- Decision Trees ← model of computation.
- Boolean functions
  $f : \{0,1\}^n \to \{0,1\}$
- Fourier (Harmonic) Analysis of Boolean functions.

- Refrences
  - Complexity Measures and DT complexity: A survey
    H. Buhrman & R. de Wolf.
  - Boolean function Complexity
    — S. Jukna. (Chapter 14)
  - Analysis of Boolean functions.
    — R. O'Donnell

→ Decision Trees.

$f : \{0,1\}^n \rightarrow \{0,1\}$ "Boolean functions"

— one of the simplest model of computation.

→ Unknown input $x_1, x_2, x_3, \dots, x_n$

→ Goal is to compute $f(x_1, \dots, x_n)$

f

0^n	0/1
0000 0..1	0/1
:	0/1
:	1
111111	0/1.

→ By asking values of these bits.

and once you have seen sufficient

bits, then may be you can answer the output value.

$\Rightarrow$ $OR_n : \{0,1\}^n \to \{0,1\}$

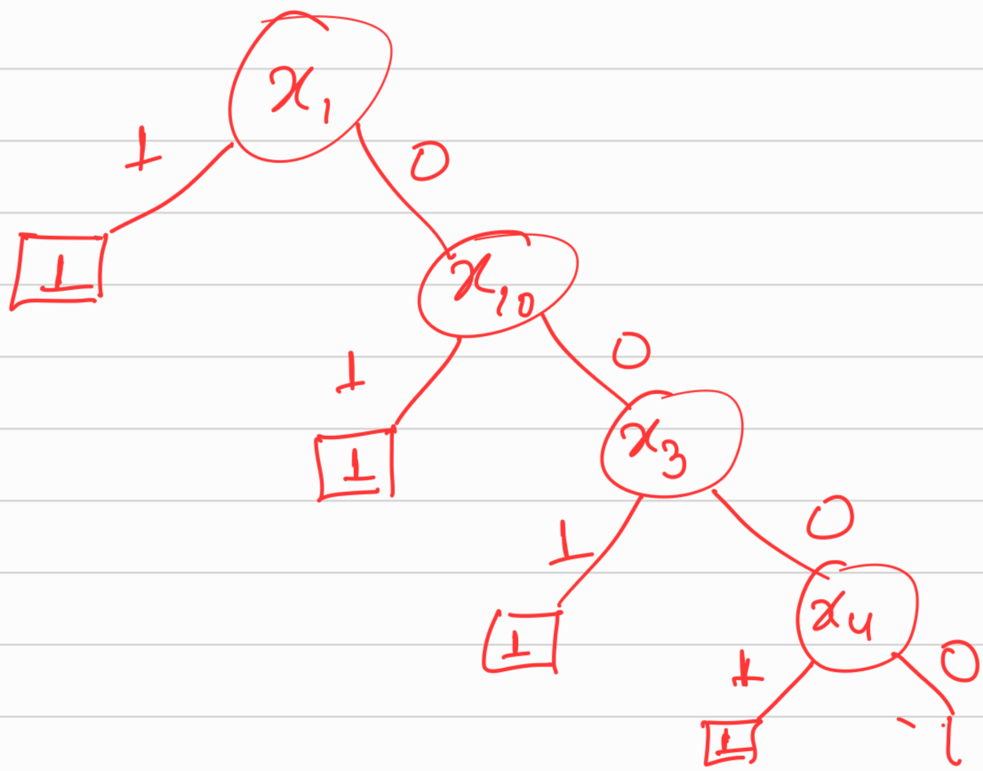
$$OR_n(x) = \begin{cases} 1 & \text{if } |x| \geq 1 \\ 0 & \text{o/w.} \end{cases}$$

