phi$, actor $\theta$ and replay buffer
Initialize target critic $\phi' \leftarrow \phi$, target actor $\theta' \leftarrow \theta$
**for** Repeat over several episodes **do**
    Initialize a random process $N$ for exploration (eg. Ornstein-Uhlenbeck process), and observe initial state $s$
    **for** Repeat over transitions **do**
        Apply action $a = \pi_\theta(s) + N_t$, observe reward $r$ and next state $s'$, and store the transition $(s, a, r, s')$ in the replay buffer
        Sample a random minibatch of transitions $(s_i, a_i, r_i, s'_i)$ from the buffer
        Compute SARSA target values $y_i = r_i + Q_{\phi'}(s'_i, \pi_{\theta'}(s'_i))$
        Update critic by minimizing MSE loss $\frac{1}{n}\sum_i (y_i - Q_\phi(s_i, a_i))^2$
        Update actor using sampled deterministic policy gradient $\frac{1}{n}\sum_i \nabla_a Q_\phi(s_i, \pi_\theta(s_i)) \nabla_\theta \pi_\theta(s_i)$
        Perform soft updates on target networks
            $\phi' \leftarrow \tau\phi + (1-\tau)\phi'$
            $\theta' \leftarrow \tau\theta + (1-\tau)\theta'$
    **end for**
**end for**