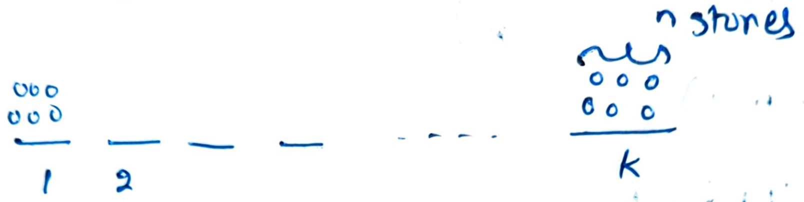
# Exam-2 (2022)

## Question. 1

Random Experiment : kattappa gives a set $S$, with $\frac{1}{2}$ prob bahubali moves all the stones in $S$ and with $\frac{1}{2}$ prob all the stones in $\overline{S}$, one position to the left:




# rounds = $r$

Given: $$n \sum_{i=0}^{k} \binom{r}{i} 2^{-r} > 1$$

for each stone $s$, we define a R.V. $X_s$ as.

$$X_s = \begin{cases} 1, & \text{if } s \text{ stays} \\ 0, & \text{otherwise} \end{cases}$$

$$E[X_s] = \sum_{i=0}^{k} \binom{r}{i} 2^{-r}$$

{ stone $s$ moves $i$ times to the left with prob $\frac{1}{2}$ and remaining $(r-i)$ times it is not moving with prob $\frac{1}{2}$. There are $\binom{r}{i}$ ways to choose $i$ rounds. $i$ can go up to $k$ because stone is still there on the board. }

Let $X$ be the R.V that denotes the number of stones remaining after $r$ rounds.

$$E[X] = \sum_{s} E[X_s] = n \sum_{i=0}^{k} \binom{r}{i} 2^{-r}$$

Exam-2 (2022)

Random experement:

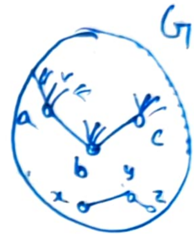
Pick a permutation independently uniformly at random from set of $n!$ permutation.

Question-2

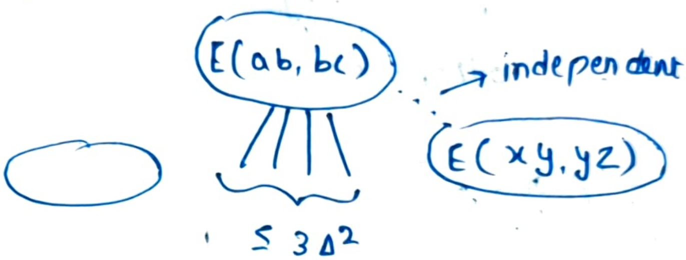
Let $G$ be a graph on $n$ vertices with maximum degree $\Delta$.

Let $E(ab, bc)$ be the "bad" event of $(ab, bc)$ pair not getting resolved in any of the $r$ permutation.

$$\Pr[E(ab, bc)] = P = \left(\frac{2}{3}\right)^r$$




Dependency graph:




by applying local Lemma,

$4 \cdot p\, d < 1$

$4 \cdot \left(\frac{2}{3}\right)^r \cdot 3\Delta^2 < 1$

$4 \cdot \frac{1}{(3/2)^r} \cdot 3\Delta^2 < 1$

Total number of permutations of a, b, c

c b a ✓
c a b
b a c
b c b
a c b
a b c ✓

good one's are those who has b in middle

bad prob $= \frac{4}{6} = \frac{2}{3}$

$\Pr[E(ab, bc)]$ is bad in all the $r$ permutation

$= \left(\frac{2}{3}\right)^r$

Jukna - Chapter-19

Question - 19.2

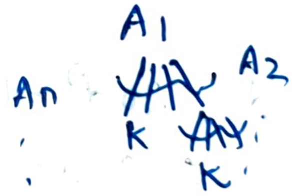
$$\text{Let } F = \{\overbrace{A_1, A_2 \dots}^{\text{k-uniform}}\}$$

$$A_i \to \{a_1 \dots a_k\}$$

Color the points of $A_i$'s independently uniformly at random with 2 colors {Red, Blue} with $1/2$ Prob each.

Bad Prob is that its set is monochromatic

$$Pr[A_i] = p = \underbrace{\frac{1}{2^k}}_{\text{all k points having Red color}} + \underbrace{\frac{1}{2^k}}_{\text{all k points having blue color}} = \frac{2}{2^k} = \frac{1}{2^{k-1}}$$




$d = k^2$

$e \cdot p\, d \leq 1$

$$e \cdot \frac{1}{2^{k-1}} \cdot k^2 < 1$$