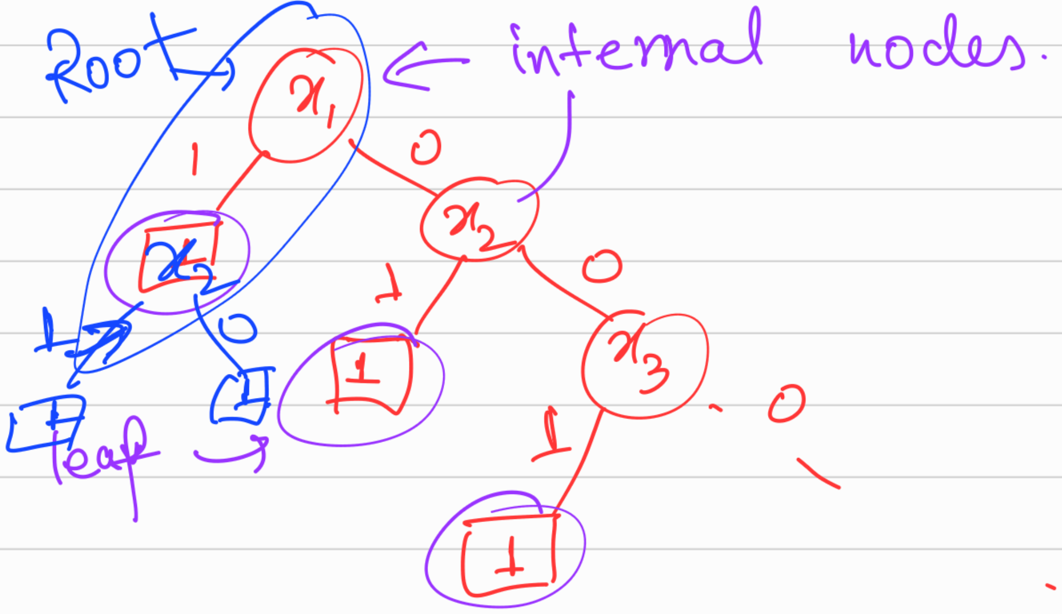
$x \to (x_1, \ldots, x_n)$

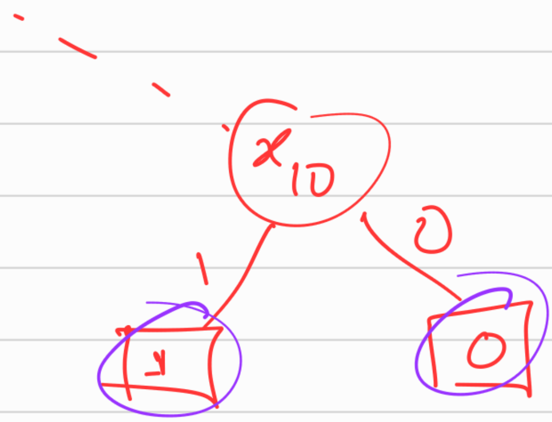
$|x| = \#$ 1's in x.

Hamming weight of x.

$$\{0,1\}^n \equiv \left\{ (x_1, x_2, \ldots, x_n) \;\middle|\; \begin{array}{l} x_i \in \{0,1\} \\ \forall\, i \in [n] \end{array} \right\}$$







Decision tree for OR_n

- It's a tree.
- Where internal nodes are labelled by input variables
- leaves are labelled by constants 0 or 1.

- When do you say that a decision tree computes a function?

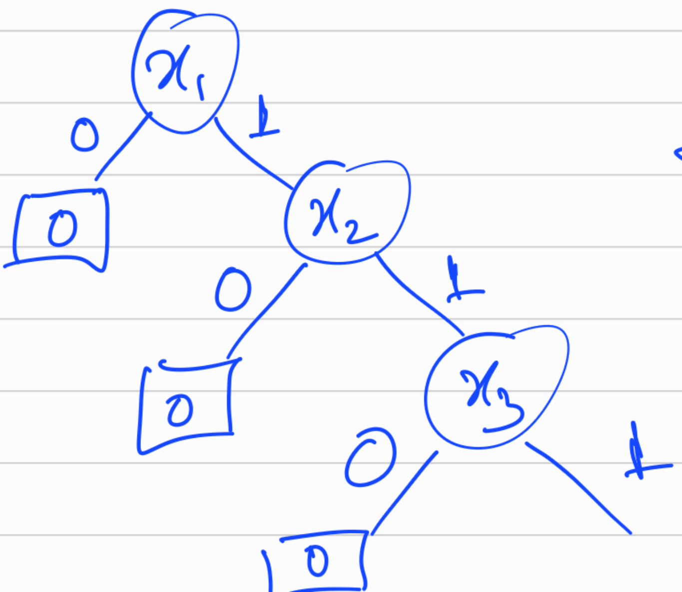
$\Rightarrow$ a decision tree T computes a function f.

$\underline{\text{iff}}$ $\quad T(x) = f(x)$

$\Rightarrow$ every input follows a unique root to leaf path.

$$AND_n : \{0,1\}^n \rightarrow \{0,1\}$$

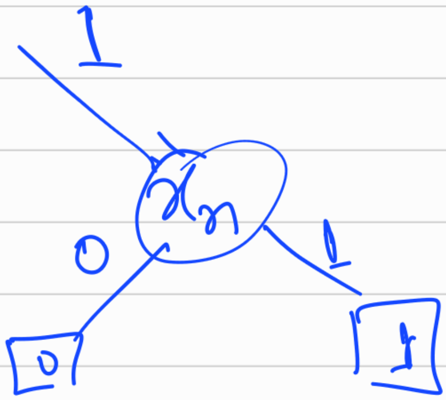
$$AND_n(x) = \begin{cases} 1 & \text{if } |x| = n \\ 0 & \text{o/w.} \end{cases}$$



for all input x

$$T(x) = f(x)$$

output of T on x.

