10/11/23

Deep Learning

- Recap
- VAE Solution
- Reparameterization Trick
- Recap: $\log \underbrace{P(X)}_{\text{unknown}} - D\left(Q(z|x) \,\|\, \underbrace{P(z|x)}_{\text{unknown}}\right) = \underbrace{E_{z\sim Q(z|x)}\left[\log \underbrace{P(x|z)}_{\text{decoder}}\right] - D\left(\underbrace{Q(z|x)}_{\text{Encoder}} \,\|\, \underbrace{P(z)}_{\text{standard normal}}\right)}$ — (1)

$$KL\left(\mathcal{N}(\mu_0, \Sigma_0) \,\|\, \mathcal{N}(\mu_1, \Sigma_1)\right)$$

- $Q(z|x) \sim \mathcal{N}\left(\mu_\phi(X), \Sigma_\phi(X)\right)$
- $P(X|z) \sim \mathcal{N}\left(\underbrace{f(z;\theta)}; \sigma^2 I\right)$

$$\mathcal{N}\left(\mu_\theta(z); \sigma^2 I\right)$$

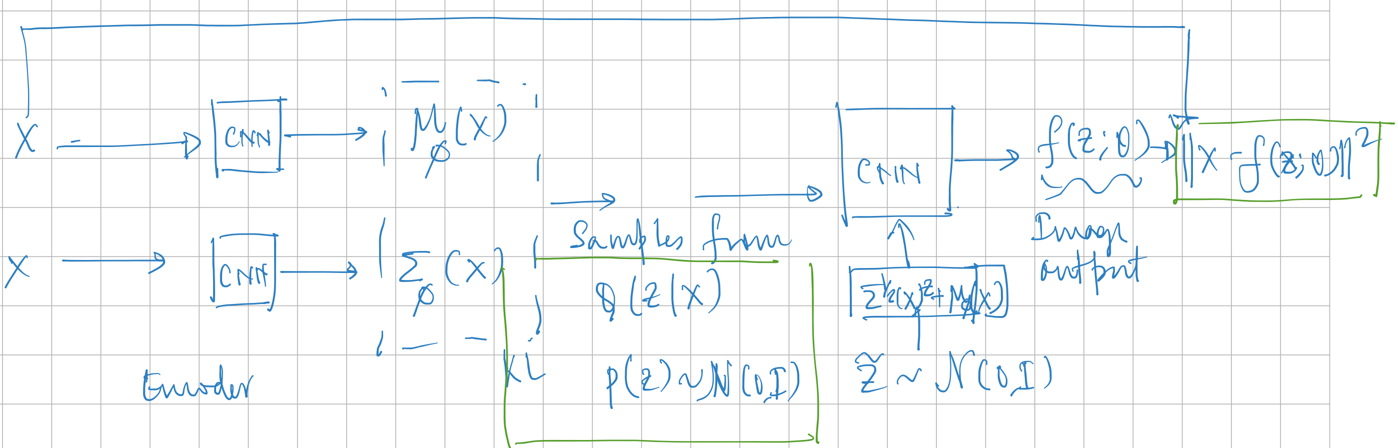
- $P(z) \sim \mathcal{N}(0, I)$
- $$D\left(\mathcal{N}(\mu_0, \Sigma_0) \,\|\, \mathcal{N}(\mu_1, \Sigma_1)\right) = \frac{1}{2}\left[tr\left(\Sigma_1^{-1}\Sigma_0\right) + (\mu_1 - \mu_0)^T \Sigma_1^{-1}(\mu_1 - \mu_0) - k + \log\left(\frac{\det \Sigma_1}{\det \Sigma_0}\right)\right]$$

$$\Rightarrow D\left(Q(z|x) \,\|\, P(z)\right) = \frac{1}{2}\left[tr\left(\Sigma_\phi(x)\right) + \mu_\phi(x)^T \cdot \mu_\phi(x) - \underbrace{k}_{\text{dim of R.V.}} + \log\frac{1}{\det \Sigma_\phi(x)}\right]$$

(Second term in (1))

$$\log P(X|z) = \log\left(k \exp\left[-\frac{\|X - f(z;\theta)\|^2}{\sigma^2}\right]\right) \quad \text{(first term in (1))}$$

$$\approx -\|X - f(z;\theta)\|^2$$



- SGD for optimization
- Note: Gradients cannot be propagated through a sampling function.
- Therefore: $z \sim \mathcal{N}(\mu_\phi(x), \Sigma_\phi(x))$ is achieved as follows

$$\tilde{z} \sim \mathcal{N}(0, I); \quad z = \Sigma_\phi^{1/2}(x) z + \mu_\phi(x).$$