22/8/23

Deep Learning

Recap

- ✓ Cauchy sequence
- ✓ Completeness, Banach space
- ✓ Inner product, inner product spaces, Hilbert space

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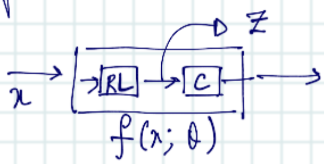
Example:

$p_{XY}(x, y)$

	1	2	3	4
1	1/8	1/16	1/32	1/32
2	1/16	1/8	1/32	1/32
3	1/16	1/16	1/16	1/16
4	1/4	0	0	0

- Probability basics: RV, ✓PMF, ✓Joint PMF, Marginal, Conditional PMF
- Entropy of a random variable
- 

$\hat{y} = f(x; \theta)$

$f_\theta(x)$

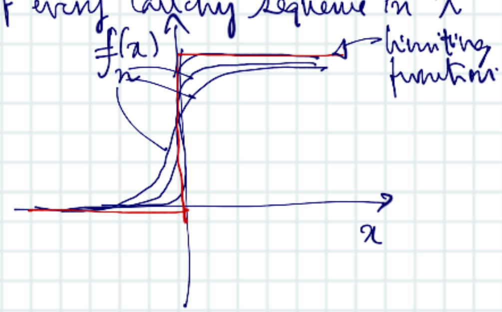
Recall: $(x, y) \sim p_{XY}(x, y)$

Recap:

- $d(x, y)$, (X, d), linear space, norm $\|\cdot\|$, normed linear space $(X, \|\cdot\|)$.
- (x_n), Convergence, Cauchy sequence
- Cauchy sequence: A sequence (x_n) defined as a map $\mathbb{N} \to \mathbb{R}$ is Cauchy if for any $\epsilon > 0$, $\exists N$ such that $\underbrace{|x_n - x_m|}_{d(x_n, x_m)} < \epsilon$ for all $n, m > N$.
- Abstract convergence & Cauchy sequence notions to any metric space (X, d)
- Completeness: A metric space (X, d) is said to be complete if every Cauchy sequence in X converges to a point in X.
- Ex: Let $X = \mathbb{Q}$, $d(x, y) = |x - y|$

  $x_n =$ the n^{th} decimal point rational approx of $\sqrt{2}$

  $(x_n) =$? So that $\lim_{n \to \infty} x_n \to e$



$(X, \|\cdot\|)$.

- Banach space: A complete normed linear space, with respect to the distance $d(x, y) = \|x - y\|$.
  Ex: Banach Wasserstein Generative Adv. Network (BWGAN)
- Inner product: An inner product defined on a linear space X over $\mathbb{R}$ is a function $\langle x, y \rangle : X \times X \to \mathbb{R}$ that satisfies:
  - $\langle x, \lambda y + \mu z \rangle = \lambda \langle x, y \rangle + \mu \langle x, z \rangle \quad \forall\ x, y, z \in X, \ \mu, \lambda \in \mathbb{R}$

- $\langle x, y\rangle = \langle y, x\rangle$ (symmetry)
- $\langle x, y\rangle = 0$ if $x \perp y$ (orthogonality)

- Ex: $\mathcal{X} = \mathbb{R}^2$; $\langle x, y\rangle = \sum_{i=1}^{2} x_i y_i$

Norm $\|x\| = \sqrt{\langle x, x\rangle}$

$d(x,y) = \|x-y\|$ $\Leftarrow$

Distance Inner product

pre-Hilbert space

- Inner product Space: A linear space $\mathcal{X}$ endowed with an inner product $(\langle \cdot, \cdot \rangle)$. $(\mathcal{X}, \langle \cdot, \cdot \rangle)$.
- Hilbert space: A complete inner product space. (w.r.t distance $d(x,y) = \| x-y \|$
$= \sqrt{\langle (x-y), (x-y)\rangle}$.

- Probability basics: (Discrete RVs)
  - Let Ω be the sample space of an experiment
- A random variable: A measurable function $X : \Omega \to \mathbb{R}$
- The probability mass function of a discrete RV X. $p_X : \mathbb{R} \to [0,1]$

Discrete RV X takes on a discrete set of values coming from say $\mathcal{X}$.

$p_X(X = x)$ is the probability that the RV X takes value x.