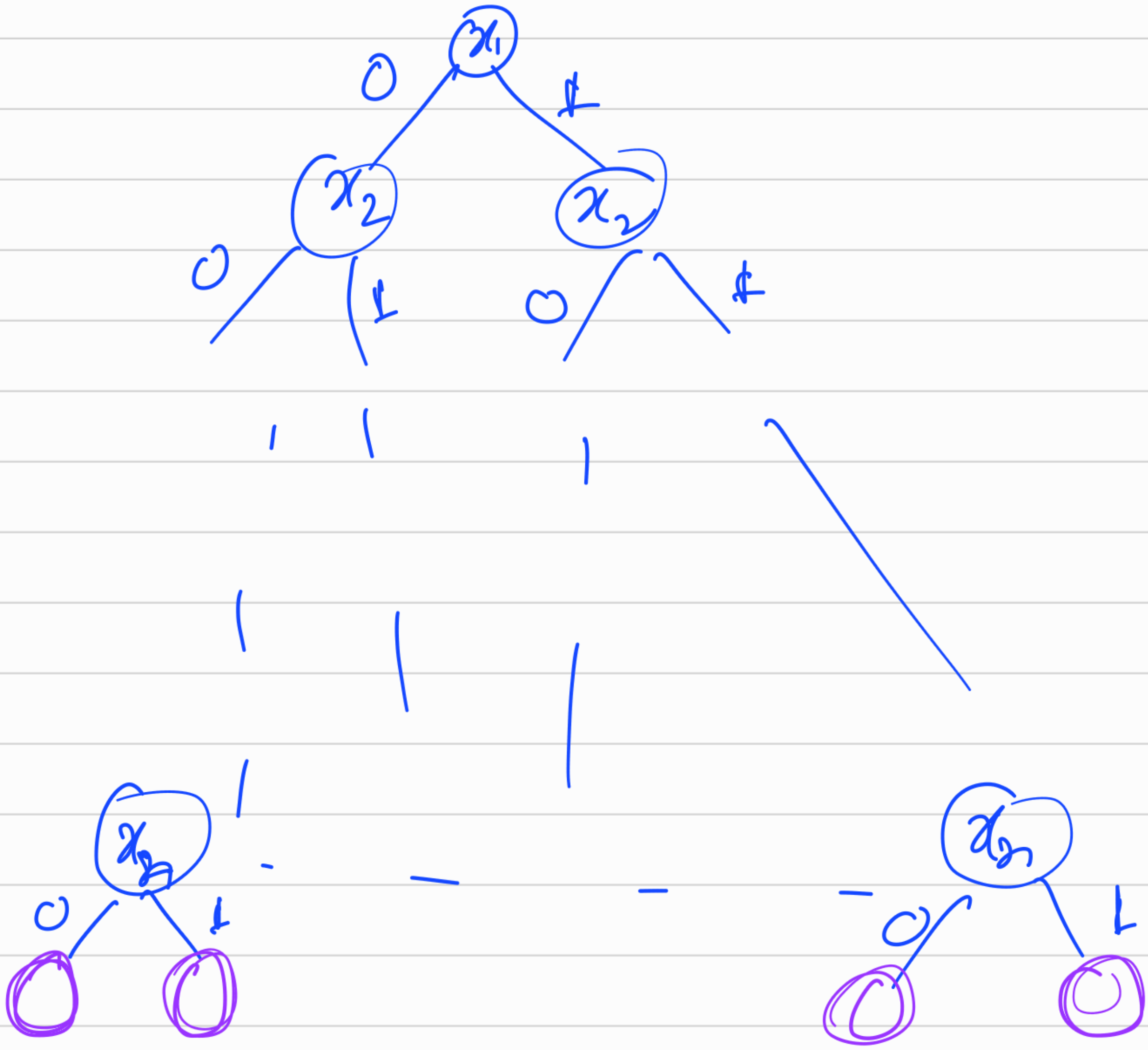


→ Cost of a T on an input x = length of the unique path that x follows on T.

$\text{Cost}(T, x)$

Cost of Tree T (depth of T) $:= \max_{x \in \{0,1\}^n} \text{Cost}(T, x)$

depth of the tree



Size = # leaves.