07/11/23

Deep Learning

- Recap: Two problems on random variables
- Variational Auto Encoder (VAE)
  - Latent variable models
  - Problem Setup

Ex 1: Let X be a continuous RV, with CDF $F_X(x)$. Let $Y = g(X)$ be a function of X. Find the PDF/CDF of Y if $\boxed{g(x) = F_X(x)}$

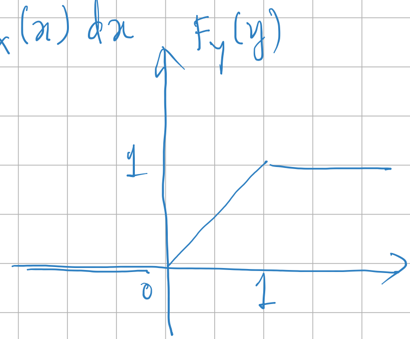
"Encoder" problem

- Let $0 \le Y \le 1$: $P(Y \le y) = P(F_X(x) \le y)$

$$= P(X \le F_X^{-1}(y))$$

$$= \int_{-\infty}^{F_X^{-1}(y)} f_X(x)\,dx$$

$$= y.$$



- $-\infty < y < 0$: $P(Y \le y) = ?$

$$= 0$$

- $1 < y < \infty$: $P(Y \le y) = 1$

Ex2: Let $X \sim U[0,1]$. Find $Y = g(X)$ such that Y satisfies a desired CDF $F_D(y)$.

"Decoder" problem

Ans: $Y = g(x) = F_D^{-1}(x)$

Find $P(Y \le y)$.

- Variational Auto Encoder (VAE).
  - Latent variables - $z \in \mathcal{Z}$
  - Target variable - $x \in \mathcal{X}$; $\theta \in \Theta$

- Mapping: $f(z, \theta) \in \mathcal{X}$
- Problem: Find $f(\cdot, \cdot) : \mathcal{Z} \times \Theta \rightarrow \mathcal{X}$ such that the likelihood of $f(z, \theta)$ being drawn from $P(X)$ is maximized.
- $$P(X) = \int p(X, z) \, dz$$

$$= \int \underbrace{p(X|z)}_{f(z;\theta)} \cdot p(z) \, dz$$

  - We are going to model $p(X|z)$ using a model $f(z; \theta)$
  - Assume $f(z; \theta) \sim \mathcal{N}(\underline{\mu}_\theta ; \sigma^2 I)$
- Revisiting the problem: Find $f(z; \theta)$ such that $P(X)$ is maximized.
- Challenge: